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Unit-1 Random variables & Probability distribution

Probablity (Introduction) → Definition of probability was given by Pierre Siemen Laplace in 1795

J. Carden an Italian Physician and a mathematician wrote the first book on probability name book of games

1. Games of chance:-

probability has been used extensively in many areas such as biology, physics etc. It also used in forecast of weather, result of election, population, earthquakes, crop production etc.

2. Random experiment:-

An experiment is said to be random if its outcome cannot be predicted that is the outcome of an experiment does not obey any rule.

Eg:-1. Tossing a coin is a random exp.

2. Throwing a die

3. Outcome:-

The result of a random experiment will be called as outcome.

4. Sample space:-

The set of all possible outcomes of an experiment are called a sample space

Ex:-1. A coin is tossed neither head or tail occurs.

2. Sample space $S = \{H, T\}$

5. Event:-

Any subset E of a sample space is called an event

Ex:- When coin is tossed getting a head.

⇒ Probability (definition):-

The No. of favourable outcomes of an event by total No. of possible outcomes of the Sample space

$$P(E) = \frac{n(E)}{n(S)}$$

Q. A class consists of 6 girls and 10 boys if a committee of 3 is chosen at random from the class. Find the probability that

i). 3 boys are selected

ii). exactly 2 girls are selected

sol: i). $n(S) = {}^{16}C_3$

$n(E) = {}^{10}C_3$

$P(E) = \frac{n(E)}{n(S)} = \frac{{}^{10}C_3}{{}^{16}C_3} = \frac{3}{14}$

ii). $n(E) = {}^{6}C_2 \times {}^{10}C_1$

6.5

$P(E) = \frac{n(E)}{n(S)}$

$= \frac{{}^{6}C_2 \times {}^{10}C_1}{{}^{16}C_3} = \frac{15}{56}$

- Random variable -

A real variable X whose values is determined by the outcome of a random experiment is called a random variable

Ex:- Consider of tossing a coin twice

$S = \{S_1, S_2, S_3, S_4\}$

where $S_1 = \{HH\}$

$S_2 = \{TH\}$

$S_3 = \{HT\}$

$S_4 = \{TT\}$

let $X(S)$ = The no. of heads

$X(S_1) = 2 \quad X(S_2) = 1 \quad X(S_3) = 1 \quad X(S_4) = 0$

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* Types of random variables
1. Discrete random variable
2. Continuous random variable

1. Discrete random variable:-
A random variable X which can take finite number of discrete values in an interval of domain is called a discrete random variable
Ex:- A random variable denoting number of students in a class

2. Continuous random
A random variable X which can take value continuously that is which take all possible values in a given interval is called Continuous random variable
Ex:- height, age, weight of individuals.

- Probability function of discrete random variables-
- for a discrete random variable X the real valued function P(x) such that $P(X=x)$ i.e, $P(X=x)=P(x)$ then $P(x)$ is called probability function or probability density function.

* properties of Probability function:-
- if $p(x)$ is probability function of random variable X then it possess following properties
1) $P(x) \geq 0 \ \forall x$
2) $\sum P(x) = 1 \ \forall x$
3) $P(x)$ cannot be negative for any value of x.

* Mean of a probability discrete distribution-
- It is denoted as a $\mu = \sum P_i x_i = E(x)$ where $E(x) \rightarrow$ Expectation

* Variants of a probability discrete distribution - denoted as σ^2 and It is defined as $\sigma^2 = \sum P_i x_i^2 - \mu^2$

* Standard Deviation - It is positive sq. root of variants. It is denoted as

$$\sigma = \sqrt{\sum P_i x_i^2 - \mu^2}$$

* Problems:

1. Two dice are thrown. Let X assign to each point (a,b) in S. The maximum of its number that is $X(a,b) = \max(a,b)$ find the prob. distrib. X is a random variable with $X(S) = \{1,2,3,4,5,6\}$ Also find mean, variants, std. deviation

sol: ~~n(S)=36~~ The total no. of cases i.e., $n(S) = 36$

The maximum number could be for $X(S) = \{1,2,3,4,5,6\}$

i.e., $X(a,b) = \max(a,b)$

The no.1 will appear maximum in one case only i.e., $(1,1)$

$\therefore P(X=1) = P(1) = \frac{1}{36}$

The no.2 will occur maximum in 3 cases i.e., $(1,2), (2,1), (2,2)$

$\therefore P(X=2) = P(2) = \frac{3}{36}$

$||^{ly}$ $P(3) = \frac{5}{36}$ $[(1,3), (3,1), (2,3), (3,2), (3,3)]$

$P(4) = \frac{7}{36}$ $[(4,1), (1,4), (2,4), (4,2), (3,4), (4,3), (4,4)]$

$P(5) = \frac{9}{36}$

$P(6) = \frac{11}{36}$

The P.D

$X = x$	1	2	3	4	5	6
$P(X=x) = P(x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

$\mu = \sum P_i x_i$

$= \frac{1}{36} + \frac{2 \times 3}{36} + \frac{3 \times 5}{36} + \frac{4 \times 7}{36} + \frac{5 \times 9}{36} + \frac{6 \times 11}{36}$

$= \frac{161}{36} = 4.47$

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$\sigma^2 = \sum P_i x_i^2 - \mu^2$

$= \left[1^2 \times \frac{1}{36} + 2^2 \times \frac{3}{36} + 3^2 \times \frac{5}{36} + 4^2 \times \frac{7}{36} + 5^2 \times \frac{9}{36} + 6^2 \times \frac{11}{36}\right] - [4.47]^2$

$= \frac{791}{36} - (4.47)^2$

$= 21.972 - (4.47)^2$

$= 1.973$

$SD = \sqrt{\text{variance}} = \sqrt{1.973} = 1.404.$

2. A random variable X has the Probability funcn.

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

i) find k

ii) Evaluate $P(X<6)$, $P(X\geq 6)$, $P(0<X<5)$, $P(0\leq X\leq 4)$

iii) if $P(X\leq t) > 1/2$

find minimum value of t

iv) determine the distribution funcn of X

v) mean, variance, S.D.

i) $\sum P(x) = 1$

$k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$10k^2 + 9k - 1 = 0$

$\boxed{k = 0.1}$

ii) 1. $P(X<6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= 0 + k + 2k + 2k + 3k + k^2$

