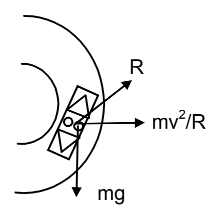
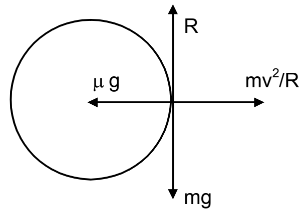
SOLUTIONS TO CONCEPTS circular motion;;
CHAPTER 7

1. Distance between Earth & Moon
   $r = 3.85 \times 10^5$ km $= 3.85 \times 10^8$m
   $T = 27.3$ days $= 24 \times 3600 \times (27.3)$ sec $= 2.36 \times 10^6$ sec
   $$v = \frac{2\pi r}{T} = \frac{2 \times 3.14 \times 3.85 \times 10^8}{2.36 \times 10^6} = 1025.42\text{m/sec}$$
   $$a = \frac{v^2}{r} = \frac{(1025.42)^2}{3.85 \times 10^8} = 0.00273\text{m/sec}^2 = 2.73 \times 10^{-3}\text{m/sec}^2$$
2. Diameter of earth = 12800km
   Radius $R = 6400$km $= 64 \times 10^5$ m
   $$V = \frac{2\pi R}{T} = \frac{2 \times 3.14 \times 64 \times 10^5}{24 \times 3600}\text{ m/sec} = 465.185$$
   $$a = \frac{V^2}{R} = \frac{(46.5185)^2}{64 \times 10^5} = 0.0338\text{m/sec}^2$$
3. $V = 2t$, $r = 1$cm
   a) Radial acceleration at t = 1 sec.
   $$a = \frac{v^2}{r} = \frac{2^2}{1} = 4\text{cm/sec}^2$$
   b) Tangential acceleration at t = 1sec.
   $$a = \frac{dv}{dt} = \frac{d}{dt}(2t) = 2\text{cm/sec}^2$$
   c) Magnitude of acceleration at t = 1sec
   $$a = \sqrt{4^2 + 2^2} = \sqrt{20}\text{ cm/sec}^2$$
4. Given that m = 150kg,
   v= 36km/hr = 10m/sec, r = 30m
   Horizontal force needed is $\frac{mv^2}{r} = \frac{150 \times (10)^2}{30} = \frac{150 \times 100}{30} = 500\text{N}$
5. in the diagram
   $R \cos\theta = mg$ ..(i)
   $R \sin\theta = \frac{mv^2}{r}$ ..(ii)
   Dividing equation (i) with equation (ii)
   $$\tan\theta = \frac{mv^2}{rmg} = \frac{v^2}{rg}$$
   v = 36km/hr = 10m/sec, r = 30m
   $$\tan\theta = \frac{v^2}{rg} = \frac{100}{30 \times 10} = (1/3)$$
   $\Rightarrow \theta = \tan^{-1}(1/3)$



6. Radius of Park = r = 10m
   speed of vehicle = 18km/hr = 5 m/sec
   Angle of banking $\tan\theta = \frac{v^2}{rg}$
   $$\Rightarrow \theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{25}{100} = \tan^{-1}(1/4)$$

7. The road is horizontal (no banking)



$\frac{mv^2}{R} = \mu N$

and N = mg

So $\frac{mv^2}{R} = \mu\, mg$ v = 5m/sec,   R = 10m

$\Rightarrow \frac{25}{10} = \mu g \Rightarrow \mu = \frac{25}{100} = 0.25$

8. Angle of banking = θ = 30°

Radius = r = 50m

$\tan\theta = \frac{v^2}{rg} \Rightarrow \tan 30° = \frac{v^2}{rg}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{v^2}{rg} \Rightarrow v^2 = \frac{rg}{\sqrt{3}} = \frac{50 \times 10}{\sqrt{3}}$

$\Rightarrow v = \sqrt{\frac{500}{\sqrt{3}}}$ = 17m/sec.

9. Electron revolves around the proton in a circle having proton at the centre.

Centripetal force is provided by coulomb attraction.

r = 5.3 →t 10^{-11}m   m = mass of electron = 9.1 × 10^{-3}kg.

charge of electron = 1.6 × 10^{-19}c.

