

Stats

Unit - I

Q1 Explain Random Variable?

- Experiment is any process that generates outcomes
- In an experiment we are interested in a particular outcome instead of that if we concentrate on the process of assigning numerical values to experimental outcomes. Such a process is called random variable
- A random variable is a numerical description of the outcome of an experiment
- A random variable from a sample space Ω into the real numbers
- i) Ex - tossing of two coins, $\Omega = \{HH, HT, TH, TT\}$
range of random variable X is $\{0, 1, 2\}$

Q2. Explain standard deviation, Variance and expectation

- a) Expectation - let x be a discrete random variable with probability distribution (x_i, p_i) the expected value of X denoted by $E(X)$ is defined as $E(X) = \sum_{i=1}^n x_i p_i = X_1 P_1 + X_2 P_2 + \dots + X_n P_n$

→ Suppose X is having a frequency distribution

| | | | | |
|--------|-------|-------|---------|-------|
| E | x_1 | x_2 | \dots | x_n |
| $f(x)$ | f_1 | f_2 | \dots | f_n |

$$\bar{x} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i} = \frac{f_1 x_1 + f_2 x_2 + \dots + f_n x_n}{N}$$

$$= \left(\frac{f_1}{N}\right)x_1 + \left(\frac{f_2}{N}\right)x_2 + \dots + \left(\frac{f_n}{N}\right)x_n$$

$$E(X) = \sum_{i=1}^n x_i p_i = x_1 p_1 + x_2 p_2 + \dots + x_n p_n$$

(n is favourable outcome)

b) variance - Mathematical expectation of a random variable provide a description of its average behaviour but it tells nothing about how much it varies let x be a discrete random variable with probability distribution (x_i, p_i) the variance of x denoted by

$$V(x) = \sigma^2 = E(X - E(x))^2 = E(X^2) - (E(x))^2$$

c) Standard deviation - The square root of variance is called standard deviation

$$S.D. = \sqrt{\sigma^2} = \sigma$$

3) Define and give properties of t distribution or chi square distribution

The chi square distribution has a single parameter called degrees of freedom - This can be any positive integer the chi-square distributed with n degrees of freedom is denoted by X_n^2 . The pdf of chi square distribution that is if χ follows X_n^2 then pd.f is given by

$$f(x) = \frac{1}{2^{n/2} J(\frac{n}{2})} x^{\frac{n}{2}-1} e^{-\frac{x}{2}}$$

Properties

- 1) If X has a gamma distribution with parameters m and B then $2PX/B$ has a chi-square distribution with $2m$ degrees of freedom
- 2) If X has a Chi-square distribution with $2m$

degrees of freedom then mean of x is $n_2 \cdot f(x)$
 $\therefore n$ & variance of $x = 2n$

3) If y/n has $y/6$ has a chi-square distribution with n degrees of freedom then the variance of y is $6y^2 = 2n^2$

4) If X and Y are independent chi-square random variables with n and p degrees of freedom then $X+Y$ is a chi-square random variable with $(n+p)$ degrees of freedom

5) Let x_1, x_2, \dots, x_n be a set of independent normal random variables with mean 0 and standard deviation s and let $S = X_1^2 + X_2^2 + \dots + X_n^2$. Then S has a chi-square distribution with n degrees of freedom

6) If X is chi-square distribution with n degrees of freedom then $Z = \frac{X - E(X)}{\sqrt{V}} = \frac{X - n}{\sqrt{2n}} \sim N(0, 1)$ as $n \rightarrow \infty$

7) Suppose that we have two independent random variables such that y follows normal with $(0, 1)$ and z follows chi-square with n degrees of freedom ($y \sim N(0, 1)$ & $z \sim \chi_n^2$) then random variable t defined by $t = \frac{y}{\sqrt{\frac{z}{n}}}$ has a t-distribution with n degrees of freedom denoted by t_n .

$$\text{Its pdf is given by } f(t) = \frac{\gamma(\frac{n+1}{2})}{\sqrt{\pi} \Gamma(\frac{n}{2})} \left(1 + \frac{t^2}{n}\right)^{-\frac{n+1}{2}} \quad -\infty < t < \infty$$

Properties of t-distribution

- 1) It is a bell shaped curve
- 2) It is symmetric about the mean

- 3) The mean, median and mode are located at the centre of distribution.
- 4) The curve never touches the x-axis.
- 5) The variance is greater than 1.
- 6) The t is a family of curves based on the concept of degrees of freedom which is related to sample size.
- 7) As the sample size increases the t distribution approaches the standard normal curve.