$= 0.81$

2. $P(X\geq 6) = \cancel{P(X=6) + P} \; 1 - P(X<6)$

$= 1 - 0.81$

$= 0.19$

3. $P(0<x<5) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= k + 2k + 2k + 3k$

$= 0.8$

4. $P(0 \le x \le 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= 0 + k + 2k + 2k + 3k$

$= 0.8$

v) Mean$(\mu) = \sum P_i x_i$

$= 1 \times k + 2 \times 2k + 3 \times 2k + 4 \times 3k + 5 \times k^2 + 6 \times 2k^2 + 7(7k^2+k)$

$= 30k + 66k^2$

$= 3.66$

Variance$(\sigma^2) = \sum P_i x_i^2 - \mu^2$

$= 1^2 \times k + 2^2 \times 2k + 3^2 \times 2k + 4^2 \times 3k + 5^2 \times k^2 + 6^2 \times 2k^2 + 7^2(7k^2+k) - (3.66)^2$

$= k + 8k + 18k + 48k + 25k^2 + 72k^2 + 343k^2 + 49k - (3.66)^2$

$= 124k + 440k^2 - (3.66)^2$

$= 3.404$

$S.D = \sqrt{\text{variance}} = \sqrt{3.404} = 1.843$

iii). $P(X \le t) > \frac{1}{2}$

$P(X \le 0) = 0 < \frac{1}{2}$

$P(X \le 1) = P(X=0) + P(X=1)$

$= k = 0.1 < \frac{1}{2}$

$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0 + k + 2k = 3k = 0.3 < \frac{1}{2}$

$P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 0 + k + 2k + 2k = 5k = 0.5 = \frac{1}{2}$

$P(X \le 4) = 0.8 > \frac{1}{2}$

$P(X \le 5) = 0.81 > \frac{1}{2}$

x	$F(x) = P(X \le x)$
0	0
1	0.1
2	0.3
3	0.5
4	0.8
5	0.81
6	0.83
7	1

iv). $t = 4$

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Q. A Sample of 4 items is selected from a box containing 12 items of which 5 are defective. find the expected number E of a defective item.

Sol. Let X denotes the no. of defective items among 4 items drawn from 12 items

$\therefore X = \{0, 1, 2, 3, 4\}$

The good items are 7

bad items are 5

$n(S) = {}^{12}C_4$

$P(X=0) = P(\text{No defective}) = \frac{{}^7C_4}{{}^{12}C_4} = 0.07$

$P(X=1) = P(1 \text{ def.}, 3 \text{ Good}) = \frac{{}^5C_1 \cdot {}^7C_3}{{}^{12}C_4} = 0.35$

$P(X=2) = P(2 \text{ def.}, 2 \text{ Good}) = \frac{{}^5C_2 \cdot {}^7C_2}{{}^{12}C_4} = 0.42$

$P(X=3) = P(3 \text{ def}, 1 \text{ Good}) = \frac{{}^5C_3 \cdot {}^7C_1}{{}^{12}C_4} = 0.14$

$P(X=4) = P(\text{all def}) = \frac{{}^5C_4}{{}^{12}C_4} = 0.01$

x	0	1	2	3	4
$P(x)$	0.07	0.35	0.42	0.14	0.01

$\mu = \sum p_i x_i$

$= 0 \times 0.07 + 1 \times 0.35 + 2 \times 0.42 + 3 \times 0.14 + 4 \times 0.01$

$= 1.65$

9. Continuous Probability distribution -

when a random variable X takes every value in an Interval it gives rise to continuous distribution of X.

- properties of probability of density funcn $f(x)$ -

1. $f(x) \geq 0 \ \forall x$
2. $$\int_{-\infty}^{\infty} f(x)\,dx = 1 \ \forall x$$
3. The probability $P(E)$ is defined as $\int_E f(x)\,dx$

=> mean (expectation) - The mean of a distribution $\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$.

if X is defined from (a,b) then $\mu = E(x) = \int_a^b x f(x)\,dx$

=> Median - median is the point which divides the entire distribution of two equal parts.

i.e, $$\int_a^M f(x)\,dx = \int_M^b f(x)\,dx = \frac{1}{2}$$

solving M we get the median

=> Mode - Mode is a value of x for which $f(x)$ is maximum that is mode is given by $f'(x)=0$ & $f''(x)<0 \ \forall \ a<x<b$.

=> Variance - It is defined as $\sigma^2 = \int_a^b x^2 f(x)\,dx - \mu^2$

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Q. The prob. density func^n of a continuous random variable is given by $f(x) = Ce^{-|x|}$ $\forall -\infty < x < \infty$ S.T $C = \frac{1}{2}$ and also find mean, variance of the distribution. Find the probability that variable lies b/w 0 & 4

sol: (i) We know that

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_{-\infty}^{\infty} ce^{-|x|}\,dx = 1$$

$$\Rightarrow 2c\int_{0}^{\infty} e^{-x}\,dx = 1$$

$$\Rightarrow 2c\left[\frac{e^{-x}}{-1}\right]_0^{\infty} = 1$$

$$\Rightarrow -2c\left[e^{-\infty} - e^{0}\right] = 1$$

$$\Rightarrow -2c[0-1] = 1$$

$$\Rightarrow c = \frac{1}{2}$$

$$\mu = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} x e^{-|x|}\,dx$$

$$= 0$$

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} x^2 e^{-|x|}\,dx$$

$$= \int_{0}^{\infty} x^2 e^{-x}\,dx$$

$\Rightarrow \left[x^2\left[\frac{e^{-x}}{-1}\right] - 2x\frac{e^{-x}}{1} + (2)\frac{e^{-x}}{-1}\right]_0^{\infty}$

$\Rightarrow \left[-x^2e^{-x} - 2xe^{-x} - 2e^{-x}\right]_0^{\infty}$

$\Rightarrow \left[-\infty^2e^{-\infty} - 2(\infty)e^{-\infty} - e^{-\infty} + 0 + 0 + 2\right]$