$\frac{mv^2}{r} = k\frac{q^2}{r^2} \Rightarrow v^2 = \frac{kq^2}{rm} = \frac{9 \times 10^9 \times 1.6 \times 1.6 \times 10^{-38}}{5.3 \times 10^{-11} \times 9.1 \times 10^{-31}} = \frac{23.04}{48.23} \times 10^{13}$

$\Rightarrow v^2 = 0.477 \times 10^{13} = 4.7 \times 10^{12}$

$\Rightarrow v = \sqrt{4.7 \times 10^{12}} = 2.2 \times 10^6$ m/sec

10. At the highest point of a vertical circle

$\frac{mv^2}{R} = mg$

$\Rightarrow v^2 = Rg \Rightarrow v = \sqrt{Rg}$

11. A celling fan has a diameter = 120cm.

$\therefore$Radius = r = 60cm = 0/6m

Mass of particle on the outer end of a blade is 1g.

n = 1500 rev/min = 25 rev/sec

$\omega = 2\,\pi\, n = 2\,\pi \times 25 = 157.14$

Force of the particle on the blade = $Mr\omega^2$ = (0.001) × 0.6 × (157.14) = 14.8N

The fan runs at a full speed in circular path. This exerts the force on the particle (inertia). The particle also exerts a force of 14.8N on the blade along its surface.

12. A mosquito is sitting on an L.P. record disc & rotating on a turn table at $33\frac{1}{3}$ rpm.

$n = 33\frac{1}{3}$ rpm $= \frac{100}{3 \times 60}$ rps

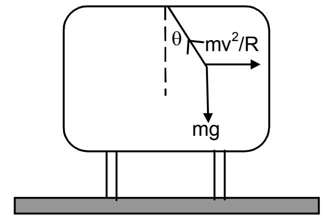
$\therefore \omega = 2\,\pi\, n = 2\,\pi \times \frac{100}{180} = \frac{10\pi}{9}$ rad/sec

r = 10cm =0.1m,   g = 10m/sec^2

$\mu mg \geq mr\omega^2 \Rightarrow \mu = \frac{r\omega^2}{g} \geq \frac{0.1 \times \left(\frac{10\pi}{9}\right)^2}{10}$

$\Rightarrow \mu \geq \frac{\pi^2}{81}$

13. A pendulum is suspended from the ceiling of a car taking a turn



r = 10m, v = 36km/hr = 10 m/sec, g = $10m/sec^2$

From the figure $T \sin\theta = \dfrac{mv^2}{r}$ ..(i)

$T \cos\theta = mg$ ..(ii)

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = \dfrac{mv^2}{rmg} \Rightarrow \tan\theta = \dfrac{v^2}{rg} \Rightarrow \theta = \tan^{-1}\left(\dfrac{v^2}{rg}\right)$

$= \tan^{-1}\dfrac{100}{10\times 10} = \tan^{-1}(1) \Rightarrow \theta = 45°$

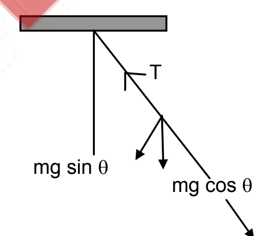
14. At the lowest pt.

$T = mg + \dfrac{mv^2}{r}$

Here m = 100g = 1/10 kg, r = 1m, v = 1.4 m/sec

$T = mg + \dfrac{mv^2}{r} = \dfrac{1}{10}\times 9.8 \times \dfrac{(1.4)^2}{10} = 0.98 + 0.196 = 1.176 = 1.2$ N

15. Bob has a velocity 1.4m/sec, when the string makes an angle of 0.2 radian.



m = 100g = 0.1kg, r = 1m, v = 1.4m/sec.

