

05. Fractionally Differentiated Features

November 2, 2018

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6.4.2 (ii) On each bootstrapped sample, sample weights are determined using the techniques from Chapter 4

```
In [1]: %load_ext watermark
        %watermark

        %load_ext autoreload
        %autoreload 2
        # import standard libs
        import warnings
        warnings.filterwarnings("ignore")
        from IPython.display import display
        from IPython.core.debugger import set_trace as bp
        from pathlib import PurePath, Path
        import sys
        import time
        from collections import OrderedDict as od
        import re
        import os
        import json
        os.environ['THEANO_FLAGS'] = 'device=cpu,floatX=float32'

        # import python scientific stack
        import pandas as pd
        pd.set_option('display.max_rows', 100)
        from dask import dataframe as dd
        from dask.diagnostics import ProgressBar
        pbar = ProgressBar()
        pbar.register()
        import multiprocessing as mp
        from multiprocessing import cpu_count
        import numpy as np
        import scipy.stats as stats
        import statsmodels.api as sm
        import numba as nb
        import math
        import pymc3 as pm
        from theano import shared, theano as tt

        # import visual tools
        import matplotlib as mpl
        import matplotlib.pyplot as plt
        import matplotlib.gridspec as gridspec
        %matplotlib inline
```

```

import seaborn as sns
import plotnine as pn

plt.style.use('seaborn-talk')
plt.style.use('bmh')
#plt.rcParams['font.family'] = 'DejaVu Sans Mono'
plt.rcParams['font.size'] = 9.5
plt.rcParams['font.weight'] = 'medium'
plt.rcParams['figure.figsize'] = 10,7
blue, green, red, purple, gold, teal = sns.color_palette('colorblind', 6)

# import util libs
import pyarrow as pa
import pyarrow.parquet as pq
from tqdm import tqdm, tqdm_notebook
import missingno as msno

from src.utils.utils import *
import src.features.bars as brs
import src.features.snippets as snp

import copyreg, types
copyreg.pickle(types.MethodType, snp._pickle_method, snp._unpickle_method)
RANDOM_STATE = 777

pdir = get_relative_project_dir('Adv_Fin_ML_Exercises')
data_dir = pdir/'data'/'processed'

print()
%watermark -p pandas,numpy,numba,pymc3,sklearn,statsmodels,scipy,matplotlib,seaborn

```

2018-10-18T16:55:10-06:00

CPython 3.6.6

IPython 6.5.0

```

compiler   : GCC 7.2.0
system     : Linux
release    : 4.15.0-36-generic
machine    : x86_64
processor   : x86_64
CPU cores  : 12
interpreter: 64bit

```

```

pandas 0.23.4
numpy 1.14.6
numba 0.41.0dev0+75.gdb0256a70
pymc3 3.5

```

```
sklearn 0.20.0
statsmodels 0.9.0
scipy 1.1.0
matplotlib 3.0.0
seaborn 0.9.0
```

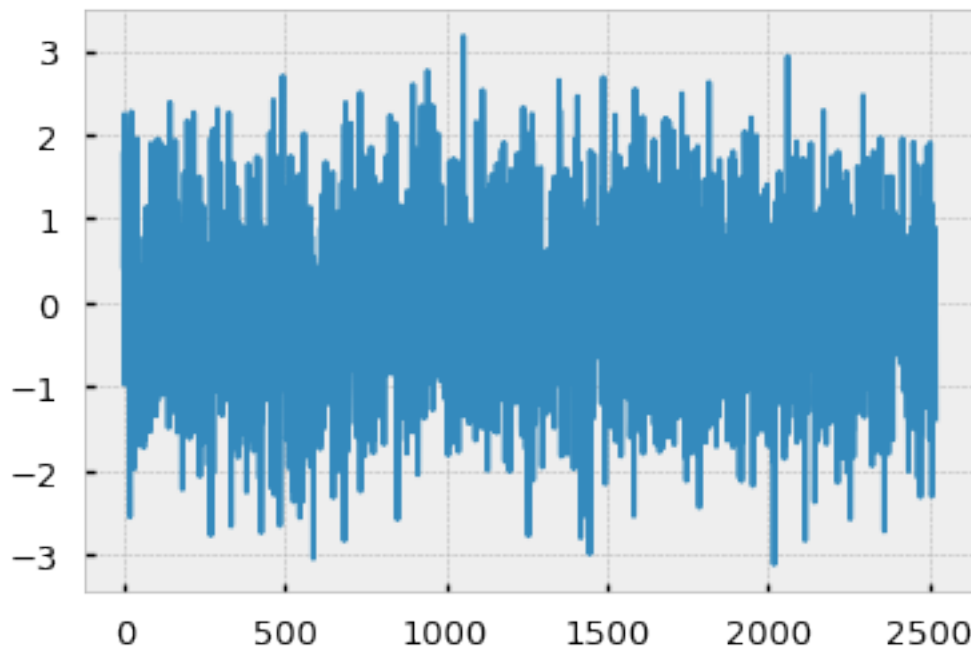
1 Chapter 5

1.1 [5.1] Generate a time series from an IID Gaussian random process. This is a memory-less, stationary series:

```
In [2]: np.random.seed(0)
```

```
N = 252*10
s = pd.Series(np.random.randn(N))
s.plot()
```

```
Out[2]: <matplotlib.axes._subplots.AxesSubplot at 0x7f4588a3cf28>
```



1.1.1 (a) Compute the ADF statistic on this series. What is the p-value?

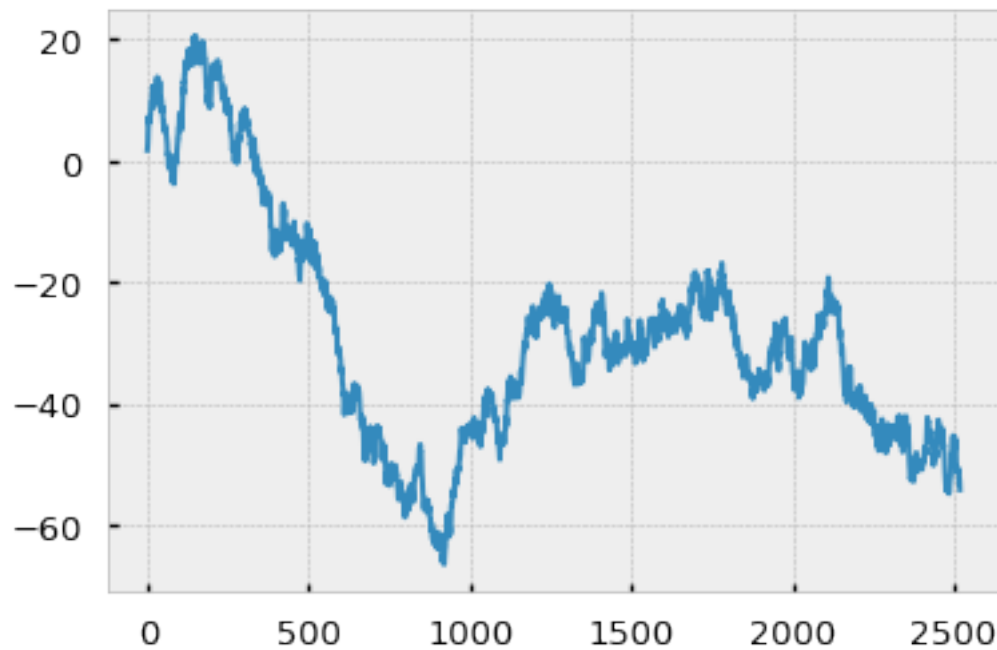
```
In [3]: adf = lambda s: sm.tsa.stattools.adfuller(s)
p_val = lambda s: sm.tsa.stattools.adfuller(s)[1]
res = adf(s); p = res[1]
res, p
```

```
Out[3]: ((-50.80332180276013,
         0.0,
         0,
         2519,
         {'1%': -3.4329486408391174,
          '5%': -2.8626880695259413,
          '10%': -2.567381161224712},
         6950.968131407137),
         0.0)
```

1.1.2 (b) Compute the cumulative sum of the observations. This is a non-stationary series w/o memory.

```
In [4]: cmsm = pd.Series(s).cumsum()
        cmsm.plot()
```

```
Out[4]: <matplotlib.axes._subplots.AxesSubplot at 0x7f4531e30f60>
```



(i) What is the order of integration of this cumulative series?

```
In [5]: orders = [0, 1, 2, 3, 4]
        for o in orders:
            diff_ = np.diff(cmsm,o)
            print('='*27)
            print(f'order: {o}, pVal: {p_val(diff_)}')
```

```

=====
order: 0, pVal: 0.5704444806659968
=====
order: 1, pVal: 0.0
=====
order: 2, pVal: 7.347529850653773e-30
=====
order: 3, pVal: 0.0
=====
order: 4, pVal: 0.0

```

(ii) Compute the ADF statistic on this series. What is the p-value?

```
In [6]: p_val(cmsm)
```

```
Out[6]: 0.5704444806659968
```