$\Rightarrow 2$

$P(E) = \int_0^4 f(x)\,dx$

$= \frac{1}{2}\int_0^4 e^{-|x|}dx$

$= \frac{1}{2}\left[\frac{e^{-x}}{-1}\right]_0^4$

$= \frac{-1}{2}\left[e^{-4} - 1\right]$

$= \frac{1}{2}\left[1 - e^{-4}\right]$

$= 0.4908.$

$-c\left[e^{-x}\right]_{-\infty}^{\infty} = 1$

$-c\left[e^{-\infty} - e^{\infty}\right] = 1$

$-c\left[0 - \right.$

$\frac{1}{e^{\infty}}$

$\frac{1}{\infty}$

$\Rightarrow 0$

$\frac{1}{e^{\infty}}$

$\frac{1}{\infty} = 0$

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Q. Probability density function of a random variable X

$$f(x) = \begin{cases} \frac{1}{2}\sin x & \text{for } 0 \le x \le \pi \\ 0 & \text{else where} \end{cases}$$

find mean, median, mode and also find the probability b/wn 0 & $\pi/2$

(i) $$\mu = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \underbrace{\int_{-\infty}^{0} x f(x)\,dx}_{\substack{\Downarrow \\ \text{odd funcn} \\ \Rightarrow 0}} + \int_{0}^{\pi} x f(x)\,dx + \int_{\pi}^{\infty} x f(x)\,dx$$

$$= \int_{0}^{\pi} x f(x)\,dx$$

$$= \frac{1}{2}\int_{0}^{\pi} x \sin x\,dx$$

$$= \frac{1}{2}\left[x(-\cos x) - (1)(-\sin x)\right]_{0}^{\pi}$$

$$\Rightarrow \frac{1}{2}\left[\pi + 0 - 0 - 0\right] \Rightarrow \frac{\pi}{2}$$

$$P(E) = \int_{0}^{\pi/2} f(x)\,dx$$

$$= \frac{1}{2}\int_{0}^{\pi/2} \sin x\,dx$$

$$= \frac{1}{2}\left[-\cos x\right]_{0}^{\pi/2}$$

$$\Rightarrow \frac{1}{2}\left[-0 + 1\right] \Rightarrow \frac{1}{2}$$

Median:- $\int_a^M f(x)dx = \int_M^b f(x)dx = \frac{1}{2}$

$\Rightarrow \frac{1}{2}\int_0^M \sin x\,dx = \frac{1}{2}\int_M^\pi \sin x\,dx = \frac{1}{2}$

$\Rightarrow \frac{1}{2}\int_0^M \sin x\,dx = \frac{1}{2} \Rightarrow \int_0^M \sin x\,dx = 1$

$\Rightarrow [-\cos x]_0^M = 1$

$\Rightarrow -\cos M + \cos 0 = 1$

$\Rightarrow \cos M = 0$

$M = \cancel{\frac{n\pi}{2}}\ (2n+1)\frac{\pi}{2}$

$\& \ M = \frac{\pi}{2}$

Mode - We know that

$f'(x) = 0 \ \& \ f''(x) < 0 \rightarrow -\frac{1}{2}\sin x$

here $f(x) = \frac{1}{2}\sin x$

$\Rightarrow \frac{1}{2}\cos x = 0$

$= -\frac{1}{2}\sin\left(\frac{\pi}{2}\right)$

$= -\frac{1}{2} < 0$

$\cos x = 0$

$\therefore x = \pi/2$ is the mode for $f(x)$

$x = \cos^{-1}(0)$

$x = \pi/2$

Q. let X denote the sum of 2 numbers that appear when a pair of fair die is tossed find distribution funcn, mean, variance

Q. A continuous random variable X has distribn funcn

$$F(x) = \begin{cases} 0, & \text{if } x \leq 1 \\ k(x-1)^4 & \text{if } 1 < x \leq 3 \\ 1 & \text{if } x > 3 \end{cases}$$

determine i) $f(x)$ ii). k iii). μ iv). σ^2

1 sol. $X = \{\cancel{0}, \cancel{1}, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$P(X=2) = \frac{1}{36}$ (1,1)

$P(X=3) = \frac{2}{36}$

$P(X=4) = \frac{3}{36}$ (3,1), (1,3), (2,2)

$P(X=5) = \frac{4}{36}$ (1,4), (2,3), (3,2) (4,1)

$P(X=6) = \frac{5}{36}$ (1,5), (2,4), (3,3) (4,2), (5,1)

$P(X=7) = \frac{6}{36}$ (1,6), (2,5), (3,4) (4,3),

$P(X=8) = \frac{5}{36}$ (2,6), (3,5), (4,4), (5,3)

$P(X=9) = \frac{4}{36}$

$P(X=10) = \frac{3}{36}$

$P(X=11) = \frac{2}{36}$

$P(X=12) = \frac{1}{36}$.

x	2	3	4	5	6	7	8	9	10	11	12
$P(x)$	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$	$3/36$	$2/36$	$1/36$

$\mu = \sum P_i x_i$

$= \frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36} + \frac{30}{36} + \frac{22}{36} + \frac{12}{36}$

$= \frac{252}{36}$

$\mu = 7$

$\sigma^2 = \sum P_i x_i^2 - \mu^2$

$= 2^2 \times \frac{1}{36} + 3^2 \times \frac{2}{36} + 4^2 \times \frac{3}{36} + 5^2 \times \frac{4}{36} + 6^2 \times \frac{5}{36} + 7^2 \times \frac{6}{36} + 8^2 \times \frac{5}{36} + 9^2 \times \frac{4}{36}$

$+ 10^2 \times \frac{3}{36} + 11^2 \times \frac{2}{36} + 12^2 \times \frac{1}{36}$

$= \frac{4}{36} + \frac{18}{36} + \frac{48}{36} + \frac{100}{36} + \frac{180}{36} + \frac{294}{36} + \frac{320}{36}$

2 sol. $f(x) = \frac{d}{dx} F(x)$

i) $f(x) = \begin{cases} 0, & \text{if } x \le 1 \\ 4k(x-1)^3, & 1 < x \le 3 \\ 0 & x > 3 \end{cases}$

ii). $\int_1^3 4k(x-1)^3 dx = 1$

$\Rightarrow 4k\left[\frac{(x-1)^4}{4}\right]_1^3 = 1$

$\Rightarrow \cancel{4}k\left[\frac{16}{\cancel{4}}\right] = 1$

$\Rightarrow k = \frac{1}{16}$

iii). $f(x) = \begin{cases} 0, & \text{if } x \le 1 \\ \frac{1}{4}(x-1)^3 & 1 < x \le 3 \\ 0 & x > 3 \end{cases}$