4) Write a short note on C.D.F.

Let x be a continuous random variable with PDF $f(x)$: The cumulative function of x or Distribution function of x denoted by $F(x)$ is defined as

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(y) dy$$

Also differentiating both sides we get

$$\frac{d}{dx} F(x) = f(x)$$

Hence, the density function is the derivative of cumulative distribution function

Properties of C.D.F

- 1) $F(x)$ is a non decreasing function of x , for $-\infty < x < \infty$ i.e. if $a < b$ then $F(a) \leq F(b)$
- 2) The cdf, $F(x)$, ranges from 0 to 1
- 3) $F(-\infty) = 0$ $F(\infty) = 1$
- 4) If X is continuous random variable then $f(x)$ is continuous function, it is smooth continuous curve

5) Write properties of expectation and variance

6) Expected value of constant is the constant itself $E(c) = c$

7) Effect of change of origin and scale of an expectation of x is given

$$E(x+b) = E(x) + b$$

$$E(ax) = a E(x)$$

$$E(a(x+b)) = a E(x) + ab$$

$$8) V(ax) = a^2 V(x)$$

$$9) V(x+b) = V(x)$$

$$10) \text{If } X \text{ and } Y \text{ are independent then } V(X+Y)$$

$$= V(X) + V(Y)$$

$$11) V(c) = 0 \text{ (where } c \text{ is constant)}$$

$$12) SD(x) = \sqrt{Var(x)}$$

6) An urn contains 6 red and 4 white balls. 3 balls are drawn at random. Obtain the probability distribution of white balls.

→ Let X be number of white balls drawn

$$X \text{ is } 0, 1, 2, 3$$

$$P(X=0) = \frac{\binom{6}{0} \binom{6}{3}}{\binom{10}{3}} = \frac{20}{120} = \frac{1}{6} = 0.1667$$

$$\binom{10}{3}$$

$$P(X=1) = \frac{\binom{6}{1} \binom{6}{2}}{\binom{10}{3}} = \frac{60}{120} = \frac{1}{2} = 0.5000$$

$$P(X=2) = \binom{4}{2} \binom{6}{1} = \frac{36}{120} = \frac{3}{10} = 0.3000 = 0.3000$$

$$P(X=3) = \binom{4}{3} \binom{6}{0} = \frac{4}{120} = \frac{1}{30} = 0.0333$$

Probability distribution of X

| X | 0 | 1 | 2 | 3 | Total |
|----------|--------|--------|--------|--------|-------|
| $P(X=x)$ | 0.1667 | 0.3000 | 0.3000 | 0.0333 | 1 |

- Q7 Define normal distribution and give its properties
 -> A random variable has a normal distribution with parameters μ and σ^2 if its density function f is given by

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2}, x \in \mathbb{R}$$

Properties

- 1) If $X \sim N(\mu, \sigma^2)$, then $\frac{X-\mu}{\sigma} \sim N(0, 1)$

Thus by subtracting the mean and dividing by the standard deviation

- 2) If $X \sim N(\mu, \sigma^2)$, then $X = \mu + \sigma Z$, with $Z \sim N(0, 1)$
 3) $E(X) = \mu$, i.e. mean of normal distribution is μ
 4) $\text{Var}(X) = \sigma^2$

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$$\text{Var}(x) = \text{Var}(N+6^2) = 6^2 \text{Var}(z)$$

$$E z^2 = \int_{-\infty}^{\infty} z^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = \int_{-\infty}^{\infty} z^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$

- 5) A bell shaped symmetric curve
 6) Symmetric, arithmetic mean, median and mode coincide