1.1.3 (c) Differentiate the series twice. What is the p-value of this over-differentiated series?

```
In [7]: diff_ = np.diff(cmsm,2)
        p_val(diff_)
```

```
Out[7]: 7.347529850653773e-30
```

1.2 [5.2] Generate a time series that follows a sinusoidal function. This is a stationary series with memory.

```
In [8]: np.random.seed(0)
```

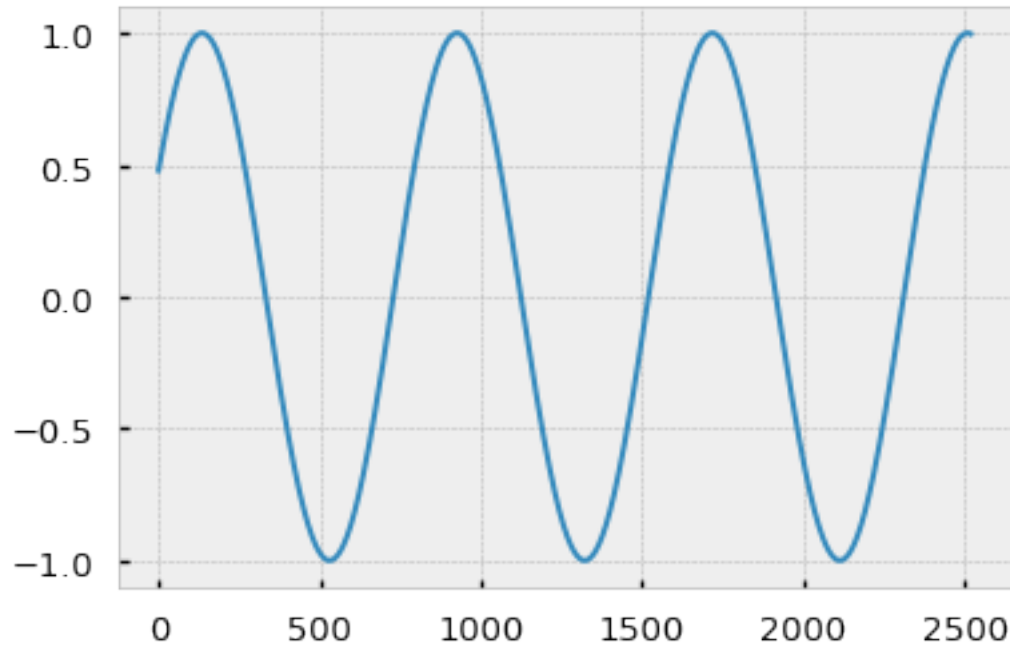
```

rand = np.random.random(N)

idx = np.linspace(0,10, N)
s = pd.Series(1*np.sin(2.*idx + .5))
s.plot()

```

```
Out[8]: <matplotlib.axes._subplots.AxesSubplot at 0x7f4531db3be0>
```



1.2.1 (a) Compute the ADF statistic on this series. What is the p-value?

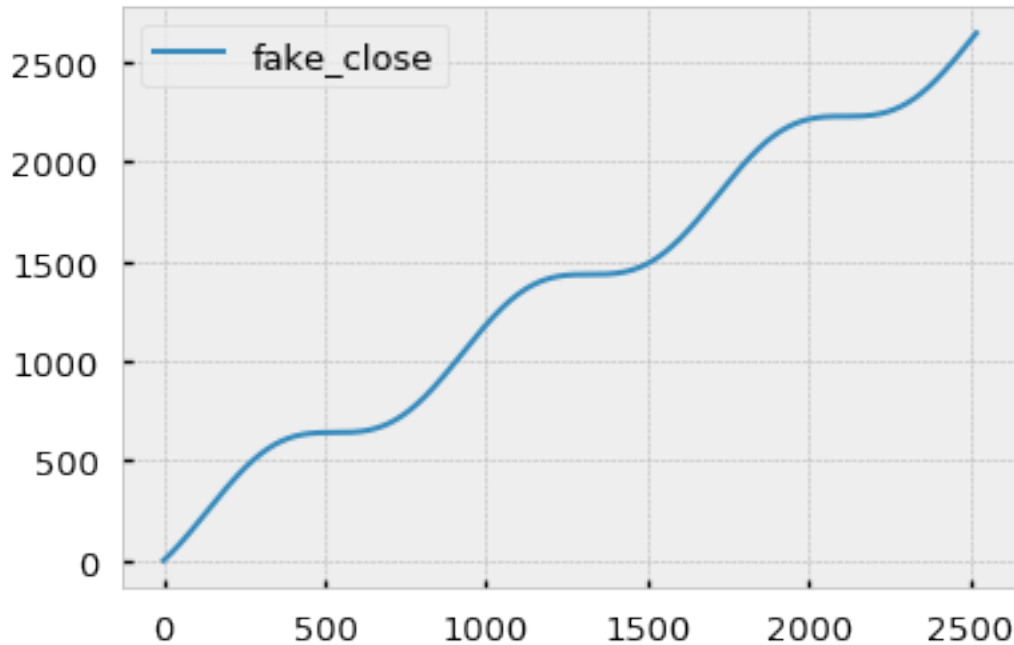
```
In [9]: p_val(s)
```

```
Out[9]: 0.0
```

1.2.2 (b) Shift every observation by the same positive value. Compute the cumulative sum of the observations. This is a non-stationary series with memory.

```
In [10]: s_ = (s + 1).cumsum().rename('fake_close').to_frame()
         s_.plot()
```

```
Out[10]: <matplotlib.axes._subplots.AxesSubplot at 0x7f4532cf7fd0>
```



(i) Compute the ADF statistic on this series. What is the p-value?

```
In [11]: adf(s_['fake_close'].dropna()), p_val(s_['fake_close'])
```

```
Out[11]: ((-0.19138785660460378,
          0.9395404066056363,
          27,
          2492,
          {'1%': -3.432976825339513,
           '5%': -2.862700515844509,
           '10%': -2.5673877878037974},
          -142670.50155880928),
          0.9395404066056363)
```

(ii) Apply an expanding window `fracdiff`, with $\tau = 1E - 2$. For what minimum d value do you get a p-value below 5%?

```
In [12]: def getWeights(d,size):
          # thres>0 drops insignificant weights
          w=[1.]
          for k in range(1,size):
              w_ = -w[-1]/k*(d-k+1)
              w.append(w_)
          w=np.array(w[::-1]).reshape(-1,1)
          return w

          #getWeights(0.1, s_.shape[0])
```



```

In [13]: def fracDiff(series, d, thres=0.01):
    '''
    Increasing width window, with treatment of NaNs
    Note 1: For thres=1, nothing is skipped
    Note 2: d can be any positive fractional, not necessarily
            bounded between [0,1]
    '''
    #1) Compute weights for the longest series
    w=getWeights(d, series.shape[0])
    #bp()
    #2) Determine initial calcs to be skipped based on weight-loss threshold
    w_=np.cumsum(abs(w))
    w_ /= w_[-1]
    skip = w_[w_>thres].shape[0]
    #3) Apply weights to values
    df={}
    for name in series.columns:
        seriesF, df_=series[[name]].fillna(method='ffill').dropna(), pd.Series()
        for iloc in range(skip, seriesF.shape[0]):
            loc=seriesF.index[iloc]
            test_val = series.loc[loc,name] # must resample if duplicate index
            if isinstance(test_val, (pd.Series, pd.DataFrame)):
                test_val = test_val.resample('1m').mean()
            if not np.isfinite(test_val).any(): continue # exclude NAs
            try:
                df_.loc[loc]=np.dot(w[-(iloc+1):,:].T, seriesF.loc[:loc])[0,0]
            except:
                continue
        df[name]=df_.copy(deep=True)
    df=pd.concat(df,axis=1)
    return df

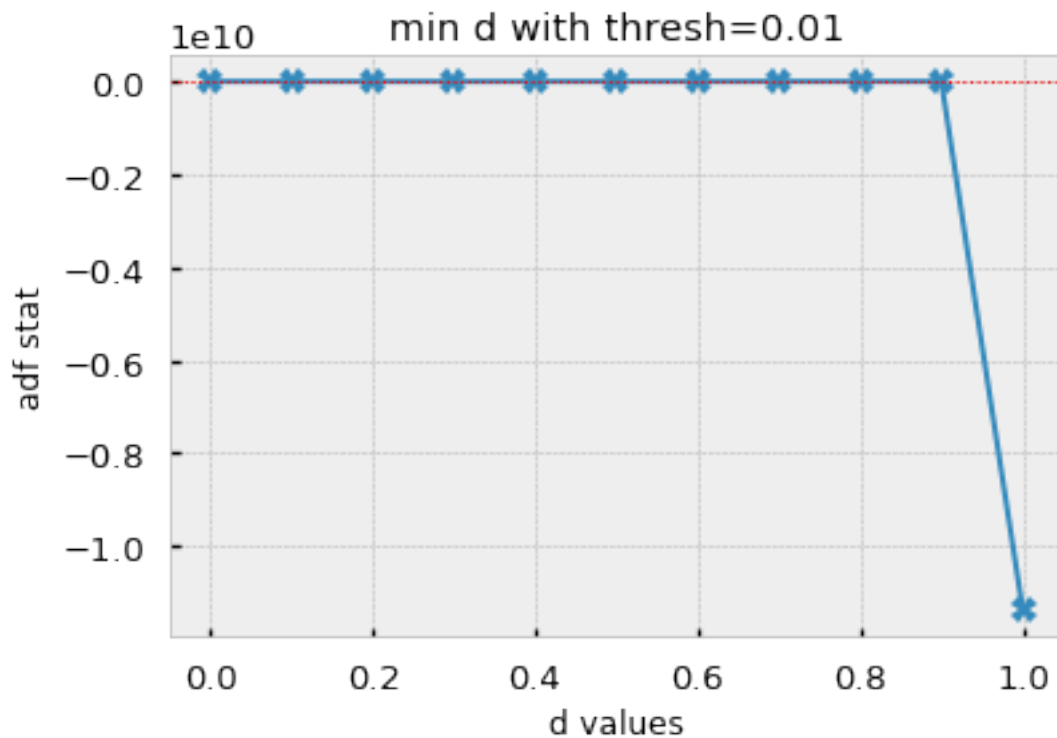
In [14]: cols = ['adfStat', 'pVal', 'lags', 'nObs', '95% conf']#, 'corr']
    out = pd.DataFrame(columns=cols)
    for d in np.linspace(0,1,11):
        try:
            df0 = fracDiff(s_,d)
            df0 = sm.tsa.stattools.adfuller(df0['fake_close'],maxlag=1,regression='c',autol
            out.loc[d]=list(df0[:4])+[df0[4]['5%']]
        except:
            break

f,ax=plt.subplots()
out['adfStat'].plot(ax=ax, marker='X')
ax.axhline(out['95% conf'].mean(),lw=1,color='r',ls='dotted')
ax.set_title('min d with thresh=0.01')
ax.set_xlabel('d values')
ax.set_ylabel('adf stat');

