Time Series Lab and assignment

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TimeSeries Lab & Assignment

Dataset: "birth" from libraty astsa, U.S. Monthly Live Births 1950-1980

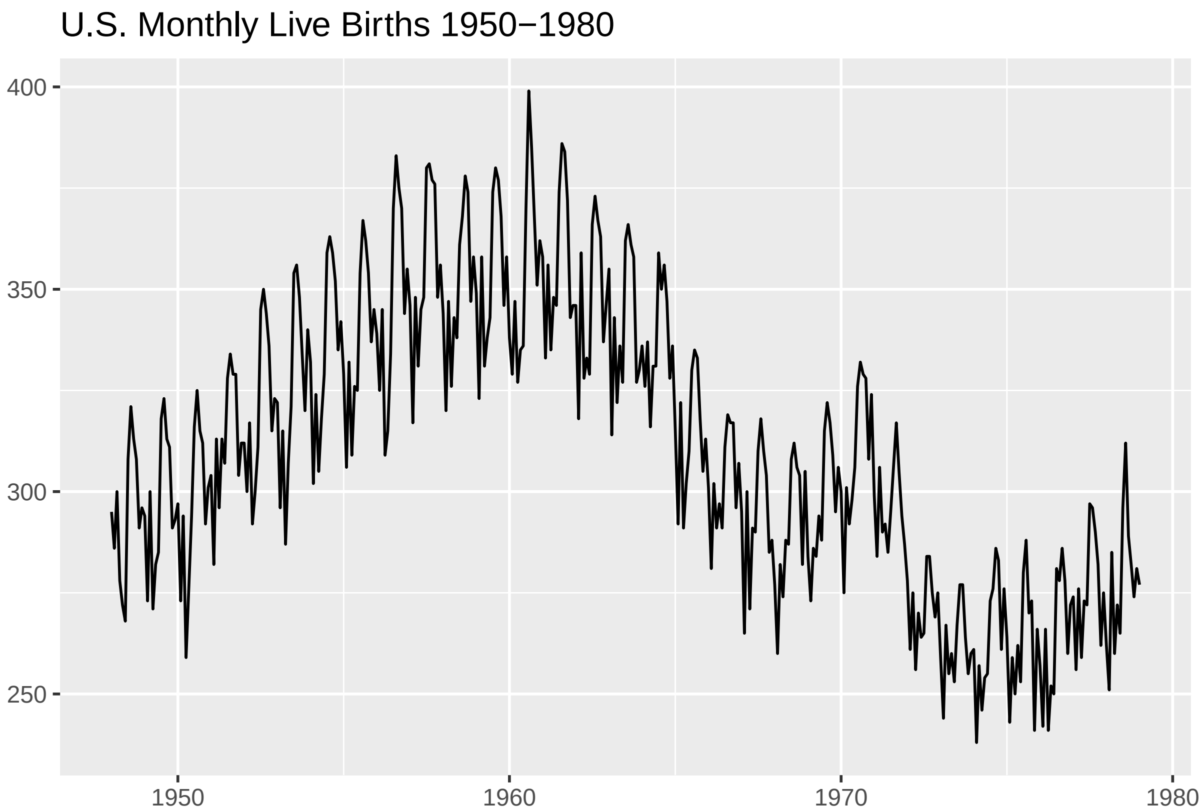
```
library(astsa)
data(birth)
head(birth)
```

```
## [1] 295 286 300 278 272 268
```

```
#plot(birth)
library(ggplot2)
library(ggfortify)
library(dplyr)
```

```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
birth %>%
  autoplot() + ggtitle("U.S. Monthly Live Births 1950-1980")
```



A seasonal plot is similar to a time plot except that the data are plotted against the individual “seasons” in which the data were observed.

```
library(forecast)
```

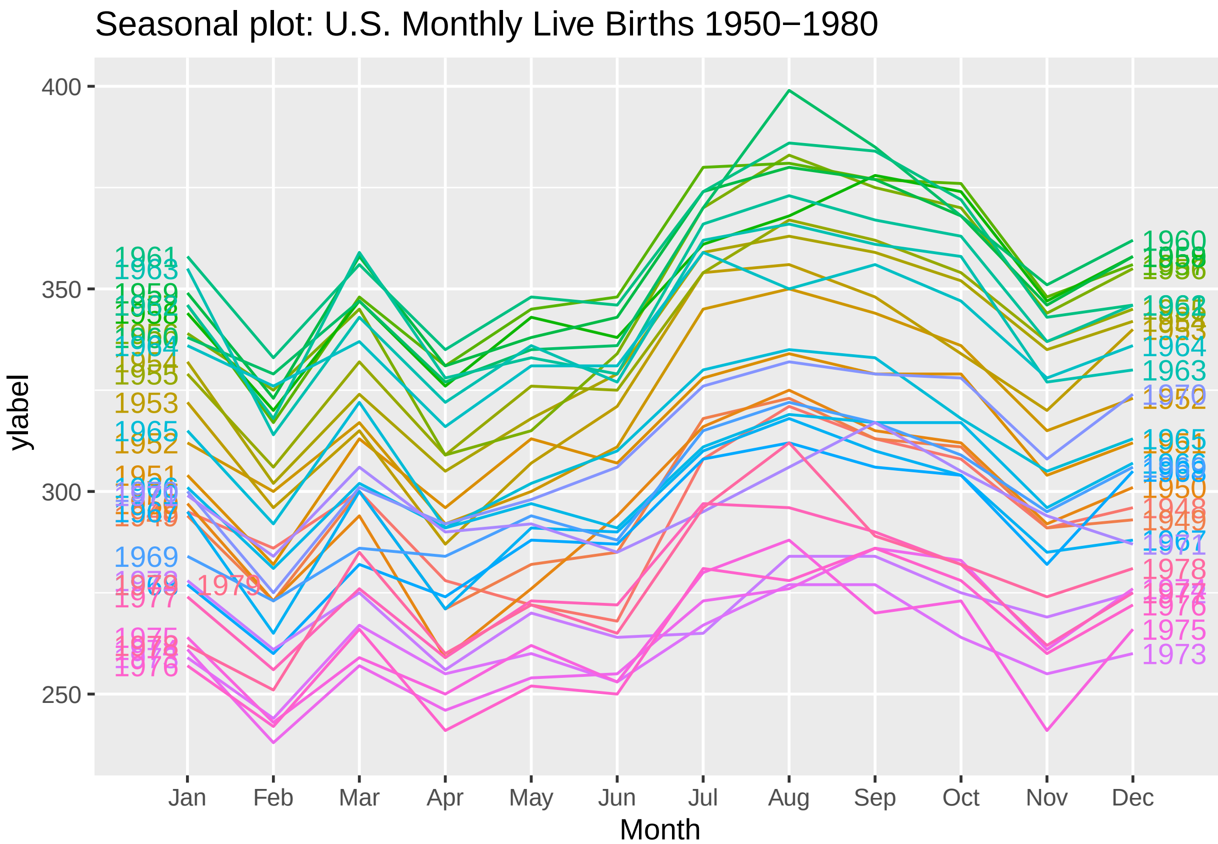
```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
## Registered S3 methods overwritten by 'forecast':
##   method                 from
##   autoplot.Arima         ggfortify
##   autoplot.acf           ggfortify
##   autoplot.ar            ggfortify
##   autoplot.bats          ggfortify
##   autoplot.decomposed.ts ggfortify
##   autoplot.ets           ggfortify
##   autoplot.forecast      ggfortify
##   autoplot.stl           ggfortify
##   autoplot.ts            ggfortify
##   fitted.ar              ggfortify
##   fortify.ts             ggfortify
##   residuals.ar           ggfortify
```

```
##
## Attaching package: 'forecast'
```

```
## The following object is masked from 'package:astsa':
##
##     gas
```

```r
ggseasonplot(birth, year.labels=TRUE, year.labels.left=TRUE) +
  ylab(" ylabel") +
  ggtitle("Seasonal plot: U.S. Monthly Live Births 1950-1980")
```



We are going to try a few things to get a feeling about the cyclical nature of the dataset.

There seems to be a yearly cycle. We can try adding monthly variables or use a sin and/or cos with the right frequency for a year repetition.

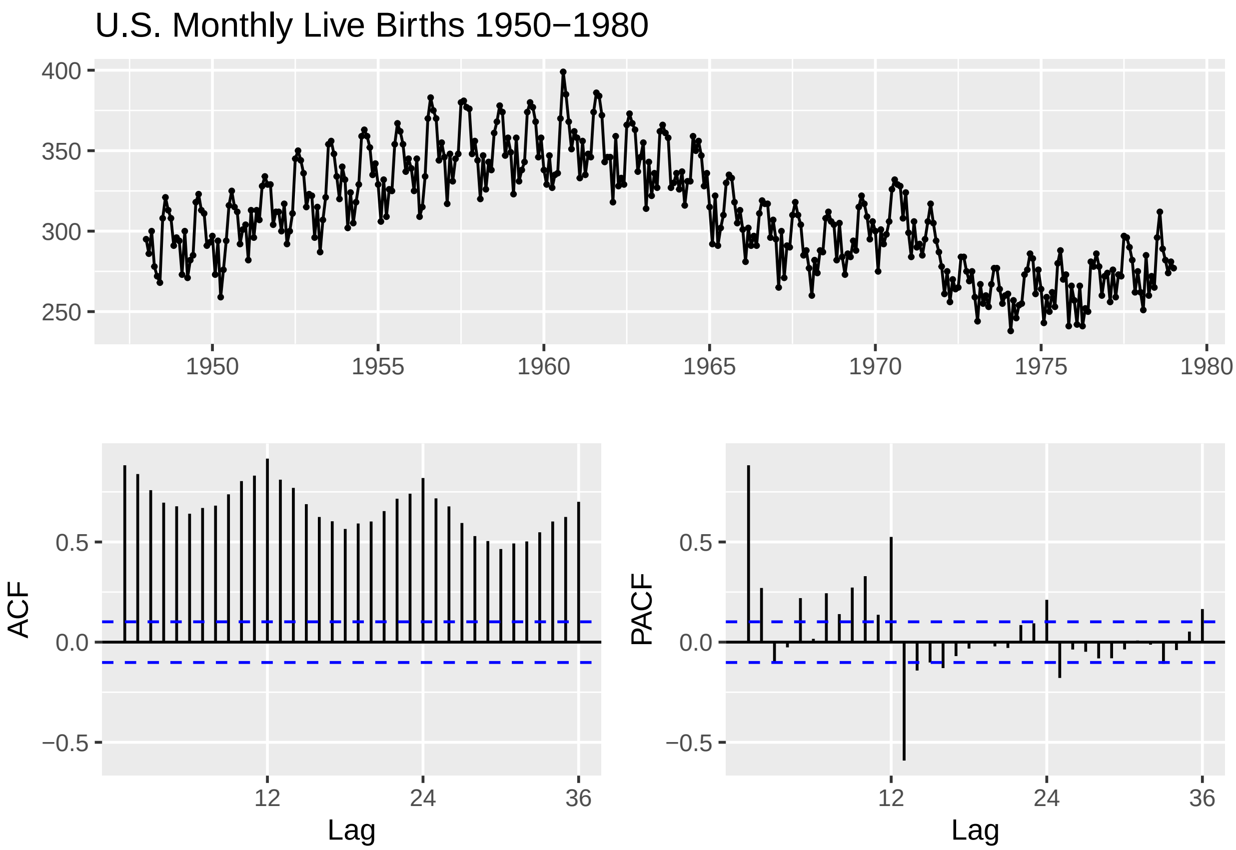
Note: I added numbers to the names of the month because otherwise r will order them alphabetically.

```r
n<-length(birth)
#n=373, n/12 = 31.08
month<-rep(c("01Jan","02Feb","03Mar","04Apr","05May","06Jun","07Jul","08Aug","09Sep","10Oct","11Nov","1
times<-1:n

# we won't use all the monthly dummy variables because Jan = when all other are 0
#X<-as.data.frame(cbind(times,Feb,Mar,Apr,May,Jun,Jul,Aug,Sep,Oct,Nov,Dec))
X<-data.frame(times=times,month=month)

# alternatively, seasons can be created with sin, cos
sint=sin(2*pi*times/12)
cost=cos(2*pi*times/12)
X_jan=data.frame(times=times,sint=sint,cost=cost,Jan=rep(c(1,0,0,0,0,0,0,0,0,0,0,0),32)[1:n])
```

Let's look at the auto correlation function and partial auto correlation functions

```r
#acf(birth)
#pacf(birth)
```

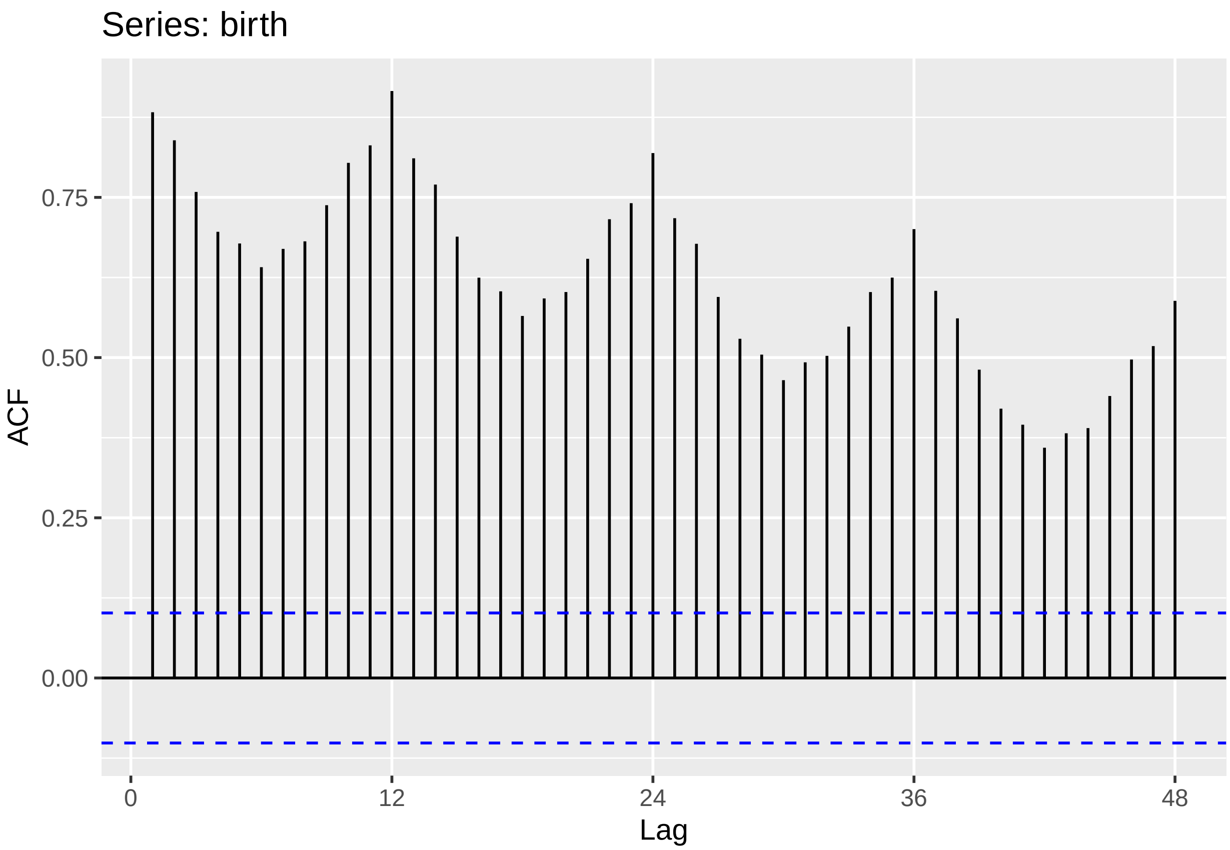
```r
birth %>% ggtsdisplay(main="U.S. Monthly Live Births 1950-1980")
```



The acf also shows the cyclic pattern.