Q8 Find $E(x)$ and $V(x)$ for the following probability distribution

| X | 0 | 1 | 2 | 3 | Total |
|------------|-----|-----|------|------|-------|
| $P(x)$ | 1/6 | 1/2 | 3/10 | 1/30 | 1 |
| $xP(x)$ | 0 | 1/2 | 6/10 | 3/30 | 5/10 |
| $x^2 P(x)$ | 0 | 1/2 | 2/5 | 6/10 | 20/10 |

$$E(x) = \sum xP(x) = 0 + 1/2 + 3/5 + 1/10 = 6/5 = 1.2$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$= 2 - (1.2)^2$$

$$= 2 - 1.44$$

$$= 0.56$$

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- Q9. An unbiased coin is tossed 5 times
Find probability of getting a) All heads
b) 3 heads. & No heads

$$\rightarrow n : \text{number of coins} = 5$$

$$p = \text{head} = 0.5$$

$$q = 1 - p = 1 - 0.5 \\ = 0.5$$

- a) All heads

$$\begin{aligned} P(X=5) &= {}^5C_5 p^5 q^{5-5} \\ &= \frac{5!}{5!} (0.5)^5 q^0 \\ &= 1 \times 0.03125 \times 1 \\ &= 0.03125 \end{aligned}$$

- b) 3 heads

$$\begin{aligned} P(X=3) &= {}^5C_3 p^3 q^{5-3} \\ &= \frac{5!}{3!2!} \times (0.5)^3 \times (0.5)^2 \\ &= 10 \times 0.125 \times 0.25 \\ &= 0.3125 \end{aligned}$$

- c) No heads

$$\begin{aligned} P(X=0) &= {}^5C_0 p^0 q^{5-0} \\ &= 1 \times 1 \times 0.03125 \\ &= 0.03125 \end{aligned}$$

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i) Define binomial distribution and give its properties.

-> Bi means two, when any experiment results into two possible outcomes it is called as Binomial.
Consider an example of tossing of n coin. If X is the random variable defined on this experiment which counts the total numbers of heads and the probability of head is p then we say X has a binomial distribution with parameters n and p and write $X \sim B(n, p)$.

Properties

- 1) If $X \sim B(n, p)$, mean of X is np
- 2) Variance of X is npq
- 3) Additive property: If $X \sim B(n_1, p)$ and $Y \sim B(n_2, p)$ are independent random variable then $X+Y \sim B(n_1 + n_2, p)$

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Unit II

- 3) What do you understand by One tailed and two tailed test?
- 3) Define different types of Hypothesis
- One Sided Hypothesis: A statistical test in which the population parameter lies entirely above or below the value specified in H_0 is a one-sided (or one-tailed) test

$$\text{eg. } H_0: \mu > 165 \text{ or } H_0: \mu < 165$$

Two sided Hypothesis: An alternative hypothesis that specifies that the parameter can lie on either side of the value specified by H_0 is called a two sided (or two-tailed test)

$$\text{eg. } H_1: \mu = 165$$

~~The null hypothesis is the null condition: no difference (means) or no relationship between variables.~~

~~The alternate hypothesis will be statement involving the same population parameters in which H_1 and H_0 cannot be true.~~

- 2) Explain different types of Errors

→ Type I error occurs when rejecting the null hypothesis when it is actually true: $\text{Type I} (\alpha)$

→ Type II error occurs when failing to reject the null hypothesis, when it is actually false. (β)

| | Null Hypothesis is true | Alternative Hypothesis is true |
|------------------------|-------------------------|--------------------------------|
| Accept Null Hypothesis | No Error | Type II Error |
| Reject Null Hypothesis | Type I Error | No Error |

(a) Define different types of hypothesis

1) null hypothesis, denotes H_0 , states that some parameter is equal to a specific value

2) The null hypothesis is the null condition - no difference between means or no relationship between variables. The null hypothesis denoted H_0 is claim about a population parameter that is assumed to be true until declared false

a) Alternate hypothesis H_1 will be a statement involving the same population parameters in such a way that H_1 and H_0 cannot be true

b) An alternative hypothesis denoted H_1 states that the value of parameter differs from the specified by the null hypothesis.

c) The mean height of 100 individuals from a population is 160. The SD is 10. Will it be reasonable to suppose that the mean height of population is 65

Hos No: 65

Q) Difference between large sample size and small sample size

| Sr No | Large Sample | Small sample |
|-------|--------------|--------------|
|-------|--------------|--------------|

| | | |
|---|------------------------------------|-------------------------------|
| 1 | The sample size is greater than 30 | The sample size is 30 or less |
|---|------------------------------------|-------------------------------|

| | | |
|---|---|--|
| 2 | Normal distribution is used for testing | Sampling distribution like t, F, etc. are used for testing |
|---|---|--|

Q) Explain the procedure for testing statistic?

