IBDP Physics Extended Essay
May 2023

Topic

Fluid Mechanics

Title

The Effect of Boat Velocity on its Wake Angle

Research Question:

"To what extent does the velocity of a boat affect the angle of its wake, causing a change in its efficiency?"

Word Count: 3958

Index

1 Introduction

1.1 Idea and Importance

The Ship wake is a specific shape a body travelling in water produces. The Ship wake is influenced by many factors such as the speed of the body travelling, viscosity and also the depth of the water[1]. These factors affect the shape of the wake which is evident in the change in angle of the wake. Understanding the physics behind a boat wake is also important in increasing the efficiency of a water body. The angle and shape of the wake provides evidence about fuel consumption. A narrower angle of a boat indicates that the boat is using less fuel in pushing itself forward. However, if the speed is changing but the angle does not vary drastically, it shows an inefficient boat model and indicates that higher speeds are not favourable due to fuel consumption.

1.2 Application

Investigating water wakes has a large application in understanding water vehicles and the best methods for transportation. The specific shapes, angles, and differing patterns in a wake can help engineers to form more efficient and helpful models for water transportation. Understanding the effect of velocity on the ship wake will also help ship captains to use less fuel and travel through water more efficiently. This same concept of ship wakes can be extended for air transportation to reduce fuel consumption.

[1] Brandslet, Steinar. "New Experiments Prove the Physics of Ship's Wakes." The Maritime Executive, 2019, accessed May 2022. https://www.maritime-executive.com/editorials/new-experiments-prove-the-physics-of-ship-s-wakes.

1.3 Physics and Rationale

Ship wake physics is a direct relation to properties of waves. To understand wakes at different depths, interference patterns and the interactions between layers of water should be known. The formula for ship wakes was first developed by Lord Kelvin and thus the almost identical shape shared between various ships is known as the Kelvin wake[2]. According to his theoretical development, Velocity and Angle are inversely proportional at shallow depths of water.[3] Thus, this investigation will explore the implications and authenticity of Kelvin Wave Theory.

1.4 Research Question

“To what extent does the velocity of a boat affect the angle of its wake, causing a change in its efficiency?”

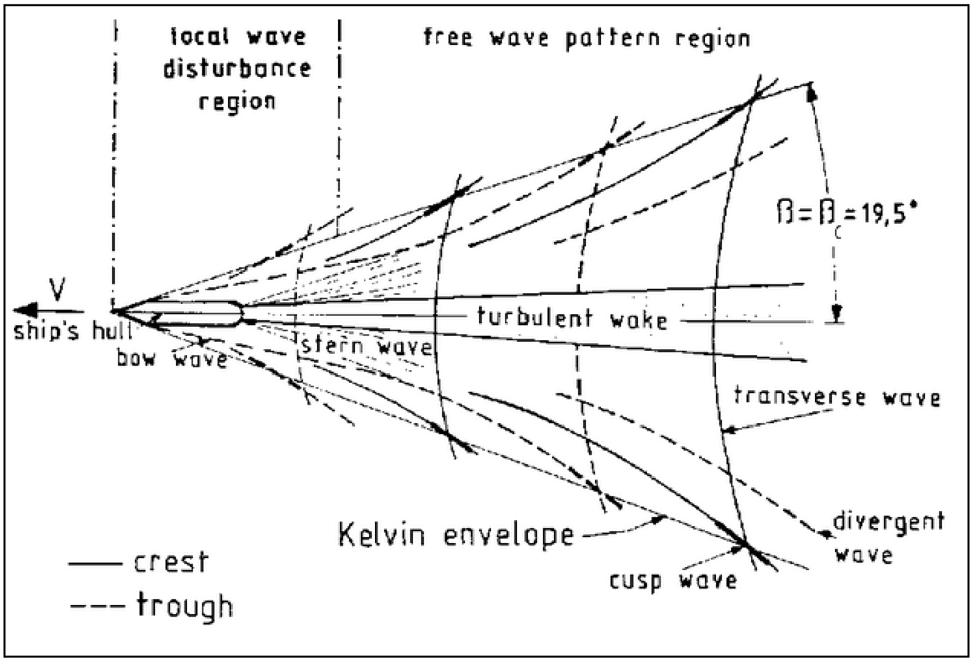
2 Background Information

2.1 Theoretical Development

Understanding Ship wakes can help reduce the losses of Water transportation as large amounts of fuel is primarily utilised to produce ship wakes. To accomplish this, the formation of a wake must be discussed first. The wake of a ship is formed due to interference of multiple water waves and contains various components such as stern waves, eddy currents or even foam. Lord Kelvin was the first one to delve into this topic and was able to identify several properties of the ship's wake. With his observations of

