Neeraj Appori
F-129
2813

Calculas Unit - 1

AL NRAO

Q1 Show that function $f(x) = x^3 - 9x^2 + 30x + 7$ is always increasing

$\rightarrow$ Let $f(x) = x^3 + 9x^2 + 30x + 7$:

$\therefore f'(x) = 3x^2 + 18x + 30$

$\therefore f'(x) > 0$

$\therefore 3x^2 + 18x + 30 > 0$, $\quad \forall x \in R$

$\therefore 3(x^2 + 6x + 10) > 0 \quad \forall x \in R$

$= (x^2 + 6x + 10) > 0 \quad \forall x \in R$

$= (x^2 + 6x + 9 + 1) > 0 \quad \forall x \in R$

$= [(x+3)^2 + 1] > 0 \quad \forall x \in R$

$\because$ The square of real number is always a non-negative $\therefore (x+3)^2 > 0$

$\therefore f(x)$ is always increasing function

Q2. Find absolute maximum and minimum value of $f(x) = (x-2)^2$ in $[1, 4]$

$f(x) = x^2 - 4x + 4$

$f'(x) = 2x - 4 \quad \forall x \in (1, 4)$

$f'(x) = 0 \Rightarrow 2x - 4 = 0$

$2(x-2) = 0$

$\therefore x - 2 = 0$

$\therefore x = 2$

at $x = 1$, $f(1) = (1)^2 - 4(1) + 4$

$= 1 - \cancel{4} + \cancel{4}$

at $x = 4$, $f(4) = (4)^2 - 4(4) + 4$

$= \cancel{16} - \cancel{16} + 4$

$= 4$

absolute maximum at 4 at $x = 4$

absolute minimum at 1 at $x = 1$

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. ALN Rao

3) Using Newton's method find the approximate root for the equation $f(x) : x . \cos x$

→ $f(x) = x \cos x$

$x_{n+1} = x_n + \frac{f(x_n)}{f(x_n)}$

$f'(x) = 1 + \sin x$

$n = 0$

$x_1 = 0 + \frac{x - \cos x}{1 + \sin x}$

$= 0 + \frac{0 - \cos 0}{1 + \sin 0}$

$= -\frac{1}{1}$

$x = -1$

$x_1 = 1$

$x_2 = x_1 + \frac{f(x_1)}{f'(x_1)} = -1 + (-1)$

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A L Nemo

i) Find relative extrema of $f(x) = 3x^5 - 5x^3$

$f(x) = 3x^5 - 5x^3$

$\therefore f'(x) = 15x^4 - 15x^2$

$\therefore f'(x) = 0$

$15x^4 - 15x^2 = 0$

$15x^2(x^2 - 1) = 0$

~~$15x^2 = 15x^2 = 0$ and $x^2 = 1$~~

~~$x = 0$~~ $\quad x = 1$

$15x^2(x-1)(x+1)$

i) C critical point $0, -1, +1$

$f''(x) = 60x^3 - 30x$

$= 30x(2x^2 - 1)$

$30x(2x^2-1)$ $\quad f''(x)$ trend

$x = -1$	-30	$-$	max
$x = 0$	0	0	incomplete
$x = 1$	30	$+$	min

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A.L.NR

5) Discuss the continuity of function $f(x) = \sqrt{4-x^2}$

$$\lim_{x\to 0} fx = \lim_{x\to 0} \sqrt{4-x^2}$$

$$= \lim_{x\to 0} (4-x^2)^{1/2}$$

$$\lim_{x\to 0} (2-x)$$

$$fx = 2$$

6) Divide 100 into two parts such that their square is maximum

→ Let x be one part of number and y be other part

$x+y = 100$ $\quad y = (100-x)$

Sum of two numbers is

$f(x) = (x^2+y^2)^2$

$= x^2 + (100-x^2)^2$

$= x^2 + 10000 - 200x + x^2$

$= 2x^2 - 200x + 10000$

$f(x) = 2x^2 - 200x + 10000$

$f'(x) = 4x - 200$

$= 4(x-200)$

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7) Show that |x| is continous everywhere