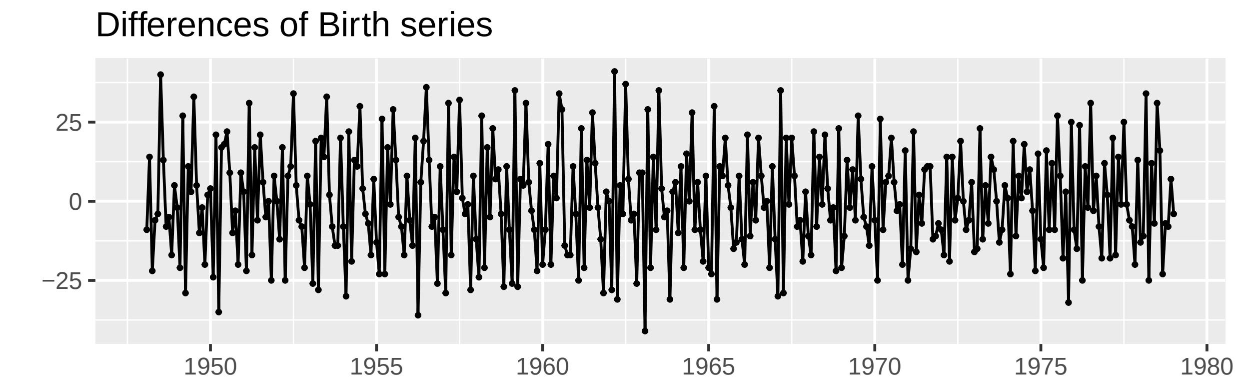
Using ggplot2

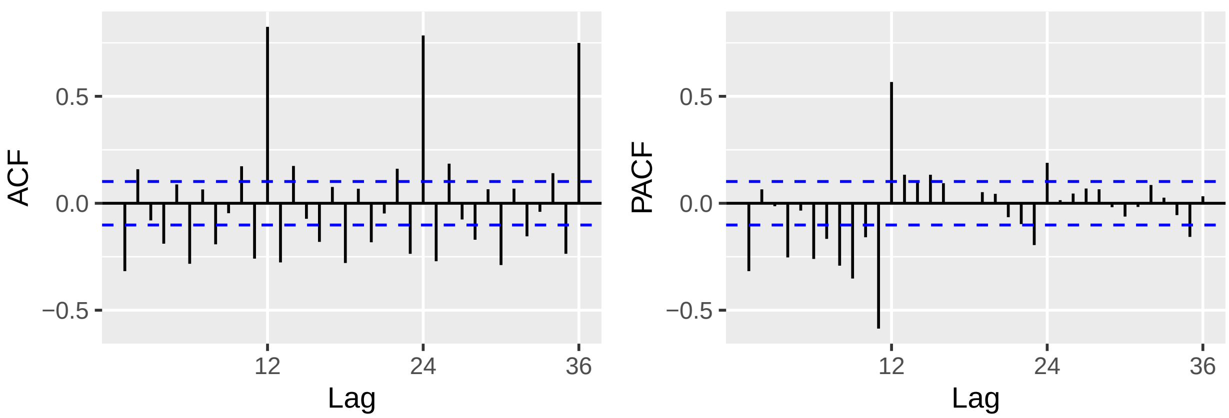
```r
ggAcf(birth,lag=48) # default is lag=24
```



Looking at the differences:

```
#acf(diff(birth,1))
birth %>% diff() %>% ggtsdisplay(main="Differences of Birth series")
```





The spikes at 12, 24, 36 tell about the cycle of 12 months. The difference seems to make the series more stable but still have issues with autocorrelation.

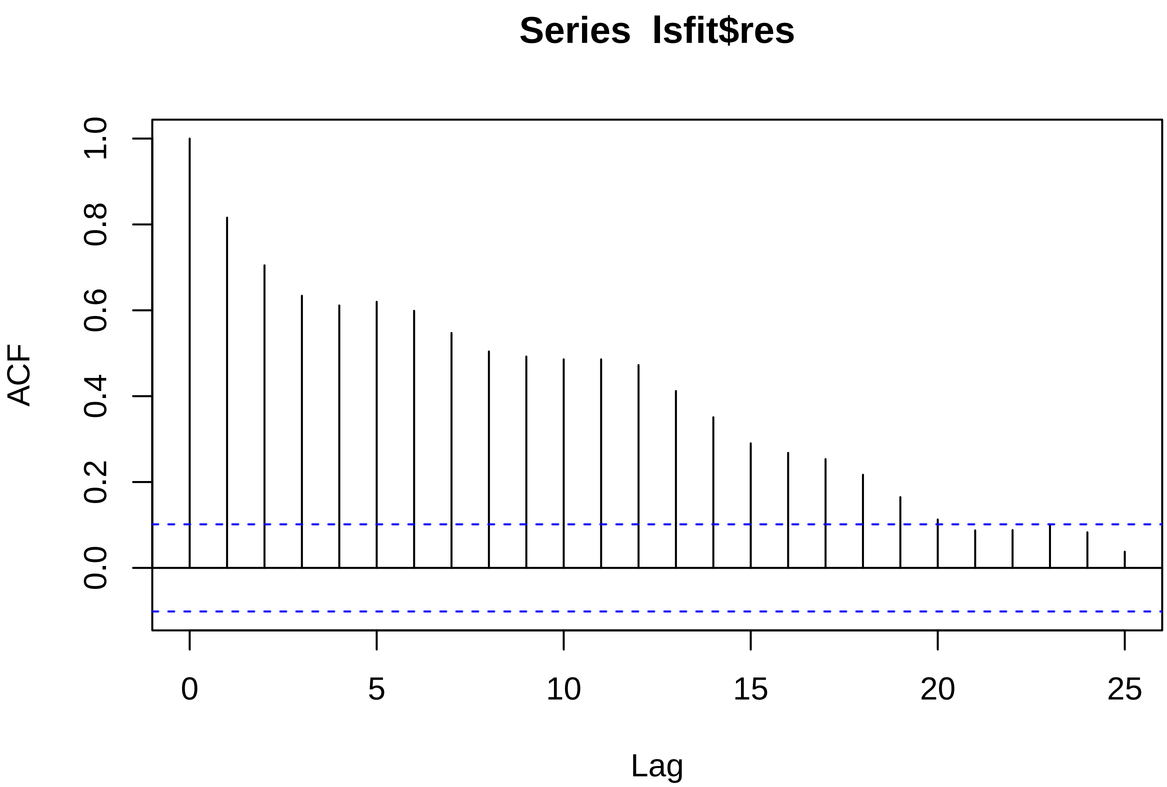
Let's fit a model with monthly dummy variables. There is a curve trend that is beyond quadratic.

```
lsfit=lm(birth~poly(times,3)+month,
 #           Feb+Mar+Apr+May+Jun+Jul+Aug+Sep+Oct+Nov+Dec,
         data=X)
summary(lsfit)
```

```
##
## Call:
## lm(formula = birth ~ poly(times, 3) + month, data = X)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -30.806  -8.521  -1.008   9.051  41.496
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)      307.478      2.164 142.119  < 2e-16 ***
## poly(times, 3)1 -356.462     12.241 -29.120  < 2e-16 ***
## poly(times, 3)2 -369.891     12.239 -30.222  < 2e-16 ***
## poly(times, 3)3  245.762     12.247  20.066  < 2e-16 ***
## month02Feb       -20.826      3.084  -6.752 5.90e-11 ***
## month03Mar         2.731      3.084   0.885   0.3766
## month04Apr       -17.837      3.084  -5.783 1.60e-08 ***
## month05May        -6.853      3.084  -2.222   0.0269 *
## month06Jun        -6.284      3.084  -2.038   0.0423 *
```

```
## month07Jul        19.869      3.084   6.442 3.79e-10 ***
## month08Aug        27.219      3.084   8.826  < 2e-16 ***
## month09Sep        23.154      3.084   7.507 4.84e-13 ***
## month10Oct        16.705      3.084   5.416 1.12e-07 ***
## month11Nov        -2.998      3.084  -0.972   0.3316
## month12Dec         6.398      3.084   2.074   0.0388 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 12.24 on 358 degrees of freedom
## Multiple R-squared:  0.884,  Adjusted R-squared:  0.8795
## F-statistic: 194.9 on 14 and 358 DF,  p-value: < 2.2e-16
```

```
acf(lsfit$res)
```



Although this looks like a good fit, we see that the residuals have autocorrelation.

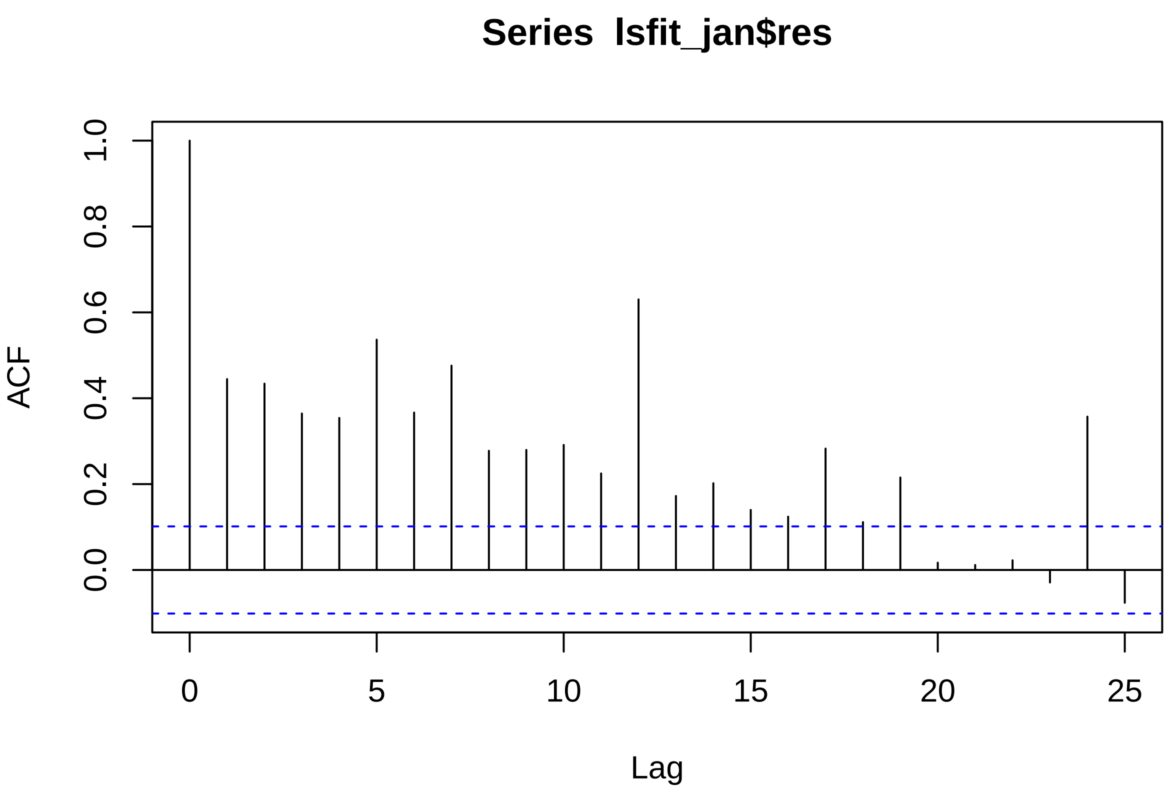
Let's also fit a model with sin and cos to model cyclical nature.

```
lsfit_jan=lm(birth~poly(times,3)+sint+cost+Jan,data=X_jan) #you remove sin/cos and do all months
summary(lsfit_jan)
```

```
##
## Call:
## lm(formula = birth ~ poly(times, 3) + sint + cost + Jan, data = X_jan)
##
## Residuals:
##    Min      1Q Median     3Q    Max
## -33.58 -11.16  -1.32  10.30  48.34
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)      310.2118     0.8069 384.462  < 2e-16 ***
## poly(times, 3)1 -356.6168    14.7680 -24.148  < 2e-16 ***
## poly(times, 3)2 -369.8896    14.7665 -25.049  < 2e-16 ***
## poly(times, 3)3  245.5296    14.7736  16.620  < 2e-16 ***
## sint             -18.0085     1.1130 -16.181  < 2e-16 ***
## cost              -2.5458     1.1695  -2.177  0.03013 *
## Jan                8.4756     3.0262   2.801  0.00537 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 14.76 on 366 degrees of freedom
## Multiple R-squared:  0.8274, Adjusted R-squared:  0.8246
## F-statistic: 292.5 on 6 and 366 DF,  p-value: < 2.2e-16
```

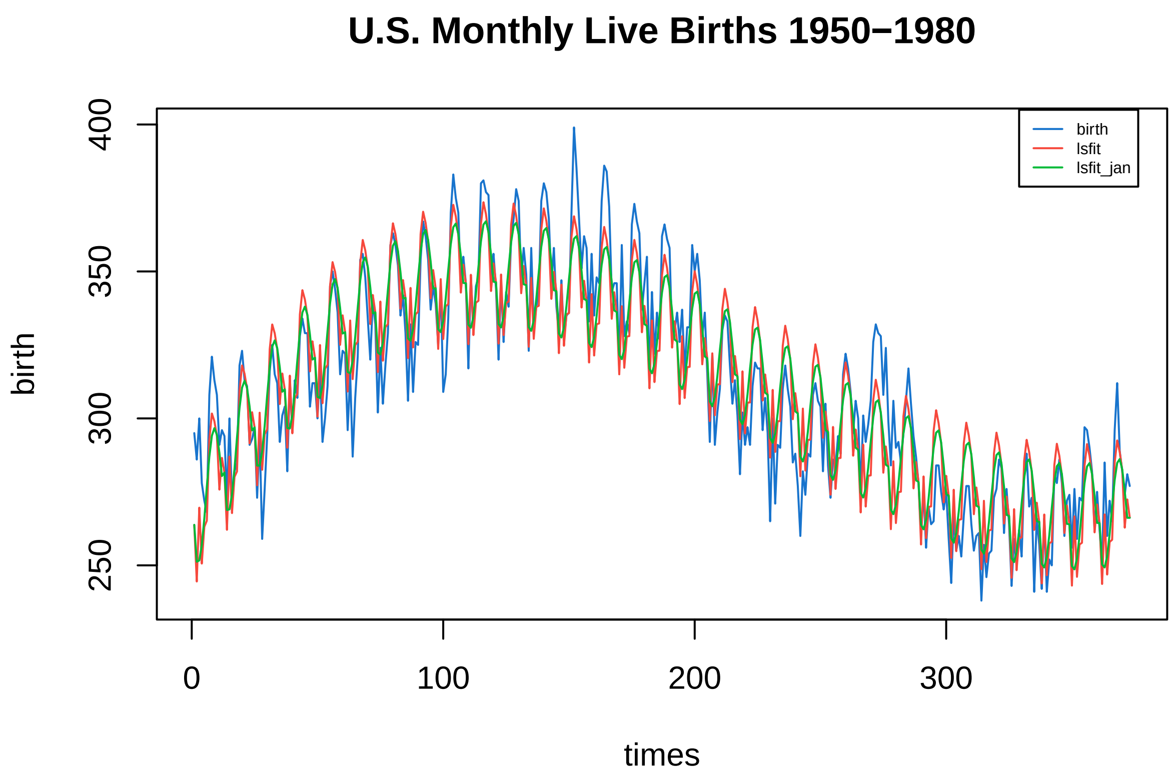
```
acf(lsfit_jan$res)
```



Same problem with this model, we also see that the residuals still have autocorrelation.