From the diagram,

$T - mg\cos\theta = \dfrac{mv^2}{R}$

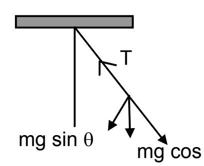
$\Rightarrow T = \dfrac{mv^2}{R} + mg\cos\theta$

$\Rightarrow T = \dfrac{0.1\times(1.4)^2}{1} + (0.1)\times 9.8 \times \left(1 - \dfrac{\theta^2}{2}\right)$

$\Rightarrow T = 0.196 + 9.8 \times \left(1 - \dfrac{(.2)^2}{2}\right)$ $\left(\because \cos\theta = 1 - \dfrac{\theta^2}{2} \text{ for small } \theta\right)$

$\Rightarrow T = 0.196 + (0.98) \times (0.98) = 0.196 + 0.964 = 1.156N \approx 1.16$ N

16. At the extreme position, velocity of the pendulum is zero.



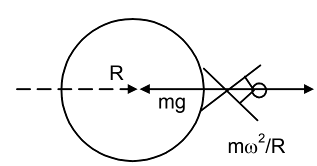
So there is no centrifugal force.

So $T = mg\cos\theta_o$

17. a) Net force on the spring balance.

$R = mg - m\omega^2 r$

So, fraction less than the true weight (3mg) is



$= \dfrac{mg - (mg - m\omega^2 r)}{mg} = \dfrac{\omega^2}{g} = \left(\dfrac{2\pi}{24\times 3600}\right)^2 \times \dfrac{6400\times 10^3}{10} = 3.5 \times 10^{-3}$

b) When the balance reading is half the true weight,

$\dfrac{mg - (mg - m\omega^2 r)}{mg} = 1/2$

$\omega^2 r = g/2 \Rightarrow \omega = \sqrt{\dfrac{g}{2r}} = \sqrt{\dfrac{10}{2\times 6400\times 10^3}}$ rad/sec

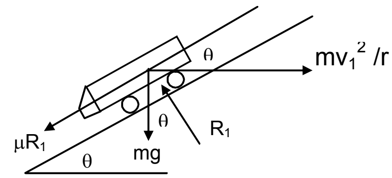
$\therefore$ Duration of the day is

$T = \dfrac{2\pi}{\omega} = 2\pi \times \sqrt{\dfrac{2\times 6400\times 10^3}{9.8}}$ sec $= 2\pi\times\sqrt{\dfrac{64\times 10^6}{49}}$ sec $= \dfrac{2\pi\times 8000}{7\times 3600}$ hr = 2hr

18. Given, $v = 36\text{km/hr} = 10\text{m/s}$, $r = 20\text{m}$, $\mu = 0.4$

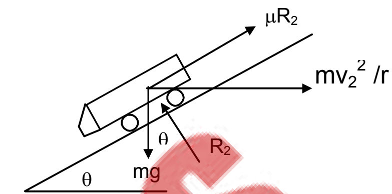
The road is banked with an angle,

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{100}{20\times 10}\right) = \tan^{-1}\left(\frac{1}{2}\right) \text{ or } \tan\theta = 0.5$$



When the car travels at max. speed so that it slips upward, μR_1 acts downward as shown in Fig.1

So, $R_1 - mg\cos\theta - \frac{mv_1^2}{r}\sin\theta = 0$ ..(i)



And $\mu R_1 + mg\sin\theta - \frac{mv_1^2}{r}\cos\theta = 0$ ..(ii)

Solving the equation we get,

$$V_1 = \sqrt{rg\frac{\tan\theta - \mu}{1+\mu\tan\theta}} = \sqrt{20\times 10\times\frac{0.1}{1.2}} = 4.082 \text{ m/s} = 14.7 \text{ km/hr}$$

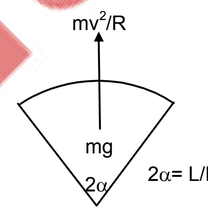
So, the possible speeds are between 14.7 km/hr and 54km/hr.

