Lab01 (Mat 565)

KR

2023-01-25

1. (15 pts) Let's replicate the example in class with dataset faithful. This dataset is part of the base distribution of R so you don't need to load any library.

a. Run the regression model of eruptions on waiting times. (replace NULL by the appropriate code)

```
model <- lm(eruptions ~ waiting, data = faithful)
summary(model)
```

```
##
## Call:
## lm(formula = eruptions ~ waiting, data = faithful)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.29917 -0.37689  0.03508  0.34909  1.19329
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.874016   0.160143  -11.70   <2e-16 ***
## waiting      0.075628   0.002219   34.09   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4965 on 270 degrees of freedom
## Multiple R-squared:  0.8115, Adjusted R-squared:  0.8108
## F-statistic:  1162 on 1 and 270 DF,  p-value: < 2.2e-16
```

< 0.05

b. Make a scatterplot of eruptions vs waiting times and overlay the regression line in red color. Put a title to your graph.

```
plot(faithful$waiting, faithful$eruptions,
     xlab = "Waiting Times",
     ylab = "Eruptions Duration",
     main = "ScatterPlot with Regression Line")
abline(model, col = "red")
```

c. Run a test of hypothesis for the slope. State the null and alternative hypothesis. State the value of the test statistic with the number of degrees of freedom. State the p-value and state the conclusion of the test.

```r
# Test of Hypothesis

coefficients <- coef(model)

summary_output <- summary(model)

t_statistic <- summary_output$coefficients["waiting", "t value"]


df <- summary_output$df[2]

p_value <- summary_output$coefficients["waiting", "Pr(>|t|)"]

summary(model)
```

```
##
## Call:
## lm(formula = eruptions ~ waiting, data = faithful)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.29917 -0.37689  0.03508  0.34909  1.19329
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.874016   0.160143  -11.70   <2e-16 ***
```

```
## waiting      0.075628   0.002219   34.09   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4965 on 270 degrees of freedom
## Multiple R-squared:  0.8115, Adjusted R-squared:  0.8108
## F-statistic:  1162 on 1 and 270 DF,  p-value: < 2.2e-16
```

```r
print(coef(summary(model))[1,1])
```

```
## [1] -1.874016
```

```r
print(coef(summary(model))[2,1])
```

```
## [1] 0.07562795
```

Whenever we perform simple linear regression, we end up with the following estimated regression equation

To determine if β_1 is statistically significant, we should perform a t-test with the following test statistic:

where se(β_1) represents the standard error of β_1.

from the model output, we can see that the estimated regression equation is:

eruptions = -1.874016 + 0.075628(waiting).

Let us check if the slope coefficient is statistically significant,

t = (β_1 - 0)/se(β_1)
t = 0.075628 / 0.002219
t = 34.082

The p-value that corresponds to this t-test statistic is shown in the column called Pr(>|t|) in the output and it turns out to be 2e-16.

H_0: "$\beta_1 = 0$ - The slope coefficient is equal to zero means no linear relationship.

H_1: $\beta_1 \neq 0$ - The slope coefficient is not equal to zero means significant linear relationship.

p-value: 2e-16 $\sim$ 0.

Conclusion: Since the p-value < 0.05, We reject the null hypothesis, There is a significant linear relationship between "Waiting Times and"Eruptions".

d. Construct, by hand, a 98% confidence interval for the slope.

98% confidence interval for β_1: $t^* =$ qt(.99,270) = 2.340238 0.98

Upper bound $= 0.075628 + 2.340238 * 0.00219 = 0.08075$
lower bound $= 0.075628 - 2.340238 * 0.3148 = 0.07050$

Verify that your computation coincides with the confidence interval that you can get using the command confint()

```r
conf_interval <- confint(model, level = 0.98)
print(conf_interval)
```

```
##                      1 %        99 %
## (Intercept) -2.24878944 -1.49924253
## waiting      0.07043603  0.08081986
```

e. Find the estimate for the model's standard deviation. Verify this computation by computing the standard deviation by hand using the residuals of the model.

```r
RSE <- summary_output$sigma
print(RSE)
```

```
## [1] 0.4965129
```

The standard deviation of the residuals is RSE $= \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-p-1}}$.