Let's plot both models:

```
plot(times,birth,type="l",main="U.S. Monthly Live Births 1950-1980",col=4)

lines(times,lsfit$fitted.values,col=2)
lines(times,lsfit_jan$fitted,col=3)
legend(329,405,c("birth","lsfit","lsfit_jan"),col=c(4,2,3),lty=1,cex=.5)
```




```
#df<-data.frame(fit=lsfit$fitted.values, times=times)
#df2<-data.frame(fit=lsfit_jan$fitted.values, times=times)
#birth %>%
#  autoplot(,col="darkgrey") +
#  ggtitle("U.S. Monthly Live Births 1950-1980") +
#  geom_line(data=df,aes(x=time(birth),y=fit),col=2)+
#  geom_line(data=df2,aes(x=time(birth),y=fit),col=3)
```

Which model performs better?

```
aic<-round(c(AIC(lsfit), AIC(lsfit_jan)),2)
bic<-round(c(BIC(lsfit), BIC(lsfit_jan)),2)
adjr2<-round(c(summary(lsfit)$ad,summary(lsfit_jan)$ad),2)
rbind(c("lsfit", "lsfit_jan"), aic,bic,adjr2)
```

```
##       [,1]      [,2]
##       "lsfit"   "lsfit_jan"
## aic   "2943.52" "3075.81"
## bic   "3006.26" "3107.19"
## adjr2 "0.88"    "0.82"
```

Now let's try the time series model with auto-regressive, integrated, moving averages and cyclic components:

```
library(forecast)
birthmod<-auto.arima(birth)
birthmod
```

```
## Series: birth
## ARIMA(0,1,2)(1,1,1)[12]
##
## Coefficients:
##           ma1      ma2     sar1      sma1
```

```
##       -0.3984  -0.1632  0.1018  -0.8434
## s.e.   0.0512   0.0486  0.0713   0.0476
##
## sigma^2 = 46.1:  log likelihood = -1204.93
## AIC=2419.86   AICc=2420.03   BIC=2439.29
```

The result is ARIMA(0,1,2)(1,1,1)[12] We also see the aic and the bic metrics and this model performed better that the ones we did earlier.

Equation corresponding to the time series model:

$$(I - sar1B^12)(I - B^12)(I - B)y_t = (I + sma1B^12)(I + ma1B + ma2B^2)w_t$$

where $\{w_t\}$ are the random errors.

Plugging in the numbers:

$$(I - 0.1018B^12)(I - B^{12})(I - B)y_t = (I - 0.8434B^{12})(I - 0.3984B - 0.1632B^2)w_t$$

Or

$$(I-B^{12})(I-B)y_t-0.1018B^{12}(I-B^{12})(I-B)y_t = (I-0.3984B-0.1632B^2)w_t-0.8434B^{12}(I-0.3984B-0.1632B^2)w_t$$

$$(I-B^{12})(y_t-y_{t-1})-0.1018B^{12}(I-B^{12})(y_t-y_{t-1}) = (w_t-0.3984w_{t-1}-0.1632w_{t-2})-0.8434(w_{t-12}-0.3984w_{t-13}-0.1632w_{t-14}$$

$$(y_t-y_{t-1})-(y_{t-12}-y_{t-13})-0.1018B^{12}((y_t-y_{t-1})-(y_{t-12}-y_{t-13})) = (w_t-0.3984w_{t-1}-0.1632w_{t-2})-0.8434(w_{t-12}-0.3984w_{t-}$$

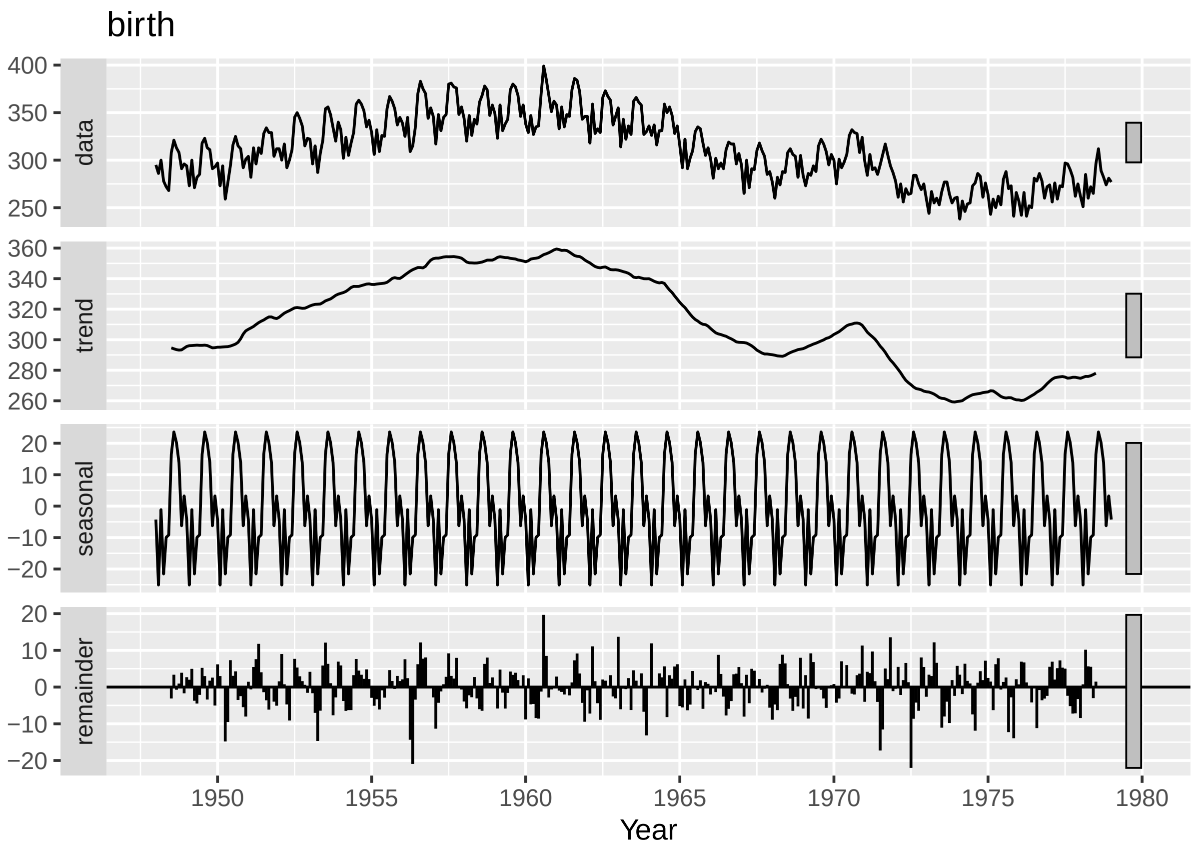
$$(y_t-y_{t-1})-(y_{t-12}-y_{t-13})-0.1018((y_{t-12}-y_{t-13})-(y_{t-24}-y_{t-25})) = w_t-0.3984w_{t-1}-0.1632w_{t-2}-0.8434w_{t-12}+0.8434*0.398$$

$$(y_t = y_{t-1}+(y_{t-12}-y_{t-13})+0.1018((y_{t-12}-y_{t-13})-(y_{t-24}-y_{t-25}))+w_t-0.3984w_{t-1}-0.1632w_{t-2}-0.8434w_{t-12}+0.8434*0.398$$

We see that this is quite a complicated structure that captures a yearly cycle plus a 2 year cycle. That seems to account for the curved patterns we observed in the plot of the values.

Let's see the decomposition of the cycles:

```r
birth %>% decompose() %>%
  autoplot() + xlab("Year") +
  ggtitle("birth")
```



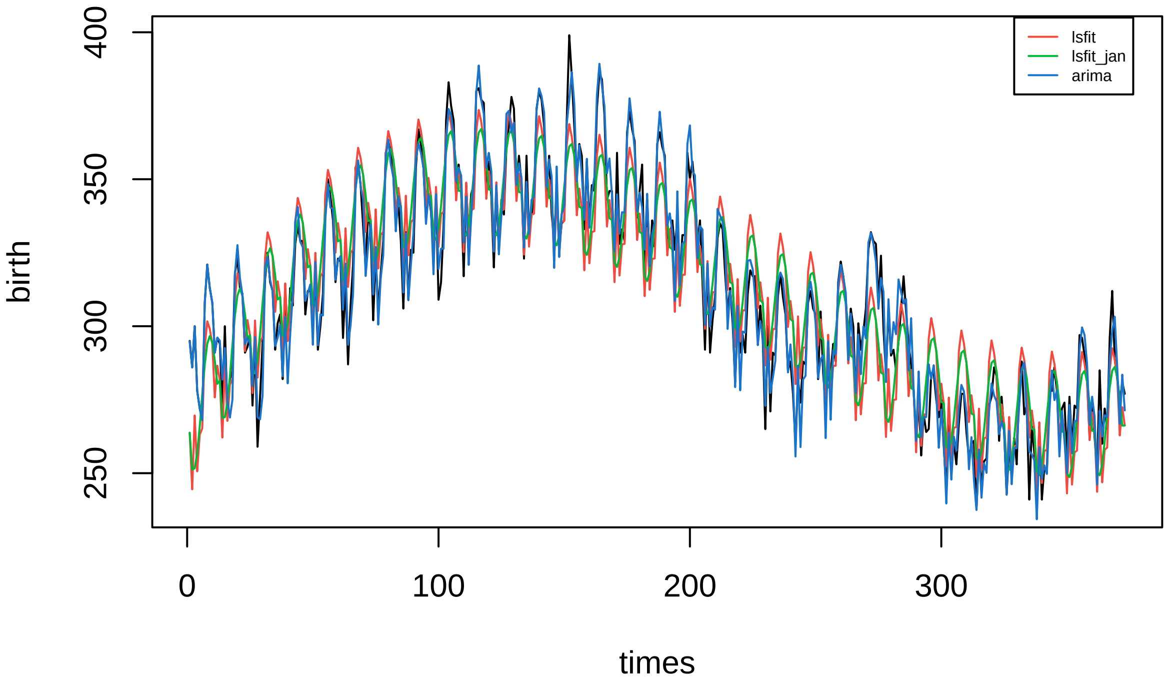

```
#dbirth<-decompose(birth)
#plot(dbirth)
```

We see the trend (2nd plot), the seasonal component (3rd plot) and the random part (4th plot). The 1st plot is the original series.

- Trend: the trend-cycle component T_t is a m-moving average, where m is the cycle. In our case of montly data, $m = 12$. (moving average = average of previous m-observations)
- Detrended series: Calculate the detrended series as $y_t - T_t$
- Seasonal component: the seasonal component for each season is the average of the detrended values for that season. This gives a series called S_t.
- Error: The remainder component is calculated by subtracting the estimated seasonal and trend-cycle components: $R_t = Y_t - T_t - S_t$.

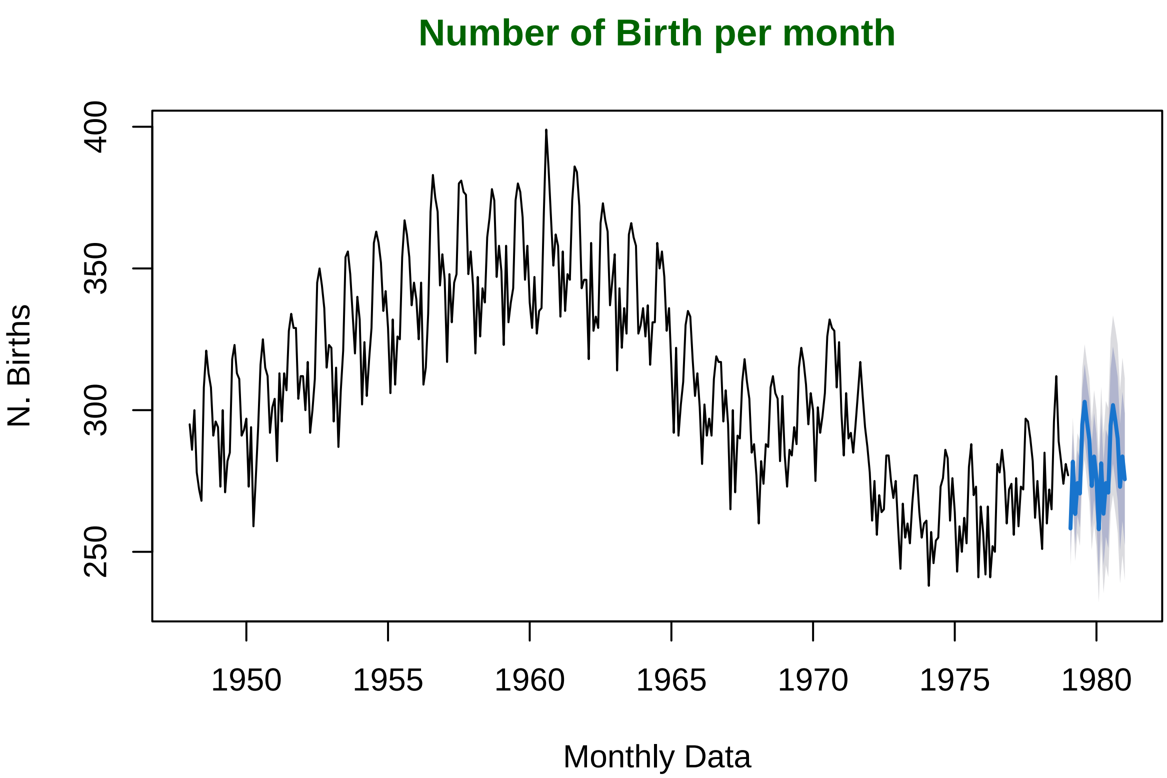
Let's plot the fitted values of the 3 models:

```
plot(times,birth,type="l") #plot on original scale
#lines(times,birth) #add lines to existing plot
lines(times,lsfit$fitted.values,col=2) #undo log for fitted model
lines(times,lsfit_jan$fitted,col=3) #undo log for fitted model
lines(times,birthmod$fitted,col=4)
legend(329,405,c("lsfit","lsfit_jan","arima"),col=c(2,3,4),lty=1,cex=.5)
```