$\mu = \int_{-\infty}^{\infty} x f(x) dx$

$= \int_{-\infty}^{1} 0\, dx + \int_1^3 x\frac{1}{4}(x-1)^3 dx + \int_3^{\infty} 0\, dx$

$= \cancel{\frac{1}{4}\left[\frac{(x-1)^4}{4}\right]_1^3}$ $\Rightarrow \frac{1}{4}\left[x\frac{(x-1)^4}{4} - (1)\frac{(x-1)^5}{20}\right]_1^3$

$= \cancel{\frac{1}{4}\left[\frac{16}{4}\right]}$ $\Rightarrow \frac{1}{4}\left[\cancel{3}\cdot\frac{\cancel{16}}{\cancel{4}} - \frac{\cancel{32}}{\cancel{20}}\right] \Rightarrow \frac{1}{4}\left[\cancel{36} - 1.6\right]$

$\mu = 1$

$\Rightarrow \cancel{8.6}\ \mu = 2.6$

iv). $\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$

$= \int_1^3 x^2\frac{1}{4}(x-1)^3 dx - (2.6)^2$

$= \frac{1}{4}\int_1^3 x^2(x-1)^3 dx - (2.6)^2$

$$= \frac{1}{4}\left[x^2\frac{(x-1)^4}{4} - 2x\frac{(x-1)^5}{20} + \frac{2(x-1)^6}{120}\right]_1^3 - (2.6)^2$$

$$= \frac{1}{4}\left[9\times\frac{16}{4} - 6\times\frac{32}{20} + 2\times\frac{64}{120} - 0\right] - (2.6)^2$$

$$\Rightarrow \frac{1}{4}\left[36 - \frac{48}{5} + \frac{16}{15}\right] - (2.6)^2$$

$\Rightarrow 5.866 \Rightarrow 0.106$

Binomial Distribution:- (B.D)

A random variable X has binomial distribution if it assumes only non-negative values and its prob. function is given by

$$P(x) = P(X=x) = {}^nC_x\, p^x q^{n-x}$$

Ex: ① The number of defective bolts in a box containing n bolts

② The number of postgraduate in a group of n men.

③ *Conditions of B.D:-

1. Trials are repeated under identical condition for a fixed number of numbers say n times.
2. There are only 2 possible outcomes
   Ex:- success, failure
3. The probability of success in each trial remains constant and does not change from trade to trade
4. Trials are independent that is probability of an event in any trade is not affected by the result of other ~~trades~~ trials.
   Ex:- Tossing a coin, Birth of baby, auditing a bill

* Mean of a binomial distribution

It is defined as $\mu = E(X) = np$

* Variance$(\sigma^2) = npq$

$q = 1-p$

$\therefore p+q = 1$

* Standard deviation of B.D $= \sqrt{npq}$

Q. 10 coins are thrown simultaneously. Find the probability of getting atleast

① 7 heads

② 6 heads

Given $n = 10$

p = Probability of getting a head $= \frac{1}{2}$

$q = 1-p = 1-\frac{1}{2} = \frac{1}{2}$

(i) 7 heads

here $r = 7$

$P(X=r) = {}^nC_r \cdot p^r q^{n-r}$

$P(X \geq 7) = P(X=7) + P(X=8) + P(X=9) + P(X=10)$

$= {}^{10}C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3 + {}^{10}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + {}^{10}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right) + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}$

$= 0.171$

(ii) 6 heads

here $r = 6$

$P(X=r) = {}^nC_r \cdot p^r \cdot q^{n-r}$

$P(X \geq 6) = P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10)$

$= {}^{10}C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4 + {}^{10}C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3 + {}^{10}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + {}^{10}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)$

$+ {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}$

$= 0.3769$

2. out of 800 families with 5 children each how many would you expect to have

a) 3 boys

b) 5 girls

c) either 2 or 3 boys

d) atleast one boy Assume equal probability for boys and girls

sol: let the number of boys in each family = x

P = prob. of each boy $= \frac{1}{2}$

q = prob. of each girl $= \frac{1}{2}$ $\quad 1-P = 1-\frac{1}{2} = \frac{1}{2}$

n = no. of children = 5

We know Binomial Distribution

$$P(X=r) = P(r) = {}^nC_r p^r q^{n-r}$$

a) $P(X=3) = {}^5C_3 \left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$

$= 0.3125$

Thus for 800 families the probability of no. of families having 3 boys

$\Rightarrow 800 \times 0.3125$

$\Rightarrow 250$

b) $P(X=5) = P(5 \text{ girls})$ or $P(X=0)$ i.e., $P(0 \text{ boys})$

$$P(X=5) = P(5) = {}^5C_5 \left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{5-5} = {}^5C_5 \left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0$$

$= 0.03125$

Thus for 800 families = 25

c) $P(X=2 \text{ or } 3) = P(X=2) + P(X=3)$

$= {}^5C_2 \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3 + {}^5C_3 \left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$

$= {}^5C_2 \left(\frac{1}{2}\right)^5 + {}^5C_3 \left(\frac{1}{2}\right)^5$

$= 0.625$

Thus for 800 families the probability of number of families having either 2 & 3 boys $= 0.625 \times 800$

$= 500$

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d. $P(X \geq 1) = 1 - P(X=0)$

$= 1 - P(X=0)$

$= 1 - {}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5$

$= 0.9687$

Thus for 800 families $= 0.9687 \times 800$

$= 775$

3. Out of 800 families with 4 children how many families would be expected to have

a) 2 boys and 2 girls
b) Atleast one boy
c) No girl
d) Atmost two girls

Sol. let the No. of boys in each family $= x$

p = probability of each boy $= \frac{1}{2}$

q = probability of each girl $= \frac{1}{2}$

n = number of children $= 4$

$P(X=r) = P(r) = {}^nC_r\, p^r q^{n-r}$

a) $P(X=2) + P(X=2) = 2P(X=2)$

$= 2 \cdot {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2$

$\Rightarrow 0.75$

Thus for 800 families Prob. $= 0.75 \times 800 = 600$

b) $P(X \geq 1) = 1 - P(X<0)$

$= 1 - P(X=0)$

$= 1 - {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4$

$= 0.9375$

Thus for 800 families prob. $= 0.9375 \times 800 = 750$

c) $P(\text{No girl}) = P(X=0)$ or $P(X=4)$ i.e., $P(4 \text{ boys})$

$= {}^4C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0 = 4\left(\frac{1}{2}\right)^4 = 0.25$