→ State the null and alternative hypothesis

• The null

Analysis of variance is a hypothesis testing technique used to test the equality of two or more population (or treatment) means by examining the variance of sample that are taking. It allows one to determine whether the difference between the sample are simply due to random error (sampling error) or whether there are systematic treatment effects that causes the mean in one group to differ from the mean in other group.

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4) The mean height of 100 individual from a population is 160. The S.D is 10. Will it be reasonable to ^{Suppose} that the mean height of population is 65?

\rightarrow Let x be the random variable that represents the height of the individual
mean = 160 and S.D = 0

↳ Mean height μ

$$\therefore \text{For } x=65, z = \frac{65-160}{10} = -14$$

$$\frac{\sigma}{\sqrt{n}}$$

$$\text{Now } z_1 = \frac{x_1 - \mu}{10}$$

\therefore probability when mean = 65
 $P(z_1 = z)$

$$= P(16 - 65)$$

$$\approx 9.5$$

\therefore The probability that the mean height of population is 65 is 9.5

Q5. Test made breaking strength of 10 pieces of a metal wire are as follows 572, 572, 570, 562, 572, 570, 570, 572, 596, 582 in kgs. Test the breaking strength of average at 577 kg.

\rightarrow Let \bar{x} be sample mean & SD (i.e. sample SD) then test if \bar{x} differs significantly from the population mean $\mu = 577$

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we take the assumed mean $M = 568 + 586 = 582$

$$d_i = x_i - M$$

$$x_i = d_i + M$$

$$\bar{x} = \frac{1}{n} \sum x_i = \frac{1}{n} \sum d_i + M$$

$$\frac{1}{10} \times (-68) + 58.2 = 575.2 \text{ (from table below)}$$

$$S^2 = \frac{1}{n} \sum d_i^2 - \left(\frac{1}{n} \sum d_i \right)^2 \text{ (from table below)}$$

$$\frac{1}{10} \times 1444 - \left(\frac{1}{10} \times (-68)^2 \right)^2$$

$$= 68.15$$

$$S = 868.15$$

Now $t = \frac{\bar{x} - M}{S / \sqrt{n-1}} = \frac{572.5752 - 577}{8.26 / \sqrt{9}}$

$$t = \frac{-0.65}{\sqrt{9}} \quad & v = n-1 \\ & = -0.65$$

| x_i | $d_i = x_i - M$ | d_i^2 |
|-------|-----------------|---------|
| 578 | -4 | 16 |
| 572 | -10 | 100 |
| 570 | -12 | 144 |
| 568 | -14 | 196 |
| 572 | -10 | 100 |
| 570 | -12 | 144 |
| 570 | -12 | 144 |
| 572 | -10 | 100 |
| 596 | 14 | 106 |
| 584 | 2 | 4 |

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$$H_0: \bar{x} = \mu \quad \text{vs} \quad H_1: \bar{x} \neq \mu$$

* Two tailed test is be used for LOS & S%
 $t = \frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{5.7 - 5.0}{0.05/\sqrt{10}} = 2.76$
since $|t| < t_{\alpha/2}$

Since $|t| < 1.05$

Therefore the mean breaking strength of the wire
can be assumed as 5.7 kg at 5%

∴ H₀- student are benefited by coaching w/o
H₁- student are not not benefited by coaching w/o

| x_i | x | d_i | d_i^2 |
|-------|-----|-----------------|-------------------|
| 70 | 67 | 2 | 4 |
| 68 | 70 | -2 | 4 |
| 56 | 52 | 4 | 16 |
| 75 | 73 | 2 | 4 |
| 80 | 75 | 5 | 25 |
| 90 | 77 | 12 | 144 |
| 63 | 80 | -17 | 144 |
| 75 | 92 | -17 | 289 |
| 656 | 54 | 2 | 4 |
| 53 | 65 | 3 | 9 |
| | | $\sum d_i = -1$ | $\sum d_i^2 = 63$ |