```

```
display(out)
```

	adfStat	pVal	lags	nObs	95% conf
0.0	2.833609e+00	1.000000e+00	1.0	2517.0	-2.862689
0.1	8.870880e+00	1.000000e+00	1.0	761.0	-2.865345
0.2	-7.366367e+00	9.213847e-11	1.0	963.0	-2.864546
0.3	-2.267608e+01	0.000000e+00	1.0	1357.0	-2.863672
0.4	-2.259792e+01	0.000000e+00	1.0	1821.0	-2.863128
0.5	-3.781556e+01	0.000000e+00	1.0	2188.0	-2.862862
0.6	-4.388734e+01	0.000000e+00	1.0	2385.0	-2.862753
0.7	-6.322546e+01	0.000000e+00	1.0	2466.0	-2.862713
0.8	-7.371512e+01	0.000000e+00	1.0	2497.0	-2.862698
0.9	-4.877829e+01	0.000000e+00	1.0	2510.0	-2.862692
1.0	-1.135810e+10	0.000000e+00	1.0	2516.0	-2.862689



(iii) Apply FFD with $\tau = 1E - 5$. For what minimum d value do you get a p-value below 5%

```
In [15]: cols = ['adfStat', 'pVal', 'lags', 'nObs', '95% conf']#, 'corr']
          out = pd.DataFrame(columns=cols)
          for d in np.linspace(0,1,11):
              try:
                  df0 = fracDiff(s_,d,thres=1e-5)
```

```

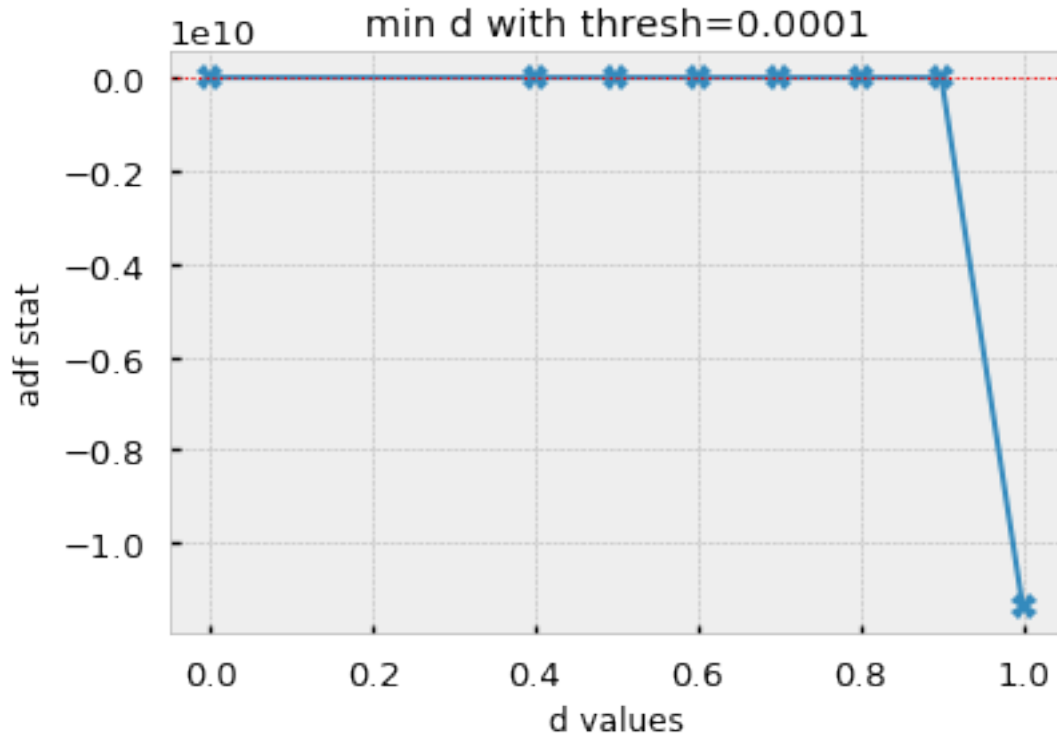
df0 = sm.tsa.stattools.adfuller(df0['fake_close'],maxlag=1,regression='c',autol
out.loc[d]=list(df0[:4])+[df0[4]['5%']]
except Exception as e:
    print(f'd: {d}, error: {e}')
    continue

f,ax=plt.subplots()
out['adfStat'].plot(ax=ax, marker='X')
ax.axhline(out['95% conf'].mean(),lw=1,color='r',ls='dotted')
ax.set_title('min d with thresh=0.0001')
ax.set_xlabel('d values')
ax.set_ylabel('adf stat');
display(out)

d: 0.1, error: maxlag should be < nobs
d: 0.2, error: maxlag should be < nobs
d: 0.30000000000000004, error: maxlag should be < nobs

```

	adfStat	pVal	lags	nObs	95% conf
0.0	2.833609e+00	1.000000	1.0	2517.0	-2.862689
0.4	-0.000000e+00	0.958532	1.0	2.0	-10.370190
0.5	-5.825592e+03	0.000000	1.0	6.0	-3.646238
0.6	-5.718052e+03	0.000000	1.0	18.0	-3.042046
0.7	-6.338667e+03	0.000000	1.0	48.0	-2.923954
0.8	-8.751121e+03	0.000000	1.0	142.0	-2.882118
0.9	-8.327682e+03	0.000000	1.0	495.0	-2.867397
1.0	-1.135810e+10	0.000000	1.0	2516.0	-2.862689



1.3 [5.3] Take the series from exercise 2.b:

1.3.1 (a) Fit the series to a sine function. What is the R-squared?