Thus for 800 families prob. $= 0.25 \times 800$

$= 200$

4. The mean and variance of a binomial distribution are 4 and $4/3$. Find $P(X \geq 1)$

Sol. Given mean and variance are

$np = 4$ — ①

$npq = \frac{4}{3}$ — ②

Solving ② ÷ ①

$\frac{npq}{np} = \frac{\frac{4}{3}}{4}$

$q = \frac{1}{3}$; $P = 1 - q$

$= 1 - \frac{1}{3}$

$P = \frac{2}{3}$

from ①

$np = 4 \Rightarrow n\left(\frac{2}{3}\right) = 4 \Rightarrow n = 6$

$P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X=0)$

$= 1 - {}^6C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^6$

$= 0.99$

5. In 8 throws of a dice the number 5 or 6 is considered as success. Find the mean and standard deviation of the number of success.

Sol. Given $n = 8$

P = prob. of 5 or 6 as a success $= \frac{2}{6} = \frac{1}{3}$

$q = 1 - P = 1 - \frac{1}{3} = \frac{2}{3}$

Mean $(\mu) = np$

$= 8\left(\frac{1}{3}\right) = 2.66$

variance $= npq$

$= 8\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)$

$= 1.77$

Standard deviation $= \sqrt{npq} = \sqrt{1.77}$

$= 1.330.$

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6. A binomial distribution to the following frequency distribution.

x	0	1	2	3	4	5	6
f	13	25	52	58	32	16	4

Sol. $n = 6$; $N = \Sigma f_i = 200$

Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = 2.67$

The mean of binomial distribution $= np$

$2.67 = 6(P)$

$P = 0.44$

$q = 1 - P = 0.56$

We know that

$(P+q)^n = p^n + {}^nC_1 p^{n-1} q + {}^nC_2 p^{n-2} q^2 + \text{-----} + {}^nC_r p^{n-r} q^r$

To fit a binomial distribution table is

$N(q+P)^n = 200[(0.56 + 0.44)^6]$

$\Rightarrow 200[(0.56)^6 + 6C_1 (0.56)^5 (0.44) + 6C_2 (0.56)^4 (0.44)^2 + 6C_3 (0.56)^3 (0.44)^3$
$+ 6C_4 (0.56)^2 (0.44)^4 + 6C_5 (0.56)^1 (0.44)^5 + 6C_6 (0.44)^6]$

$\Rightarrow 200[0.030 + 0.1453 + 0.285 + 0.299 + 0.1763 + 0.055 + 0.0072]$

$\Rightarrow \cancel{199.36} \Rightarrow [6 + 29.06 + 57 + 59.8 + 35.2 + 11 + 1.44]$

The binomial distribution table is

X	0	1	2	3	4	5	6
Frequency	13	25	52	58	32	16	4
Theoretical values	6	29	57	60	35	11	2

* Poisson Distribution -

It is a rare distribution of rare events i.e., the events whose prob. of is very small but the no. of trials which could lead to the occurence of the events very large

- A random variable 'X' is said to follow a poisson distribution if it assumes only non-negative values and the function is given by

$$P(x,\lambda) = P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad \forall x=0$$

where $\lambda>0$ is called the parameter of a distribution.

Ex:- 1) Consider the number of persons born blind per year in a large city.

2) The number of Telephone calls per minute at a switch board.

* Conditions of Poisson distribution -

-

- The occurence are rare
- The number of trials is large
- The probability of success is very small
- $np=\lambda$ is finite

=> Mean of a poisson distribution -

$$\mu = E(x) = \lambda \quad [\because np=\lambda]$$

Variance :- It is defined as mean of the distribution

$$V(x)=\lambda$$

hence, the standard deviation is $\sqrt{\lambda}$

* Recurrence relation for poisson distribution is

$$P(x+1) = \left[\frac{\lambda}{x+1}\right] P(x)$$

or

$$P(x) = \frac{\lambda}{x} P(x-1)$$

Problems

1. A car hire firm has 2 cars which it hides out day by day the number of demand for a car each day is distributed with a mean 1.5. Calculate:

(i) on which there is no demand

(ii) $P(x>2)$

sol. Given mean $= \lambda = 1.5$

W.K.T

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

(i) $P(X=0) = P(0) = \frac{e^{-1.5}(1.5)^0}{0!}$

$= 0.2231$

(ii) $P(x>2) = 1 - P(X \le 2)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \left[0.2231 + \frac{e^{-1.5}(1.5)^1}{1!} + \frac{e^{-1.5}(1.5)^2}{2!}\right]$

$= 0.1911$

2. If the prob. that an individual suffers a bad reaction from a certain infection is 0.001. Determine the probability that out of 2000 individuals.

(i) exactly 3

(ii) More than 2 individuals

(iii) None

(iv) More than one individual suffers a bad reaction

sol. Given $P = 0.001$, $n = 2000$

$\lambda = np$

$= (0.001)(2000)$

$\lambda = 2$

(i) $P(X=3) = \frac{e^{-2}2^3}{3!}$

$= 0.1804$

(ii) $P(X>2) = 1 - P(X\leq 2)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \left[\frac{e^{-2}2^0}{0!} + \frac{e^{-2}2}{1!} + \frac{e^{-2}2^2}{2!}\right]$

$= 1 - \left[e^{-2} + 2e^{-2} + \frac{4e^{-2}}{2}\right]$

$= 1 - [5e^{-2}]$

$P(X>2) = 0.3233$

(iii) $P(X=0) = \frac{e^{-2}2^0}{0!}$

$= 0.1353$

(iv) $P(X>1) = 1 - P(X\leq 1)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - \left[0.1353 + \frac{e^{-2}2^1}{1!}\right]$

$= 1 - [0.1353 + 0.2706]$

$= 0.5941.$

3. In a random variable has a poisson distribution such that P(1) = P(2) find

(i) mean

(ii) P(4)

(iii) $P(X \geq 1)$

(iv) $P(1 < X < 4)$

Sol. Given, $P(1) = P(2)$

(i) W.k.T $P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ i.e., $\frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$