[2] McAlpine, Lauren. “The Fascinating Physics of Boat Wakes.” TED, 2021, accessed May 2022. https://ed.ted.com/best_of_web/kEQUGWqB#digdeeper.
[3] Fitzpatrick, Richard. “Ship Wakes” Ship Wakes, 2013, accessed May 2022. https://farside.ph.utexas.edu/teaching/315/Waveshtml/node70.html.

the ship wake, Kelvin was able to theorise the appearance of a wake with its



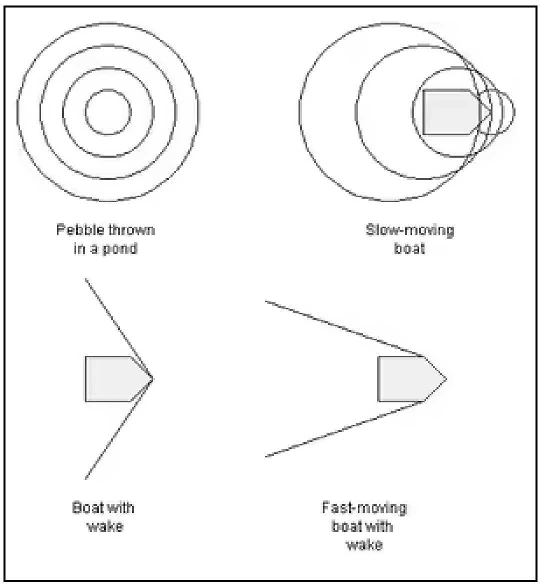
components[4]:

- Figure 1 *(diagram of components of a boat wake)*

As seen in this appearance, the wake is composed of multiple water waves. The resulting angle observed is created by the interference of all these waves. Over the wake, curved lines can also be observed which are labelled as transverse waves in the diagram. However, the ship wake is composed of two main elements as observed in the diagram. One is the divergent wave (or the Kelvin wake) of subsequent crests and troughs while the other is the turbulent wake which arises due to turbulence in the water or foam. However, the visibility of the various components of a wake depends on the water conditions and also the body travelling through it. Wakes are best observed in still water with little disturbances and with ships of large hulls and faster velocity.

[4] Christian Melsheimer, "Ship Wakes Observed with ERS and Spot," Centre for Remote Imaging, Sensing and Processing, CRISP (CRISP, 2001), https://crisp.nus.edu.sg/~research/shipwakes/shipwakes.htm.

As observed in the diagram, the shape of a ship's wake is similar to a sonic boom[5]. A sonic boom occurs when a body travelling through air exceeds the speed of sound leading to a shockwave due to the intense pressure caused by the compression of the soundwaves. This comparison between boat wakes and sonic booms provides an idea about the origins of boat wakes. Essentially, similar to bodies travelling in air, bodies travelling in water must exceed a certain speed (speed of the water waves) to create a defined wake trail. This leads to a similar interference pattern as the water waves are superimposed, creating the familiar wake.



- Figure 2 *(change in the wake shape with change in velocity of the boat)*

However, Kevlin's wake theory has not stayed foolproof. Simen Ådnøy Ellingsen is a scientist who has tried to go deeper into ship wake physics with this ideology of saving fuel losses[6]. He has conducted simulations and theoretical research to conclude

[5] "What Causes a Sonic Boom?" 1 April 2000. HowStuffWorks.com. <https://science.howstuffworks.com/question73.htm> December 2022
[6] Brandslet, Steinar. "New Experiments Prove the Physics of Ship's Wakes." The Maritime Executive, 2019, accessed May 2022. https://www.maritime-executive.com/editorials/new-experiments-prove-the-physics-of-ship-s-wakes.

the accuracy of Kelvin’s theory of ship wakes. Several experiments of ship wakes were done to test Ellingsen’s works and proved his conclusions. According to Ellingsen, Kelvin’s theory is not completely accurate as they show that all ship wakes should look identical. He points out that this is not the case due to many factors affecting the interference pattern. However, he also points out that Kelvin’s theory is not disproved with these experimentations as they still apply in shallow depths of water.