→ The give function is a Modulu's function It that has two values

x for all the numbers >0
$-x$ for all the numbers <0

This is a continous function because
$\frac{dy}{dx}$ of $(x>0)$ is 1

whereas for $(x<0)$ is -1
Since LHS and RHS limit of this function

8) A garden is to be laid out in rectangular area and protected by a chicken fence. What is the largest possible area of garden. It only 72 ~~feet of~~ running feet of chicken wire is available for fence).
→ let x, y, z and p be the lenght, breadth and ~~area~~ area and perimeter respectively

$$P = xy = 72 \text{ feet} \qquad y = \frac{72}{x}$$

∴ Perimeter of rectangle $= 2(l+b)$
$$P = 2(x+y)$$
$$P = 2\left(x + \frac{72}{x}\right)$$
$$= 2x + \frac{144}{x}$$

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$$\frac{dp}{dx} = 2x + \frac{144}{x}$$

$$= 2 + 144(-1)x^{-2}$$

$$\therefore \frac{dp}{dx} = 2 - \frac{144}{x^2}$$

for p minimum $\frac{dp}{dx} = 0$

$$2 - \frac{144}{x^2}$$

$$2\left(1 - \frac{72}{x^2}\right) = 0$$

$$1 - \frac{72}{x^2} = 0 \Rightarrow x^2 = 72$$

$$x^2 = 72$$

$$x = \sqrt{72}$$

$$= \sqrt{9 \times 8}$$

$$x = 6\sqrt{2}$$

$$\frac{d^2p}{dx^2} = 2 - 144x^2$$

$$= -144 \cdot -2 \cdot x^{-3}$$

$$= \frac{288}{x^3}$$

$\therefore$ at $x = 6\sqrt{2}$

$$= \frac{288}{(6\sqrt{2})^3}$$

$$= \frac{288}{216 ...}$$

$= \frac{4}{\sqrt{2}} > 0$

$\therefore P$ is minimum when $x = 6\sqrt{2}$ by equation (1)

$y = \frac{72}{6\sqrt{2}} = \frac{12}{\sqrt{2}} = 3\sqrt{2}.$

Thus the perimeter of rectangle is less

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9) Find the asymptotes of the function $y = \frac{x}{(x+1)(x+2)^2}$

$\rightarrow$ $y = \frac{x}{(x+1)(x+2)^2}$

$x \rightarrow -1$ $y \rightarrow -\infty$

$x \rightarrow -2$ $y \rightarrow -\infty$

$\therefore x = -1$ and $x = -2$

$$\lim_{x \rightarrow \infty} \frac{x}{(x+1)(x+2)^2}$$

$$= \lim_{x \rightarrow \infty} \frac{x}{x\left(1+\frac{1}{x}\right)x^2\left(1+\frac{2}{x}\right)^2}$$

$$= \lim_{x \rightarrow \infty} \frac{1}{\left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)^2 x^2} = 0$$

$$= \lim_{x \rightarrow -\infty} \frac{x}{(x+1)(x+2)^2}$$

$$= \lim_{x \rightarrow -\infty} \frac{x}{x\left(1+\frac{1}{x}\right)(x+2)^2}$$

$$= \lim_{x \rightarrow -\infty} \frac{1}{\left(1+\frac{1}{x}\right)(x+2)^2} = 0$$

$y = 0$ is a horizontal asymptote

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10) Determine whether the following limit exists.

i) $$\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2}$$

$$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2}$$

In this case the function is not continous at the point and can't just play in the middle

$$\lim_{(x,y)\to(0,0)} \left(\frac{x^2-y^2}{x^2+y}\right)^2$$

$$\lim_{(x,y)\to(0,0)} \frac{0-0}{0+0} = 0$$

~~That~~ Thus limit does not exist.

Navnoor Appu

Unit - II

1) If $f(x) = x^3$ over the interval $[-3, 2]$ find the area.

$$y = x^3$$

$$\left[\frac{x^4}{4}\right]_{-3}^{2}$$

$$\left[\frac{x^4}{4}\right]_0^2 = \frac{16}{4} - 0, \quad \left[\frac{x^4}{4}\right]_{-3}^{0} = 0 - \frac{81}{4}$$

$$\frac{16}{4} + \frac{81}{4} = \frac{97}{4} = 24.25 \text{ sq. units}$$

2) $\frac{dy}{dx} = 1 - y$; $y(0) = 0$, find $y(0.1)$ and $y(0.2)$ using Euler's method taking $h = 0.1$