$\lambda = 2$

(ii) $P(4) = P(X=4)$

$= \frac{e^{-2}2^4}{4!}$

$P(4) = 0.0902$

(iii) $P(X \geq 1) = 1 - P(X<1)$

$= 1 - P(X=0)$

$= 1 - \left[\frac{e^{-2}2^0}{0!}\right]$

$= 1 - [e^{-2}]$

$P(X \geq 1) = 0.8646$

(iv) $P(1<X<4) = P(X=2) + P(X=3)$

$= \frac{e^{-2}2^2}{2!} + \frac{e^{-2}2^3}{3!}$

$= e^{-2}.2 + \frac{e^{-2}.\cancel{8}^4}{\cancel{6}3}$

$P(1<X<4) = 0.4511.$

5. Fit the Poisson distribution for the following data

x	0	1	2	3	4
$f(x)$	109	65	22	3	1

Sol. Let $N = \sum f_i$

$= 200$

Mean $= \dfrac{\sum f_i x_i}{\sum f_i} = 0.61$

$\lambda = 0.61$

To fit a P.D is $N \cdot P(x)$

$200\, P(X=x)$ here $x = 0, 1, 2, 3, 4$

when $x=0$

$\Rightarrow NP(0) = 200 \dfrac{e^{-0.61} (0.61)^0}{0!} = 108.6$

when $x=1$

$NP(1) = 200 \dfrac{e^{-0.61} (0.61)^1}{1!} = 66.28$

when $x=2$

$NP(2) = 200 \dfrac{e^{-0.61} (0.61)^2}{2!} = 20.21$

when $x=3$

$NP(3) = 200 \dfrac{e^{-0.61} \cdot (0.61)^3}{3!}$

$= 4.11$

when $x=4$

$NP(4) = 200 \dfrac{e^{-0.61} (0.61)^4}{4!} = 0.62$

x	0	1	2	3	4
$f(x)$	109	65	22	3	1
Expected	109	66	20	4	1

Q. A random variable X has the following probability function

x	-3	-2	-1	0	1	2	3
$P(x)$	k	0.1	k	0.2	$2k$	0.4	$2k$

Find i) k

ii). Mean

iii). Variance

Q. A function is defined by $f(x) = \begin{cases} 0, & x < 2 \\ \frac{1}{18}(2x+3); & 2 \le x \le 4 \\ 0, & x > 4 \end{cases}$

S.T $f(x)$ is a density function or not and also find for the interval $2 \le x \le 3$

Q. If 10% of rivets produced by a machine are defective. Find the prob. that out of 5 rivets chosen at random

i) None will be defective

ii). One will be defective

iii). atmost two will be defective

Q. 20% of items produced from a factory are defective. Find the prob. that sample of 5 chosen at random (i) 1

(ii) None

(iii) $P(1 < X < 4)$

Q. It is found that 2% of the tools produced by certain machines are defective. What is the prob. that in a shipment of 400 such tools

i) 3% or more

ii) 2% or less will be defective

Q. If 2 cards are drawn from a pack of 52 cards which are diamonds use Poisson distrib. to find Prob. of getting two diamonds atleast 3 times in 51 consecutive trials of 2 cards drawn each trial.

- The distributions binomial and poisson are discrete distributions whereas normal distribution is continuous distribution.
- The random variable X is said to have normal distribution if its density function is defined $z = \frac{x-\mu}{\sigma}$ where $\mu \to$ mean
$\sigma \to$ S.D of x

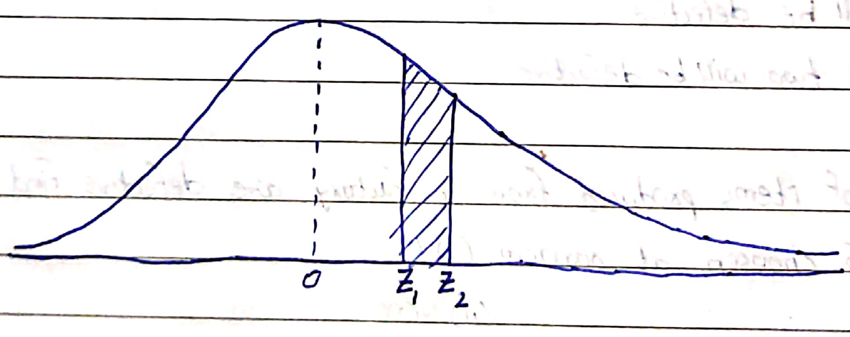
* To find the probability density of normal curve:-
- The probability that normal variate X with a mean (μ), S.D (σ) lies b/w two specific values x_1, x_2

Step-I - The change of scale $z = \frac{x-\mu}{\sigma}$ and find Z_1, Z_2 corresponding to the values of x_1 and x_2

Step 2 -

1. To find $P(x_1 \le X \le x_2) = P(Z_1 \le Z \le Z_2)$

case(i) - If both Z_1 and Z_2 are positive or both are negative then
$P(x_1 \le X \le x_2) = |A(Z_2) - A(Z_1)|$

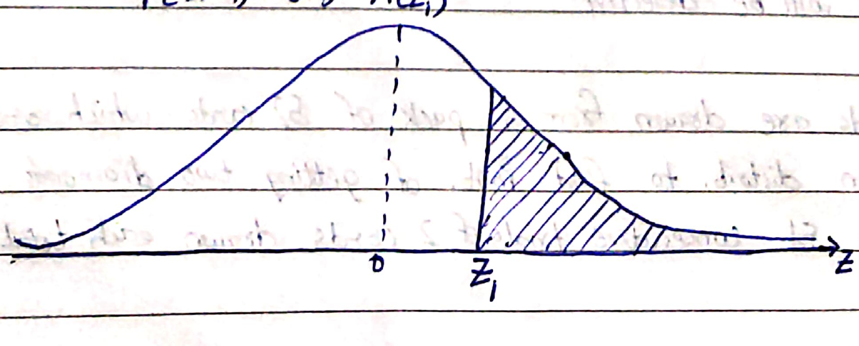


case(ii) - If both $Z_1 < 0$, $Z_2 > 0$ or viceversa then
$P(x_1 \le X \le x_2) = |A(Z_2) + A(Z_1)|$