1. Extract the residuals from the model.

```r
residuals <- resid(model)
print(residuals[1:100])
```

```
##            1            2            3            4            5            6
## -0.500591902 -0.409893203 -0.389452162 -0.531916787 -0.021359589  0.597478849
##            7            8            9           10           11           12
## -0.081243433 -0.954359589 -0.033009359 -0.204359589 -0.376893203 -0.561731642
##           13           14           15           16           17           18
##  0.175036046  0.069502433  0.296896306  0.108362693 -1.064916787  0.321268358
##           19           20           21           22           23           24
## -0.458637307  0.149408098 -0.183009359  0.069502433 -0.574963954 -0.277312422
##           25           26           27           28           29           30
##  0.810547838 -0.803103694 -0.318521151  0.209291942 -0.174963954  0.332408098
##           31           32           33           34           35           36
##  0.653175786  0.517663994  0.249571422 -0.143219850  0.110547838 -0.041637307
##           37           38           39           40           41           42
##  0.110874485  0.656780150 -0.755032943 -0.149499329  0.173780150 -0.629404995
##           43           44           45           46           47           48
##  0.088268358 -0.762404995  0.886175786 -1.086103694  0.866827317 -0.034265255
##           49           50           51           52           53           54
##  0.305524254 -0.588032943  1.001919890 -0.216499329 -0.376893203  0.656780150
##           55           56           57           58           59           60
## -0.476893203  0.479896306  0.221431682 -1.299172683  0.617663994  0.065152202
##           61           62           63           64           65           66
## -0.355032943  0.021268358 -0.006125515  0.472524254 -0.846660891 -0.683755225
##           67           68           69           70           71           72
##  0.142036046  0.675036046 -0.974800630  1.053175786 -0.294475746 -0.394149099
##           73           74           75           76           77           78
##  0.399408098  0.504431682 -0.831916787  1.193291942 -0.646660891  0.542036046
##           79           80           81           82           83           84
##  0.009291942 -0.803103694  0.334919890  0.005524254  0.680059630 -0.408800630
##           85           86           87           88           89           90
##  0.420175786  0.151756567  0.076291942  0.340780150  0.410874485 -0.629987537
##           91           92           93           94           95           96
## -0.463660891 -0.599499329 -0.040381411  0.792036046 -1.057544735  0.728803734
##           97           98           99          100
##  0.188268358 -0.048080110 -0.116009359  0.572524254
```

2. calculate the mean squared residuals

```
mean_squared_residuals <-mean(residuals^2)
print(mean_squared_residuals)
```

```
## [1] 0.2447124
```

The above value we obtained is the mean of the squared residuals.

3. Taking the root over the mean_squared_residuals will give us RSE value.

$\sqrt{0.2447124} = 0.4956$

This will be the RSE value.

f. Find the model's R^2. What is the interpretation of the R^2?

```
R_squared <- summary_output$r.squared
print(paste("R^2 value:",R_squared))
```

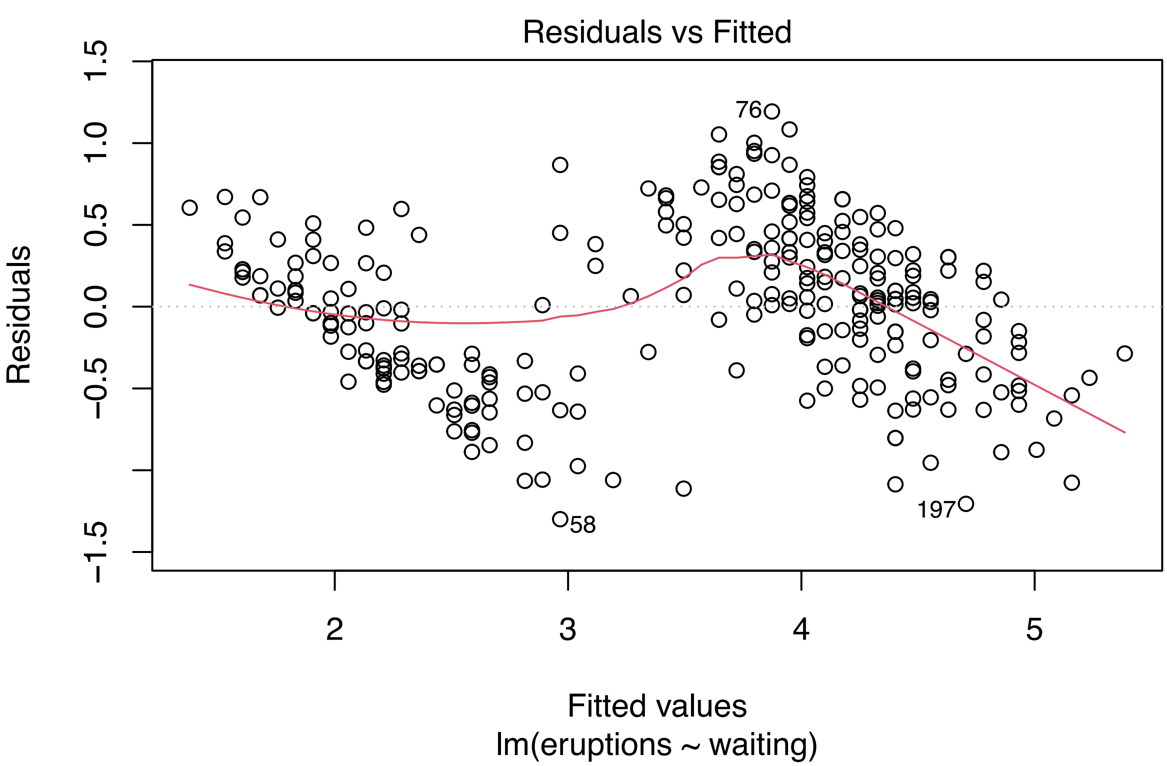
```
## [1] "R^2 value: 0.811460760973309"
```