$x_0 = 0$

$x_1 = x_0 + h = 0 + 0.1 = 0.1$

$x_2 = x_1 + h = 0.1 + 0.1 = 0.2$

$x_3 = x_2 + h = 0.2 + 0.1 = 0.3$

$y_0 = y(0) = 0$

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$y_1 = y_0 + hf(x_0, y_0)$
$= 0 + 0.1 f(x_0, y_0)$
$= 0 + 0.1 \times 1$
$= 0.1$

$f(x_0, y_0) = \cancel{x + y + xy}\ 1-y$
$= \cancel{0 + 0 + 0}$
$= \cancel{0}\ 1$

$y_2 = y_1 + hf(x_1, y_1)$
$= 0 + 0.1 f(x_1, y_1)$
$= 0 + 0.1 \times 0.9$
$= 0.9$

$f(x_1, y_1) = \cancel{x + y + xy}\ 1-y_1$
$= \cancel{0.1 + 0 + 0.1 \times 0}$
$= \cancel{0.1}\ 1-1 = 0$
0.9

$y_3 = y_2 + hf(x_2, y_2)$
$= 0.1 + 0.1 f(x_2, y_2)$
$= 0.1 + (0.1 \times \cancel{0.5})\ 0.1$
$= \cancel{0.1 + 0.5}\ 0.1 + 0.1$
$= \cancel{0.6}\ \ 0.2$

$f(x_2, y_2) = \cancel{x + y + xy}\ 1-y_2$
$= \cancel{0.3}$
$= 0.2 + 0.1 + 0.2$
$= 0.5\ \ 1-0.9$
$= 0.1$

3) Solve differential equation $\frac{dy}{dx} = -xy$

$\cancel{\frac{dy}{dx} + Py = Q}$

$\frac{dy}{dx} + xy = 0$ By applying product rule

$\frac{d}{dx}(-xy) = -x\frac{dy}{dx} + y\frac{dx}{dx}$

$= -x\frac{dy}{dx} - y$

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(6) $x(x+y)dy - y^2dx = 0$

$\Rightarrow x(x+y)dy = y^2dx$

$x^2+xy\,dy = y^2dx$

$\frac{x^2 y\,dy}{y^2} = \frac{dx}{x^2}$

$\frac{y\,dy}{y^2} = \frac{1}{x^3}dx$

$\frac{1}{y}dy = \frac{1}{x^3}dx$

$\log y \cdot x^3 dx$ $\quad \log y = \frac{x^{-2}}{-2} + C$

4) Find the approximate value $\int_1^2 \frac{1}{x^2}dx$ using Simpson's rule with $n=10$

$\Rightarrow h = \frac{b-a}{n}$

$= \frac{2-1}{10} = \frac{1}{10} = 0.1$

	0	1	2	3	4	5	6	7
x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
y	0	$\frac{1}{0.01}$	$\frac{1}{0.04}$	$\frac{1}{0.09}$	$\frac{1}{0.16}$	$\frac{1}{0.25}$	$\frac{1}{0.36}$	$\frac{1}{0.49}$

8	9	10
0.8	0.9	1.0
$\frac{1}{0.64}$	$\frac{1}{0.81}$	1

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R.1 MR80

$y_0 = \frac{1}{x^2} = \frac{1}{0} = x^{-2} = 0$ $y_1 = x^{-2} = (0.1)^{-2} = \frac{1}{0.01}$

$y_2 = \frac{1}{x^2} = \frac{1}{(0.2)^2} = \frac{1}{0.04}$ $y_3 = \frac{1}{x^2} = \frac{1}{(0.3)^2} = \frac{1}{0.09}$

$y_4 = \frac{1}{x^2} = \frac{1}{(0.4)^2} = \frac{1}{0.16}$ $y_5 = \frac{1}{x^2} = \frac{1}{(0.5)^2} = \frac{1}{0.25}$

$y_6 = \frac{1}{x^2} = \frac{1}{(0.6)^2} = \frac{1}{0.36}$ $y_7 = \frac{1}{x^2} = \frac{1}{(0.7)^2} = \frac{1}{0.49}$

