Correction

Tuesday, September 15, 2020 9:18 PM

$X_t = 0.01 + 0.2\, X_{t-2} + a_t$

$\{a_t\}$ is gaussian noise with mean = 0, variance = 0.02

Acc to AR(2) model :

$X_t = a_0 + a_1 X_{t-1} + a_2 X_{t-2} + e_t$

$a_0 = 0.01$

$a_1 = 0$

$X_{t-1} = 0$

$a_2 = 0.2$

$e_t = a_t$

Part (a) : Mean & Variance of X_t

Taking expectation on both sides of Equation (2)

$$E[X_t] = a_0 + a_1 E[X_{t-1}] + a_2 E[X_{t-2}] + E[e_t]$$
$$= 0.01 + 0.2\, E[X_{t-2}]$$

Under stationary conditions :

$E[X_t] = E[X_{t-1}] = E[X_{t-2}] = \mu$

$\mu = 0.01 + 0.2\mu$

$\mu - 0.2\mu = 0.01$

$0.8\mu = 0.01$

$\mu = \frac{0.01}{0.8}$

$= \frac{1}{80} = 0.0125$

$\mu = a_0 + \mu a_1 + \mu a_2$

$a_0 = \mu - \mu a_1 - \mu a_2$

$a_0 = \mu(1 - a_1 - a_2)$

$$\boxed{\mu = \frac{a_0}{1 - a_1 - a_2}} \Rightarrow \text{mean}$$

Variance :

$a_0 = \mu(1 - a_1 - a_2)$

$X_t = \mu(1 - a_1 - a_2) + a_1 X_{t-1} + a_2 X_{t-2} + a_t$

$X_t = \mu - a_1\mu - a_2\mu + a_1 X_{t-1} + a_2 X_{t-2} + e_t$

$X_t - \mu = a_1(X_{t-1} - \mu) + a_2(X_{t-2} - \mu) + e_t$ — (1)

Squaring & taking Expectation

$$Var(X_t) = a_1^2\, Var(X_{t-1}) + a_2^2\, Var(X_{t-2}) + \sigma_e^2$$

Under Stationary Condition :

$Var(X_t) = Var(X_{t-1}) = Var(X_{t-2})$

$Var(X_t) - a_1^2\, Var(X_{t-1}) - a_2^2\, Var(X_{t-2}) = \sigma_e^2$

$Var(X_t)[1 - a_1^2 - a_2^2] = \sigma_e^2$

$$\boxed{Var(X_t) = \frac{\sigma_e^2}{1 - a_1^2 - a_2^2}} = \frac{0.02}{1 - 0 - (0.2)^2} = \frac{0.02}{0.96} = \frac{2}{96} = \boxed{0.020833}$$

Part (b): Lag 1 and Lag 2 of auto-correlation of X_t

Multiplying Eq (1) by $(X_{t-1} - \mu)$

$$(X_{t-1} - \mu)(X_t - \mu) = a_1 (X_{t-1} - \mu)(X_{t-1} - \mu) + a_2 (X_{t-2} - \mu)(X_{t-1} - \mu) + e_t (X_{t-1} - \mu)$$

Taking Expectation, where,

$$E[(X_{t-1} - \mu)\, e_t] = 0$$

$$V_l = a_1 V_{l-1} + a_2 V_{l-2}$$

when divided by V_0 =>

$$\frac{V_l}{V_0} = a_1 \frac{V_{l-1}}{V_0} + a_2 \frac{V_{l-2}}{V_0}$$

$$r_l = a_1 r_{l-1} + a_2 r_{l-2} \qquad [l > 0]$$

for lag = 1

$$r_1 = a_1 r_0 + a_2 r_{(-1)}$$

$$r_1 = a_1 + a_2 (r_{(-1)})$$

$$r_1 = \frac{a_1}{1 - a_2} = \frac{0}{1 - 0.2} = 0$$

$$r_2 = a_1 r_1 + a_2 r_0$$

$$r_2 = 0 + 1(0.2)$$

$$r_2 = 0.2$$

Part (c)

Equation: $X_t = a_0 + a_1 X_{t-1} + a_2 X_{t-2} + e_t$

$t = 101$

$$X_{101} = a_0 + a_1 X_{100} + a_2 X_{99} + e_t$$
$$= 0.01 + 0 \to 0.2\,(0.02) + e_t$$
$$= 0.01 + 0.2\,(0.02) + e_t$$
$$= 0.01 + 0.004 + e_t$$
$$= 0.014 + e_t$$

$$X_{102} = a_0 + a_1 X_{101} + a_2 X_{100} + e_t$$
$$= 0.01 + 0 + 0.2(-0.01) + e_t$$
$$= 0.01 + (-0.002) + e_t$$
$$= (8 \times 10^{-3}) + e_t$$