Interpretation of the R^2: In the output, the R^2 value is 0.811. This means that 81.1% variation in eruption duration is accounted for by th linear relationship between waiting time and eruption duration in the model. The remaining 18.9% of the variation in eruption duration is attributed to other factors or random variation not captured by the model.

g. Look at the first 2 assessment plots plots

Residual plot

```
plot(model,1)
```



```r
model <- lm(eruptions ~ waiting, data = faithful)
summary(model)
```

```
##
## Call:
## lm(formula = eruptions ~ waiting, data = faithful)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.29917 -0.37689  0.03508  0.34909  1.19329
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.874016   0.160143  -11.70   <2e-16 ***
## waiting      0.075628   0.002219   34.09   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4965 on 270 degrees of freedom
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## F-statistic:  1162 on 1 and 270 DF,  p-value: < 2.2e-16
```

Interpret the meaning of the coefficient β_1.

For every each increase in the value of waiting time in the data faithful, the eruption duration between waiting times is said to increase by 0.075628 times according to the regression model.

b. Let's re-scale the waiting times by subtracting its mean and dividing by its standard deviation.

```r
wait_time_scaled <- (faithful$waiting - mean(faithful$waiting))/sd(faithful$waiting)
print(wait_time_scaled[1:100])
```

```
##   [1]  0.596024774 -1.242890136  0.228241792 -0.654437365  1.037364352
##   [6] -1.169333540  1.258034141  1.037364352 -1.463559925  1.037364352
##  [11] -1.242890136  0.963807756  0.522468177 -1.757786311  0.890251159
##  [16] -1.390003329 -0.654437365  0.963807756 -1.390003329  0.596024774
##  [21] -1.463559925 -1.757786311  0.522468177 -0.139541190  0.228241792
##  [26]  0.890251159 -1.169333540  0.375354985  0.522468177  0.596024774
##  [31]  0.154685195  0.448911581 -0.360210979  0.669581370  0.228241792
##  [36] -1.390003329 -1.684229714  0.669581370 -0.875107154  1.405147334
##  [41]  0.669581370 -0.948663750  0.963807756 -0.948663750  0.154685195
##  [46]  0.890251159 -0.507324172 -1.316446732  0.816694563 -0.875107154
##  [51]  0.301798388  1.405147334 -1.242890136  0.669581370 -1.242890136
##  [56]  0.890251159  0.007572003 -0.507324172  0.448911581  0.743137966
##  [61] -0.875107154  0.963807756 -1.684229714  0.816694563 -0.801550558
##  [66]  1.552260527  0.522468177  0.522468177 -0.433767576  0.154685195
##  [71]  0.816694563 -1.095776943  0.596024774  0.007572003 -0.654437365
##  [76]  0.375354985 -0.801550558  0.522468177  0.375354985  0.890251159
##  [81]  0.301798388  0.816694563 -0.065984594 -0.433767576  0.154685195
##  [86]  1.258034141  0.375354985  0.669581370 -1.684229714  1.110920948
##  [91] -0.801550558  1.405147334 -1.537116522  0.522468177 -0.580880769
##  [96]  0.081128599  0.963807756  0.301798388 -1.463559925  0.816694563
```

c. Run the regression model of eruptions on wait_time_scaled and interpret the meaning of the coefficient β_1.

```r
model_scaled <- lm(eruptions ~ wait_time_scaled, data = faithful)
summary(model_scaled)
```

```
##
## Call:
## lm(formula = eruptions ~ wait_time_scaled, data = faithful)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.29917 -0.37689  0.03508  0.34909  1.19329
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)       3.48778    0.03011  115.85   <2e-16 ***
## wait_time_scaled  1.02816    0.03016   34.09   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4965 on 270 degrees of freedom
## Multiple R-squared:  0.8115, Adjusted R-squared:  0.8108
## F-statistic:  1162 on 1 and 270 DF,  p-value: < 2.2e-16
```

The value of β_1 gives a value of 1.02816 of the coefficient wait_time_scaled. For every increase in the value of wait_time_scaled, the value of eruptions increase by β_1 times.