19. R = radius of the bridge

L = total length of the over bridge

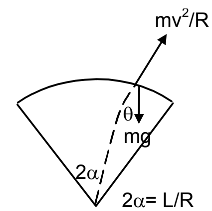
a) At the highest pt.

$$mg = \frac{mv^2}{R} \Rightarrow v^2 = Rg \Rightarrow v = \sqrt{Rg}$$



b) Given, $v = \frac{1}{\sqrt{2}}\sqrt{Rg}$

suppose it loses contact at B. So, at B, $mg\cos\theta = \frac{mv^2}{R}$



$\Rightarrow v^2 = Rg\cos\theta$

$$\Rightarrow \left(\sqrt{\frac{Rv}{2}}\right)^2 = Rg\cos\theta \Rightarrow \frac{Rg}{2} = Rg\cos\theta \Rightarrow \cos\theta = 1/2 \Rightarrow \theta = 60° = \pi/3$$

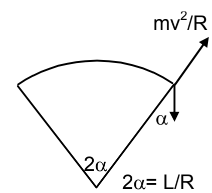
$$\theta = \frac{\ell}{r} \to \ell = r\theta = \frac{\pi R}{3}$$

So, it will lose contact at distance $\frac{\pi R}{3}$ from highest point

c) Let the uniform speed on the bridge be v.

The chances of losing contact is maximum at the end of the bridge for which $\alpha = \frac{L}{2R}$.

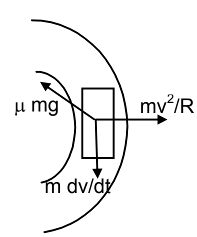
$$\text{So, } \frac{mv^2}{R} = mg\cos\alpha \Rightarrow v = \sqrt{gR\cos\left(\frac{L}{2R}\right)}$$

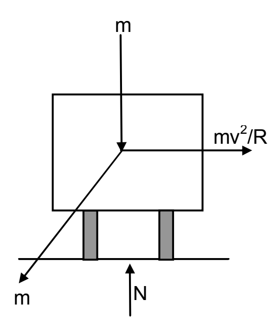


20. Since the motion is nonuniform, the acceleration has both radial & tangential component

$$a_r = \frac{v^2}{r}$$

$$a_t = \frac{dv}{dt} = a$$





$$\text{Resultant magnitude} = \sqrt{\left(\frac{v^2}{r}\right)^2 + a^2}$$

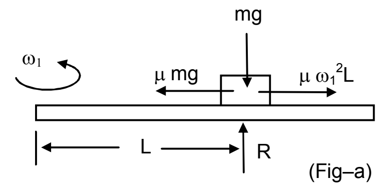
$$\text{Now } \mu N = m\sqrt{\left(\frac{v^2}{r}\right)^2 + a^2} \Rightarrow \mu mg = m\sqrt{\left(\frac{v^2}{r}\right)^2 + a^2} \Rightarrow \mu^2 g^2 = \left(\frac{v^4}{r2}\right) + a^2$$

$$\Rightarrow v^4 = (\mu^2 g^2 - a^2)\, r^2 \Rightarrow v = [(\mu^2 g^2 - a^2)\, r^2]^{1/4}$$

21. a) When the ruler makes uniform circular motion in the horizontal plane, (fig–a)

μ mg = $m\omega_1^2 L$

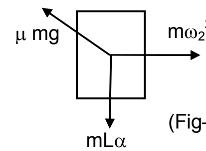
$\omega_1 = \sqrt{\dfrac{\mu g}{L}}$



(Fig–a)

b) When the ruler makes uniformly accelerated circular motion,(fig–b)

$$\mu \text{ mg} = \sqrt{(m\omega_2^2 L)^2 + (mL\alpha)^2} \Rightarrow \omega_2^4 + \alpha^2 = \frac{\mu^2 g^2}{L^2} \Rightarrow \omega_2 = \left[\left(\frac{\mu g}{L}\right)^2 - \alpha^2\right]^{1/4}$$



(Fig–b)

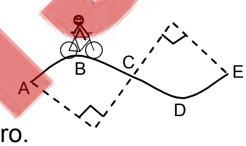
(When viewed from top)

22. Radius of the curves = 100m

Weight = 100kg

Velocity = 18km/hr = 5m/sec

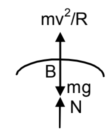
a) at B mg – $\dfrac{mv^2}{R}$ = N ⇒ N = (100 × 10) – $\dfrac{100 \times 25}{100}$ = 1000 – 25 = 975N



At d, N = mg + $\dfrac{mv^2}{R}$ = 1000 + 25 = 1025 N

b) At B & D the cycle has no tendency to slide. So at B & D, frictional force is zero.