2. To find $P(Z > Z_1)$

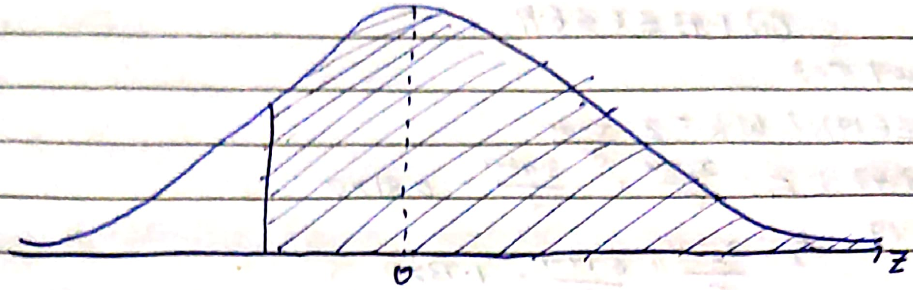
case(i) - If $Z_1 > 0$ then

$P(Z > Z_1) = 0.5 - A(Z_1)$



case (ii) - if $Z_1 < 0$ then probability

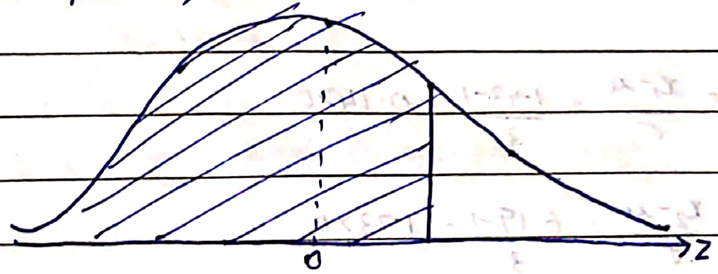
$$P(Z > Z_1) = 0.5 + A(Z_1)$$



3. To find $P(Z < Z_1)$

case (i) - if $Z_1 > 0$ then

$$P(Z < Z_1) = 0.5 + A(Z_1)$$



case (ii) if $Z_1 < 0$ then

$$P(Z < Z_1) = 0.5 - A(Z_1)$$

Q: for a normally distributed variate with a mean = 1 and S.D = 3 find the probabilities that (i) $3.43 \le x \le 6.19$

(ii) $1.43 \le x \le 6.19$

Given that $\mu = 1$ $\sigma = 3$

(i) $P(3.43 \le x \le 6.19) = ?$ W.K.T $Z = \frac{x-\mu}{\sigma}$

here $x_1 = 3.43$; $Z_1 = \frac{x_1-\mu}{\sigma} = \frac{3.43-1}{3} = 0.81 > 0$

$x_2 = 6.19$; $Z_2 = \frac{x_2-\mu}{\sigma} = \frac{6.19-1}{3} = 1.73 > 0$

$P(3.43 \le x \le 6.19) = |A(Z_2) - A(Z_1)|$

$= |A(1.73) - A(0.81)|$

$= |0.4582 - 0.2910|$

$= 0.1672$

(ii) $P(1.43 \le x \le 6.19) = ?$

here $x_1 = 1.43$, $Z_1 = \frac{x_1-\mu}{\sigma} = \frac{1.43-1}{3} = 0.14 > 0$

$x_2 = 6.19$; $Z_2 = \frac{x_2-\mu}{\sigma} = \frac{6.19-1}{3} = 1.73 > 0$

$P(1.43 \le x \le 6.19) = |A(Z_2) - A(Z_1)|$

$= |A(1.73) - A(0.14)|$

$= |0.4582 - 0.0948|$

$= 0.4025$

Q. If X is a normal variant with $\mu = 30$, $\overset{(S.D)}{\sigma} = 5$ find

(i) $P(26 \le x \le 40)$

(ii) $P(x \ge 45)$

(i) W.k.T $Z = \frac{x-\mu}{\sigma}$

here $x_1 = 26$; $Z_1 = \frac{x_1 - \mu}{\sigma} = -0.8 < 0$

$x_2 = 40$; $Z_2 = \frac{x_2 - \mu}{\sigma} = 2 > 0$

$P(26 \le x \le 40) = |A(Z_2) + A(Z_1)|$

$= 0.7653$

(ii) $P(x \ge 45) = ?$

here $x = 45$ $Z = \frac{x-\mu}{\sigma} = \frac{45-30}{5} = 3 > 0$

$P(x \ge 45) = 0.5 - A(Z)$

$= 0.5 - A(3)$

$= 0.5 - 0.4987$

$= 0.0013$

Q. Find the mean and standard deviation of a normal distribution in which 7% of items are under 35 and 89% are under 63

Sol. $\mu = ?$, $\sigma = ?$

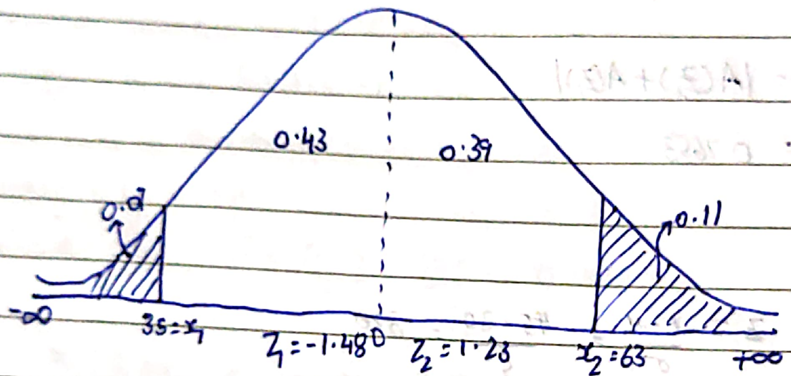
$P(X < 35) = 0.07$

$P(X < 63) = 0.89$

$\Rightarrow P(X > 63) = 1 - P(X < 63)$

$= 1 - 0.89$

$= 0.11$



when $x_1 = 35$, $z_1 = -1.48$ [from the table]

$x_2 = 63$, $z_2 = 1.23$ [from the table]