3. OLS (10pts) In the following two cases, find the OLS estimate of the beta parameter and the unbiased estimate for the variance. How many degrees of freedom does it have. (Hint: how many betas are being estimated?)

a) $Y_i = \beta_0 + \epsilon_i$

In ordinary Least Squares (OLS) regression, we aim to estimate the beta parameters and the unbiased estimate for the variance.
$Y_i = \beta_0 + \epsilon_i$

$Q_1(\beta_0) = \sum_{i=1}^{n}(Y_i - \beta_0)^2$

$\frac{\partial Q_1}{\partial \beta_0} = -2 * \sum_{i=1}^{n}(Y_i - \beta_0) = 0$

$\bar{Y} = \hat{\beta}_0$

We know that,
$\sigma^2(\beta_0) = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \beta_0)^2$

$= \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$

$\sigma^2(\beta_0) = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$

The degrees of freedom for the equation $Y_i = \beta_0 + \epsilon_i$

In this equation, there is only one parameter to be estimated: the intercept (β_0). Therefore, the degrees of freedom $df = n - 1$.

b) $Y_i = \beta_1 x_i + \epsilon_i$

The linear regression model is: $Y_i = \beta_1 x_i + \epsilon_i$

To estimate β_1 using ordinary least squares (OLS), we minimize the sum of squared residuals. The residuals e_i are the differences between the observed values Y_i and the fitted values $\hat{Y}_i$:

$e_i = Y_i - \hat{Y}_i$

$\hat{Y}_i = \hat{\beta}_1 x_i$

The OLS estimator for β_1 is obtained by minimizing the sum of squared residuals.

$SSR = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$

substituting $\hat{y}_i = \hat{\beta}_1 x_i$

$SSR = \sum_{i=1}^{n} (Y_i - \hat{\beta}_1 x_i)^2$

To find the value of $\hat{\beta}_1$ that minimizes SSR, we differentiate SSR with respect to $\hat{\beta}_1$ and set it equal to zero:

$\frac{dSSR}{d\hat{\beta}_1} = -2 * \sum_{i=1}^{n} x_i (Y_i - \hat{\beta}_1 x_i) = 0$

Expanding and rearranging, we get $\sum_{i=1}^{n} x_i y_i - \hat{\beta}_1 \sum_{i=1}^{n} x_i^2 = 0$

Now, isolate $\hat{\beta}_1$, $\hat{\beta}_1 \sum_{i=1}^{n} x_i y_i$

$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}$

$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i}$

$\hat{\beta}_1 = \frac{\bar{Y}}{\bar{X}}$

The mean squared Residual estimator is defined as $\hat{\sigma^2} = \frac{1}{n-1} \sum_{i=1}^{n} e_i^2$

where $e_i = Y_i - \hat{Y}_i$, substituting this equation in the above definition gives,
$\hat{\sigma^2} = \frac{1}{n-1} \sum_{i=1}^{n} (Y_i - \hat{\beta}_1 x_i)^2$

This gives us the expression for the mean squared residual estimator for σ^2.

The regression degrees of freedom represent the number of parameters estimated in the regression model. In simple linear regression, there are two parameters estimated: the slope (β_1) and the intercept (β_0) if it is included in the model. Therefore the $df = n - 2$

4. (10pts) Summary Statistics Can Hide Important Relationships. Call the dataset in R called Anscombe (1973). The purpose of this exercise is to demonstrate how plotting data can reveal important information that is not evident in numerical summary statistics.

a. Compute the averages and standard deviations of each column of data. Check that the averages and standard deviations of each of the x columns are the same, within two decimal places, and similarly for each of the y columns.

```
data("anscombe")
head(anscombe)
```

```
##   x1 x2 x3 x4   y1   y2    y3   y4
## 1 10 10 10  8 8.04 9.14  7.46 6.58
## 2  8  8  8  8 6.95 8.14  6.77 5.76
## 3 13 13 13  8 7.58 8.74 12.74 7.71
## 4  9  9  9  8 8.81 8.77  7.11 8.84
## 5 11 11 11  8 8.33 9.26  7.81 8.47
## 6 14 14 14  8 9.96 8.10  8.84 7.04
```

```
x_stats <- sapply(anscombe[, c("x1", "x2", "x3", "x4")], function(x) {
  c(mean = mean(x), sd = sd(x))
})


y_stats <- sapply(anscombe[, c("y1", "y2", "y3", "y4")], function(y) {
  c(mean = mean(y), sd = sd(y))
})

print(x_stats)
```

```
##            x1       x2       x3       x4
## mean 9.000000 9.000000 9.000000 9.000000
## sd   3.316625 3.316625 3.316625 3.316625
```

```
print(y_stats)
```

```
##            y1       y2       y3       y4
## mean 7.500909 7.500909 7.500000 7.500909
## sd   2.031568 2.031657 2.030424 2.030579
```