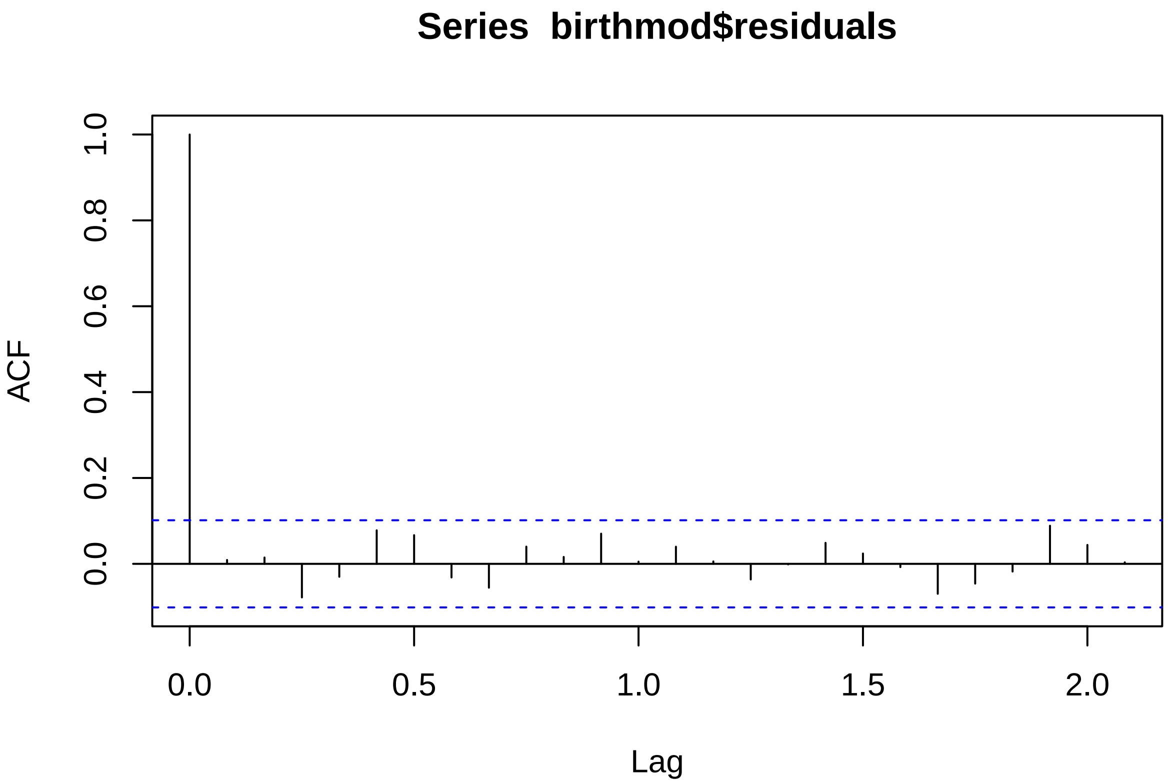
Now let's use our arima model to do forecasts:

```
plot(forecast(birthmod, 24), xlab ="Monthly Data",
     ylab ="N. Births",
     main ="Number of Birth per month", col.main ="darkgreen")
```



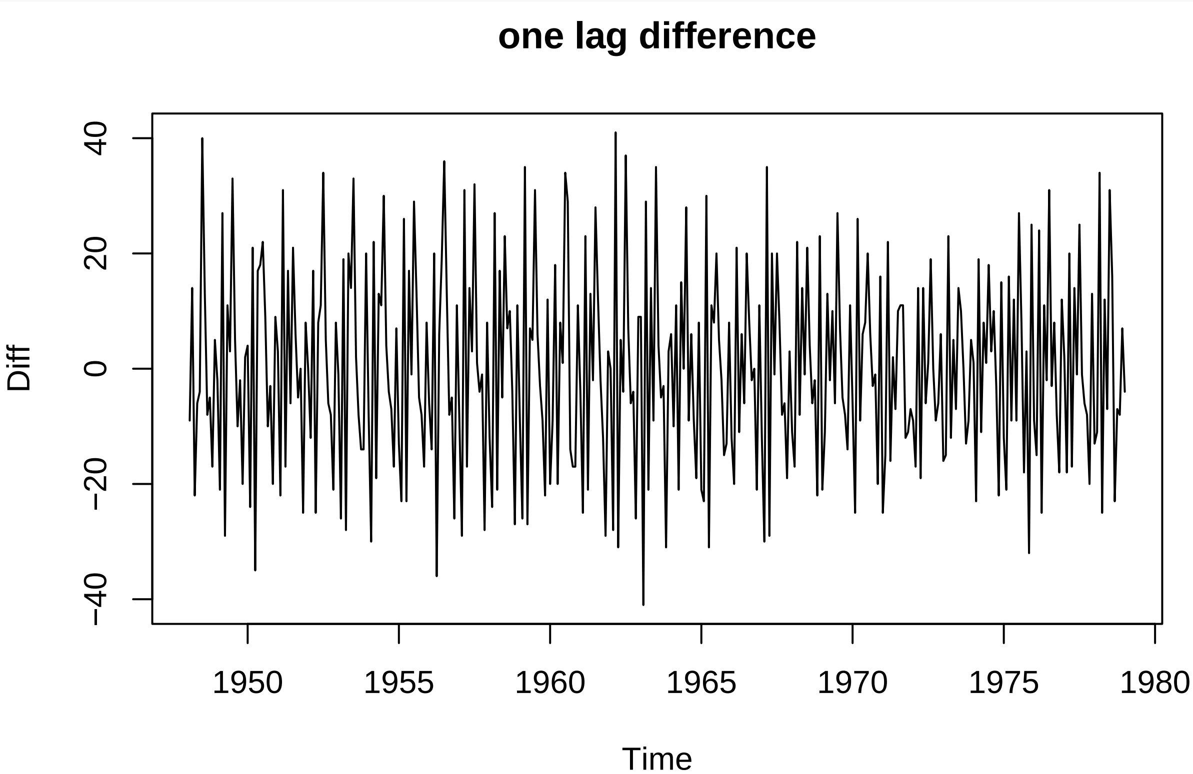
Let's check that the errors do not have any auto-correlation:

```r
acf(birthmod$residuals)
```

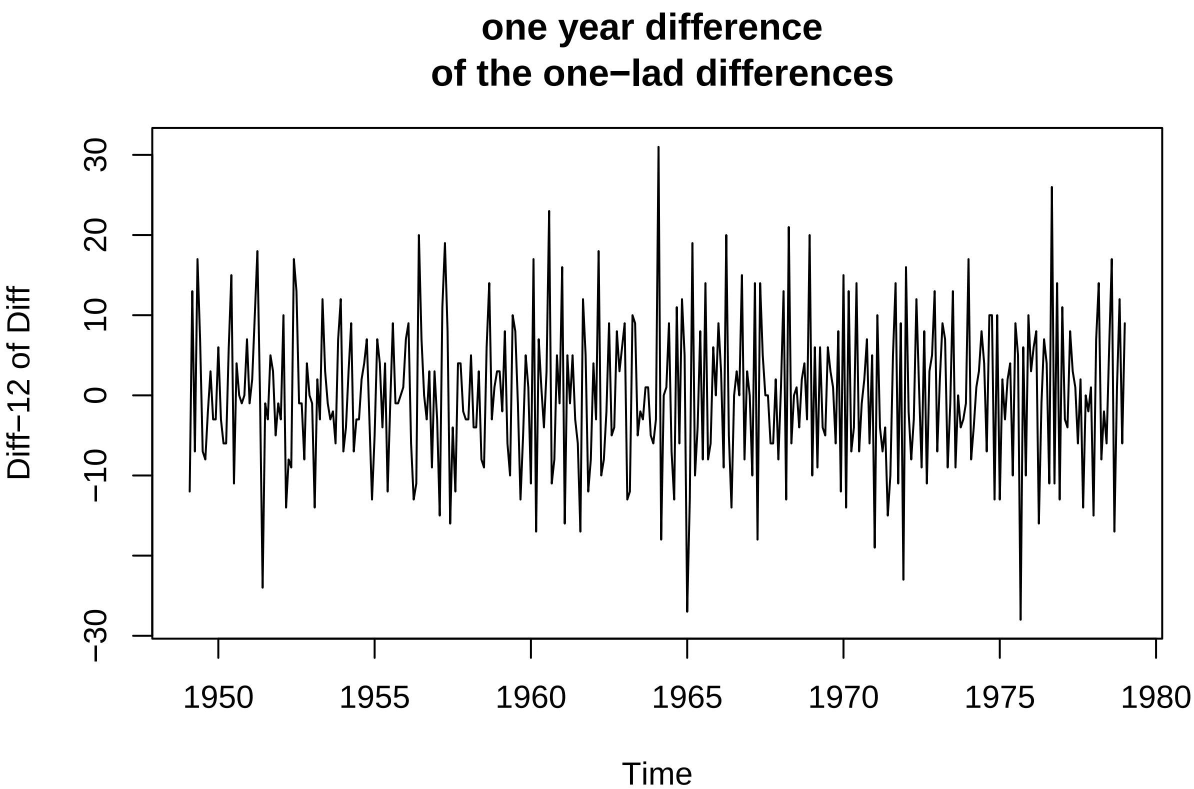


Just for the heck of it, let's look at the differences involved in the arima model:

```r
plot(diff(birth,1),main="one lag difference",ylab="Diff")
```



```
plot(diff( diff(birth,1) ,12),main="one year difference \n of the one-lad differences",ylab="Diff-12 of
```



Time Series Assignment

We will fit a model to the log of the Australian wine sales.

- Plot wine and log(wine).
- Plot the auto correlation and partial auto correlation functions for log(wine).
- Just as we did for "birth", fit a model allowing for a term for each month and time.
- Just as we did for "birth", fit a model using sin and cos to model seasonality and time.
- compute the aic, bic and adjusted r^2 corresponding to both models.
- Use auto.arima() to obtain the arima model.
- compare the aic and bic of the arima model to the previous 2 models.
- Write down the equation corresponding to the arima model.
- Plot the decomposition of the series.
- Plot the fitted values of all 3 models over the values of wine. Remember that your models were for log(wine) but you are plotting wine, so you need to adjust your fitted values.
- plot the predicted values for the next 12 months.
- auto.arima does not work with covariates. But we can use the structure it developed to add one or several covarites. Consider the models:
  - Arima(y, order = c(1,1,1), xreg = X) and

– Arima(y, order = c(1,0,1), xreg = X) where X is the data frame with times and the monthly dummy variables

```
wine=c(
.46400E+03,
.67500E+03,
.70300E+03,
.88700E+03,
.11390E+04,
.10770E+04,
.13180E+04,
.12600E+04,
.11200E+04,
.96300E+03,
.99600E+03,
.96000E+03,
.53000E+03,
.88300E+03,
.89400E+03,
.10450E+04,
.11990E+04,
.12870E+04,
.15650E+04,
.15770E+04,
.10760E+04,
.91800E+03,
.10080E+04,
.10630E+04,
.54400E+03,
.63500E+03,
.80400E+03,
.98000E+03,
.10180E+04,
.10640E+04,
.14040E+04,
.12860E+04,
.11040E+04,
.99900E+03,
.99600E+03,
.10150E+04,
.61500E+03,
.72200E+03,
.83200E+03,
.97700E+03,
.12700E+04,
.14370E+04,
.15200E+04,
.17080E+04,
.11510E+04,
.93400E+03,
.11590E+04,
.12090E+04,
.69900E+03,
.83000E+03,
```

```
.99600E+03,
.11240E+04,
.14580E+04,
.12700E+04,
.17530E+04,
.22580E+04,
.12080E+04,
.12410E+04,
.12650E+04,
.18280E+04,
.80900E+03,
.99700E+03,
.11640E+04,
.12050E+04,
.15380E+04,
.15130E+04,
.13780E+04,
.20830E+04,
.13570E+04,
.15360E+04,
.15260E+04,
.13760E+04,
.77900E+03,
.10050E+04,
.11930E+04,
.15220E+04,
.15390E+04,
.15460E+04,
.21160E+04,
.23260E+04,
.15960E+04,
.13560E+04,
.15530E+04,
.16130E+04,
.81400E+03,
.11500E+04,
.12250E+04,
.16910E+04,
.17590E+04,
.17540E+04,
.21000E+04,
.20620E+04,
.20120E+04,
.18970E+04,
.19640E+04,
.21860E+04,
.96600E+03,
.15490E+04,
.15380E+04,
.16120E+04,
.20780E+04,
.21370E+04,
.29070E+04,
```

```
.22490E+04,
.18830E+04,
.17390E+04,
.18280E+04,
.18680E+04,
.11380E+04,
.14300E+04,
.18090E+04,
.17630E+04,
.22000E+04,
.20670E+04,
.25030E+04,
.21410E+04,
.21030E+04,
.19720E+04,
.21810E+04,
.23440E+04,
.97000E+03,
.11990E+04,
.17180E+04,
.16830E+04,
.20250E+04,
.20510E+04,
.24390E+04,
.23530E+04,
.22300E+04,
.18520E+04,
.21470E+04,
.22860E+04,
.10070E+04,
.16650E+04,
.16420E+04,
.15250E+04,
.18380E+04,
.18920E+04,
.29200E+04,
.25720E+04,
.26170E+04,
.20470E+04)
```