Note: Is there a simpler way to do this?

```
In [16]: ## fitting function taken from stackoverflow
## https://stackoverflow.com/questions/16716302/how-do-i-fit-a-sine-curve-to-my-data-
import numpy, scipy.optimize

def fit_sin(tt, yy):
    '''Fit sin to the input time sequence, and return fitting parameters "amp", "omega"
    tt = numpy.array(tt)
    yy = numpy.array(yy)
    ff = numpy.fft.fftfreq(len(tt), (tt[1]-tt[0])) # assume uniform spacing
    Fyy = abs(numpy.fft.fft(yy))
    guess_freq = abs(ff[numpy.argmax(Fyy[1:])+1]) # excluding the zero frequency "peak"
    guess_amp = numpy.std(yy) * 2.**0.5
    guess_offset = numpy.mean(yy)
    guess = numpy.array([guess_amp, 2.*numpy.pi*guess_freq, 0., guess_offset])

    def sinfunc(t, A, w, p, c): return A * numpy.sin(w*t + p) + c
    popt, pcov = scipy.optimize.curve_fit(sinfunc, tt, yy, p0=guess)
```

```

    A, w, p, c = popt
    f = w/(2.*numpy.pi)
    fitfunc = lambda t: A * numpy.sin(w*t + p) + c
    return {"amp": A, "omega": w, "phase": p, "offset": c, "freq": f, "period": 1./f, "

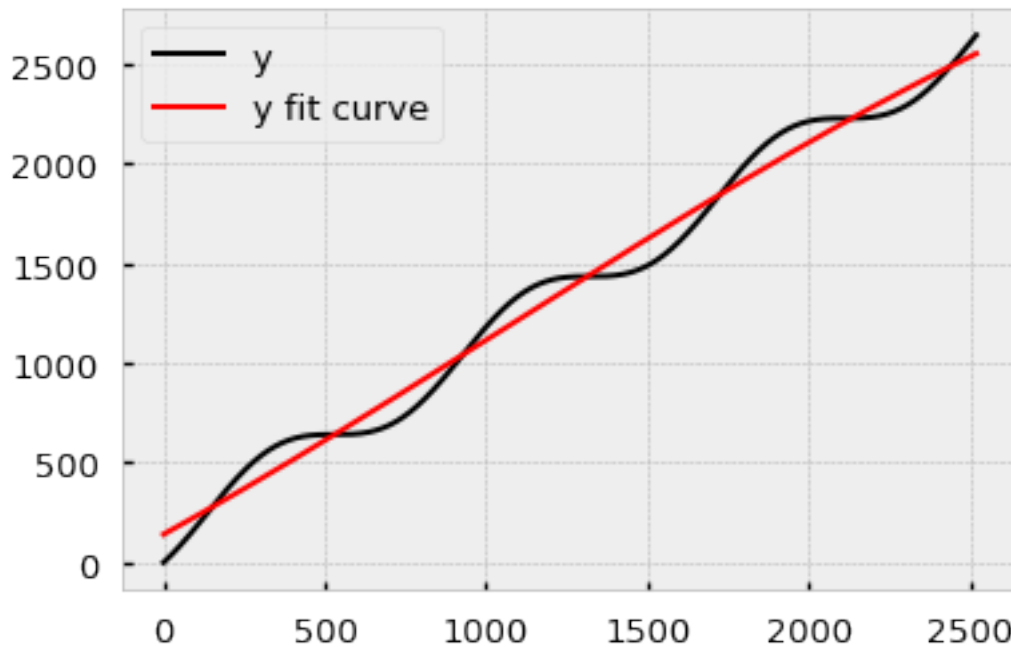
In [17]: res = fit_sin(s_.index.values, s_.values.ravel())
        res

Out[17]: {'amp': -2126.284918105075,
          'omega': 0.0004797999293735519,
          'phase': 2.6281709304906165,
          'offset': 1186.5758476117003,
          'freq': 7.636253045494306e-05,
          'period': 13095.42774502529,
          'fitfunc': <function __main__.fit_sin.<locals>.<lambda>(t)>,
          'maxcov': 20695.22970332341,
          'rawres': (array([1.01493055e+03, 2.49332750e-03, 0.00000000e+00, 1.36801775e+03]),
                    array([-2.12628492e+03, 4.79799929e-04, 2.62817093e+00, 1.18657585e+03]),
                    array([[ 2.06952297e+04, 4.98650324e-03, -7.28185170e+00,
                           3.99478720e+03],
                          [ 4.98650324e-03, 1.20345366e-09, -1.74718083e-06,
                           9.43037058e-04],
                          [-7.28185170e+00, -1.74718083e-06, 2.76112152e-03,
                           -1.81976202e+00],
                          [ 3.99478720e+03, 9.43037058e-04, -1.81976202e+00,
                           1.64645703e+03]]))})

In [18]: xx = s_.index.values
        yy = s_.values.ravel()

        plt.plot(xx, yy, "-k", label="y", linewidth=2)
        #plt.plot(tt, yynoise, "ok", label="y with noise")
        plt.plot(xx, res["fitfunc"](xx), "r-", label="y fit curve", linewidth=2)
        plt.legend(loc="best")
        plt.show()

```



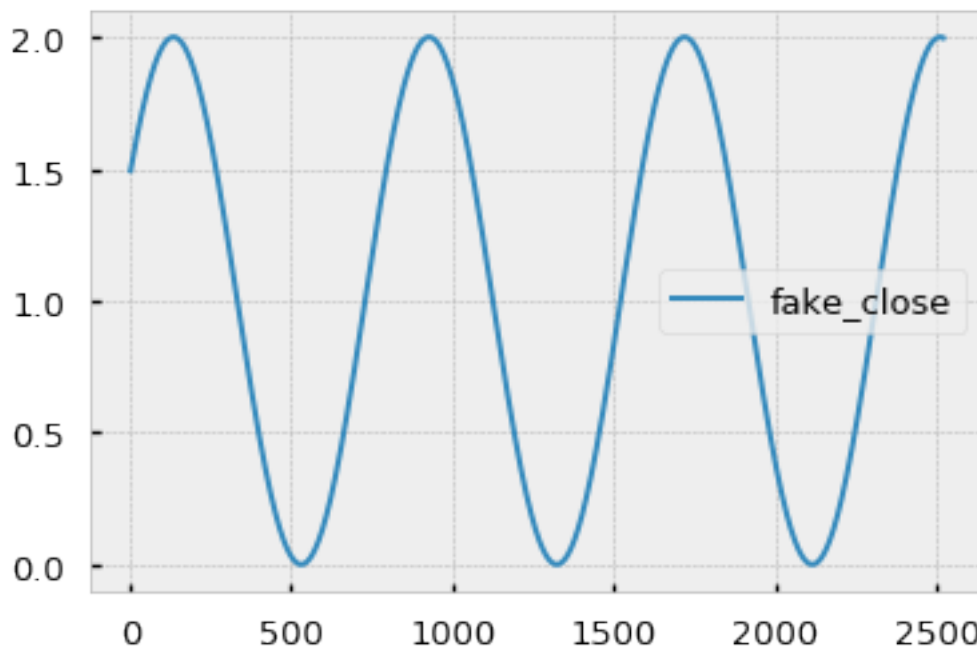
```
In [19]: slope, intercept, r_value, p_value, std_err = scipy.stats.linregress(yy, res["fitfunc"])
         r_value**2
```

```
Out[19]: 0.9859147406461111
```

1.3.2 (b) Apply FFD($d = 1$). Fit the series to a sine function. What is the R-squared?

```
In [20]: #cols = ['adfStat', 'pVal', 'lags', 'nObs', '95% conf']#, 'corr']
         #out = pd.DataFrame(columns=cols)
         df1 = fracDiff(s_, d=1)
         #df1 = sm.tsa.stattools.adfuller(df0['fake_close'], maxlag=1, regression='c', autolag=None)
         #out.loc[d]=list(df0[:4])+[df0[4]['5%']]
         df1.plot()
```

```
Out[20]: <matplotlib.axes._subplots.AxesSubplot at 0x7f452c31d160>
```



```
In [21]: xx = df1.index.values
         yy = df1.values.ravel()

         res = fit_sin(xx, yy)
         slope, intercept, r_value, p_value, std_err = scipy.stats.linregress(yy, res["fitfunc"])
         r_value**2
```

Out[21]: 1.0

1.3.3 (c) What value of d maximizes the R-squared of a sinusoidal fit on $\text{FFD}(d)$? Why?

In []:

1.4 5.4

Take dollar bar series on E-mini S&P 500 futures. Using the code in Snippet 5.3, for some d in $[0, 2]$, compute $\text{fracDiff_FFD}(\text{fracDiff_FFD}(\text{series}, d))$. What do you get? Why?

Note: for some reason this never finishes computing in my notebook

```
In [22]: def getWeights_FFD(d, thres):
         w, k = [1.], 1
         while True:
             w_ = -w[-1]/k*(d-k+1)
             if abs(w_) < thres: break
             w.append(w_); k+=1
         return np.array(w[:-1]).reshape(-1, 1)
```

```

#-----
def fracDiff_FFD(series,d,thres=1e-5):
    # Constant width window (new solution)
    w = getWeights_FFD(d,thres)
    width = len(w)-1
    df={}
    for name in series.columns:
        seriesF, df_=series[[name]].fillna(method='ffill').dropna(), pd.Series()
        for iloc1 in range(width,seriesF.shape[0]):
            loc0,loc1=seriesF.index[iloc1-width], seriesF.index[iloc1]
            test_val = series.loc[loc1,name] # must resample if duplicate index
            if isinstance(test_val, (pd.Series, pd.DataFrame)):
                test_val = test_val.resample('1m').mean()
            if not np.isfinite(test_val).any(): continue # exclude NAs
            #print(f'd: {d}, iloc1:{iloc1} shapes: w:{w.T.shape}, series: {seriesF.loc[loc0:loc1].shape}')
            try:
                df_.loc[loc1]=np.dot(w.T, seriesF.loc[loc0:loc1])[0,0]
            except:
                continue
        df[name]=df_.copy(deep=True)
    df=pd.concat(df,axis=1)
    return df

```

```

In [23]: def dask_resample(ser, freq='L'):
        dds = dd.from_pandas(ser, chunksize=len(ser)//100)
        tdf = (dds
                .resample(freq)
                .mean()
                .dropna()
                ).compute()
        return tdf

```

```

infp=PurePath(data_dir/'clean_IVE_fut_prices.parquet')
df = pd.read_parquet(infp)

dv_rs = dask_resample(df, '1s')
cprint(dv_rs)

dbars = brs.dollar_bar_df(dv_rs, 'dv', 1_000_000)
cprint(dbars)