b. Run four regressions, (i) y_1 on x_1, (ii) y_2 on x_2, (iii) y_3 on x_3, and (iv) y_4 on x_4.

```
model_y1_x1 <- lm(y1 ~ x1, data = anscombe)
model_y2_x2 <- lm(y2 ~ x2, data = anscombe)
model_y3_x3 <- lm(y3 ~ x3, data = anscombe)
model_y4_x4 <- lm(y4 ~ x4, data = anscombe)
summary(model_y1_x1)
```

```
##
## Call:
## lm(formula = y1 ~ x1, data = anscombe)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.92127 -0.45577 -0.04136  0.70941  1.83882
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   3.0001     1.1247   2.667  0.02573 *
## x1            0.5001     0.1179   4.241  0.00217 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 1.237 on 9 degrees of freedom
## Multiple R-squared:  0.6665, Adjusted R-squared:  0.6295
## F-statistic: 17.99 on 1 and 9 DF,  p-value: 0.00217
```

```r
summary(model_y2_x2)
```

```
##
## Call:
## lm(formula = y2 ~ x2, data = anscombe)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.9009 -0.7609  0.1291  0.9491  1.2691
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3.001      1.125   2.667  0.02576 *
## x2             0.500      0.118   4.239  0.00218 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.237 on 9 degrees of freedom
## Multiple R-squared:  0.6662, Adjusted R-squared:  0.6292
## F-statistic: 17.97 on 1 and 9 DF,  p-value: 0.002179
```

```r
summary(model_y3_x3)
```

```
##
## Call:
## lm(formula = y3 ~ x3, data = anscombe)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.1586 -0.6146 -0.2303  0.1540  3.2411
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   3.0025     1.1245   2.670  0.02562 *
## x3            0.4997     0.1179   4.239  0.00218 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.236 on 9 degrees of freedom
## Multiple R-squared:  0.6663, Adjusted R-squared:  0.6292
## F-statistic: 17.97 on 1 and 9 DF,  p-value: 0.002176
```

```r
summary(model_y4_x4)
```

```
##
## Call:
## lm(formula = y4 ~ x4, data = anscombe)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -1.751 -0.831  0.000  0.809  1.839
```

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   3.0017     1.1239   2.671  0.02559 *
## x4            0.4999     0.1178   4.243  0.00216 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.236 on 9 degrees of freedom
## Multiple R-squared:  0.6667, Adjusted R-squared:  0.6297
## F-statistic:    18 on 1 and 9 DF,  p-value: 0.002165
```

Verify, for each of the four regressions fits, that $b_0 \sim 3.0$, $b_1 \sim 0.5$, $s \sim 1.237$, and $R^2 \sim 0.666$, within two decimal places.

As we can see the summary of those four regression models,