###############. solutions

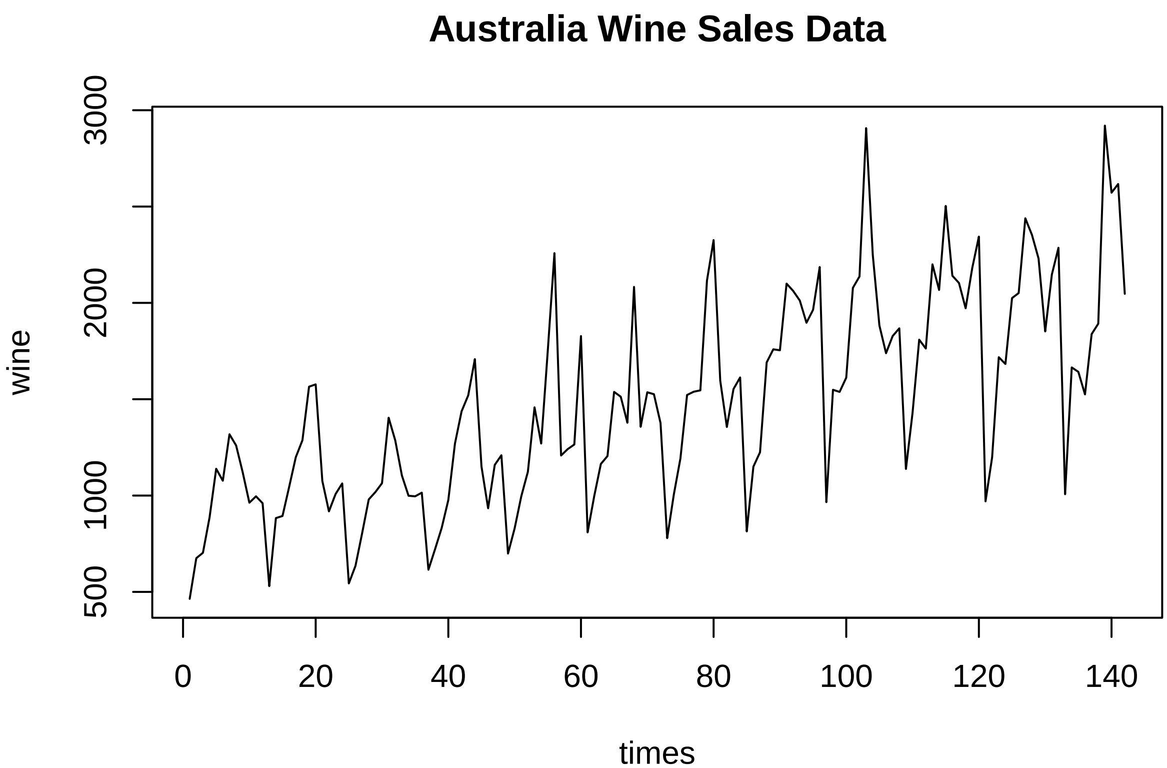
Plot wine and log(wine). Warning

```
wine<-ts(wine,frequency = 12)
library(ggplot2)
library(ggfortify)
library(dplyr)
```

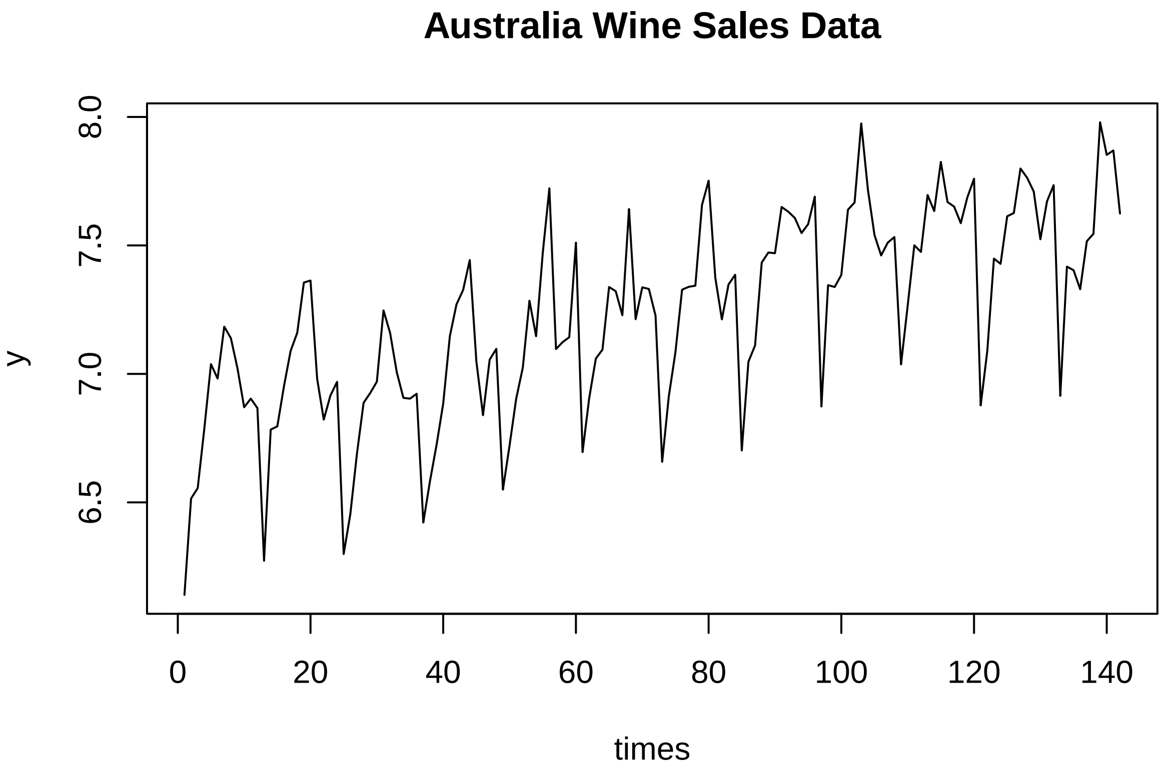
```
y=log(wine)
times=1:142

Jan=rep(c(1,0,0,0,0,0,0,0,0,0,0,0),12)[1:142]
Feb=rep(c(0,1,0,0,0,0,0,0,0,0,0,0),12)[1:142]
Mar=rep(c(0,0,1,0,0,0,0,0,0,0,0,0),12)[1:142]
```

```
Apr=rep(c(0,0,0,1,0,0,0,0,0,0,0,0),12)[1:142]
May=rep(c(0,0,0,0,1,0,0,0,0,0,0,0),12)[1:142]
Jun=rep(c(0,0,0,0,0,1,0,0,0,0,0,0),12)[1:142]
Jul=rep(c(0,0,0,0,0,0,1,0,0,0,0,0),12)[1:142]
Aug=rep(c(0,0,0,0,0,0,0,1,0,0,0,0),12)[1:142]
Sep=rep(c(0,0,0,0,0,0,0,0,1,0,0,0),12)[1:142]
Oct=rep(c(0,0,0,0,0,0,0,0,0,1,0,0),12)[1:142]
Nov=rep(c(0,0,0,0,0,0,0,0,0,0,1,0),12)[1:142]
Dec=rep(c(0,0,0,0,0,0,0,0,0,0,0,1),12)[1:142]
sint=sin(2*pi*times/12)
cost=cos(2*pi*times/12)
X=data.frame(times=times,Feb=Feb,Mar=Mar,Apr=Apr,May=May,Jun=Jun,Jul=Jul,Aug=Aug,
             Sep=Sep,Oct=Oct,Nov=Nov,Dec=Dec)  #sin and cos and constant for Jan;
X_jan=data.frame(times=times,sint=sint,cost=cost,Jan=Jan)  #sin and cos and constant for Jan;
```

```
plot(times, wine, type = "l", main = "Australia Wine Sales Data")
```

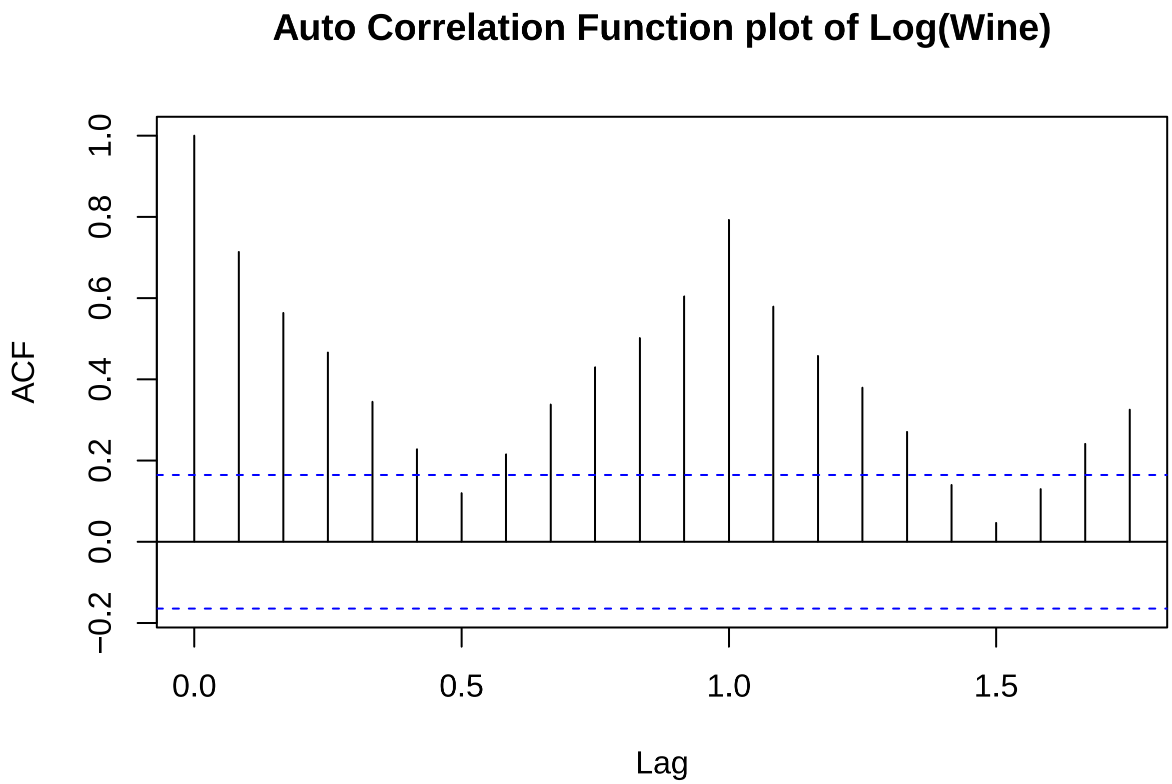



```
plot(times, y, type = "l", main = "Australia Wine Sales Data")
```

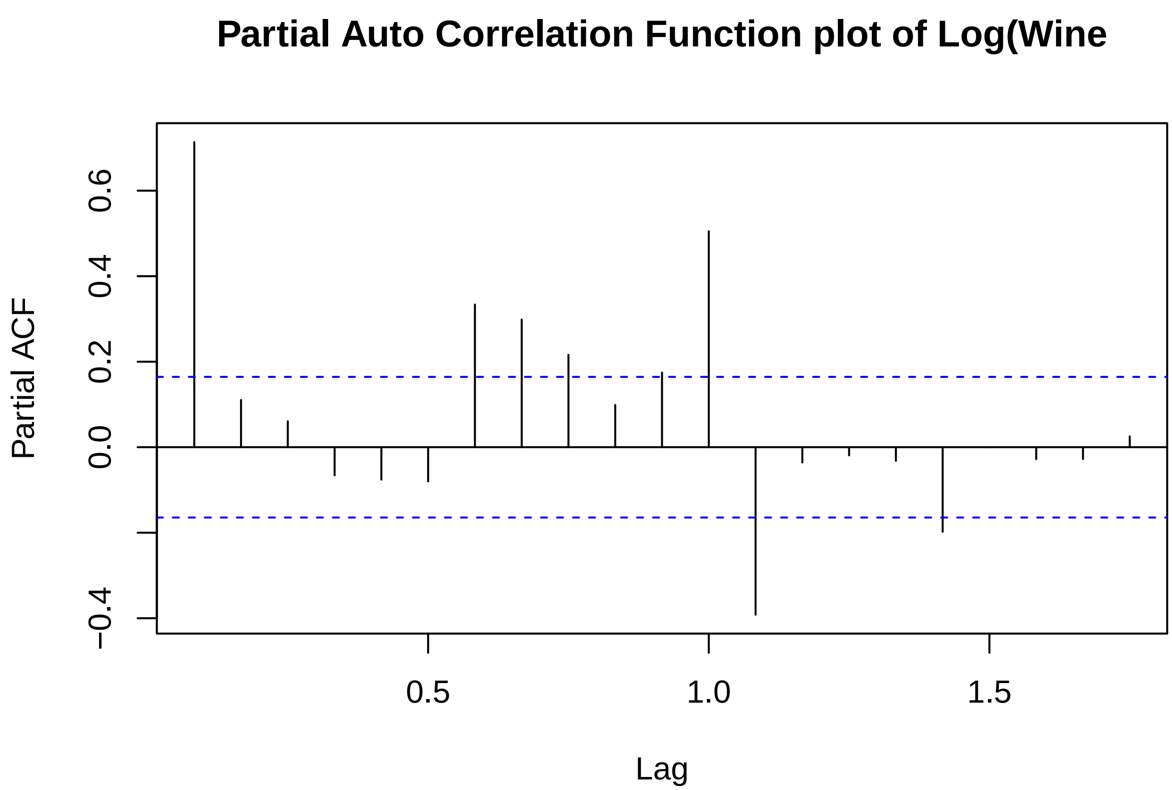


- Plot the auto correlation and partial auto correlation functions for log(wine) correlation functions