$$d = \frac{\sum d_i}{n} = \frac{-1}{10} = -0.1$$

$$\bar{d} \pm S_d$$

Test statistic
 $t = \frac{\bar{d} - \mu}{S_d / \sqrt{n}}$

$$\pm S_d$$

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$$Sd^2 = \frac{Sd^2 - \bar{n}d^2}{n-1}, \frac{643-10}{9} : \frac{633}{9} : 70.333$$

$$\begin{aligned} t &= \frac{-1\sqrt{10}}{\sqrt{70.333}} \\ &= -1 \times 3.1627 \\ &= -3.1627 = -0.3771 \\ &\quad 8.3864 \end{aligned}$$

Ans

Critical region

Reject H_0 if $|t_{cal}| > t_{crit}$
 $t_{0.01, 12} = 3.256$

∴ we accept H_0 as $|t_{cal}| < t_{crit}$

The student are happy by coaching

| | A | B | C | D | Total |
|-------|----|----|----|----|-------|
| P | 6 | 4 | 8 | 6 | 24 |
| Q | 7 | 6 | 6 | 9 | 28 |
| R | 8 | 5 | 10 | 9 | 32 |
| Total | 21 | 15 | 24 | 24 | 84. |

Column

$$P: 24/4 = 6 = \bar{Y}_1$$

$$Q: 28/4 = 7 = \bar{Y}_2$$

$$R: 32/4 = 8 = \bar{Y}_3$$

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$$P = 2113 - 7 = \sqrt{7}$$

$$B = 1513 - 5 = \sqrt{5}$$

$$L = 2413 - 8 = \sqrt{8}$$

$$D = 2413 - 8 = \sqrt{16}$$

$$\bar{y} = \frac{P + Q + R + A + B + C + D}{12} = \frac{68}{12} = 4$$

$$x = \bar{y} - \bar{y} = 6 - 4 = 2$$

$$x_1 = 6 - 14 = -8 \quad : B_1 = y_1 - \bar{y} = 6 - 4 = 2$$

$$x_2 = 7 - 14 = -7 \quad : B_2 = 8 - 4 = -4$$

$$x_3 = 8 - 14 = -6 \quad : B_3 = 14 - 4 = -10$$

$$\begin{aligned} SSE &= (6-14)^2 + (7-14)^2 + (8-14)^2 + (4-14)^2 + (6-14)^2 + (5-14)^2 \\ &\quad + (8-14)^2 + (6-14)^2 + (0-14)^2 + (6-14)^2 + (8-14)^2 + (9-14)^2 \\ &= (8)^2 + (-7)^2 + (-6)^2 + (-10)^2 + (-2)^2 + (-9)^2 + (-10)^2 + (-1)^2 + (-3)^2 + (-4)^2 \\ &\quad + (-2)^2 + (-5)^2 + (-5)^2 = -324 \end{aligned}$$

$$\begin{aligned} SSE &= (6-24)^2 + (4-24)^2 + (8-24)^2 + (6-24)^2 + (7-24)^2 + (0-24)^2 \\ &\quad + (6-24)^2 + (9-24)^2 + (8-32)^2 + (5-32)^2 + (110-32)^2 + (9-32)^2 \\ &\quad (6-24)^2 + (7-24)^2 + (2-24)^2 + (4-24)^2 + (6-14)^2 + (3-14)^2 \\ &\quad (8-44)^2 + (6-24)^2 + (10-24)^2 + (6-24)^2 + (9-24)^2 + (9-24)^2 \\ &= (-18)^2 + (-20)^2 + (-16)^2 + (-18)^2 + (-2)^2 + (-20)^2 + (-2)^2 + (-19)^2 \\ &\quad + (-24)^2 + (-20)^2 + (-22)^2 + (-23)^2 + (-18)^2 + (-15)^2 + (-13)^2 + (-12)^2 \\ &\quad + (-11)^2 + (-9)^2 + (-10)^2 + (6)^2 + (1-15)^2 + (-11)^2 + (-1)^2 + (-1)^2 \\ &= -433 \end{aligned}$$

$$SSD = 500 + 500 + 500$$

$$= 1500 (-24 + -43)$$

$$= -663$$

2 way var

| | Sos | Pof | Masa | Fas |
|-------|------|-----|-------|------|
| A | -84 | 3 | -22 | 42 |
| B | -24 | 3 | -42 | 42 |
| Masa | -435 | 63 | -6390 | 6390 |
| Total | -603 | 74 | | |

