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Tutorial-4

(1) $T(n) = 3T(n/2) + n^2$

Ans $a = 3, b = 2$ $F(n) = n^2$

$n^{\log_b a} = n^{\log_2 3}$

Comparing $n^{\log_2 3}$ and n^2

$n^{\log_2 3} < n^2$ (Case 3)

∴ according to master's Theorem $T(n) = \Theta(n^2)$

(2) $T(n) = 4T(n/2) + n^2$

$a = 4$ $b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2 = F(n)$ (Case 2)

∴ according to master's theorem $T(n) = \Theta(n^2 \log n)$

(3) $T(n) = T(n/2) + 2^n$

$a = 1, b = 2$

$n^{\log_2 1} = n^0 = 1$

$1 < 2^n$ (Case 3)

∴ Acc to masters theorem $T(n) = \Theta(2^n)$

(4) $T(n) = 2^n T(n/2) + n^n$

∴ Master's theorem is not applicable as a is function of n

(5) $T(n) = 16T(n/4) + n$

$a = 16$ $b = 4$ $F(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 > F(n)$ (Case 1)

$T(n) = \Theta(n^2)$

6. $T(n) = 2T(n/2) + n\log n$

$a = 2, \quad b = 2 \quad F(n) = n\log n$

$n^{\log_b a} = n^{\log_2 2} = n$

Now $F(n) > n$

Acc to master's $T(n) = \Theta(n\log n)$

(7) $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

$a = 2, b = 2 \quad F(n) = \frac{n}{\log n}$

$n^{\log_b a} = n^{\log_2 2} = n$

$n > F(n)$

$\therefore$ Acc. to master theorm $T(n) = \Theta(n)$

(8) $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a = 2, \quad b = 4, \quad F(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

$n^{0.5} < F(n)$

$\therefore$ Acc. to master Theorm $T(n) = \Theta(n^{0.5})$

(9) $T(n) = 0.5T(n/2) + \frac{1}{n}$

$\therefore$ Master's Not Applicable as $a < 1$

(10) $T(n) = 16T(n/4) + n$

$a = 16, \quad b = 4 \quad F(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 < n!$

$\therefore$ Acc to master $T(n) = \Theta(n!)$

$T(n) = 4T\left(\frac{n}{2}\right) + \log n$

$F(n) = \log n$

As: $a = 4$, $b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > F(n)$

$\therefore$ Acc. to masters $T(n) = \Theta(n^2)$

(12) $T(n) = \text{sqrt}\ T(n/2) + \log n$

$\therefore$ master's Not applicable as it is not Constant

(13) $T(n) = 3T(n/2) + n$

Ans $a = 3$ $b = 2$ $F(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > F(n)$

$\therefore$ Acc. to master's Theorem $T(n) = \Theta(n^{\log_2 3})$

(14) $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3$ $b = 3$ $F(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$n > \sqrt{n}$

$\therefore$ Acc. to masters theorem $T(n) = \Theta(n)$

(15) $T(n) = 4T(n/2) + Cn$

$a = 4$ $b = 2$ $F(n) = C*n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > C*n$

$\therefore$ Acc to masters theorem $T(n) = \Theta(n^2)$

16 $T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4 \quad f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n\log n$

$\therefore$ Acc to master's theorem $T(n) = \Theta(n\log n)$

17 $T(n) = 3T(n/3) + n/2$

$a = 3 \quad b = 3 \quad f(n) = n/2$

$n^{\log_b a} = n^{\log_3 3} = n$

$\Theta(n) = \Theta\left(\frac{n}{2}\right)$

$\therefore$ Acc to master's theorem

$T(n) = \Theta(n\log n)$

18 $T(n) = 6T(n/3) + n^2\log n$

$a = 6 \quad b = 3 \quad f(n) = n^2\log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$n^{1.63} < n^2\log n$

$\therefore$ Acc to master's theorem $T(n) = \Theta(n^2\log n)$

19 $T(n) = 4T(n/2) + n/\log n$

$a = 4,\ b = 2 \quad f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > n/\log n$

Acc to master's theorem $T(n) = \Theta(n^2)$

(20) $T(n) = 64T(n/8) - n^2 \log n$

Master's theorm is not applicabl as $F(n)$ is not increasing Function.

(21) $T(n) = 7T(n/3) + n^2$

$a = 7 \quad b = 3 \quad f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} \; n^{1.7}$

$n^{1.7} < n^2$

$\therefore$ Acc to Master's $T(n) = \Theta(n^2)$

(22) $T(n) = T(n/2) + n(2 - \cos n)$

Master's Theorem is not applicable sice regularity condition is isolated in Case 3.