$y_8 = \frac{1}{x^2} = \frac{1}{(0.8)^2} = \frac{1}{0.64}$ $y_9 = \frac{1}{x^2} = \frac{1}{(0.9)^2} = \frac{1}{0.81}$

$y_{10} = \frac{1}{x^2} = \frac{1}{(1)^2} = 1$

By simpson rule

$S_n = \frac{\Delta x}{3}\left(y_0 + y_{10} + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8)\right)$

$= \frac{0.1}{3}\left(0 + 4\left(\frac{1}{0.01} + \frac{1}{0.09} + \frac{1}{0.25} + \frac{1}{0.49} + \frac{1}{0.81}\right) + 2\left(\frac{1}{0.04} + \frac{1}{0.16} + \frac{1}{0.36} + \frac{1}{0.64}\right) + 1\right)$

$= 100 + 11.11 + 4 + 2.04 + 1.23 = 113.?$

$25 + (25 + 2.77 + 1.56) = 35.?$

$= \frac{0.1}{3}(0 + 473.52 + 71.16 + 1)$

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D.L.N.R.A.C

5) Find the area of the region that is enclo[sed] between two curves $y = x^2$ and $y = x + 6$

$\rightarrow$ $y = x^2$

$y = x + 6$

$x^2 = x + 6$ $\quad \because y_1 = y_2$

$x^2 - x + 6$

$x^2 - 3x + 2x - 6$

$x(x-3) + 2(x-3)$

$(x-3)(x+2)$

$x = 3, \ x = -2$

$$\int_{-2}^{3} x + 6 - x^2$$

$$= \left[x + 6 - x^2\right]_{-2}^{3}$$

$$= \left[\frac{x^2}{2} + 6x - \frac{x^3}{3}\right]_{-2}^{3}$$

$$= \left[\frac{3^2}{2} + 6(3) - \frac{3^3}{3}\right] - \left[\frac{-2^2}{2} + 6(-2) - \frac{(-2)^3}{3}\right]$$

$$= \left[\frac{9}{2} + 18 - \frac{27}{3}\right] - \left[\frac{4}{2} - 12 + \frac{8}{3}\right]$$

$$= \left[\frac{9}{2} + 9\right] - \left[2 - 12 + \frac{8}{3}\right]$$

$$= \left[\frac{9 + 18}{2}\right] - \left[\frac{-10 + 8}{3}\right]$$

$$= \frac{27}{2} - \left[\frac{-22}{3}\right]$$

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A. L. N. RAO

$= 13.5 + 7.33$
$= 20.833$

1) If $f(x) = 3x^3$, find the area on the interval ~~[1, 5]~~ $(2, 3)$

→ $\Delta x = \frac{b-a}{n} = \frac{1-0}{n} = \frac{1}{n}$ $\quad \frac{3-2}{n} = \frac{1}{n}$

$x_k^* = a + k\Delta x$
$= 0 + k\left(\frac{1}{n}\right) \quad 2\left(\frac{1}{n}\right)$
$= \frac{k}{n} \left(\frac{2k}{n}\right)$

$\lim_{n \to \infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$

$f(x_k^*) = \left(\frac{2k}{n}\right)^2$

$\lim_{n \to \infty} \sum_{k=1}^{n} \left(\frac{2k}{n}\right)^2 \cdot \frac{1}{n}$

$\lim_{n \to \infty} \frac{4}{n^3} \sum_{k}^{n} k^2$

$\lim_{n \to \infty} \frac{4}{n^3} \frac{n(n+1)(2n+1)}{6}$

$= \frac{k^2}{b^3} \lim_{n \to \infty} \frac{n}{n} \frac{(n+1)}{n} \frac{(2n+1)}{n}$

$= \frac{2}{3}(1+0)(2+0)$

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P.L NRAO

$= \frac{4}{3}$

6) Find the area of the region bounded along by $y = x+6$, bounded below $y = x^2$ and bounded on side by $x=0$ and $x=2$