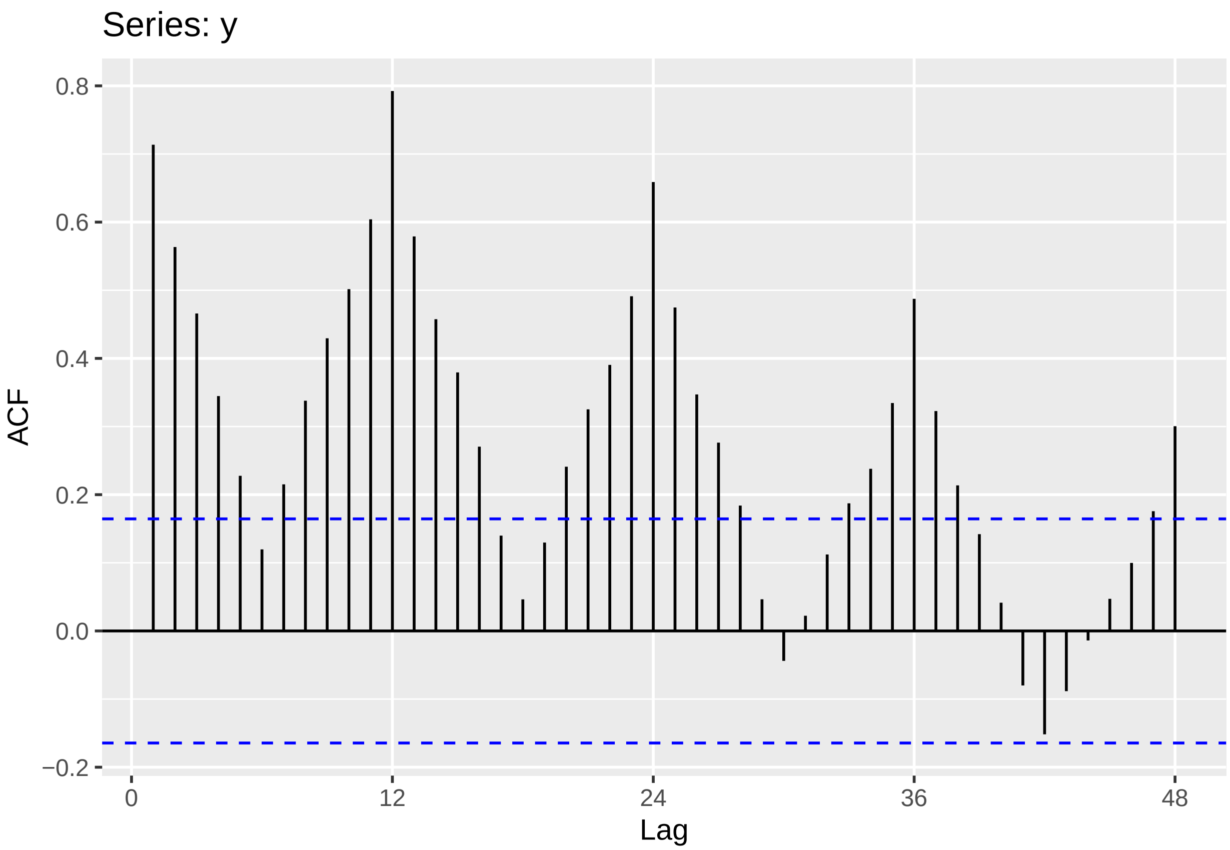
```
library(forecast)

# ACF and PACF plots for log transformed data
acf(y, main = "Auto Correlation Function plot of Log(Wine)")
```




```r
pacf(y, main = "Partial Auto Correlation Function plot of Log(Wine")
```




```r
ggAcf(y, lag = 48)
```



- Just as we did for “birth”, fit a model allowing for a term for each month and time by using the data frame X.

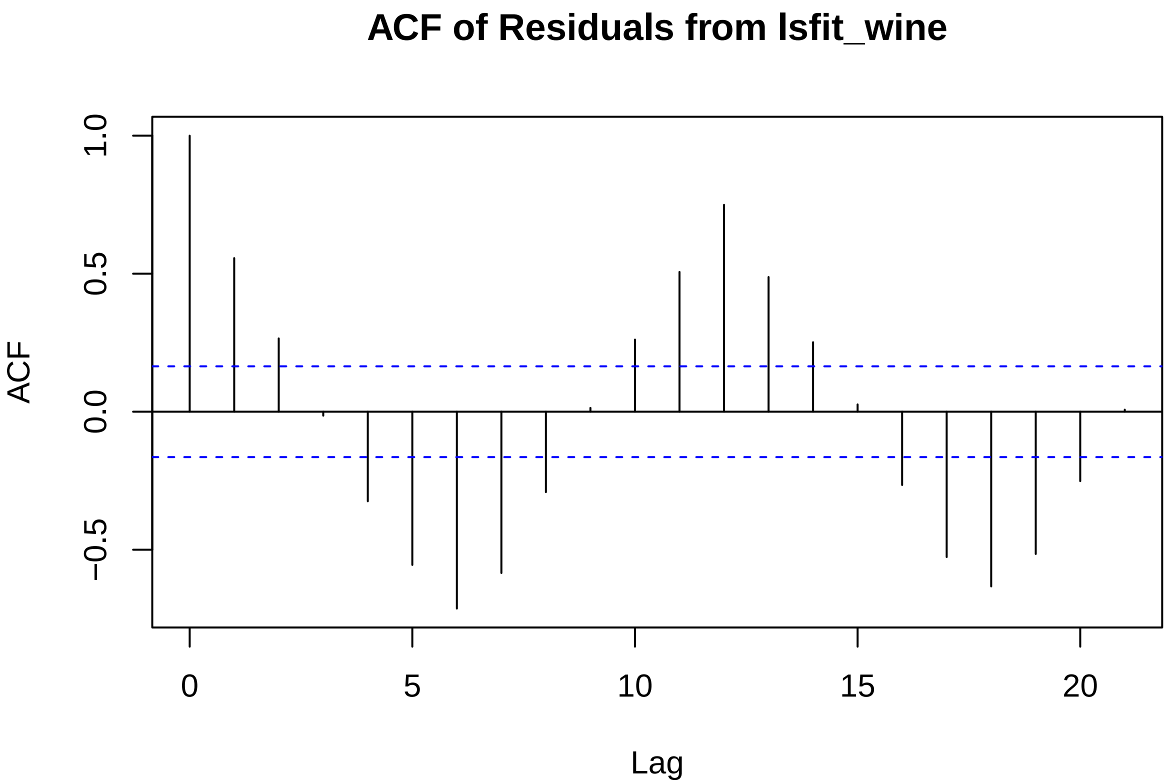
```
# Assuming y is the name of the dataset column containing wine sales
n <- length(y)

# Creating the times variable
times <- 1:n

# Generating a sequence of months to match the length of the data
months <- rep(c("01Jan", "02Feb", "03Mar", "04Apr", "05May", "06Jun",
                "07Jul", "08Aug", "09Sep", "10Oct", "11Nov", "12Dec"),
              each = ceiling(n / 12))[1:n]

# Assuming X is another variable you want to include in the model
 X_df <- data.frame(months, X)

# Creating a data frame with month and times variables
X_wine <- data.frame(times = times, month = months)

# Fitting a model with terms for each month and time
# Assuming y and X_wine are part of the same observation set
lsfit_wine <- lm(y ~ poly(times, 3) + month, data = X_wine)
summary(lsfit_wine)
```

```
##
## Call:
## lm(formula = y ~ poly(times, 3) + month, data = X_wine)
```

```
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.42543 -0.16557  0.00536  0.14626  0.50724
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)      10.1533     0.3899  26.037  < 2e-16 ***
## poly(times, 3)1  25.2902     2.8255   8.951 3.65e-15 ***
## poly(times, 3)2   0.6795     1.1926   0.570  0.56985
## poly(times, 3)3   0.4122     0.7954   0.518  0.60522
## month02Feb       -0.5036     0.1624  -3.101  0.00238 **
## month03Mar       -1.2010     0.2474  -4.854 3.49e-06 ***
## month04Apr       -1.6736     0.3139  -5.332 4.30e-07 ***
## month05May       -2.1087     0.3713  -5.680 8.73e-08 ***
## month06Jun       -2.6422     0.4277  -6.178 8.15e-09 ***
## month07Jul       -3.1581     0.4857  -6.502 1.64e-09 ***
## month08Aug       -3.6435     0.5426  -6.715 5.64e-10 ***
## month09Sep       -4.1924     0.5936  -7.063 9.52e-11 ***
## month10Oct       -4.7895     0.6354  -7.538 7.94e-12 ***
## month11Nov       -5.5298     0.6707  -8.244 1.78e-13 ***
## month12Dec       -6.1842     0.7101  -8.709 1.40e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2169 on 127 degrees of freedom
## Multiple R-squared:  0.7088, Adjusted R-squared:  0.6767
## F-statistic: 22.08 on 14 and 127 DF,  p-value: < 2.2e-16
```

The to see the auto correlation of the residuals of the model (acf)

```r
# Calculate residuals from the model
residuals_lsfit_wine <- residuals(lsfit_wine)

# Plot the autocorrelation function of the residuals
acf(residuals_lsfit_wine, main = "ACF of Residuals from lsfit_wine")
```



- Just as we did for "birth", fit a model using sin and cos to model seasonality and time using the data frame X_jan

```r
n <- length(y)

# Creating sine and cosine terms for seasonality
sint <- sin(2 * pi * times / 12) # Frequency for yearly seasonality
cost <- cos(2 * pi * times / 12)

# Creating the Jan indicator (assuming January as the starting point)
Jan <- rep(c(1,0,0,0,0,0,0,0,0,0,0,0),ceiling(n / 12))[1:n]

# Fitting a model with sine, cosine and Jan indicator
lsfit_wine_jan <- lm(y ~ poly(times, 3) + sint + cost + Jan, data = X_jan)
summary(lsfit_wine_jan)
```

```
##
## Call:
## lm(formula = y ~ poly(times, 3) + sint + cost + Jan, data = X_jan)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.31228 -0.09852 -0.00531  0.09645  0.46227
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)       7.26388    0.01226 592.512  < 2e-16 ***
## poly(times, 3)1   3.08221    0.13837  22.275  < 2e-16 ***
## poly(times, 3)2 -0.20972    0.13825  -1.517 0.131622
## poly(times, 3)3 -0.51123    0.13879  -3.683 0.000332 ***
```

```
## sint            -0.17542    0.01682 -10.428  < 2e-16 ***
## cost            -0.13436    0.01789  -7.508 7.31e-12 ***
## Jan             -0.41833    0.04629  -9.037 1.49e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1381 on 135 degrees of freedom
## Multiple R-squared:  0.8745, Adjusted R-squared:  0.8689
## F-statistic: 156.8 on 6 and 135 DF,  p-value: < 2.2e-16
```

- compute the aic, bic and adjusted r2 corresponding to both models.

```
aic <- round(c(AIC(lsfit_wine), AIC(lsfit_wine_jan)),2)
bic <- round(c(BIC(lsfit_wine), BIC(lsfit_wine_jan)),2)
adjr2 <- round(c(summary(lsfit_wine)$ad,summary(lsfit_wine_jan)$ad))
rbind(c("lsfit_wine", "lsfit_wine_jan"), aic,bic,adjr2)
```

```
##       [,1]         [,2]
##       "lsfit_wine" "lsfit_wine_jan"
## aic   "-14.93"     "-150.44"
## bic   "32.37"      "-126.79"
## adjr2 "1"          "1"
```

- Use auto.arima() to obtain the arima model.

```
winemod <- auto.arima(y)
summary(winemod)
```

```
## Series: y
## ARIMA(1,0,1)(0,1,1)[12] with drift
##
## Coefficients:
##          ar1      ma1     sma1   drift
##       0.8930  -0.6841  -0.7372  0.0062
## s.e.  0.0785   0.1249   0.0951  0.0008
##
## sigma^2 = 0.0125:  log likelihood = 97.74
## AIC=-185.48   AICc=-185   BIC=-171.14
##
## Training set error measures:
##                          ME      RMSE        MAE         MPE     MAPE      MASE
## Training set -7.957843e-05 0.1053301 0.07939004 -0.01674237 1.090572 0.6036391
##                    ACF1
## Training set 0.02659543
```

- compare the aic and bic of the arima model to the previous 2 models.

```
aic <- round(c(AIC(lsfit_wine), AIC(lsfit_wine_jan), AIC(winemod)),2)
bic <- round(c(BIC(lsfit_wine), BIC(lsfit_wine_jan), BIC(winemod)),2)
rbind(c("lsfit_wine", "lsfit_wine_jan","winemod"), aic,bic)
```

```
##     [,1]         [,2]             [,3]
##     "lsfit_wine" "lsfit_wine_jan" "winemod"
## aic "-14.93"     "-150.44"        "-185.48"
## bic "32.37"      "-126.79"        "-171.14"
```

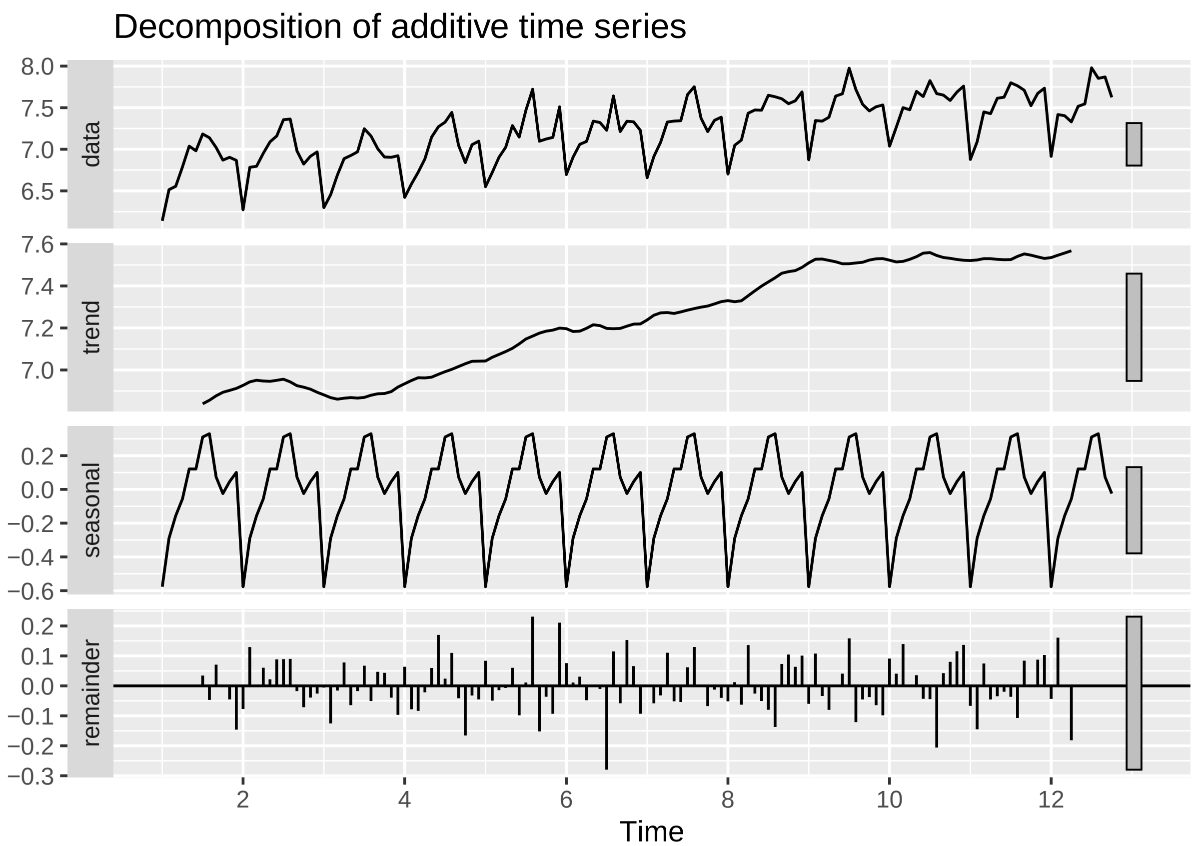
- Write down the equation corresponding to the arima model.