$\Rightarrow$ when $x_1 = 35$

$$z_1 = \frac{x_1 - \mu}{\sigma}$$

$$-1.48 = \frac{35 - \mu}{\sigma}$$

$$\mu + 1.48\sigma = 35 \quad \text{—} ①$$

when $x_2 = 63$

$$z_2 = \frac{x_2 - \mu}{\sigma}$$

$$1.23 = \frac{63 - \mu}{\sigma} \quad \cancel{②}$$

$$\mu + 1.23\sigma = 63 \quad \text{—} ②$$

By solving ① & ② $\mu = 50$

$\sigma = 10$

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Q. ~~In~~ With the mass of 300 students are normally distributed 68 kgs and S.D 3 kg how many students have the mass (i) greater than 72 kg
(ii) ≤ 64 kgs
(iii) b/w 65 and 71 kg

sol. $\mu = 68, \sigma = 3$

(i) $P(X>72)$

$x_1 = 72; \; Z_1 = \frac{x_1 - \mu}{\sigma}$

$= \frac{72-68}{3}$

$= 1.33 > 0$

$P(X>72) = 0.5 - A(Z_1)$

$= 0.5 - A(1.33)$

$= 0.0918$

(ii) $P(X \leq 64)$

$x_1 = 64, \; Z_1 = -1.33 < 0$

$P(X \leq 64) = 0.5 - A(Z_1)$

$= 0.0918$ $[\because A(-Z) = A(Z)]$

for 300 student $= 800 \times 0.0918$

$= 27.5$

$\cong 28$ $(Z_1 < 0; Z_2 > 0)$

(iii) $P(65 < X < 71) \Rightarrow A(Z_1) + A(Z_2) \Rightarrow A(1) + A(1)$

$\Rightarrow 0.3413 + 0.3413$

$\Rightarrow 0.6826$

for 300 students $= 800 \times 0.6826$

$= 204.78$

$= 205$

$x_1 = 65; \; Z_1 = \frac{x - \mu}{\sigma}$

$= \frac{65-68}{3}$

$Z_1 \Rightarrow -1$

$x_2 = 71; \; Z_2 = \frac{x - \mu}{\sigma}$

$= \frac{71-68}{3}$

$= \frac{3}{3}$

$Z_2 = 1$

* Moments - moments are used to describe the various characteristics of frequency distribution. Kurtosis enables us to study the shape of the top of
* the curve
* skewness - is a measure refers to the extent of symmetry as symmetry distribution.

→ Moments is defined as arithmetic mean of various power of deviation of items from their means (assume or natural) will give the required powers of moments the distribution.

- If the deviations of the items are taken from the arithmatic mean is known as central moments
- The moments are denoted by 'μ'

* central moments (or) det moments about actual mean:

1. central moments for individual series - let $\bar{X}$ be the mean of the individual series

let x be the deviation of X from its mean

$$X = d = x - \bar{x}$$

let N be the total number of items or observation

$$\mu_1 = \frac{\Sigma d}{N}$$

$$\mu_2 = \frac{\Sigma d^2}{N}$$

$$\mu_3 = \frac{\Sigma d^3}{N} \quad ||^{ly} \quad \mu_r = \frac{\Sigma d^r}{N}$$

2. Central moments for frequency distribution:-

let n be the observations of $x_1, x_2, \text{-----}, x_n$ occuring with the frequencies $f_1, f_2, \text{------} f_n$ with A.M of a frequency is defined as

$$\bar{X} = \frac{\Sigma f_i x_i}{N} \quad \text{where } N = \sum_{i=1} f_i$$

- The deviation is $d = r$ taken as $d = x - \bar{X}$ and the moments are defined as

$$\mu_1 = \frac{\Sigma f d}{N} \qquad \mu_3 = \frac{\Sigma f d^3}{N}$$

$$\mu_2 = \frac{\Sigma f d^2}{N} \qquad ||^{ly} \quad \mu_r = \frac{\Sigma f d^r}{N}$$

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* Properties of

1. The first moment about mean is always zero i.e, $\mu_1 = 0$
2. The second moment about mean measures variance i.e., $\mu_2 = \sigma^2$ (or) $\sigma = \pm\sqrt{\mu_2}$
3. The third moment about mean measures skewness

case 1: if $\mu_3 > 0$ the distribution is positively skewed

case 2: if $\mu_3 < 0$ the distribution negatively skewed

case 3: if $\mu_3 = 0$ the distribution symmetrical

4. The fourth moment about mean measures the kurtosis

* Raw moments about the origin

- Moments about the arbitrary origin are called raw moments are denoted by (μ_r')
- The moments are defined as $\mu_1' = \frac{\Sigma fd}{N}$ $\mu_3' = \frac{\Sigma fd^3}{N}$

$\mu_2' = \frac{\Sigma fd^2}{N}$ //ly $\mu_r' = \frac{\Sigma fd^r}{N}$

The moments about the origin are defined as

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - (\mu_1')^2$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$

efined as

$$)^2 - 3(\mu_1')^4$$

Q. find the first four moments for the set of numbers 2,4,6,8

sol. Given $x = 2, 4, 6, 8$ $N = 4$

$\bar{x} = \frac{\Sigma x}{N} = \frac{20}{4} = 5$

x	$d = x - \bar{x}$	d^2	d^3	d^4
2	-3	9	-27	81
4	-1	1	-1	1
6	1	1	1	1
8	3	9	27	81
	$\Sigma d = 0$	$\Sigma d^2 = 20$	$\Sigma d^3 = 0$	$\Sigma d^4 = 164$

$\mu_1 = \frac{\Sigma d}{N} = 0$

$\mu_2 = \frac{\Sigma d^2}{N} = 5$

$\mu_3 = \frac{\Sigma d^3}{N} = 0$

$\mu_4 = \frac{\Sigma d^4}{N} = 41.$

Q. Calculate the first 4 moments for the following distribution and also calculate β_1 and β_2

x	f	fx	$d = x - \bar{x}$	fd^2	fd^3	fd^4	fd
0	1	0	-4	16			
1	8	8	-3	72			
2	28	56	-2	112			
3	56	168	-1	56			
4	70	280	0	0			
5	56	280	1	56			
6	28	168	2	112			
7	8	56	3	72			
8	1	8	4	16			
		$\Sigma fx = 1024$	$\Sigma d = 0$				

$\therefore$ $N = \Sigma f = 256$

$\bar{x} = \frac{\Sigma f_i x_i}{N} = 3.9 \cong 4$

$\mu_1 = 0$

$\mu_2 = \frac{\Sigma f d^2}{N}$

$\mu_3 = \frac{\Sigma f d^3}{N}$

$\mu_4 = \frac{\Sigma f d^4}{N}$

$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$

$\beta_2 = \frac{\mu_4}{\mu_2^2}$