$\rightarrow$ $y_1 = x+6$, $y_2 = x^2$

$= \int_0^b y_2 - y_1$

$= \int_0^2 \{ \cancel{x+6} - (x^2 - x+6) \, dx$

$= \int_0^2 \left| \frac{x^3}{3} - \frac{x^2}{2} + 6x \right]_0^2$

$= \left| \frac{2^3}{3} - \frac{2^2}{2} + 6(2) \right\} - 0$

$= \frac{8}{3} - \frac{4}{3} + 12$

$= \cancel{\frac{8}{3} - \frac{4+36}{3}}$ $\frac{8}{3} + 12 - \frac{4}{3}$

$= \cancel{\frac{8}{3} - \frac{40}{-3}}$ $\frac{8+36}{3} - \frac{4}{3}$

$= \frac{44}{3} - \frac{4}{3}$

$= \frac{40}{3}$ Sq. units

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$Q.1 \times RK$

1) Using Euler's method $\frac{dy}{dx} = y - x$, $y(0) = 2$
0.2

→ $f(x,y) = y - x$

$y(0) = 2$ $x_0 = 0$ $y_0 = 2$ step size is 0.1

$x_1 = 0 + h$
$= 0.1$

$x_2 = 0.1 + 0.1$
$= 0.2$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1

$y_1 = y_0 + 0.1(x_0, y_0)$
$= 2 + 0.2 = 2.2$

$f(x,y) = 2 - 0 = 2$

$f(x,y) = 2.2 - 0.1 = 2.1$

$y_2 = y_1 + 0.1(x_1, y_1)$
$= 2.2 + 0.1(2.1) = 2.4$

$n = 2$
$y_3 = 2.41 + 0.1(2.41) = 2.631$

2) Evaluate $\int \frac{1}{(9\cos 2x + 4\sin 2x)} dx$

$\int \frac{1}{(9\cos 2x + 4\sin 2x)}$

$\int \frac{1}{9\cos 2x} + \int \frac{1}{4\sin 2x}$

$\int \frac{1}{9(\cos^2 x - \sin^2 x)} + \int \frac{1}{4(2\sin x \cos x)}$

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9) $xy' = (1-x)y$ prove $y = xe^{-x}$

$x\frac{dy}{dx} - (1-x)y = 0$

$P = (1-x) \quad Q = 0$

$y \cdot e^{\int P dx} = \int Q \cdot e^{\int P dx} dx + C$

$y \cdot e^{\int (1-x) dx} = \int 0 \cdot e^{(1-\ln x)} dx + C$

$y \cdot e^{-x} = 0 + C$

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III unit - Calculus

AL.MCRAE

Q1 find an equation of the tangent plane to the surface $x^2 + 4y^2 + z^2 = 18$ at the point (1,2,1). Also find the parameter equation to the line that is normal to the surface at the point (1,2,1)

$\rightarrow$ $x^2 + 4y^2 + z^2 = 18$

$\therefore f_x = x^2$ $f_y = 4y^2$ $f_z = z^2$

$f_x = 2x$ $f_y = 4 \times 2y$ $f_z = 2z$

$f_x = 2(1)$ $f_y = 8(2)$ $f_z = 2(1)$

$= 2$ $= 16$ $= 2$

$\therefore$ The equation of tangent plane is

$$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$$

$$2(x-1) + 16(y-2) + 2(z-1) = 0$$

$$2x - 2 + 16y - 32 + 2z - 2$$

$$\therefore 2x + 6y + 2z - 36 = 0$$

The equation of the normal is

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$\frac{x - 1}{2} = \frac{y - 2}{16} = \frac{z - 1}{2}$$

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A.L.N.RAO

2) Find all relative extreme and saddle points of $f(x,y) = 4xy - x^4$

$f_x = 4y - \cancel{3x}\,4x^3$; $f_y = 4x$

$4y - 4x^3 = 0$; $4x = 0$

$\cancel{4(y - x^3) = 0}$

$x = 0$

$4y - 0 = 0$

$y = 0$

∴ The critical points are $(0,0)$ and $(1,1)$ and $(-1,1)$

∴ There is no relative extrema

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3. Find $f_x(1,3)$ and $f_y(1,3)$ for the function $f(x,y) = 2x^3y^2 + 2y + 4x$