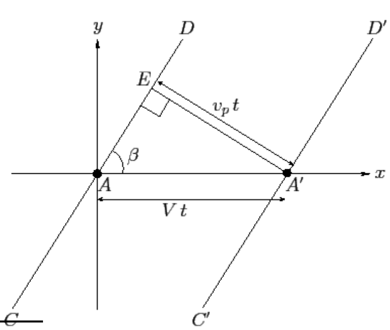
The wake angle has been a method of finding efficiency of bodies travelling in water. This is known as the wake fraction[7]. However, the wake fraction is not a direct application of the wake physics but instead relates to the turbulence caused by the propeller and how to optimise it.

2.2 Formulae

To derive the Kelvin formula for shallow water, the concept of gravitational waves must be used. This formula explains the speed of the water waves at a shallow depth of water. This is given by the formula:

$$v_p = (g\,d)^{1/2}$$ - Equation 1

Where v_p is the speed of the water waves, g is the acceleration due to gravity and d is the depth of the water.



- Figure 3 *(Simplified boat wake)*

[7] Artyszuk, J. “Wake Fraction and Thrust Deduction during Ship Astern Manoeuvres.” WIT Press, 2003, accessed July 2022. www.witpress.com.

From the Diagram of a ship wake, concepts of trigonometry can be used to identify the formula for the angle of the wake.

In this diagram the distance travelled by the ship is given by Vt where V is the velocity of the boat and the length of the wavefront is given by the $v_p t$ where v_p is the speed of the waves. From this, the trigonometric ratio, sine, can be used to identify angle in terms of Velocity of the boat:[8]

$$\beta = \sin^{-1}\left[\frac{(g\,d)^{1/2}}{V}\right]$$ - Equation 2

A caveat to keep in mind here is that V should always be greater than the critical speed which is the speed of the water waves. This is because when V is smaller than the critical speed, the ship wake does not form at the tail of the boat but near the centre.

2.3 Hypothesis

If the Velocity of the boat increases the angle of the wake will decrease. Hence they are inversely proportional to each other as per the second equation given above.

The efficiency of the boat increases with velocity. This is because a narrower angle of the wake will decrease the drag force which in turn increases efficiency. Therefore, the efficiency will increase with velocity at first but start to decrease as the change in angle will become insignificant with subsequent changes in velocities.

[8] Fitzpatrick, Richard. "Ship Wakes" Ship Wakes, 2013, accessed May 2022. https://farside.ph.utexas.edu/teaching/315/Waveshtml/node70.html.

2.4 Variables

Independent Variable: Speed of the Boat (v) (21.4 cms^{-1}, 35.3 cms^{-1}, 52.2 cms^{-1}, 75.0 cms^{-1}, 92.3 cms^{-1} and 120 cms^{-1})

Using a motor to pull the boat at different speeds

Dependent Variable: Angle of Boat Wake (θ)

By using a camera, the footage of the wake can be filmed from above the model boat to identify the angle of the wake later.

Controlled Variables:

Variable	Why is it Controlled?	How is it Controlled?
Depth of Fluid (10 cm)	Due to the depth of the fluid, the influence of the boat's velocity on the wake will change as the fluid will behave differently with increased depth and pressure changes.	The depth is controlled by keeping a fixed measure of 10cm throughout the experiment. This value is calculated based on the formula of a boat wake.
Viscosity of Fluid (0.01 Nsm^{-2})	At different viscosities, the properties of the fluid will change causing differences in the angle and shape of the boat wake.	To keep constant viscosity, the same source and type of liquid can be used. For this experiment, water is used to imitate real life conditions of a boat wake which has viscosity of 0.01 Nsm^{-2}.
Power Output of the Motor	The motor should have a constant power output to ensure that the change in velocity is consistent with the change in wake angle.	To control this, the overall resistance of the circuit and the motor is kept constant giving a power output of 15 Watts.
Shape of the Boat	Due to the shape of the hull of the model boat, the	To keep this constant, the same boat model must be

	shape of the wake will change and will affect its angle leading to an error.	used throughout the experiment to ensure a consistent shape of the wake.