```r
round(coef(model_y1_x1)[1],2)
```

```
## (Intercept)
##           3
```

```r
round(coef(model_y2_x2)[1],2)
```

```
## (Intercept)
##           3
```

```r
round(coef(model_y3_x3)[1],2)
```

```
## (Intercept)
##           3
```

```r
round(coef(model_y4_x4)[1],2)
```

```
## (Intercept)
##           3
```

```r
round(coef(model_y1_x1)[2],2)
```

```
##  x1
## 0.5
```

```r
round(coef(model_y2_x2)[2],2)
```

```
##  x2
## 0.5
```

```r
round(coef(model_y3_x3)[2],2)
```

```
##  x3
## 0.5
```

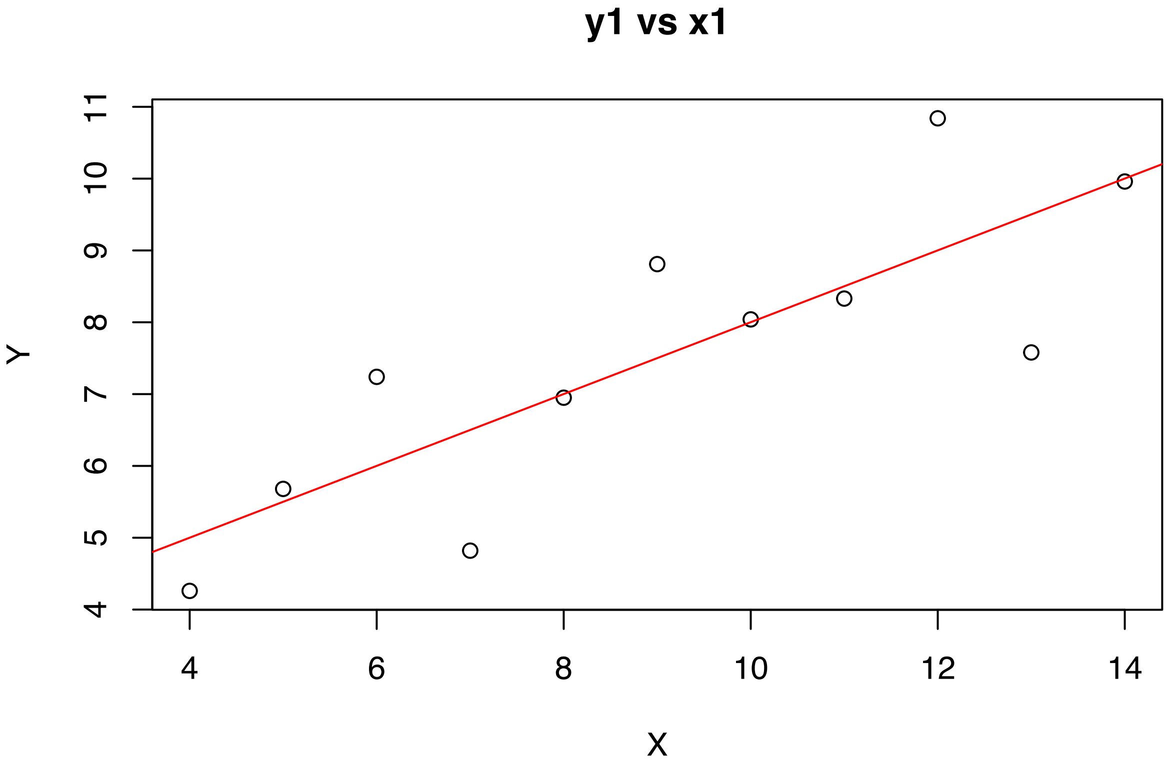
```r
round(coef(model_y4_x4)[2],2)
```