(ii) short side formula

$$A = B \cdot C$$

$$12 \cdot m = 6$$

$$12 \cdot 2 = 12$$

$$12 \cdot 3 = 12$$

$$12 \cdot 4 = 16$$

$$12 \cdot 5 = 10 \quad x = 12$$

$$N = 8 \cdot 12 = 12 + 2 + 16 + 2 + 10 + 7 + 8 + 12 +$$

$$N = 80$$

$$10^2 = 100$$

$$SSD = \sum (x_{ij} - \bar{x})^2 = 13(80)^2 + 3(12+2+16+2+10+7+8+12+10^2 - 24)$$

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$$\begin{aligned} SSE &= \sum (y_{ij} - \bar{y}_{ij})^2 = ((4-9)^2 + (1-7)^2 + (12-2)^2 + (2-2)^2 \\ &\quad + (6-10)^2 + (8-10)^2 + (10-10)^2 + (3-10)^2 + (12-12)^2 + (14-13)^2 + (16-10)^2 \\ &= 36 + 16 + 16 + 36 + 16 + 4 + 0 + 0 + 36 + 0 + 0 \\ &= 184 \end{aligned}$$

$$SST = 3SSTR + SSE = 24 + 184 = 208$$

$$MSSTR = \frac{SSTR}{C-1} = \frac{24}{2} = 12$$

$$MSE = \frac{SSE}{n-1} = \frac{184}{24-3} = 184/21$$

$$s^2 = MSAE = \frac{12}{184} = 0.06521$$

| Source | SOS | Df | Mean | F value |
|---------------|-----|----|-----------------|---------|
| Between Group | 24 | 2 | 12 | |
| Within group | 184 | 21 | 184/21 = 8.7619 | 0.06521 |
| Total Group | 208 | 23 | | |

- Q1) When do we use Non-parametric test?
- > Generally one need to use parametric test unless and until clear identification of non parametric test
 - b) It is used for small sample size
 - c) Non It is used when we have nominal or ordinal data
 - d) It is used for qualitative or ranked data

Q2) Write a difference between parametric and non parametric test?

- >
- a) Non-parametric tests
 - 1) Generally one need to use parametric test unless and until clear identification of non parametric test
 - 2) Non parametric test are less accurate with compare to parametric test
 - 3) Non parametric test are used for small samples size
 - 4) Non parametric methods are used when we have nominal or ordinal data
 - 5) Non parametric test are used for qualitative or ranked data
- b) Parametric tests
 - 1) Parametric follows standard distribution
 - 2) Parametric test are more accurate than non parametric test
 - 3) Parametric test used for large sample size
 - 4) Parametric method are generally used for ratio or interval data
 - 5) Parametric test are used for quantitative data

(Q2)

Q2 Explain Wilcoxon sign test and Sign test procedure.

- > 1) This is an extension of sign test takes into account magnitude and direction also
- 2) It is powerful and more accurate than sign test
- 3) We assign rank for ordinal data

Q3) Steps

- a) We need to compute the difference scores
- b) Next step is to rank the difference scores
- c) We order the absolute value of difference scores and assign rank from 1 to n
- d) Add + or - to Ranks.
- e) We use $\frac{n(n+1)}{2}$
- f) We find critical by using table of critical values of T
- g) By critical value we determine whether to reject or accept H_0

Q4) A study is run to evaluate the effectiveness of an exercise program in reducing systolic blood pressure in patients with pre-hypertension. A total of 15 patients were involved.