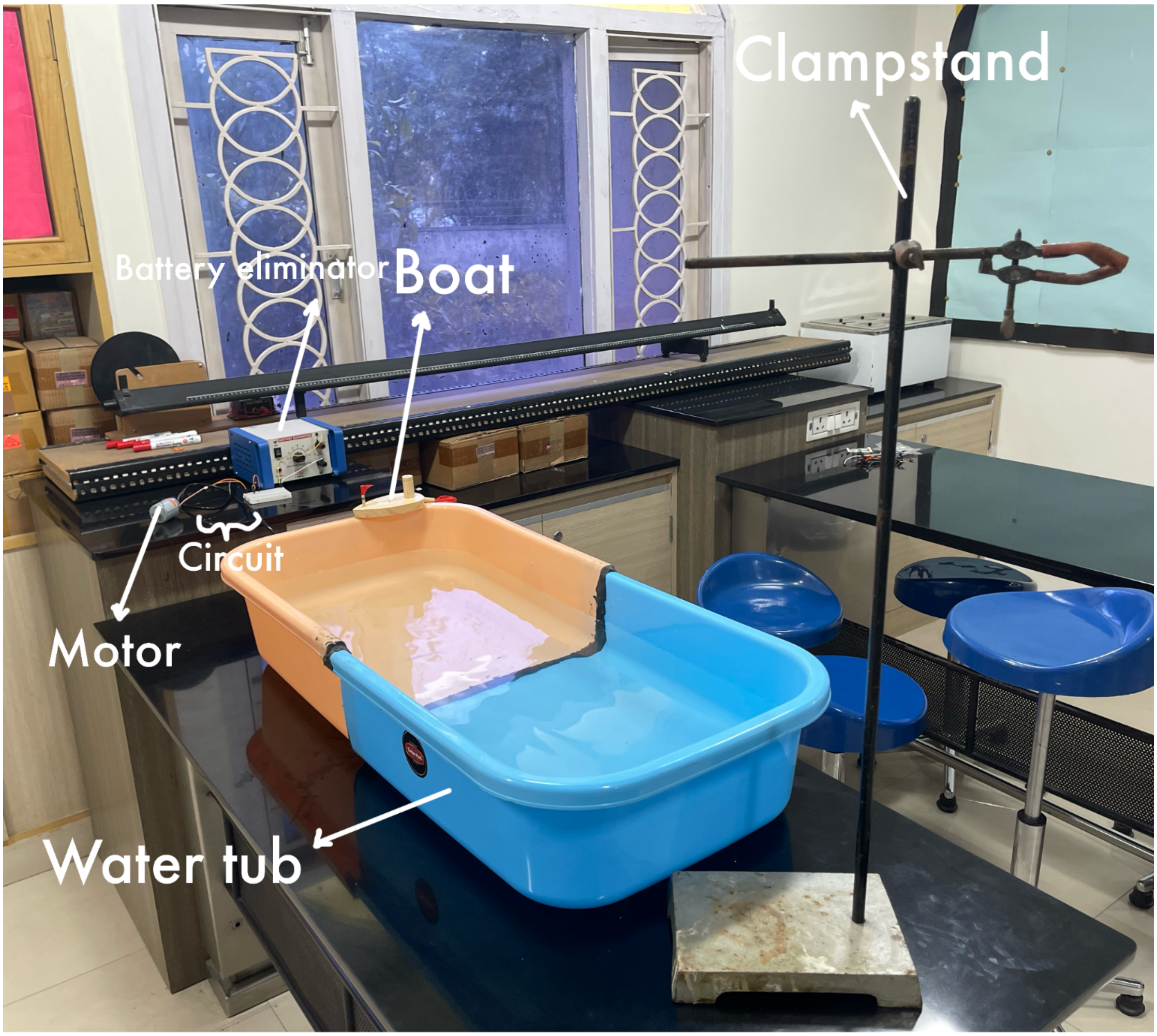
3 Experiment

3.1 Materials

Apparatus	Specifications	Uncertainty
Model Boat - 1	Necessary for producing the boat wake. At least 5 cm wide with an aerodynamic hull to produce an evident boat wake.	NA
Brushed DC Motor - 1	Necessary for accelerating the boat to a constant velocity. It should have a variable RPM (DC motor)	NA
Battery Eliminator - 1	Necessary to power the motor at different RPMs Has a range from 0V to 12V	Uncertainty: ± 2 V
Breadboard - 1	Necessary to build the circuit between the battery eliminator and the motor	NA
Resistor - 1	Necessary to prevent overload Must have resistance of 220 ohms	NA
Wires - 4	Necessary to build the circuit	NA
Camera - 1	Necessary to identify angle of the wake. Should record at 24 frames/second to allow quality investigation	Uncertainty: ± 1/60 seconds
String - 1	Attached to the boat and the motor.	NA
Water Tub - 1	Should have a height of at least 15 cm to accommodate the necessary water depth.	Uncertainty: ± 0.5 cm

Protractor - 1	A digital protractor is necessary to measure the angle of the wake. Should be	Uncertainty: ± 0.5 degrees
Clamp Stand - 1	Necessary to hold the Camera above the experimental setup	NA

3.2 Experimental Setup



- Figure 4 *(Experimental Setup of the tub, boat, clamp stand and circuit)*

3.3 Procedure to find Angle of the Boat Wake

1. Set up the apparatus by pouring water into the water tub for 10 cm depth
2. Attach the axle of the motor to the boat using the string and place the boat in the water tub
3. Set the Voltage supply to 2V using the battery eliminator
4. Place the boat at the end of the tub and record the boat wake with the camera
5. Record the wake with the same speed two more times
6. Measure the Angle of the wake by placing a digital protractor on top of the footage
7. Repeat the steps 3 to 6 for the voltages 4V, 6V, 8V, 10V, 12V

