Homework - 2 A20545853 MATH 564.

3) Given that,

In statistics we follow:

- Sum of squares of derivations of x values

$$S_{xx} = \sum_{i=0}^{n} (x_i - \bar{x})^2 \quad -①$$

- Sum of squares of derivations of y values.

$$S_{yy} = \sum_{i=0}^{n} (y_i - \bar{y})^2. \quad -②$$

- Sum of the product of derivations of x and y values

$$S_{xy} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \quad -③.$$

we have to proove, these sum of square can be also written as.

(i) $$S_{xx} = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$$

Let us solve ①, we get

$$S_{xx} = \sum_{i=0}^{n} (x_i - \bar{x})^2$$

$$= \sum_{i=0}^{n} (x_i^2 - 2x_i\bar{x} + \bar{x}^2$$

$$= \sum_{i=0}^{n} x_i^2 - \sum_{i=0}^{n} 2x_i\bar{x} + \sum_{i=0}^{n} \bar{x}^2.$$

$$= \sum_{i=0}^{n} x_i^2 - 2\bar{x}\sum_{i=0}^{n} x_i + \bar{x}^2\sum_{i=0}^{n} 1 .$$

$$\left(\because \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \text{, then } \sum_{i=1}^{n} x_i = n\bar{x} \right)$$

$$= \sum_{i=0}^{n} x_i^2 - 2\bar{x}(n\bar{x}) + n\bar{x}^2$$

$$= \sum_{i=0}^{n} x_i^2 - 2n\bar{x}^2 + n\bar{x}^2$$

we get,

$$Sxx = \sum_{i=0}^{n} x_i^2 - n\bar{x}^2 .$$

$\boxed{\text{Hence proved.}}$

(ii) $$Syy = \sum_{i=1}^{n} y_i^2 - n\bar{y}^2$$

solving (2), we get

$$Syy = \sum_{i=0}^{n} (y_i - \bar{y})^2$$

$$= \sum_{i=0}^{n} (y_i^2 - 2y_i\bar{y} + \bar{y}^2)$$

$$= \sum_{i=0}^{n} y_i^2 - \sum_{i=0}^{n} 2y_i\bar{y} + \sum_{i=0}^{n} \bar{y}^2$$

$$= \sum_{i=0}^{n} y_i^2 - 2\bar{y}\sum_{i=0}^{n} y_i + \bar{y}^2\sum_{i=0}^{n} 1$$

$\left(\because \bar{y} = \frac{\sum_{i=0}^{n} y_i}{n} \right.$ then, $\left. \sum_{i=0}^{n} y_i = n\bar{y} \right)$

$$= \sum_{i=0}^{n} y_i^2 - 2\bar{y}n\bar{y} + n\bar{y}^2$$

$$= \sum_{i=0}^{n} y_i^2 - 2n\bar{y}^2 + n\bar{y}^2$$

$$S_{yy} = \sum_{i=0}^{n} y_i^2 - n\bar{y}^2$$

$\boxed{\text{Hence proved}}$

(iii) $S_{xy} = \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}$.

Let's solve ③, we get.

$$S_{xy} = \sum_{i=0}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

$$= \sum_{i=0}^{n} (x_i y_i - x_i\bar{y} - \bar{x}y_i + \bar{x}\bar{y})$$

$$= \sum_{i=0}^{n} x_i y_i - \sum_{i=0}^{n} x_i\bar{y} - \sum_{i=0}^{n} \bar{x}y_i + \sum_{i=0}^{n} \bar{x}\bar{y}$$

$$= \sum_{i=0}^{n} x_i y_i - \bar{y}\sum_{i=0}^{n} x_i - \bar{x}\sum_{i=0}^{n} y_i + \bar{x}\bar{y}\sum_{i=1}^{0} 1$$

$\left(\because \text{from above we know, } n\bar{x} = \sum_{i=0}^{n} x_i;\ n\bar{y} = \sum_{i=0}^{n} y_i \right)$

$$= \sum_{i=0}^{n} x_i y_i - \bar{y}.n\bar{x} - \bar{x}.n\bar{y} + \bar{x}\bar{y}n.$$

$$S_{xy} = \sum_{i=0}^{n} x_i y_i - n\bar{x}\bar{y}$$

$\boxed{\text{Hence proved}}$