$\rightarrow$ $f_x(x,y) = \frac{\partial}{\partial x}(2x^2y^2 + 2y + 4x)$

$= 6x^2y^2 + 4$

$f_x(1,3) = 6(1)^2(3)^2 + 4$

$= 6 \times 1 \times 9 + 4$

$= 54 + 4$

$= 58$

$f_y(x,y) = \frac{\partial}{\partial y}(2x^3y^2 + 2y + 4x)$

$= 4x^3y + 2$

$f_y(1,3) = 4(1)^3(3) + 2$

$= 4 \times 1 \times 3 + 2$

$= 12 + 2$

$= 14$

4. Evaluate $\lim_{(x,y)(0,0)} y \log(x^2+y^2)$ by converting into polar coordinates

$\rightarrow$ $\lim_{(x,y)(0,0)} = y \log(x^2+y^2)$

$= y \log(x^2+y^2)$

$= 0 - \log(0^2+0^2)$

$= 0 \log 0$

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Date: A.L. NIRMO

6 Find the directional derivative of $f(x,y) = e^{xy}$ at $(2,0)$ in the direction of unit vector that makes an angle of $\pi/3$ with positive x-axis

$\Rightarrow$ $f(x,y) = e^{xy}$

$f(x) = e^{xy} y = ye^{xy}$

$f(y) = e^{xy} x = xe^{xy}$

$f(2,0) = e^{xy} y = 0$
$f(2,0) = e^{20} 0 = 2$

$D_i f = \cos\phi\, i + \sin\theta$

$= \cos\frac{\pi}{3} + 2\sin\frac{\pi}{3}$

$= 0 + 2\frac{\sqrt{3}}{2}$

$= \sqrt{3}$

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Date: D.L.N.RAO

1] Find second order derivatives of $f(x,y) = y^2 e^x + y$

$f(x,y) = y^2 e^x + y$

$f_x = y^2 e^x$ $\quad$ $f_y = 2y.e^x + 1$

So $f_x(x,y) = y^2 e^x + y$ $\quad$ $f(x,y) = 2y.e^x + 1$

So

$f_{xx}(x,y) = \frac{d^2 f}{dx^2}$

$= \frac{d}{dx}\left(\frac{df}{dx}\right)$

$= \frac{d(y^2.e^x)}{dx}$

$= 2e^x$

$f_{xy}(x,y) = \frac{d^2 f}{dy.dx}$

$= \frac{d}{dy}\left(\frac{df}{dx}\right)$

$= \frac{d(y^2.e^x)}{dx}$

$= 2y.e^x$

$f_{yx}(x,y) = \frac{d^2 f}{dx.dy}$

$= \frac{d}{dx}\left(\frac{df}{dy}\right)$

$= \frac{d(2y.e^x + 1)}{dx}$

$2y.e^x$

$f_{yy}(xy) = \frac{d^2 f}{dy^2}$

$= \frac{d}{dy}\left(\frac{df}{dy}\right)$

$= \frac{d(2ye^x + 1)}{dy}$

$2e^x$

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Date: P.L.MRAG

8) Find the directional derivative of $(x,y,z) = x^2y - y^3z^2$ at the point $(1,2,0)$ in the direction of the vector $a = 2i + j - 2k$

→ Unit vector along $\bar{v}$ is $\frac{\bar{v}}{|\bar{v}|}$

$$= \frac{2i + j - 2k}{\sqrt{2^2 + 1^2 + (-2)^2}}$$

$$= \frac{2i + j - 2k}{\sqrt{9}}$$

$$= \frac{2i + j - 2k}{\sqrt{9}}$$

$$u(1,2,0) = (1)^2(2) - 2(0)^3 + 0$$

$$= 2 - 0 + 0 = 2$$

$$u + hv = (1,2,0) + h\left(\frac{2i}{\sqrt{9}}, \frac{j}{\sqrt{9}}, \frac{-2k}{\sqrt{9}}\right)$$

$$\cancel{f(u+hv) = \left(1 + \frac{2h}{\sqrt{9}}, 2 + \frac{j}{\sqrt{9}}, -0 + \frac{(-2k)}{\sqrt{9}}\right)}$$

$$f(u+hv) = \left(1 + \frac{2h}{\sqrt{9}}\right)^2\left(2 + \frac{h}{\sqrt{9}}\right) - \left(2 + \frac{jh}{\sqrt{9}}\right)\left(\frac{-2k}{\sqrt{9}}\right)^3 + \cancel{\left(0 - \frac{2h}{\sqrt{9}}\right)}$$

$$+ \left(0 - \frac{2h}{\sqrt{9}}\right)$$

$$\Rightarrow 3$$

$$= 1 + \frac{4h}{\sqrt{9}} + \frac{4h^2}{9} \times 2 + \frac{h}{\sqrt{9}} - 2 + \frac{h}{\sqrt{9}} + 8h^2$$