At 'C', mg sin θ = F ⇒ F = 1000 × $\dfrac{1}{\sqrt{2}}$ = 707N



c) (i) Before 'C' mg cos θ – N = $\dfrac{mv^2}{R}$ ⇒ N = mg cos θ – $\dfrac{mv^2}{R}$ = 707 – 25 = 683N

(ii) N – mg cos θ = $\dfrac{mv^2}{R}$ ⇒ N = $\dfrac{mv^2}{R}$ + mg cos θ = 25 + 707 = 732N

d) To find out the minimum desired coeff. of friction, we have to consider a point just before C. (where N is minimum)

Now, μ N = mg sin θ ⇒ μ × 682 = 707

So, μ = 1.037

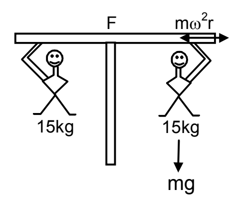
23. d = 3m ⇒ R = 1.5m

R = distance from the centre to one of the kids

N = 20 rev per min = 20/60 = 1/3 rev per sec

$\omega = 2\pi r = 2\pi/3$

m = 15kg



∴ Frictional force F = $mr\omega^2$ = 15 × (1.5) × $\dfrac{(2\pi)^2}{9}$ = 5 × (0.5) × $4\pi^2$ = $10\pi^2$

∴ Frictional force on one of the kids is $10\pi^2$

24. If the bowl rotates at maximum angular speed, the block tends to slip upwards. So, the frictional force acts downward.

Here, r = R sin θ

From FBD –1

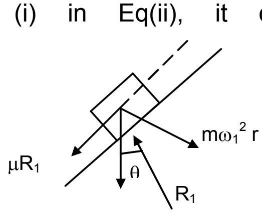
R_1 – mg cos θ – $m\omega_1^2$ (R sin θ) sin θ = 0 ..(i) [because r = R sin θ]

and μR_1 mg sin θ – $m\omega_1^2$ (R sin θ) cos θ = 0 ..(ii)

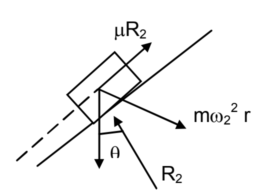
Substituting the value of R_1 from Eq (i) in Eq(ii), it can be found out that

$$\omega_1 = \left[\frac{g(\sin\theta + \mu\cos\theta)}{R\sin\theta(\cos\theta - \mu\sin\theta)}\right]^{1/2}$$

Again, for minimum speed, the frictional force μR_2 acts upward. From FBD–2, it can be proved that,



(FBD – 1)



(FBD – 2)

$$\omega_2 = \left[\frac{g(\sin\theta - \mu\cos\theta)}{R\sin\theta(\cos\theta + \mu\sin\theta)}\right]^{1/2}$$

$\therefore$ the range of speed is between ω_1 and ω_2

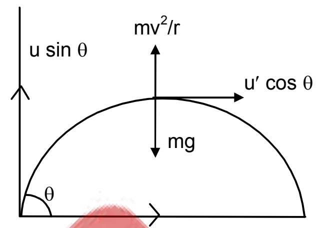
25. Particle is projected with speed 'u' at an angle θ. At the highest pt. the vertical component of velocity is '0'

So, at that point, velocity = u cos θ

centripetal force = $m\,u^2 \cos^2\left(\frac{\theta}{r}\right)$

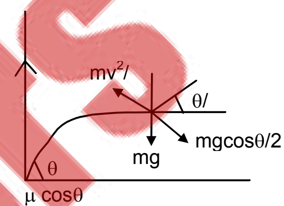
At highest pt.