```

```

[#####] | 100% Completed | 31.4s

```

```

-----
dataframe information
-----

```

```

price      bid      ask      size      v      dv

```



```

dates
2018-02-26 15:59:59 115.35 115.34 115.36      412.5      412.5 4.758188e+04
2018-02-26 16:00:00 115.35 115.34 115.35     5362.0     5362.0 6.185067e+05
2018-02-26 16:10:00 115.35 115.22 115.58        0.0        0.0 0.000000e+00
2018-02-26 16:16:14 115.30 114.72 115.62    778677.0    778677.0 8.978146e+07
2018-02-26 18:30:00 115.35 114.72 117.38        0.0        0.0 0.000000e+00
-----

```

```

0%|          | 0/941297 [00:00<?, ?it/s]

```

```

<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 941297 entries, 2009-09-28 09:30:00 to 2018-02-26 18:30:00
Data columns (total 6 columns):
price      941297 non-null float64
bid        941297 non-null float64
ask        941297 non-null float64
size       941297 non-null float64
v          941297 non-null float64
dv         941297 non-null float64
dtypes: float64(6)
memory usage: 50.3 MB
None
-----

```

```

100%|| 941297/941297 [00:00<00:00, 2822315.09it/s]

```

```

-----
dataframe information
-----

```

		price	bid	ask	size \
dates					
2018-02-26 15:31:06	115.29	115.280000	115.290000	2022.000000	
2018-02-26 15:40:15	115.41	115.400000	115.410000	723.000000	
2018-02-26 15:49:42	115.20	115.176667	115.186667	4487.166667	
2018-02-26 15:59:04	115.27	115.260000	115.270000	300.000000	
2018-02-26 16:16:14	115.30	114.720000	115.620000	778677.000000	

		v	dv
dates			
2018-02-26 15:31:06	2022.000000	2.331164e+05	
2018-02-26 15:40:15	723.000000	8.344143e+04	
2018-02-26 15:49:42	4487.166667	5.171190e+05	
2018-02-26 15:59:04	300.000000	3.458100e+04	
2018-02-26 16:16:14	778677.000000	8.978146e+07	

```

<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 30861 entries, 2009-09-28 09:53:49 to 2018-02-26 16:16:14
Data columns (total 6 columns):
price      30861 non-null float64
bid        30861 non-null float64
ask        30861 non-null float64
size       30861 non-null float64
v          30861 non-null float64
dv         30861 non-null float64
dtypes: float64(6)
memory usage: 1.6 MB
None

```

```

In [24]: d = 0.5
        sel = dbars[['price']].iloc[:100]
        #val = fracDiff_FFD(fracDiff_FFD(sel, d), -d) # Never finishes don't run
        #val

```

1.5 5.5

Take the dollar bar series on E-mini S&P 500 futures.

1.5.1 (a) Form a new series as a cumulative sum of log-prices

```

In [25]: x = np.log(dbars.price).cumsum()
        cprint(x)

        x.plot()

```

```

-----
dataframe information
-----

```

	price
dates	
2018-02-26 15:31:06	136219.267867
2018-02-26 15:40:15	136224.016358
2018-02-26 15:49:42	136228.763027
2018-02-26 15:59:04	136233.510305
2018-02-26 16:16:14	136238.257842

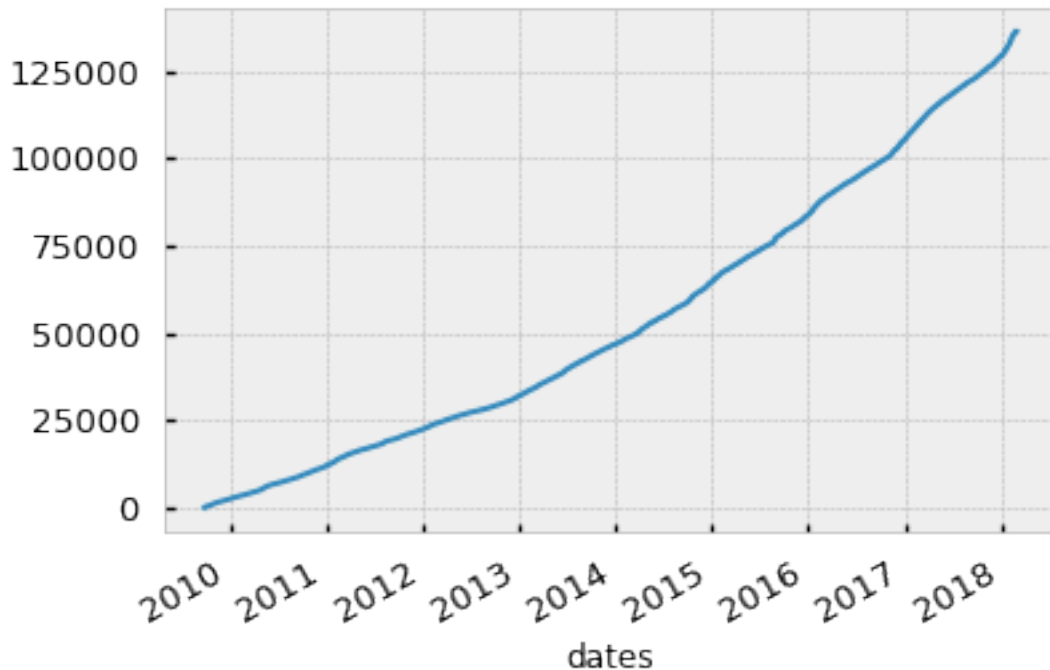
```

-----
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 30861 entries, 2009-09-28 09:53:49 to 2018-02-26 16:16:14
Data columns (total 1 columns):
price      30861 non-null float64
dtypes: float64(1)
memory usage: 482.2 KB

```

None

Out[25]: <matplotlib.axes._subplots.AxesSubplot at 0x7f452c294c18>



1.5.2 (b) Apply FFD, with $\tau = 1E - 5$. Determine for what minimum $d \in [0,2]$ the new series is stationary

```
In [26]: def get_optimal_ffd(ds, t=1e-5):

    cols = ['adfStat', 'pVal', 'lags', 'nObs', '95% conf']#, 'corr']
    out = pd.DataFrame(columns=cols)

    for d in tqdm_notebook(ds):
        try:
            #dfx = fracDiff(x.to_frame(), d, thres=1e-5)
            dfx = fracDiff_FFD(x.to_frame(), d, thres=t)
            dfx = sm.tsa.stattools.adfuller(dfx['price'], maxlag=1, regression='c', autol
            out.loc[d]=list(dfx[:4])+[dfx[4]['5%']]
        except Exception as e:
            print(f'{d} error: {e}')
            break
    return out
```

```

=====
ds = [0.25,0.5,1,1.5,1.8,1.9,1.999,2]
thres = 1e-5
out = get_optimal_ffd(ds, thres) # takes 15 minutes to run on ~44k points

HBox(children=(IntProgress(value=0, max=8), HTML(value='')))

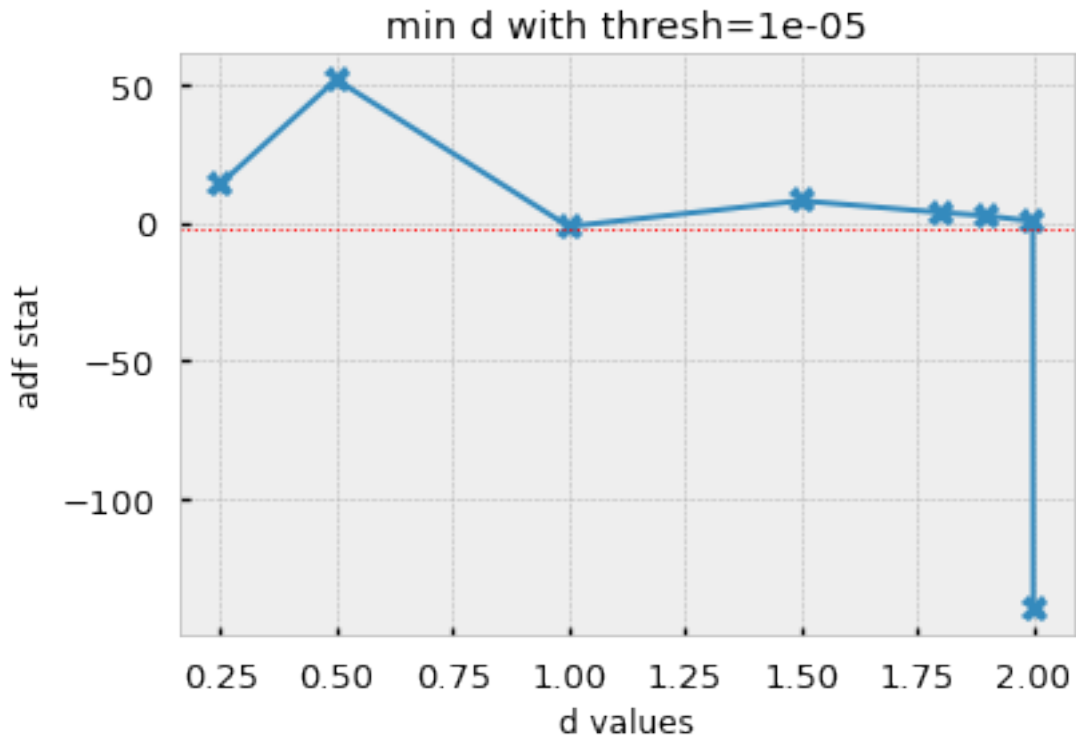
```

```

In [27]: f,ax=plt.subplots()
         out['adfStat'].plot(ax=ax, marker="X", markersize=10)
         ax.axhline(out['95% conf'].mean(),lw=1,color='r',ls='dotted')
         ax.set_title(f'min d with thresh={thres}')
         ax.set_xlabel('d values')
         ax.set_ylabel('adf stat');
         display(out)