Ans) The ARIMA (1,1,1) model can be written in the following form:

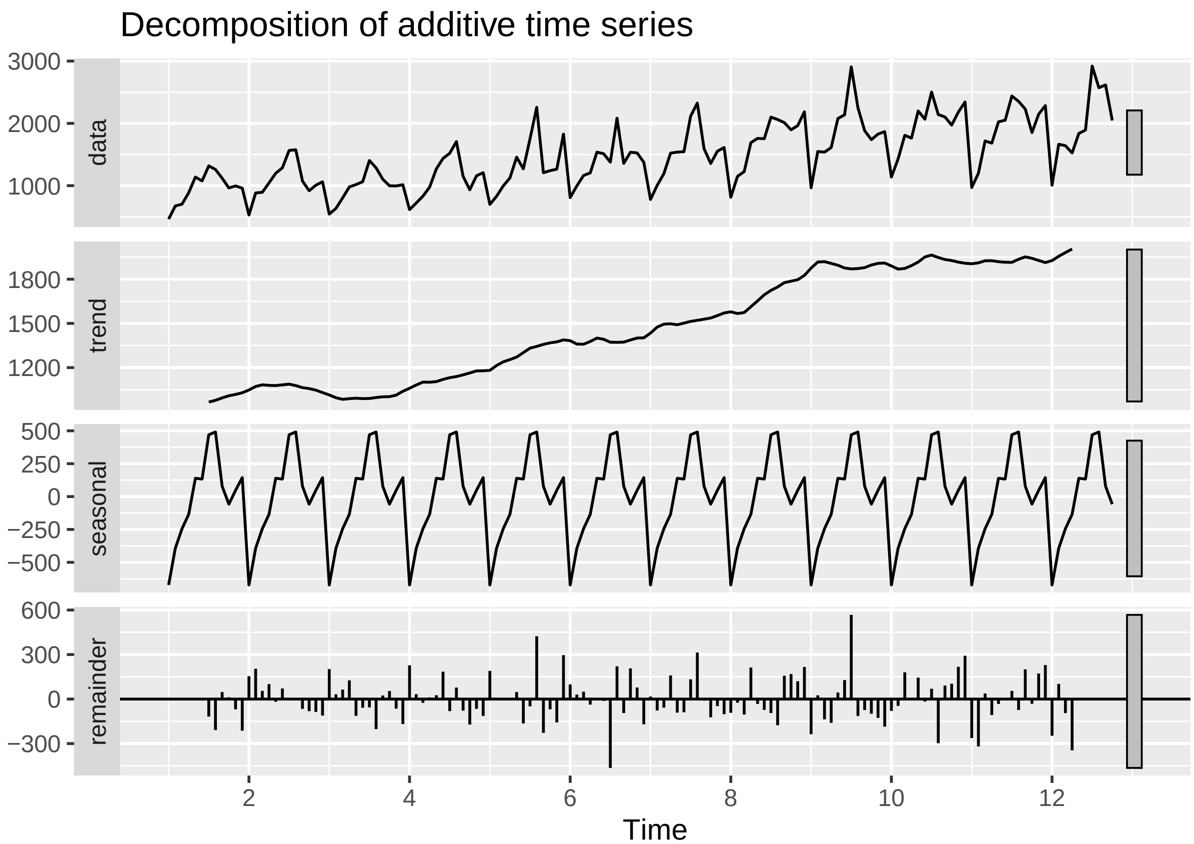
$(I - ar1B)(I - B)yt = (I + ma1B)wt$
Equation corresponding to the above ARIMA model is
$(I - 0.8930B)(I - B)yt = (I - 0.6841B)wt$

- Plot the decomposition of the series.

```
wine_ts <- ts(y, frequency = 12)

# Try decomposing the time series
decomposed <- decompose(wine_ts)

# Plot the decomposition if successful
autoplot(decomposed)
```

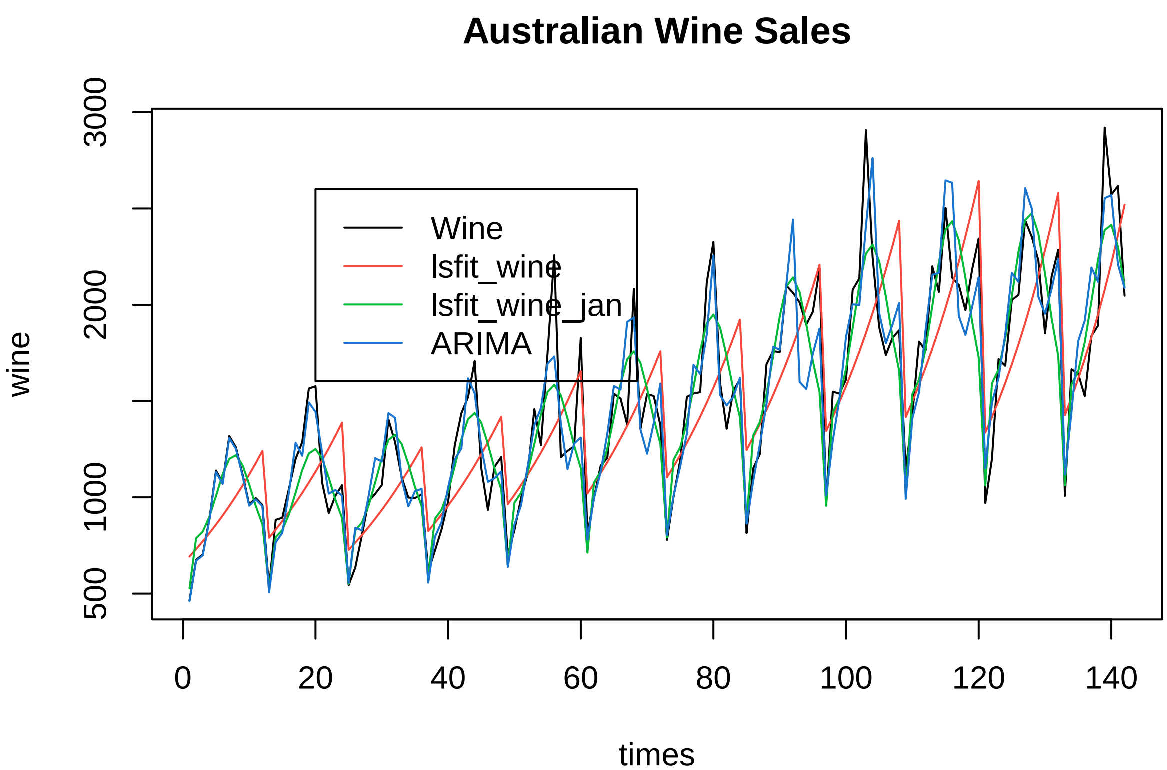



```
wine_ts <- ts(wine, frequency = 12)

# Try decomposing the time series
decomp_wine <- decompose(wine_ts)

# Plot the decomposition
autoplot(decomp_wine)
```



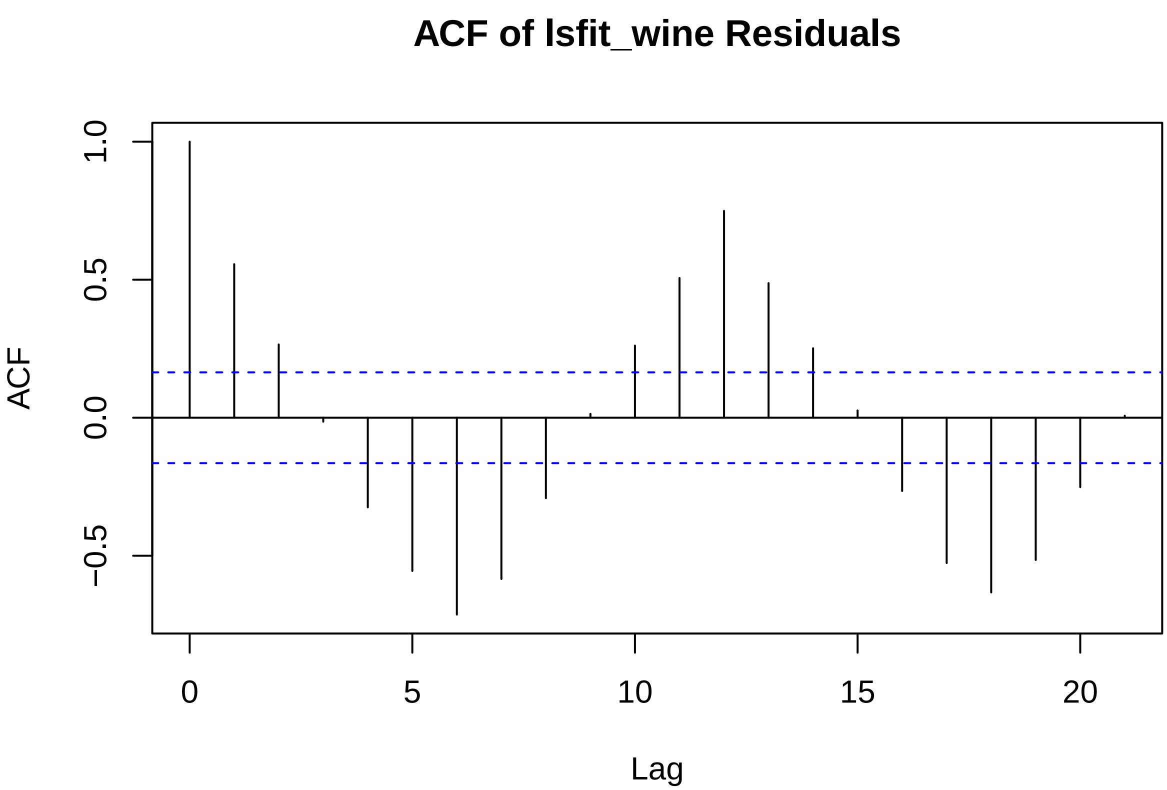
- Plot the fitted values of all 3 models over the values of wine. Remember that your models were for log(wine) but you are plotting wine, so you need to adjust your fitted values.

```
plot(times, wine, type = "l", main = "Australian Wine Sales")
lines(times, exp(lsfit_wine$fitted.values), col = 2)
lines(times, exp(lsfit_wine_jan$fitted), col = 3)
lines(times, exp(winemod$fitted), col = 4)
legend(20,2600, c("Wine", "lsfit_wine","lsfit_wine_jan", "ARIMA"), col = c(1,2,3,4),lty = 1)
```

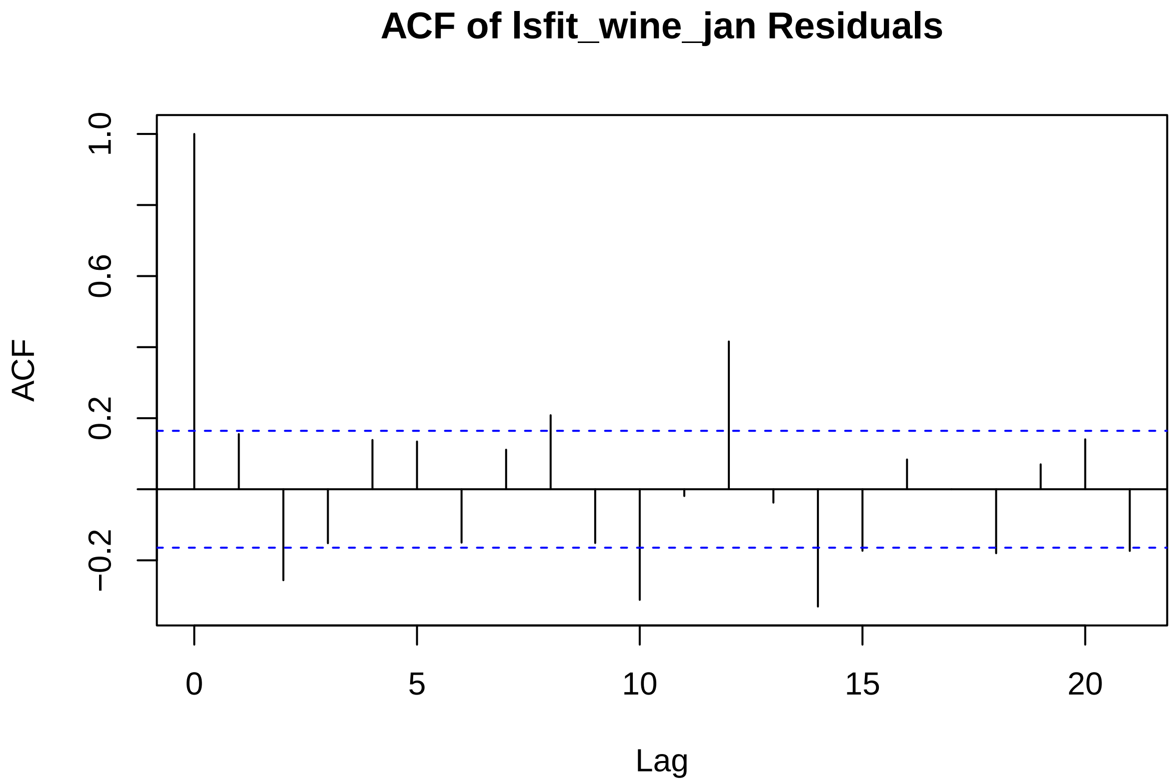


Check that the errors do not have any auto-correlation (acf)