$mg = \frac{mv^2}{r} \Rightarrow r = \frac{u^2\cos^2\theta}{g}$



26. Let 'u' the velocity at the pt where it makes an angle $\theta/2$ with horizontal. The horizontal component remains unchanged

So, $v\cos\theta/2 = \omega\cos\theta \Rightarrow v = \frac{u\cos\theta}{\cos\left(\frac{\theta}{2}\right)}$ ...(i)



From figure

$mg\cos(\theta/2) = \frac{mv^2}{r} \Rightarrow r = \frac{v^2}{g\cos(\theta/2)}$

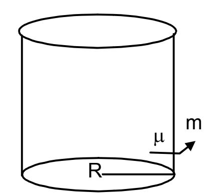
putting the value of 'v' from equn(i)

$r = \frac{u^2\cos^2\theta}{g\cos^3(\theta/2)}$

27. A block of mass 'm' moves on a horizontal circle against the wall of a cylindrical room of radius 'R' Friction coefficient between wall & the block is μ.

a) Normal reaction by the wall on the block is = $\frac{mv^2}{R}$

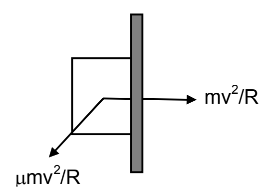
b) $\therefore$ Frictional force by wall = $\frac{\mu mv^2}{R}$



c) $\frac{\mu mv^2}{R} = ma \Rightarrow a = -\frac{\mu v^2}{R}$ (Deceleration)

d) Now, $\frac{dv}{dt} = v\frac{dv}{ds} = -\frac{\mu v^2}{R} \Rightarrow ds = -\frac{R}{\mu}\frac{dv}{v}$

$\Rightarrow s = -\frac{R\mu}{} \ln V + c$



At s = 0, $v = v_0$

Therefore, $c = \frac{R}{\mu}\ln V_0$

so, $s = -\frac{R}{\mu}\ln\frac{v}{v_0} \Rightarrow \frac{v}{v_0} = e^{-\mu s/R}$

For, one rotation $s = 2\pi R$, so $v = v_0 e^{-2\pi\mu}$

28. The cabin rotates with angular velocity ω & radius R

$\therefore$ The particle experiences a force $mR\omega^2$.

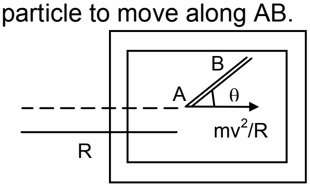
The component of $mR\omega^2$ along the groove provides the required force to the particle to move along AB.

$\therefore mR\omega^2\cos\theta = ma \Rightarrow a = R\omega^2\cos\theta$

length of groove = L

$L = ut + ½at^2 \Rightarrow L = ½R\omega^2\cos\theta\, t^2$

$\Rightarrow t^2 = \frac{2L}{R\omega^2\cos\theta} = \Rightarrow t = 1\sqrt{\frac{2L}{R\omega^2\cos\theta}}$



29. v = Velocity of car = 36km/hr = 10 m/s
r = Radius of circular path = 50m
m = mass of small body = 100g = 0.1kg.
μ = Friction coefficient between plate & body = 0.58
a) The normal contact force exerted by the plate on the block

$$N = \frac{mv^2}{r} = \frac{0.1 \times 100}{50} = 0.2N$$

b) The plate is turned so the angle between the normal to the plate & the radius of the road slowly increases

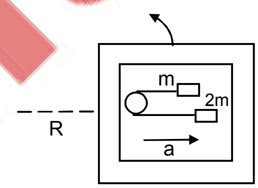
$$N = \frac{mv^2}{r} \cos\theta \quad ..(i)$$

$$\mu N = \frac{mv^2}{r} \sin\theta \quad ..(ii)$$

Putting value of N from (i)

$$\mu \frac{mv^2}{r} \cos\theta = \frac{mv^2}{r} \sin\theta \Rightarrow \mu = \tan\theta \Rightarrow \theta = \tan^{-1}\mu = \tan^{-1}(0.58) = 30°$$

30. Let the bigger mass accelerates towards right with 'a'.
From the free body diagrams,

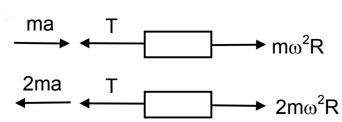


$T - ma - m\omega^2 R = 0 \quad ...(i)$

$T + 2ma - 2m\omega^2 R = 0 \quad ...(ii)$

Eq (i) – Eq (ii) $\Rightarrow 3ma = m\omega^2 R$

$$\Rightarrow a = \frac{m\omega^2 R}{3}$$



Substituting the value of a in Equation (i), we get $T = 4/3\, m\omega^2 R$.

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