```

	adfStat	pVal	lags	nObs	95% conf
0.250	13.729830	1.000000	1.0	28056.0	-2.861643
0.500	51.436628	1.000000	1.0	29933.0	-2.861637
1.000	-1.552027	0.507636	1.0	30858.0	-2.861634
1.500	7.740002	1.000000	1.0	30788.0	-2.861634
1.800	3.516861	1.000000	1.0	30818.0	-2.861634
1.900	2.326653	0.998971	1.0	30829.0	-2.861634
1.999	0.342193	0.979200	1.0	30853.0	-2.861634
2.000	-139.096911	0.000000	1.0	30857.0	-2.861634



```
In [28]: min_ffd = out[out.pVal < 0.05].iloc[0].name
         min_ffd
```

```
Out[28]: 2.0
```

1.5.3 (c) Compute the correlation of the fracdiff series to the original (untransformed) series

```
In [29]: dfx2 = fracDiff_FFD(x.to_frame(),min_ffd,thres=thres)
         cprint(dfx2)

         joined = dfx2.join(x.rename('original'), how='left')
         joined.corr()
```

```
-----
dataframe information
-----
```

```

           price
2018-02-26 15:31:06 -0.000575
2018-02-26 15:40:15  0.001040
2018-02-26 15:49:42 -0.001821
2018-02-26 15:59:04  0.000607
2018-02-26 16:16:14  0.000260
-----
```

```
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 30859 entries, 2009-09-28 10:19:50 to 2018-02-26 16:16:14
Data columns (total 1 columns):
price      30859 non-null float64
dtypes: float64(1)
memory usage: 482.2 KB
None
```

```
Out[29]:
```

	price	original
price	1.000000	-0.002704
original	-0.002704	1.000000

1.5.4 (d) Apply Engel-Granger cointegration test on the original and fracdiff series. Are they cointegrated? Why?

```
In [30]: sm.tsa.stattools.coint(joined.price, joined.original)
```

```
Out[30]: (-27.177877591790516, 0.0, array([-3.89679495, -3.33632801, -3.04458745]))
```

1.5.5 (e) Apply a Jarque-Bera normality test on the fracdiff series.

```
In [31]: np.random.seed(0)
stats.jarque_bera(df2)
```

```
Out[31]: (4067589070.053132, 0.0)
```

1.6 5.6 Take the fracdiff series from exercise 5

1.6.1 (a) Apply a CUSUM filter (Chapter 2), where h is twice the standard deviation of the series.

```
In [32]: tEvents = snp.getTEvents(df2,h=df2.std().iat[0]*2)
display(tEvents)
```

```
100%|| 8417/8417 [00:03<00:00, 2430.45it/s]
```

```
DatetimeIndex(['2009-09-28 11:34:21', '2009-09-28 11:53:11',
               '2009-09-28 12:30:58', '2009-09-28 14:44:10',
               '2009-09-28 15:39:46', '2009-09-29 12:53:47',
               '2009-09-29 13:59:56', '2009-09-30 12:40:13',
               '2009-09-30 13:31:12', '2009-10-01 09:29:51',
               ...,
               '2018-02-26 12:09:16', '2018-02-26 12:48:18',
               '2018-02-26 13:06:34', '2018-02-26 13:21:31',
               '2018-02-26 13:48:20', '2018-02-26 13:58:17',
               '2018-02-26 14:09:45', '2018-02-26 15:20:10',
```

```

        '2018-02-26 15:24:32', '2018-02-26 16:16:14'],
dtype='datetime64[ns]', length=8341, freq=None)

```

1.6.2 (b) Use the filtered timestamps to sample a features' matrix. Use as one of the features the fracDiff value.

```

In [33]: dbars_feat = dbars.price.loc[tEvents]
        frac_diff_feat = dfx2.loc[tEvents]
        ftMtx = (pd.DataFrame()
                .assign(dbars=dbars_feat,
                        frac_diff_feat=frac_diff_feat)
                .drop_duplicates().dropna())
        cprint(ftMtx)

```

```

-----
dataframe information
-----

```

	dbars	frac_diff_feat
2018-02-26 13:58:17	115.1200	0.000391
2018-02-26 14:09:45	115.1501	0.000261
2018-02-26 15:20:10	115.3200	0.000694
2018-02-26 15:24:32	115.3563	0.000315
2018-02-26 16:16:14	115.3000	0.000260

```

-----
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 8329 entries, 2009-09-28 11:34:21 to 2018-02-26 16:16:14
Data columns (total 2 columns):
dbars                8329 non-null float64
frac_diff_feat       8329 non-null float64
dtypes: float64(2)
memory usage: 195.2 KB
None
-----

```

1.6.3 (c) Form labels using the triple-barrier method, with symmetric horizontal barriers of twice the daily standard deviation, and a vertical barrier of 5 days

```

In [34]: dailyVol = snp.getDailyVol(ftMtx.dbars)
        t1 = snp.addVerticalBarrier(tEvents, ftMtx.dbars, numDays=5)

        pts1 = [1,1]
        #pts1 = [daily]
        target=dailyVol*2
        # select minRet
        minRet = 0.01

```

```
# get cpu count - 1
cpus = cpu_count() - 1
events = snp.getEvents(ftMtx.dbars,tEvents,ptsl,target,minRet,cpus,t1=t1)
cprint(events)
```

2018-10-18 17:06:56.495423 100.0% applyPtSl0nT1 done after 0.02 minutes. Remaining 0.0 minutes..

```
-----
dataframe information
-----
```

```

          t1      trgt
2018-02-26 13:58:17 NaT  0.023527
2018-02-26 14:09:45 NaT  0.023438
2018-02-26 15:20:10 NaT  0.023394
2018-02-26 15:24:32 NaT  0.023354
2018-02-26 16:16:14 NaT  0.023291
-----
```

```
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 6948 entries, 2009-09-30 12:40:13 to 2018-02-26 16:16:14
Data columns (total 2 columns):
t1      6914 non-null datetime64[ns]
trgt     6948 non-null float64
dtypes: datetime64[ns](1), float64(1)
memory usage: 162.8 KB
None
-----
```

In [35]: *## Example*

```
close=ftMtx.dbars
numCoEvents = snp.mpPandasObj(snp.mpNumCoEvents,('molecule',events.index),
                               cpus,closeIdx=close.index,t1=events['t1'])
numCoEvents = numCoEvents.loc[~numCoEvents.index.duplicated(keep='last')]
numCoEvents = numCoEvents.reindex(close.index).fillna(0)
out=pd.DataFrame()
out['tW'] = snp.mpPandasObj(snp.mpSampleTW,('molecule',events.index),
                            cpus,t1=events['t1'],numCoEvents=numCoEvents)

cprint(out)
```

2018-10-18 17:06:58.956595 100.0% mpNumCoEvents done after 0.01 minutes. Remaining 0.0 minutes..

```
-----
dataframe information
-----
```

```

          tW
2018-02-26 13:58:17 NaN
-----
```



```
2018-02-26 14:09:45 NaN
2018-02-26 15:20:10 NaN
2018-02-26 15:24:32 NaN
2018-02-26 16:16:14 NaN
```

```
-----
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 6948 entries, 2009-09-30 12:40:13 to 2018-02-26 16:16:14
Data columns (total 1 columns):
tW      6914 non-null float64
dtypes: float64(1)
memory usage: 108.6 KB
None
-----
```

```
2018-10-18 17:06:59.900933 100.0% mpSampleTW done after 0.01 minutes. Remaining 0.0 minutes.
```

```
In [36]: ## example ##
         out['w']=snp.mpPandasObj(snp.mpSampleW,('molecule',events.index),cpus,
                                t1=events['t1'],numCoEvents=numCoEvents,close=close)
         out['w']*=out.shape[0]/out['w'].sum()
         cprint(out)
```

```
2018-10-18 17:07:01.247596 100.0% mpSampleW done after 0.01 minutes. Remaining 0.0 minutes.
```

```
-----
dataframe information
-----
```

	tW	w
2018-02-26 13:58:17	NaN	0.0
2018-02-26 14:09:45	NaN	0.0
2018-02-26 15:20:10	NaN	0.0
2018-02-26 15:24:32	NaN	0.0
2018-02-26 16:16:14	NaN	0.0

```
-----
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 6948 entries, 2009-09-30 12:40:13 to 2018-02-26 16:16:14
Data columns (total 2 columns):
tW      6914 non-null float64
w       6948 non-null float64
dtypes: float64(2)
memory usage: 162.8 KB
None
-----
```

```

In [37]: # get labels
         labels = snp.getBins(events, ftMtx.dbars)
         #cprint(labels)

         clean_labels = snp.dropLabels(labels)
         cprint(clean_labels)

dropped label:  0.0 0.0011570726063060456
-----
dataframe information
-----
           ret  bin
2018-02-21 09:47:52  0.009430  1.0
2018-02-21 11:10:00  0.008097  1.0
2018-02-21 12:53:48  0.012764  1.0
2018-02-21 13:19:05  0.012619  1.0
2018-02-21 14:12:30  0.011240  1.0
-----
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 6906 entries, 2009-09-30 12:40:13 to 2018-02-21 14:12:30
Data columns (total 2 columns):
ret      6906 non-null float64
bin      6906 non-null float64
dtypes: float64(2)
memory usage: 161.9 KB
None
-----