```
##  x4
## 0.5
```

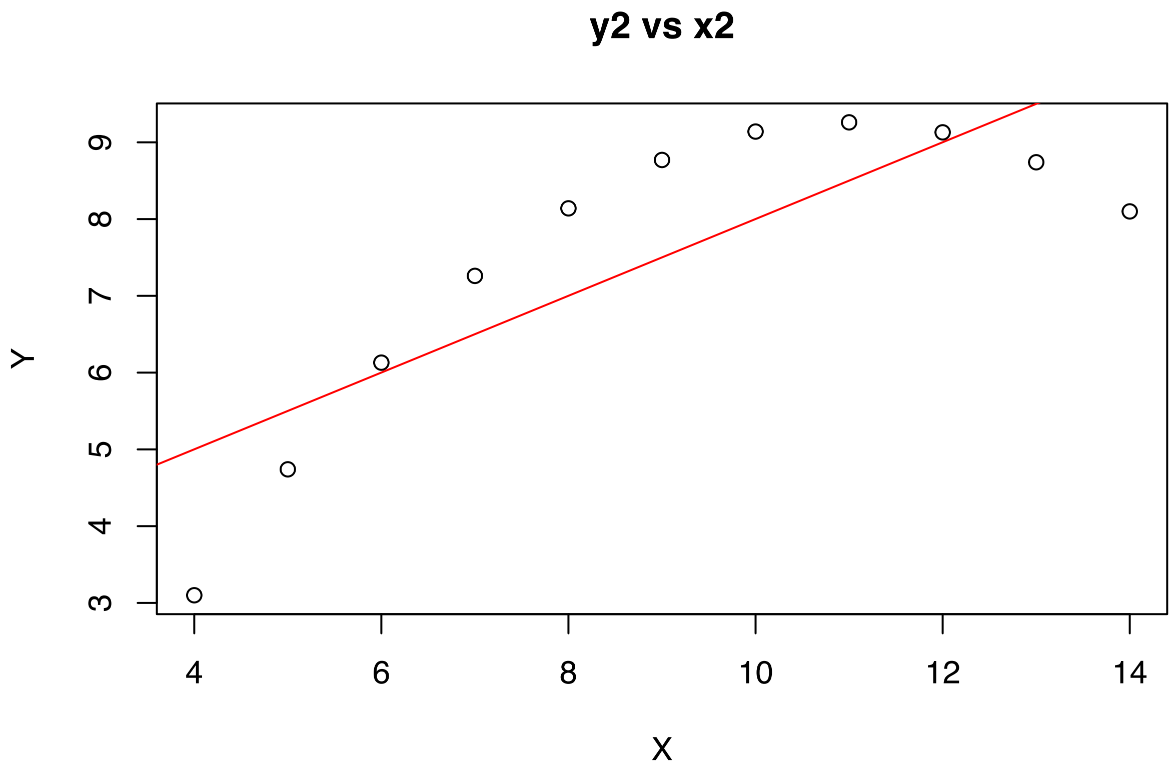
As we examine, the values of $s \sim$ and $R^2 \sim$ are 1.237 and 0.666

c. Produce scatter plots for each of the four regression models that you fit in part (b) and add the regression line in red.

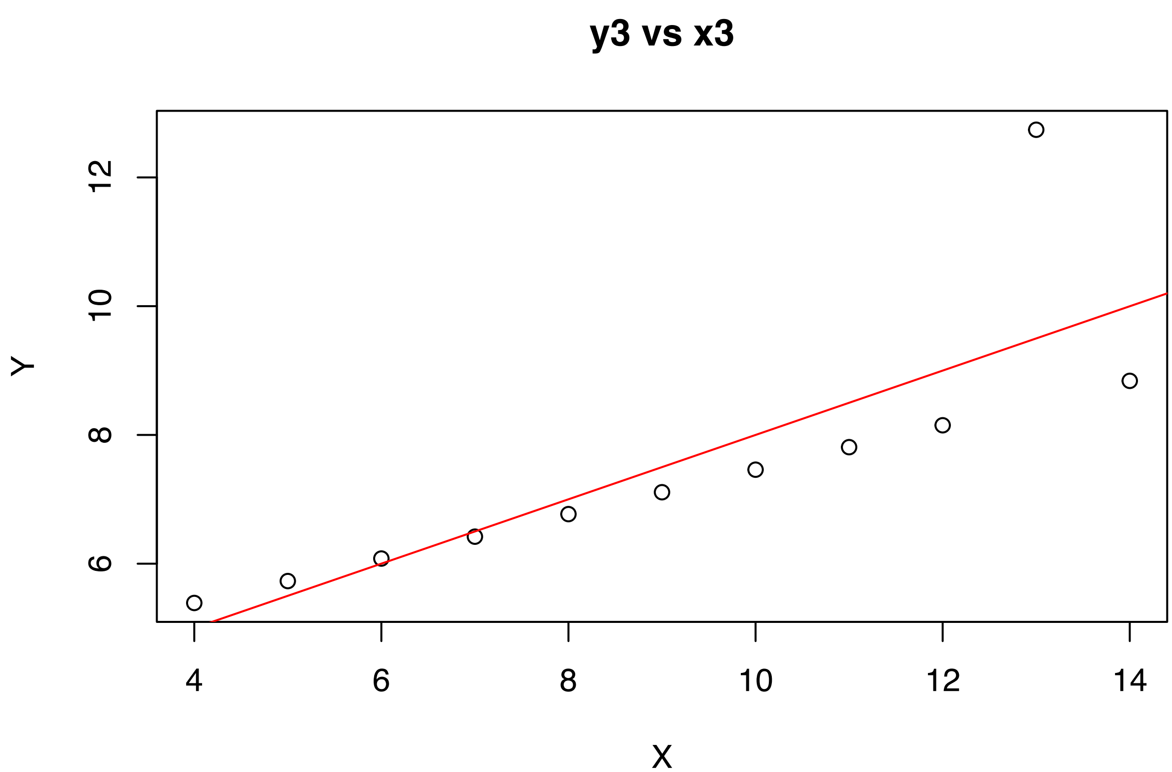
```r
plot_with_regression_line <- function(x, y, model, title) {
  plot(x, y, main = title, xlab = "X", ylab = "Y")
  abline(model, col = "red")
}
plot_with_regression_line(anscombe$x1, anscombe$y1, model_y1_x1, "y1 vs x1")
```



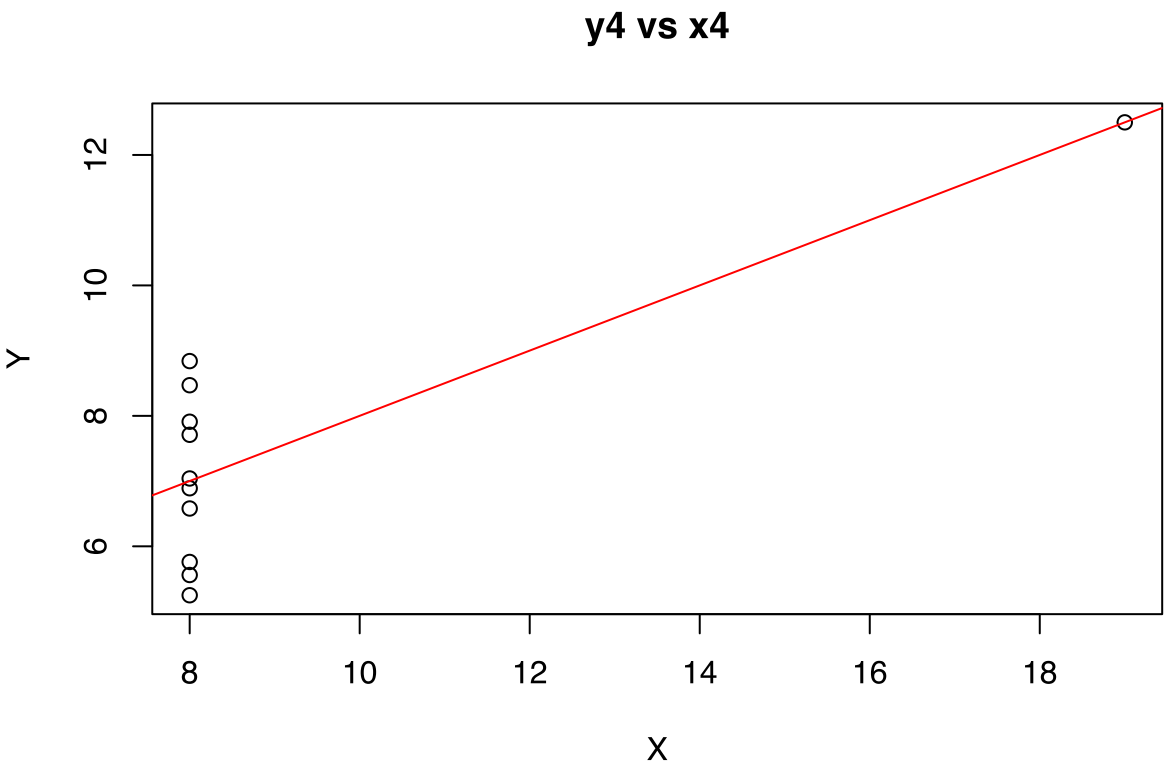

```r
plot_with_regression_line(anscombe$x2, anscombe$y2, model_y2_x2, "y2 vs x2")
```




```r
plot_with_regression_line(anscombe$x3, anscombe$y3, model_y3_x3, "y3 vs x3")
```




```r
plot_with_regression_line(anscombe$x4, anscombe$y4, model_y4_x4, "y4 vs x4")
```



d. Discuss the fact that the fitted regression models produced in part
   (b) imply that the four datasets are similar, though the four scatter plots produced in part (c) yield a dramatically different story.

Ans. The scatter plots in part c depicts a relationship between two numerical values. On the other hand, the regression analysis in part b displays a straight line representing the connection between dependent and independent variables. Consequently, when we perform a regression analysis, it establishes the connection between the x and y variables, assuming their similarity. However, when we create scatter plots, we treat these two x and y values as independent and plot them based solely on their respective values, disregarding their inherent relationship.

5. (5 pts) Effects of an Unusual Point. You are analyzing a data set of size $n = 100$. You have just performed a regression analysis using one predictor variable and notice that the residual for the 10th observation is unusually large.

a. Suppose that, in fact, it turns out that $e_{10} = 8s$. What percentage of the sum of squares errors, SSE, is due to the 10th observation?

Ans. Given data set size, n = 100

Given that the residual for the 10th observation is unusually large.

e_{10} = 8s

SSE = $\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$

$= \sum_{i=1}^{n} e_i^2$

also, we know that $s^2 = \frac{SSE}{n-k}$

rewriting the SSE in terms of s^2, SSE $= s^2(n-k)$

Given n = 100, k = 1

SSE $= \frac{8s^2}{\sum_{i=1}^{n} e_i^2}$

$= \frac{64s^2}{\sum_{i=1}^{n} e_i^2}$

$= \frac{64s^2}{s^2(100-1)}$

$= 0.64646465$

SSE in terms of percentage $= 0.64646465 * 100$

$= 64.64\%$

b. Suppose that $e_{10} = 4s$. What percentage of the sum of squares errors, SSE, is due to the 10th observation?

Ans. Given $e_{10} = 4s$

SSE $= \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$

$= \sum_{i=1}^{n} e_i^2$

also, we know that $s^2 = \frac{SSE}{n-k}$

rewriting the SSE in terms of s^2

SSE $= s^2(n-k)$

Given n = 100, k = 1

SSE $= \frac{(4s)^2}{\sum_{i=1}^{n} e_i^2}$

$= \frac{16s^2}{\sum_{i=1}^{n} e_i^2}$

$= \frac{16s^2}{s^2(100-1)}$

$= 0.16161616$

SSE in terms of percentage $= 0.16161616 * 100$

$= 16.16\%$

c. Suppose that you reduce the dataset to size $n = 20$. After running the regression, it turns out that we still have $e_{10} = 4s$. What percentage of the error sum of squares, Error SS, is due to the 10th observation?

Ans. Given data set size, n = 20

Error SS = $\frac{(4s)^2}{s^2(20-1)}$

$=\frac{16s^2}{s^2(19)}$

$= \frac{16}{19}$

$= 0.8421$

Error SS in percentage $= 0.8421 * 100$

$= 84.21\%$