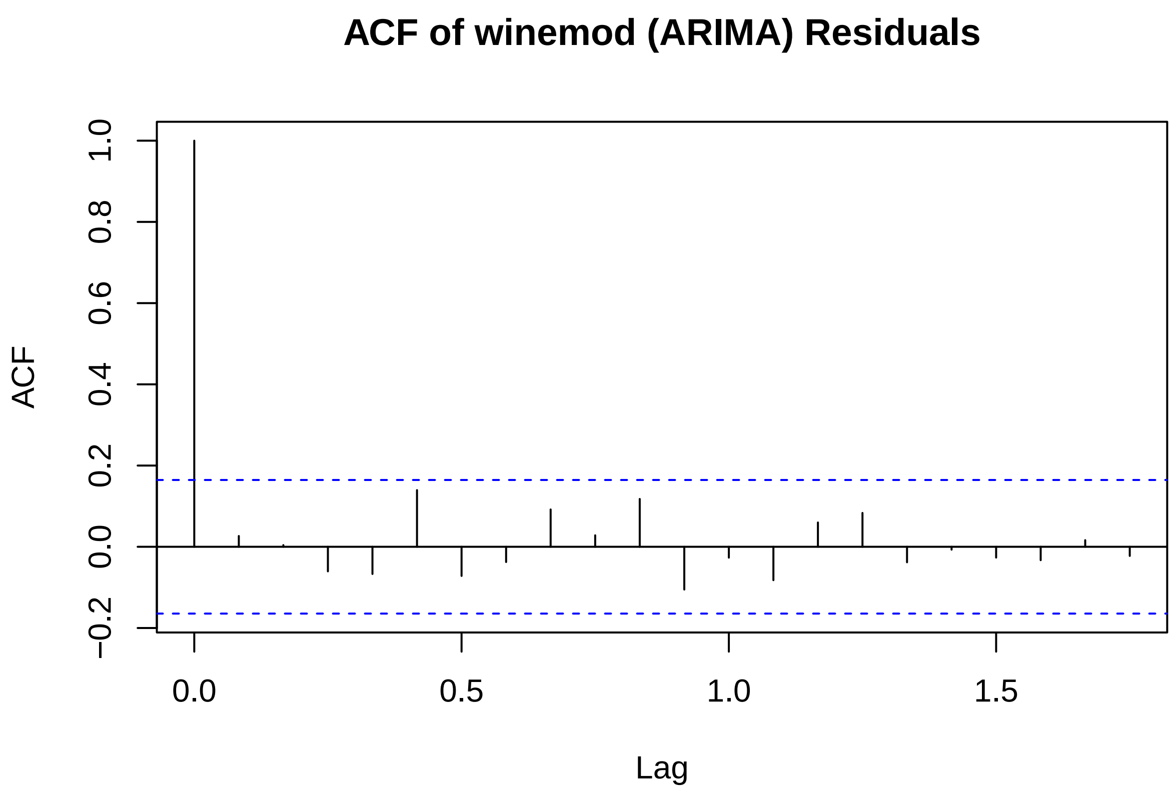
```
# Check autocorrelation in residuals of lsfit_wine
acf(residuals(lsfit_wine), main = "ACF of lsfit_wine Residuals")
```



```r
# Check autocorrelation in residuals of lsfit_wine_jan
acf(residuals(lsfit_wine_jan), main = "ACF of lsfit_wine_jan Residuals")
```



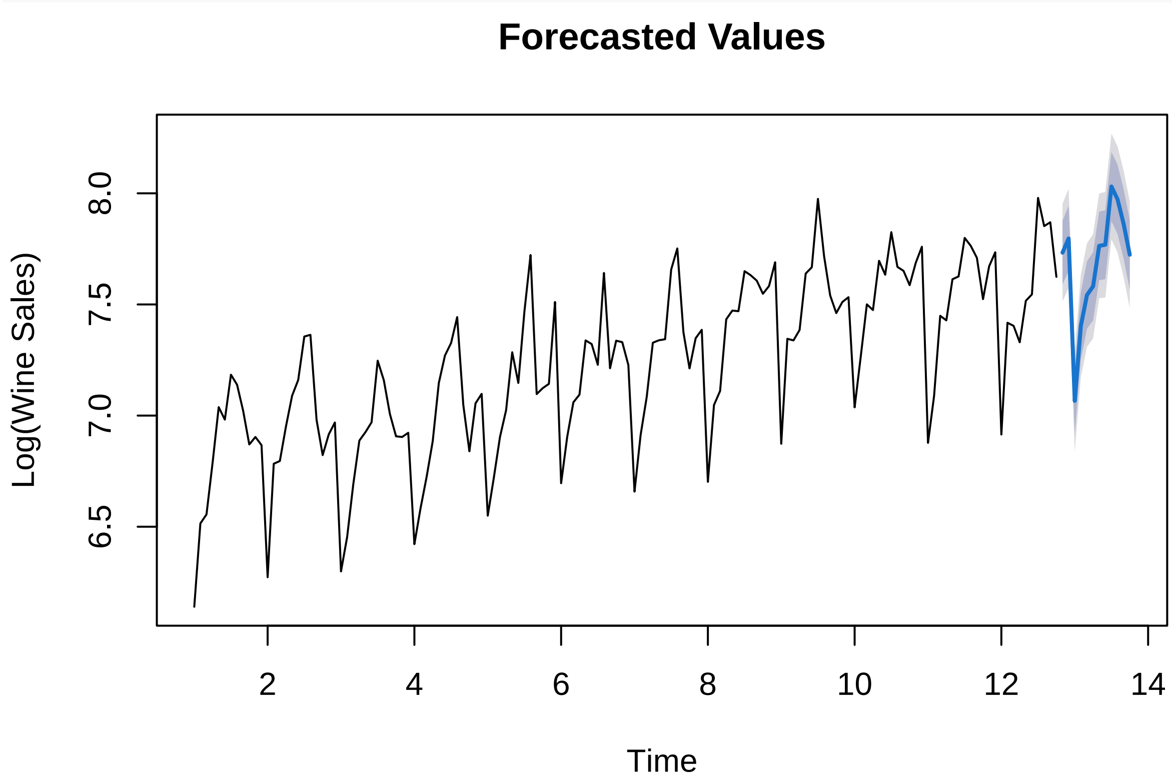

```r
# Check autocorrelation in residuals of winemod (ARIMA model)
acf(residuals(winemod), main = "ACF of winemod (ARIMA) Residuals")
```



- plot the predicted values for the next 12 months.

Now let's use our arima model to do forecasts:

```r
plot(forecast(winemod, h = 12), xlab = "Time", ylab = "Log(Wine Sales)", main = "Forecasted Values")
```



- auto.arima does not work with covariates. But we can use the structure it developed to add one or several covarites. Consider the models: – Arima(y, order = c(1,1,1), xreg = X) and – Arima(y, order = c(1,0,1), xreg = X) where X is the data frame with times and the monthly dummy variables

```
# Convert all columns of X to numeric if they are not already
X <- data.frame(lapply(X, function(col) as.numeric(as.character(col))))
X_matrix <- data.matrix(X)
tsmod2<-Arima(y, order = c(1,1,1), xreg = data.matrix(X))
tsmod3<-Arima(y, order = c(1,0,1), xreg = data.matrix(X))
summary(tsmod2)
```

```
## Series: y
## Regression with ARIMA(1,1,1) errors
##
## Coefficients:
##          ar1      ma1   times     Feb     Mar     Apr     May     Jun     Jul
##       0.0985  -0.8350  0.0058  0.2940  0.4195  0.5300  0.6984  0.6957  0.9033
## s.e.  0.1093   0.0668  0.0016  0.0362  0.0382  0.0386  0.0387  0.0389  0.0389
##          Aug      Sep     Oct     Nov     Dec
##       0.9107   0.6771  0.5659  0.6322  0.6841
## s.e.  0.0389   0.0389  0.0388  0.0391  0.0372
##
## sigma^2 = 0.01128:  log likelihood = 122.98
## AIC=-215.97   AICc=-212.13   BIC=-171.74
##
## Training set error measures:
##                       ME     RMSE       MAE         MPE     MAPE      MASE
## Training set 0.001359156 0.100425 0.0790184 0.005170204 1.087867 0.6008134
##                     ACF1
## Training set -0.00813603
```

```
summary(tsmod3)
```

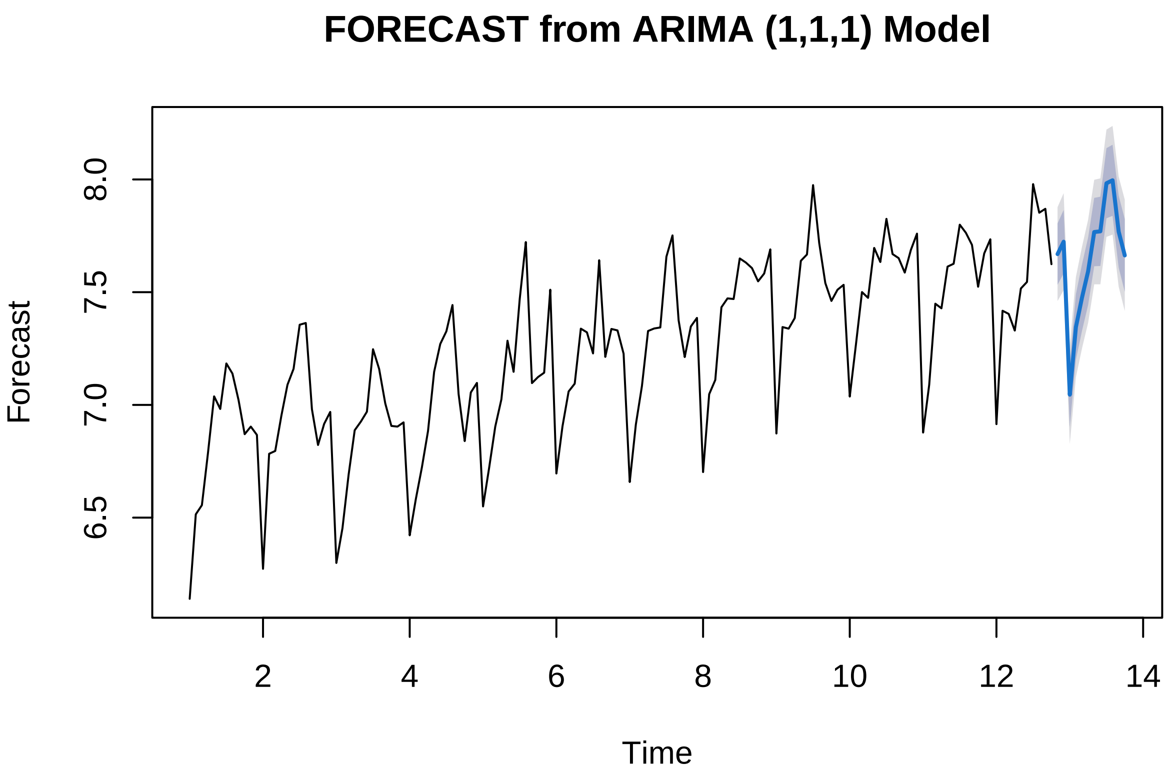
```
## Series: y
## Regression with ARIMA(1,0,1) errors
##
## Coefficients:
##          ar1      ma1  intercept   times     Feb     Mar    Apr     May     Jun
##       0.8694  -0.6558     6.1964  0.0063  0.2936  0.4189  0.529  0.6969  0.6938
## s.e.  0.0891   0.1412     0.0474  0.0005  0.0363  0.0373  0.038  0.0385  0.0388
##          Jul      Aug     Sep     Oct     Nov     Dec
##       0.9009   0.9078  0.6736  0.5619  0.6308  0.6832
## s.e.  0.0390   0.0389  0.0387  0.0383  0.0382  0.0373
##
## sigma^2 = 0.01092:  log likelihood = 127.02
## AIC=-222.05   AICc=-217.7   BIC=-174.75
##
## Training set error measures:
##                        ME       RMSE        MAE         MPE     MAPE      MASE
## Training set 0.0003300068 0.09881831 0.07768568 -0.01376188 1.070394 0.5906801
##                    ACF1
## Training set 0.01067203
```

```
newdata<-cbind(times=143:154,Feb=c(0,0,0,1,rep(0,8)),
                   Mar=c(0,0,0,0,1,rep(0,7)),
                   Apr=c(0,0,0,0,0,1,rep(0,6)),
```

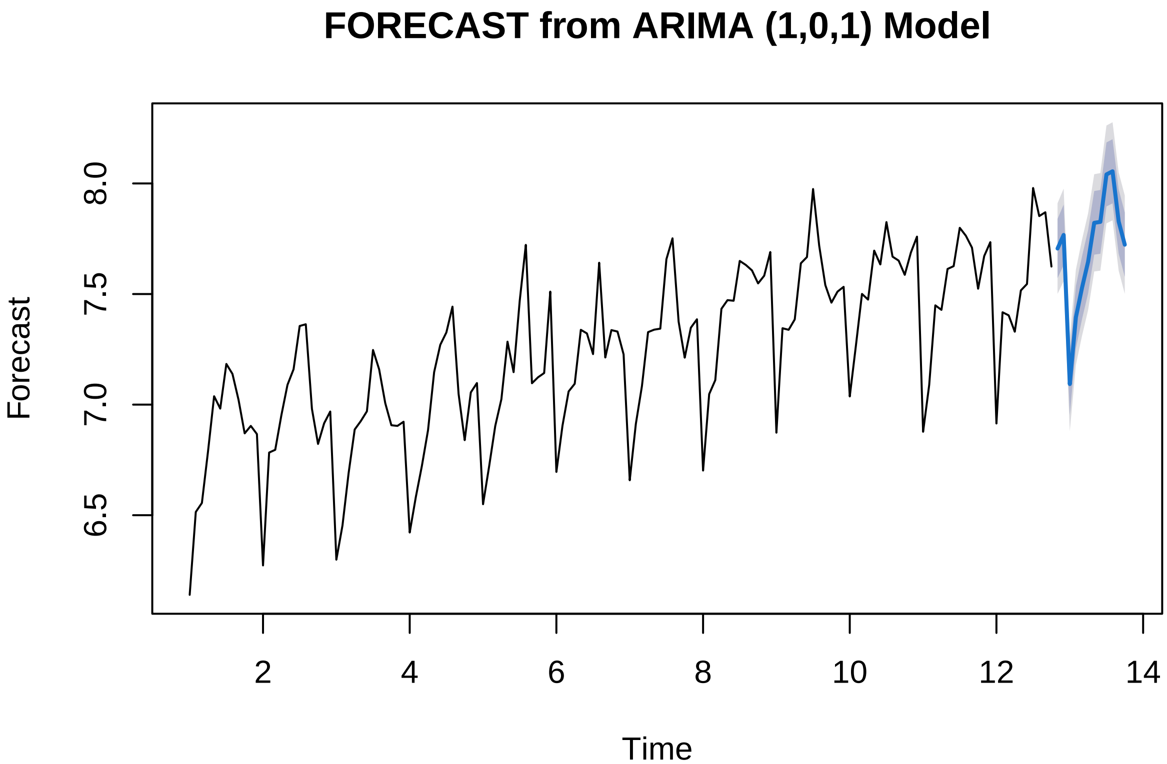
```
                    May=c(rep(0,6),1,rep(0,5)),
                    Jun=c(rep(0,7),1,rep(0,4)),
                    Jul=c(rep(0,8),1,rep(0,3)),
                    Aug=c(rep(0,9),1,rep(0,2)),
                    Sep=c(rep(0,10),1,0),Oct=c(rep(0,11),1),
                    Nov=c(1,rep(0,11)),Dec=c(0,1,rep(0,10)))
```

PLot forecast for the two models using newdata. Put titles in your plots.

```
forecast_tsmod2 <- forecast(tsmod2, xreg = newdata)
plot(forecast_tsmod2, main = "FORECAST from ARIMA (1,1,1) Model",
     xlab = "Time", ylab = "Forecast")
```




```
forecast_tsmod3 <- forecast(tsmod3, xreg = newdata)
plot(forecast_tsmod3, main = "FORECAST from ARIMA (1,0,1) Model",
     xlab = "Time", ylab = "Forecast")
```


FORECAST from ARIMA (1,0,1) Model
Forecast
Time
6.5
7.0
7.5
8.0
2
4
6
8
10
12
14