```

1.6.4 (d) Fit a bagging classifier of decision trees where:

(i) The observed features are bootstrapped using the sequential method from chapter 4. Note: must use multiprocessing of some kind as seqBootstrap is very slow

```

In [38]: @nb.njit
         def func(arr,i):
             col = arr[i]
             mask = np.where(col>0)
             return np.mean(col[mask])

         @nb.njit
         def njit_getAvgUniqueness(indM):
             # Average uniqueness from indicator matrix
             c=indM.sum(axis=1).reshape(-1,1) # concurrency
             u=np.divide(indM,c) # uniqueness
             avgU = np.zeros(len(u.T)) # avg. uniqueness
             i = 0

```

```

        for i in range(len(u.T)):
            avgU[i] = func(u.T,i)
            i+=1
        return avgU

@nb.jit
def jit_seqBootstrap(indM,sLength=None):
    # Generate a sample via sequential bootstrap
    if sLength is None:sLength=indM.shape[1]
    phi=[]
    while len(phi)<sLength:
        avgU=pd.Series()
        for i in indM:
            indM_=indM[phi+[i]] # reduce indM
            avgU.loc[i]=njit_getAvgUniqueness(indM_.values)[-1]
        prob=avgU/avgU.sum() # draw prob
        phi+= [np.random.choice(indM.columns,p=prob)]
    return phi
#-----

def split_t1(t1, partitions):
    return np.array_split(t1, partitions)

def mp_func(indM):
    # jit funcs about 2x as fast
    phi = jit_seqBootstrap(indM)
    seqU = njit_getAvgUniqueness(indM[phi].values).mean()
    #phi = snp.seqBootstrap(indM)
    #seqU= snp.getAvgUniqueness(indM[phi])
    return seqU

def main_mp(t1, partitions=100, cpus=8):
    jobs = []
    splits = split_t1(t1,partitions=100)
    for part_t1 in splits:
        indM = snp.getIndMatrix(part_t1.index, part_t1)
        job = {'func':mp_func,'indM':indM}
        jobs.append(job)
    if cpus==1: out=snp.processJobs_(jobs)
    else: out=snp.processJobs(jobs,numThreads=cpus)
    return pd.DataFrame(out)

```

In [39]: seqUs = main_mp(t1)

2018-10-18 17:09:25.339624 100.0% mp_func done after 2.29 minutes. Remaining 0.0 minutes.

In [40]: seqUs.describe()

```
Out[40]:
0
count    100.000000
mean      0.074860
std       0.022629
min       0.029593
25%       0.062652
50%       0.071346
75%       0.089933
max       0.133672
```

```
In [41]: # get avg uniqueness for bootstrapping
avgU = seqUs.mean()[0]
avgU
```

```
Out[41]: 0.07485970931182089
```

(ii) On each bootstrapped sample, sample weights are determined using the techniques from Chapter 4 Note: alternative implementations are welcome.

```
In [42]: from sklearn.model_selection import train_test_split
from sklearn.tree import DecisionTreeClassifier
from sklearn.ensemble import RandomForestClassifier, RandomForestRegressor, BaggingClassifier
```

```
In [43]: def evaluate(X,y,clf):
    from sklearn import metrics
    # The random forest model by itself
    y_pred_rf = clf.predict_proba(X)[: , 1]
    y_pred = clf.predict(X)
    fpr_rf, tpr_rf, _ = metrics.roc_curve(y, y_pred_rf)
    print(metrics.classification_report(y, y_pred))

    plt.figure(figsize=(9,6))
    plt.plot([0, 1], [0, 1], 'k--')
    plt.plot(fpr_rf, tpr_rf, label='clf')
    plt.xlabel('False positive rate')
    plt.ylabel('True positive rate')
    plt.title('ROC curve')
    plt.legend(loc='best')
    plt.show()
```

```
In [44]: trgt = clean_labels.bin
trgt
```

```
Out[44]: 2009-09-30 12:40:13    -1.0
2009-09-30 13:31:12    -1.0
2009-10-01 09:29:51    -1.0
2009-10-02 10:18:42     1.0
2009-10-02 10:35:05     1.0
2009-10-02 10:37:29     1.0
```

2009-10-02	13:20:04	1.0
2009-10-05	10:41:50	1.0
2009-10-05	11:50:49	1.0
2009-10-05	12:25:18	1.0
2009-10-05	13:11:27	1.0
2009-10-06	09:29:52	1.0
2009-10-06	10:16:02	1.0
2009-10-06	11:32:02	1.0
2009-10-06	15:35:49	1.0
2009-10-07	15:34:00	1.0
2009-10-07	15:53:56	1.0
2009-10-08	09:29:51	1.0
2009-10-08	12:07:39	1.0
2009-10-08	12:52:50	1.0
2009-10-09	11:21:48	1.0
2009-10-09	14:58:12	1.0
2009-10-09	15:28:04	1.0
2009-10-12	09:31:02	1.0
2009-10-12	10:14:38	1.0
2009-10-12	11:12:35	1.0
2009-10-12	14:08:45	1.0
2009-10-14	09:29:52	1.0
2009-10-14	11:59:09	1.0
2009-10-14	12:25:52	1.0
2009-10-14	14:12:01	1.0
2009-10-15	11:02:54	-1.0
2009-10-15	14:07:36	-1.0
2009-10-15	14:52:32	-1.0
2009-10-15	15:18:10	-1.0
2009-10-15	15:57:25	-1.0
2009-10-16	13:37:21	-1.0
2009-10-16	15:40:15	-1.0
2009-10-19	10:46:14	-1.0
2009-10-19	11:39:38	-1.0
2009-10-19	12:34:14	-1.0
2009-10-19	13:35:25	-1.0
2009-10-19	14:32:08	-1.0
2009-10-21	09:41:06	-1.0
2009-10-21	10:11:57	-1.0
2009-10-21	10:57:04	-1.0
2009-10-22	11:36:37	-1.0
2009-10-22	12:20:25	-1.0
2009-10-22	13:04:37	-1.0
2009-10-22	15:17:46	-1.0
	...	
2018-02-12	11:17:12	1.0
2018-02-12	11:22:33	1.0
2018-02-12	11:55:37	1.0

2018-02-12 12:12:51	1.0
2018-02-12 12:23:34	1.0
2018-02-12 13:57:53	1.0
2018-02-12 15:14:17	1.0
2018-02-13 11:48:45	1.0
2018-02-13 12:01:57	1.0
2018-02-13 13:43:37	1.0
2018-02-14 10:09:34	1.0
2018-02-14 10:19:18	1.0
2018-02-14 10:30:48	1.0
2018-02-14 11:41:53	1.0
2018-02-14 12:28:36	1.0
2018-02-14 12:58:13	1.0
2018-02-14 13:14:57	1.0
2018-02-14 13:36:02	1.0
2018-02-14 13:53:59	1.0
2018-02-14 14:08:28	1.0
2018-02-14 14:48:01	1.0
2018-02-14 15:00:14	1.0
2018-02-14 15:08:14	1.0
2018-02-15 09:31:56	1.0
2018-02-15 09:56:14	1.0
2018-02-15 11:22:26	1.0
2018-02-15 11:29:54	1.0
2018-02-15 13:21:31	-1.0
2018-02-15 14:51:30	-1.0
2018-02-15 15:58:36	-1.0
2018-02-15 15:59:47	-1.0
2018-02-15 16:00:00	-1.0
2018-02-16 10:42:42	-1.0
2018-02-16 10:59:17	-1.0
2018-02-16 11:11:50	-1.0
2018-02-16 12:05:18	-1.0
2018-02-16 12:32:45	-1.0
2018-02-16 13:46:07	-1.0
2018-02-16 14:56:08	-1.0
2018-02-16 15:03:29	-1.0
2018-02-16 15:10:45	-1.0
2018-02-20 09:50:13	1.0
2018-02-20 11:36:52	1.0
2018-02-20 12:35:02	1.0
2018-02-20 15:50:16	1.0
2018-02-21 09:47:52	1.0
2018-02-21 11:10:00	1.0
2018-02-21 12:53:48	1.0
2018-02-21 13:19:05	1.0
2018-02-21 14:12:30	1.0