- > H_0 = The difference is zero
- 1. The difference is not zero $\alpha = 0.05$

| Patient | Before | After | Difference | Ranks | Unadjusted Ranks |
|---------|--------|-------|------------|-------|------------------|
| 1 | 125 | 118 | 7 | 1 | 1 - 1 |
| 2 | 132 | 134 | -2 | 2.5 | 2 - 2.5 |
| 3 | 138 | 130 | 8 | 2.5 | 2 - 2.5 |
| 4 | 120 | 124 | -4 | 4 | 3 - 4 |
| 5 | 125 | 105 | 20 | 6 | 4 - 6 |
| 6 | 127 | 130 | -3 | 6 | 4 - 6 |
| 7 | 136 | 136 | 0 | 6 | 4 - 6 |
| 8 | 139 | 132 | 7 | 8 | 6 - 8 |
| 9 | 131 | 123 | 8 | 10 | 7 - 10 |
| 10 | 132 | 120 | 4 | 10 | 7 - 10 |
| 11 | 135 | 126 | 9 | 10 | 7 - 10 |
| 12 | 136 | 140 | -4 | 12.5 | 8 - 13.5 |
| 13 | 128 | 135 | -7 | 12.5 | 8 - 13.5 |
| 14 | 127 | 126 | 1 | 14 | 9 - 14 |
| 15 | 130 | 132 | 2 | 15 | 20 - 15 |
| 16 | 133 | | | | |

$$W+ = 89 \text{ and } W- = 31$$

$$IN = 89 + 31 = 120, \quad \frac{n(n+1)}{2} = \frac{15(16)}{2} = 120$$

Test Statistic
 $W_1, \text{Min}(+IN, -W) = \text{Min}(89, 31) = 31$

at $n=15$ and $\alpha=0.05$
 we do not reject null hypothesis
 \therefore There is no difference between the two

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- 5) Explain the RUN test with an example.
- 1) The runs test is used to determine for serial randomness: whether or not observation occurs in a sequence in time or over space.
- 2) In geographic studies, the run test is most often used to determine whether observations are random.

~~HABBAAB~~

Ex

AA BB AA BBBB AAA BBBB HA BBB

H_0 : The distribution of success is random
 H_1 : The distribution of success is not random

$n_1 = 9$ ← there 9 occurrences of the value A

$n_2 = 14$ ← there 14 occurrences of the value B

$u = 8$ ← there are 8 runs

$\alpha = 0.05$

← there are two critical values using top upper and lower value

$U_{critical} = 6, 17$

Since $6 < 8 < 17$ accept Null hypothesis H_0

6) In Mumbai University a student appeared for an examination in 12 Paper result is given below. Does students performed well or no?

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P | P | P | F | F | F | F | F | P | P | F | P |
| 1 | 2 | 3 | H | 1 | 2 | 3 | 4 | S | S | 6 | 6 |
| | | | | | | | | | | 5 | 5 |

H_0 : Students have performed well
 H_1 : Students have not performed well

$$n_1 \rightarrow 6 = (P)$$

$$n_2 \rightarrow 6 = F$$

$$u = 5$$

$$\alpha = 0.05$$

There are two critical value using run test table

$$U_{\text{critical}} = U_{\text{critical}} = 3, 1 \\ 3 < 5 < 11$$

Thus accept Null hypothesis H_0

7) The known Wilcoxon test us if the difference between the groups are so large that they are unlikely to have occurred by chance

7) Sample

$$\begin{array}{c} \text{No} \quad 20 \quad 60 \\ \text{No} \quad 23 \end{array}$$

In This outcome $n_1 = 8$ $n_2 = 8$ $n_3 = 3$

Null hypothesis : There is difference (f)

Alternative hypothesis : There is no difference (D)

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| Sample | | | Ordered | | |
|--------|-------|-------|---------|-------|-------|
| No | 20min | 60min | No | 20min | 60min |
| 23 | 22 | 59 | 22 | | |
| 26 | 27 | 66 | 23 | | |
| 51 | 39 | 38 | 26 | | |
| 49 | 29 | 49 | 27 | | |
| 58 | 46 | 56 | 37 | | |
| 37 | 48 | 60 | | | |
| 29 | 49 | 65 | 39 | | |
| 24 | 65 | 62 | 44 | | |
| | | | 46 | | |
| | | | 49 | 49 | 49 |

| Sample | | | Ordered | | | Ranks | | |
|--------|-------|-------|---------|-------|-------|-------|-------|-------|
| No | 20min | 60min | No | 20min | 60min | No | 20min | 60min |
| 23 | 22 | 59 | 22 | | | 1 | | |
| 26 | 27 | 66 | 23 | | | 2 | | |
| 51 | 39 | 38 | 26 | | | 3 | | |
| 49 | 29 | 49 | 27 | | | 4 | | |
| 58 | 46 | 56 | 29 | 29 | | 5.5 | 5.5 | |
| 37 | 48 | 60 | 37 | | | 7 | | |
| 29 | 49 | 65 | | | | 8 | | |
| 24 | 65 | 62 | 39 | | | | | |
| | | | 44 | | | 9 | | |
| | | | | 46 | | | 10 | |
| | | | | 48 | | | 11 | |
| | | | 49 | 49 | 49 | 12 | 12 | |
| | | | 51 | | | 14 | 14 | 14 |
| | | | | 56 | | 16 | | |
| | | | 58 | | | 18 | 18 | |
| | | | 59 | | | 19 | | |
| | | | 60 | | | 20 | 20 | |

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| | | | | | |
|--|----|------|------|------|----|
| | | 62 | | | 12 |
| | 65 | 65 | 22.5 | 22.5 | |
| | 66 | 24.5 | | | |

$$R_1 = 75.5 + \cancel{79} \quad R_2 = \cancel{79} \quad R_3 = 145.5 \\ = 145.5$$

$$R_1 = 75.5$$

$$n(n+1) = \frac{25(26)}{2} = 325$$

$$= \frac{24(25)}{2} = 300 \quad \textcircled{1}$$

$$\therefore R_1 + R_2 + R_3 = 75.5 + 79 + 145.5 \\ = 300 \quad \textcircled{2}$$

$$\textcircled{1} = \textcircled{2}$$

$$H = \left\{ \frac{12}{24(25)} \left| \frac{(75.5)^2}{8} + \frac{(79)^2}{8} + \frac{(145.5)^2}{8} \right. \right\} - 3(24+1)$$

8) Define Duncans test and Mann Whitney-U

$$= 0.02 (712.53 + 780.12 + 2646.28) - 3(25) \\ = 0.02 (4138.93) - 75 \\ = 82.7786 - 75 \\ = 7.7786$$

$$K=3 \quad (\text{number of groups}) \\ H = K-1 = 3-1 = 2$$

Critical value

$H = 7.7786 > 5.66$ therefore we reject null hypothesis

Thus there is difference.

8) Define Duncan's test and Mann Whitney U test

→ a) **Duncan's multiple range test** for multiple comparison where error rate is reduced on an experimental basis no. on a per-comparison basis

$$M = q (\bar{y}_r, N - q) \times \sqrt{\frac{MS_{WMS}}{n}}$$

where c is before, n is the number of observation per treatment groups; N is total number of observation from g treatment groups; MS_{WMS} is with (residual) mean square derived from ANOVA Table.

b) **Mann Whitney U test** is used to compare two independent samples like unpaired t-test it is parametric test. A popular non-parametric test to compare outcome between two independent group is the Mann Whitney U test.

9) The distribution of Kruskal-Wallis test
Explain the procedure and limitation of Kruskal-Wallis test

→ 1) The distribution of the Kruskal-Wallis test statistic approximates a chi-square distribution with $k-1$ degrees of freedom.
2) If the calculated value of the Kruskal-Wallis test is less than critical ch-square value, then the null hypothesis cannot be rejected.

3) The procedure for the test involves pooling the observation from k samples into one combined sample, keeping track of which sample each observation comes from, and then ranking lowest to highest from 1 to N , where $N = n_1 + n_2 + \dots + n_k$

(ii) What are the advantages of Non-parametric test?

→ If the sample size is very small, there may be non-parametric statistical test unless the nature of the ~~popula~~ population distribution is known exactly

2) Non parametric test typically make fewer assumption about the data and may be more relevant in a particular situation

3) Non parametric test typically make few assumptions as a method is available to treat data which are simply categorical i.e. are measured in money scale.

4) Non-parametric statistical tests typically are much easier to learn and to apply than one parametric tests. In addition, their interpretation often is more direct than the interpretation of parametric tests