Logistic Regression

$$P = \frac{1}{1+e^{-y}}$$

$$y = \beta_0 + \beta_1 x$$

$$\ln\left(\frac{P}{1-P}\right) = y = \beta_0 + \beta_1 x$$

No: of hours of Study	Probability of Pass %	$y = \ln\left(\frac{P}{1-P}\right)$
29	0.248	−6
27	0.00454	−10
33	88.08	2
28	0.0336	−8
36	99.966	+8

for Actual probability divide by 100

$$X = \begin{bmatrix} 1 & 29 \\ 1 & 27 \\ 1 & 33 \\ 1 & 28 \\ 1 & 36 \end{bmatrix} \qquad y = \begin{bmatrix} -6 \\ -10 \\ 2 \\ -8 \\ 8 \end{bmatrix}$$

$$x'x = \begin{bmatrix} 5 & 153 \\ 153 & 4739 \end{bmatrix} \qquad x'y = \begin{bmatrix} -14 \\ -314 \end{bmatrix} \qquad \begin{bmatrix} 4739/286 & -153/286 \\ -153/286 & 5/286 \end{bmatrix}$$

$$x'x^{-1} = \begin{bmatrix} 16.569 & -0.534 \\ -0.534 & 0.01784 \end{bmatrix}$$

$$(x'x^{-1})(x'y) = \begin{bmatrix} -64.29 \\ 2.0124 \end{bmatrix}$$

$$\beta = (x^T x)^{-1}$$

a) Probability of pass for the student who studied 34 hours.

$$y = \beta_0 + \beta_1 x$$

$$= -64.29 + 2.012(34)$$

$$= 4.118$$

$$P = \frac{1}{1 + e^{-y}}$$

$$= \frac{1}{1 + e^{-4.118}}$$

$$= 0.9839 \quad \text{(98.39\% of pass) for student who studied 34 hrs}$$

(b) Atleast how many hours a student should study so that he pass the cours with probability of more than 75%

$x \rightarrow$ no: of hours

$$P = 0.75$$

$$y = \ln\left(\frac{P}{1-P}\right)$$

$$= 1.098$$

$$y = \beta_0 + \beta_1 x$$

$$x = \frac{y - \beta_0}{\beta_1}$$

$$= \frac{1.098 + 64.29}{2.0129}$$

$$= 32.492$$

minimum 32.5 hrs has to study