Name: bin, Length: 6906, dtype: float64

```
In [45]: # model data
        #data = ftMtx.join(out,how='left').join(trgt,how='left').iloc[phi].dropna()
        data = ftMtx.join(out,how='left').join(trgt,how='left').dropna()
        X = data.iloc[:, :-1].values
        y = data.iloc[:, -1].values.reshape(-1,1)
```

```
In [46]: cprint(data)
```

```
-----
dataframe information
-----
```

	dbars	frac_diff_feat	tW	w	bin
2018-02-21 09:47:52	113.4700	0.002338	0.056174	0.355414	1.0
2018-02-21 11:10:00	113.6200	0.000304	0.056076	0.196611	1.0
2018-02-21 12:53:48	113.6000	0.000969	0.052932	0.258432	1.0
2018-02-21 13:19:05	113.6163	0.000143	0.051662	0.270087	1.0
2018-02-21 14:12:30	114.0382	0.001476	0.048293	0.326394	1.0

```
-----
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 6906 entries, 2009-09-30 12:40:13 to 2018-02-21 14:12:30
Data columns (total 5 columns):
dbars                6906 non-null float64
frac_diff_feat       6906 non-null float64
tW                   6906 non-null float64
w                    6906 non-null float64
bin                  6906 non-null float64
dtypes: float64(5)
memory usage: 323.7 KB
None
-----
```

```
In [47]: X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3, random_state=R
```

```
In [48]: base_clf = DecisionTreeClassifier(criterion='entropy',max_features='auto',
                                           class_weight='balanced')
        bc = BaggingClassifier(base_estimator=base_clf,n_estimators=1000,
                               max_samples=avgU,max_features=1.,random_state=RANDOM_STATE)
```

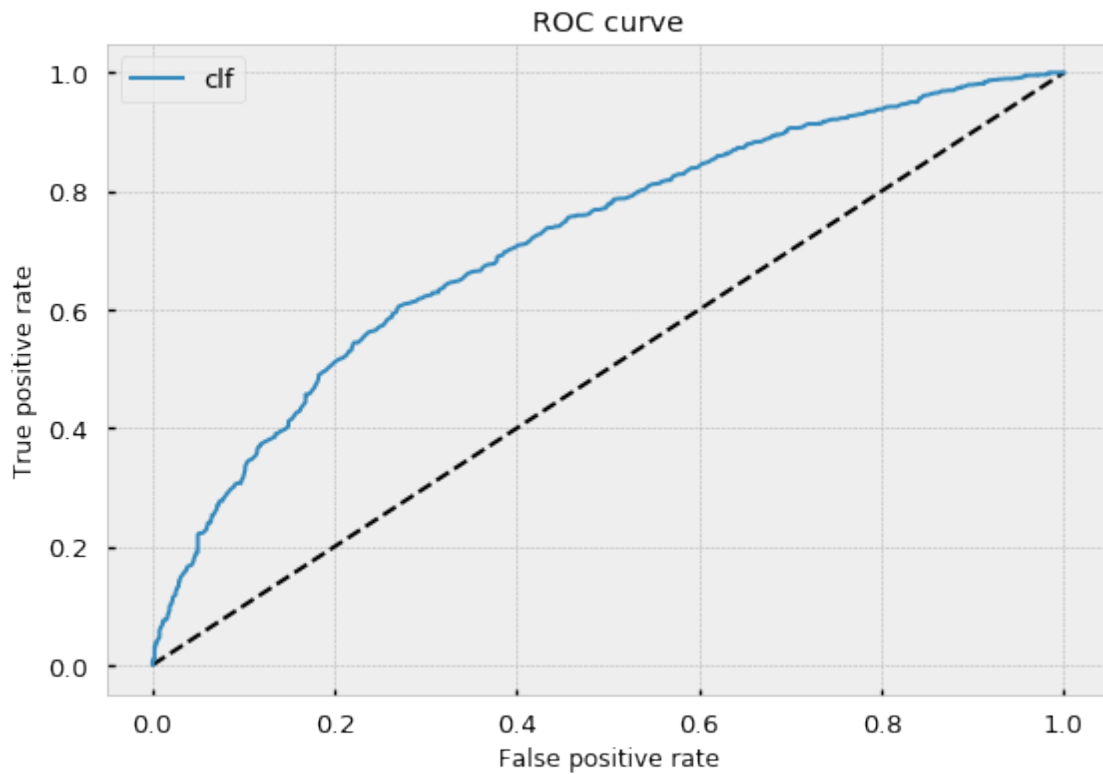
```
In [49]: fit = bc.fit(X_train,y_train)
```

```
/media/bcr/HDD/anaconda3/envs/bayes_dash/lib/python3.6/site-packages/sklearn/ensemble/bagging.py
y = column_or_1d(y, warn=True)
```

```
In [50]: evaluate(X_test,y_test,fit)
```

```
precision    recall  f1-score   support
```

-1.0	0.65	0.44	0.53	905
1.0	0.65	0.81	0.72	1167
micro avg	0.65	0.65	0.65	2072
macro avg	0.65	0.63	0.63	2072
weighted avg	0.65	0.65	0.64	2072

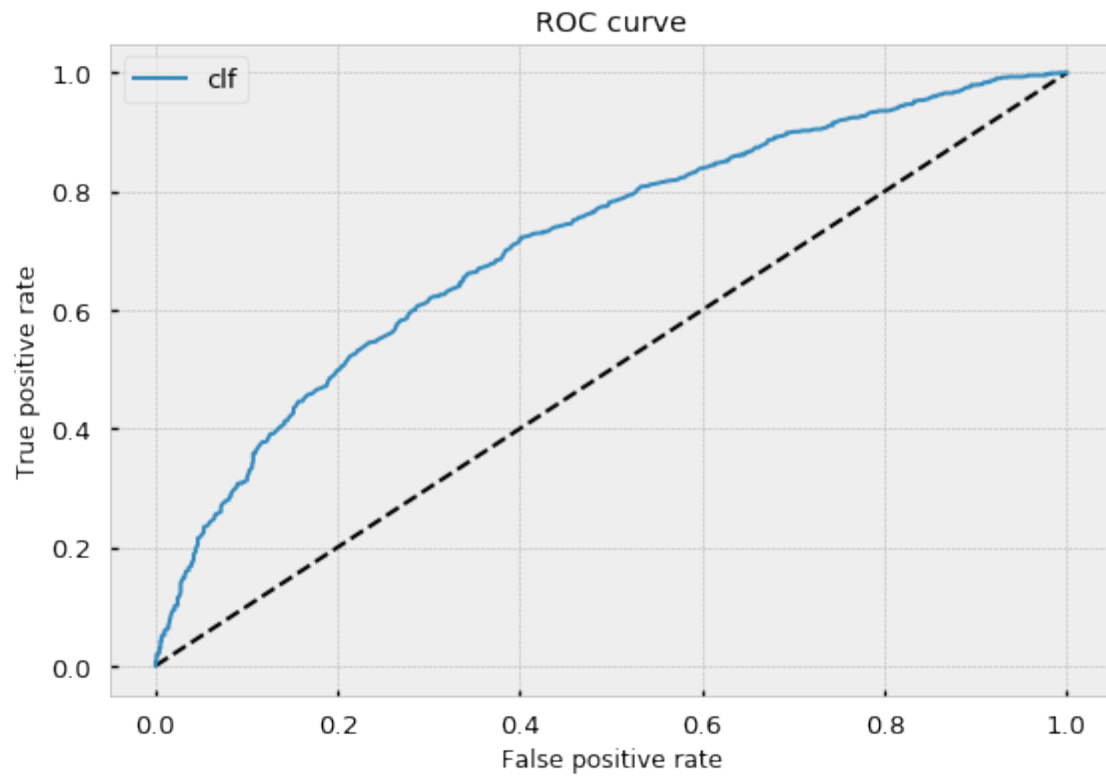


```
In [51]: rf_clf = RandomForestClassifier(n_estimators=1,
                                         class_weight='balanced_subsample',
                                         criterion='entropy',
                                         bootstrap=False)
bc_rf = BaggingClassifier(base_estimator=rf_clf, n_estimators=1000,
                          max_samples=avgU, max_features=1.,
                          random_state=RANDOM_STATE)

fit = bc_rf.fit(X_train,y_train)
evaluate(X_test,y_test,fit)
```

```
/media/bcr/HDD/anaconda3/envs/bayes_dash/lib/python3.6/site-packages/sklearn/ensemble/bagging.py
y = column_or_1d(y, warn=True)
```


	precision	recall	f1-score	support
-1.0	0.65	0.45	0.53	905
1.0	0.66	0.81	0.73	1167
micro avg	0.65	0.65	0.65	2072
macro avg	0.65	0.63	0.63	2072
weighted avg	0.65	0.65	0.64	2072



In []: