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Experimentally obtaining the Speed of Light and Investigating its error

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Abstract

In this report, the speed of light was measured using a light transmitter, receiver and an oscilloscope connected to both components. This experiment involved reflecting incoming collimated light from the transmitter onto the receiver to observe the time delay in their electrical signals. The distance of the mirror was varied to find the change in the time delay. From this data, the value for speed of light was calculated from each trials, their average and the slope of a distance against time plot. The result with the highest accuracy obtained from these was $(1.67 \pm 0.39) \times 10^8$ ms^{-1} indicating significant errors in the experimental setup.

1 Introduction

This experiment aims to measure the two-way speed of light with the use of an oscilloscope and other electrical components. The two-way speed of light 'c' is the measurement of one round trip taken by light (where it is reflected by a mirror to change direction).

The first person to measure c was a Danish astronomer named Ole Roemer. He has made detailed observations of one of Jupiter's moons, Io and observed that during its eclipses, it took longer time intervals when Jupiter was further from Earth's orbit. This led to the conclusion that the speed of light was finite, and its speed was calculated to be 214,000 km/s.

The first non-astronomical measurement of c was made by Armand Fizeau who used a spinning cogwheel whose rotational velocity was varied such that incoming light would enter through one tooth and reflect through another. This method provided a value of 315,000 km/s. The method was later improved by Leon Foucault who replaced the cogwheel with rotating mirrors such that the change in angle of light provides information of its speed. The value obtained was 298,000 km/s (Froome and Essen, 1969).

After the development of Electromagnetism through Maxwell's research, numerous methods have been created involving electrical circuits to obtain values for c. In this investigation, the value of c was measured using an electrical circuit by simply measuring the time delay in the signal between a light transmitter and a receiver where both components send electrical signals. The delay in the signal is the time taken for light to make the round-trip. Hence the speed of light can be calculated with:

$$c = \frac{2L}{\Delta t} \tag{1}$$

Where L is the distance between the transmitter and the mirror (which is equal to the distance between the receiver and the mirror) and Δt is the delay in the signals observed.

To measure this delay in the signal, an oscilloscope can be used. The oscilloscope is an instrument which can measure electrical signals to a high precision compared to most other electrical measurement devices. This instrument can be connected to both the transmitter and the receiver at the same time and the time delay can be seen visually represented by peaks on the screen.

2 Methods

In this investigation a light transmitter was collimated and then reflected onto a receiver. The transmitter and the receiver were connected to the oscilloscope in the following way:

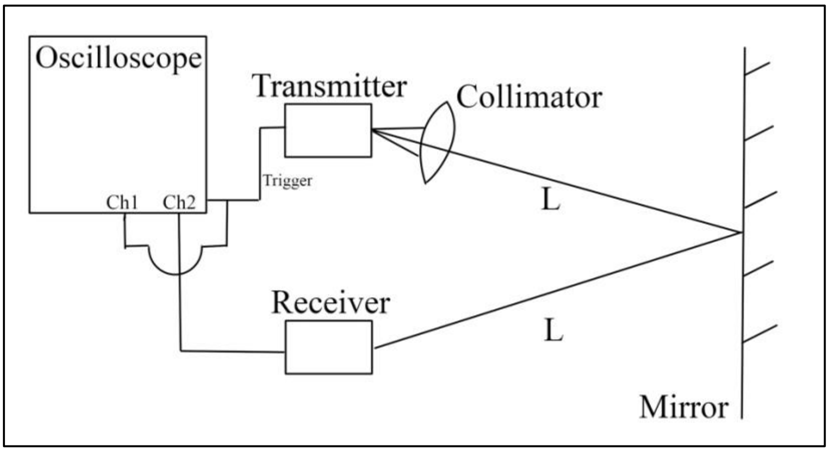

Figure 1: Diagram of Experimental Setup

The oscilloscope's accuracy was tested by calculating the frequency of the pulse of light which was set to 1 MHz. By changing the voltage per division and time per division, the signals were adjusted to view the differences clearly. Then, measurements of the delay were taken from both peak-to-peak and the base-to-base of the waves when the mirror was 0.5 m away from the transmitter.

After selecting the higher accuracy measurement, the time delay was measured for distances 0.55, 0.6, 0.65, 0.7, 0.75 and 0.8 m away from the mirror. After the measurements, the time delay was also measured when the transmitter and the receiver were not separated by any distance. This is to identify the time delay inherently present in the oscilloscope's and the receiver's mechanism.

3 Results

The values obtained from peak-to-peak and base-to-base are presented below (including compensation of oscilloscope's delay of 50 ± 1 ns):

	Peak-to-Peak	**Base-to-Base**
Time delay (Δt_i) (ns ± 1)	105	56
Time delay Corrected (Δt) (ns ± 1)	55	6
Speed of Light (*c*) (ms^{-1} x 10^8)	0.18	1.67
Error (%)	94	44

Table 1: Comparison between Peak-to-Peak and Base-to-Base measurement of signal delay

The percentage error was calculated using the theoretical value of *c*: 299,792,458 ms^{-1} which is the internationally recognized value used for the SI units. This shows that the base-to-base measurements yield more accuracy compared to the peak-to-peak measurements. Hence it was decided that only base-to-base measurements will be used for the rest of this investigation.

The results of the investigation are provided below (including compensation of oscilloscope's delay of 50 ± 1 ns):

Distance to Mirror (L) (cm ± 0.05)	**Time delay (Δt_i) (ns ± 1)**	**Time delay Corrected (Δt) (ns ± 1.4)**	**Speed of Light (*c*) (ms^{-1} x 10^8)**
50	56	6	1.67 ± 0.39
55	58	8	1.38 ± 0.24
60	60	10	1.20 ± 0.17
65	61	11	1.18 ± 0.15
70	63	13	1.08 ± 0.12
75	65	15	1.00 ± 0.09
80	66	16	1.00 ± 0.09

Table 2: Time Delay, Time Delay compensated observed in the signals and speed of light with varying distance

The compensated time delay and its error were calculated using the following equations:

$$\Delta t = \Delta t_i - 50 \tag{2.1}$$

$$\Delta(\Delta t) = \sqrt{1^2 + 1^2} \tag{2.2}$$

Where $\Delta(\Delta t)$ is the uncertainty in Δt (compensated time delay).

Using equation (1), the speed of light for different distances was calculated. The error for this value was obtained through the equation (Ku, 1966):

$$\Delta c = \sqrt{\left(\frac{\partial c}{\partial \Delta t}\right)^2 \left(\Delta(\Delta t)\right)^2 + \left(\frac{\partial c}{\partial L}\right)^2 (\Delta L)^2}$$

$$\Delta c = c\sqrt{\left(\frac{\Delta(\Delta t)}{\Delta t}\right)^2 + \left(\frac{\Delta L}{L}\right)^2} \tag{3}$$

From table 2 the average value of speed of light can be calculated with its error using error for small sets:

$$\Delta c_{avg} = \frac{c_{max} - c_{min}}{2\sqrt{N}} \tag{4}$$

Where Δc_{avg} is the uncertainty in the average value of c, c_{max} is the maximum value obtained, c_{min} is the minimum value obtained and N is the number of trials taken.

The average value for c obtained was:

$c = (1.21 \pm 0.12) \times 10^8\ ms^{-1}$ (59% error with theoretical value)

Another method to calculate *c* from the data of the investigation is to plot the distance (total distance) against the time delay to obtain the value of *c* from the slope:

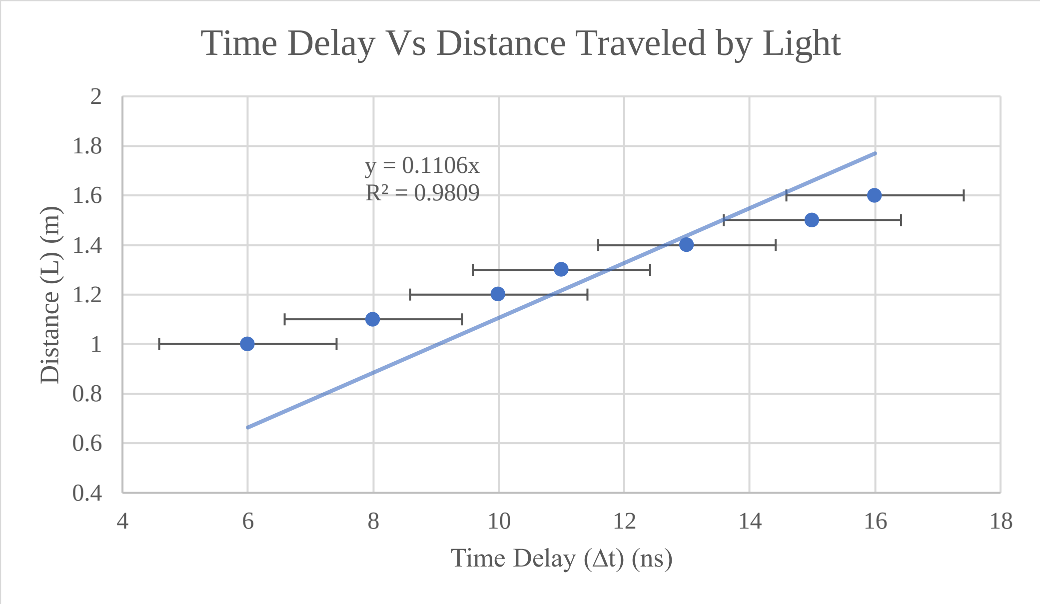

Figure 2: Distance Traveled plotted against Time delay to obtain speed of light through the slope (vertical error bars were drawn but not visible)

The value of the slope and its error were obtained from least squares method where the y-intercept was fixed to be 0 (which is the theoretical value as distance and time are directly proportional). The value of *c* obtained was:

$c = (1.11 \pm 0.06) \times 10^8$ ms^{-1} (63% error with theoretical value)

Though this value is more precise than previous values obtained for *c*, it is also less accurate. It is also seen that the error bars on the time delay are significantly overlapping, this makes the data recorded less reliable. Overall, the above plot suggests that the experiment has multiple sources of error which will be discussed further in the next section of the report.

4 Discussion

From the results, it is clear that there were significant errors present in the experiment cause a large discrepancy with the theoretical value of *c*. It is also seen in figure 2 that the line of best fit does not pass through most of the points (when intercept is set to 0) indicating discrepancy with the theoretical linear relationship. The largest source of error leading to inaccuracy in the results is present in the instruments used to measure the signals itself.

Firstly, the oscilloscope's least count is extremely high. This source of error arises from the fact that the signal will inherently have noise. Though the screen allows for further increasing the temporial resolution to identify the delay between the signals more carefully, it also reduces the clarity of the defined curves and more noise is introduced disrupting the measurement. When decreasing resolution to larger time scales, the peaks are more defined, and it is easier to identify the bases of the waves and the peaks of the waves.

This led to a trade-off between precision and definition of the time delay where the 1 ns time scale was decided to be a balance of both. The issue with this high least count is evident in figure 2 where there is a significant overlap between error bars, indicating that the data values are not fully reliable.

Secondly, the increments of distance are too minute to observe meaningful change in the time delay. This source of error arises from the fact that the oscilloscope has a limited resolution. To obtain reliable data from the device, it is key that the peaks are separated by a large time delay and to obtain an accurate value of c from the slope it is required that the change in time delay is also great. This would allow for the data to be more reliable as unlike in figure 2, the error bars would not overlap as significantly.

This shows that the amount by which the distance was changed was too little. A change in distance of 10 cm would only cause the time delay to increase by 0.3 ns which is too fine for the oscilloscope to measure with defined peaks.

Lastly, the beam of light becomes fainter with larger distance even when its collimated. This causes the receiver signal to become fainter and less defined with larger distances creating more uncertainty and inaccuracy in the measurement of the time delay. The receiver has also received some background radiation making the signal less defined. The signal of the receiver was observed to be very inconsistent and slight changes to the positions of the mirror, transmitter or collimator changed the signal significantly. As the distance was increased the peak and base of the signal has also become more spread making it more difficult to measure the time delay accurately.

These sources of errors can be fixed by taking larger distances, larger increments of distances and stronger light source or a larger surface for the receiver. It is also vital to conduct the experiment in a darker environment to reduce noise from background light spilling through. The first two improvements help to compensate the issues with the resolution of the oscilloscope, mainly by reducing the effects of the high least count and allowing for more accurate measurements of the slope by increasing the distance by a larger amount (which changes the time delay by a more significant amount). The stronger light source or larger receiver help with the issues regarding the clarity of the signal by making it more defined and thus the time delay easier to measure.

Therefore, future investigations should focus on how to produce a stronger signal of light for larger distances between the transmitter and the receiver to obtain values, with more accuracy, for the speed of light c.

5 Conclusions

This experiment delves into a simple and logical method to measure the speed of light using an oscilloscope. With a relatively simple setup, it allows to easily obtain values of c and helps highlight major issues present within the method and effectively shows the requirement of high precision when measuring extremely high velocities.

The most accurate value of *c* obtained in this investigation (as well as the least precise) is $(1.67 \pm 0.39) \times 10^8$ ms^{-1} which has an error of 44% with the theoretical value. Through some careful consideration of the experimental setup, the sources behind this high discrepancy were identified allowing for a wide range of improvements which future investigations can implement to find more reliable results.

In summary, this experiment is not successful as it has failed to obtain a value for *c* with a high degree of precision or accuracy. However, this investigation did highlight the disadvantages present when using an oscilloscope to measure the speed of light and how to minimise their effects.

References

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