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Page No.
Date A.Thakur

9) $f(x,y) = x^3 + 2xy^2$. Evaluate it at $(-3,-4)$

$f(x,y) = x^3 + 2y\,xy^2$

$\frac{\partial f}{\partial x} = 3x^2 + 2y^2$ and $\frac{\partial f}{\partial y} = 4xy$

$\therefore$ grad $f(x,y) = (3x^2 + 2y^2, 4xy)$
Hence
grad $f(-3,-4) = \nabla f(-3,-4)$
$= (3(-3)^2 + 2(-4)^2, 4(-3)(-4))$
$= (3\times 9 + 2\times 16, 4\times 12)$
$= (27 + 32, 48)$
$\nabla f(-3,-4) = (59, 48)$

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1. Locate all relative extrema and saddle points of $f(x,y) = x^3 + 2y^3 - 3x^2 - 24y + 16$

→ Given $f(x,y) = x^3 + 2y^3 - 3x^2 - 24y + 16$

$\therefore f_x = 3x^2 - 6x \qquad f_y = 6y^2 - 24$

$f_x = 0, \; f_y = 0$

$3x^2 - 6x = 0$

$6y^2 - 24 = 0$

$3x^2 - 6x = 0 \quad -(1)$

$3x(x-2) = 0$

$x(x-2) = 0 \quad \therefore x = 0, x = 2$

$6y^2 - 24 = 0 \quad -(2)$

$\cancel{y^2 - 24} = 0 \quad 6(y^2 - 4) = 0$

$y^2 - 4 = 0$

$y^2 = 4 \quad \therefore y = \pm 2$

$\therefore$ The critical points

$(0,2)\;(0,-2),\;(2,2)$ and $(2,-2)$

Now $r = f_{xx} = 6x - 6 \qquad s = f_{xy} = 0$ and $t = f_{yy} = 12y$

~~Now we~~ di Extreme values

$r = f_{xx}(0,2) = -6 < 0$

$s = f_{xy}(0,2) = 0$

$t = f_{yy}(0,2) = 12(2) = 24$

$\therefore$ at $(0,2)$

$rt - s^2 = (-6)(24) - 0$

$= -144 < 0$

Hence f has neither local maximum nor minimum at (0, 2)

$\therefore$ (0, 2) is a saddle point of f at (0, -2)

$r = f_{xx}(0,-2) = -6 < 0$ $\quad$ $s = f_{xy}(0,2) = 0$

$t = f_{yy}(0,-2) = 12(-2) = -24$ $\quad$ $\therefore rt - s^2 = (-6)(-24) - 0 = 144 > 0$

$\therefore r < 0$ and $rt - s^2 > 0$

The function has local maximum and the local maximum value is

$f(0,-2) = (0) + 2(-2)^3 - 3(0) - 24(-2) + 16$

$= -16 + 48 + 16$

$= 48$

$r = f_{xx}(2,2) = 12 - 6 = 6 > 0$ $\quad$ $s = f_{xy}(2,2) = 0$

$t = f_{yy}(2,2) = 12(2) = 24$ $\quad$ $\therefore rt - s^2 = 6 \times 24 = 144 > 0$

$\therefore r > 0$ and $rt - s^2 > 0$

The function has local minimum at (2, 2) and the minimum value is

$f(2,2) = (2)^3 + 2(2)^3 - 3(2)^2 - 24(2) + 16$

$= 8 + 16 - 12 - 48 + 16$

$= 40 - 60$

$= -20$

$r = f_{xx}(2,-2) = 12 - 6 = 6 > 0$

$t = f_{yy}(2,-2) = 12(-2) = -24$

$s = f_{xy}(2,-2) = 0$

$\therefore rt - s^2 = 6(-24) - 0 = -144 < 0$

$\therefore r > 0$ and $rt - s^2 < 0$

$\therefore$ (2, -2) is a ~~sadd~~ saddle point

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