3.4 Procedure to find Velocity of the Boat

1. Place the boat in the same apparatus and create two markings on the side of the boat 10 cm apart
2. Set the voltage to 2V and record the boat travelling
3. Measure the velocity of the boat at this velocity by identifying the time taken for the boat to travel 10 cm
4. Repeat steps 1 to 3 for Voltages 4V, 6V, 8V, 10V and 12V

Voltage Vs Velocity:

Voltage (V ± 1)/Volts	Velocity (v ± 11.86)/(cm/s)	
	Value	Uncertainty
2	21.40	±0.87
4	35.30	±2.25
6	52.20	±4.80
8	75.00	±9.75
10	92.30	±14.66
12	120.00	±24.60

3.5 Safety, Environmental and Ethical Considerations

- Safety: As the procedure requires electrical circuits to be in close proximity to water, it is important to prevent the circuit from ever touching the tub as this can cause an electrical shock. This could ruin the equipment and can cause harmful damage to anyone in contact with either the water or the circuit by electrocution. This can be avoided by creating proper insulation between the motor and the setup and also for the wires. Insulated gloves can be used to avoid electrocution by contact and shoes can be worn to avoid earthing.

- Environmental: The experiment uses a plastic tub which can be reused and recycled to avoid any environmental harm

- Ethical: The experiment requires no ethical considerations

3.6 Data Collection and Processing

Table 1: Raw Data Table

Velocity (v ± 11.86)/(cm/s)	Angle (θ ± 1)/degrees		
	Trial 1	Trial 2	Trial 3
21.40	109	108	110
35.30	94	95	92
52.20	78	77	78
75.00	73	70	74
92.30	65	66	63
120.00	54	51	56

To calculate the uncertainty, the least count of the protractor can be used. This provides an uncertainty of ±1.0. However, when $\frac{max - min}{2}$ formula is used, the uncertainty value is much higher. Therefore, the larger value of uncertainty is used instead.

Example of Data Collection Using Protractor:

Figure 5.1 *(Picking a Frame from the Footage)*

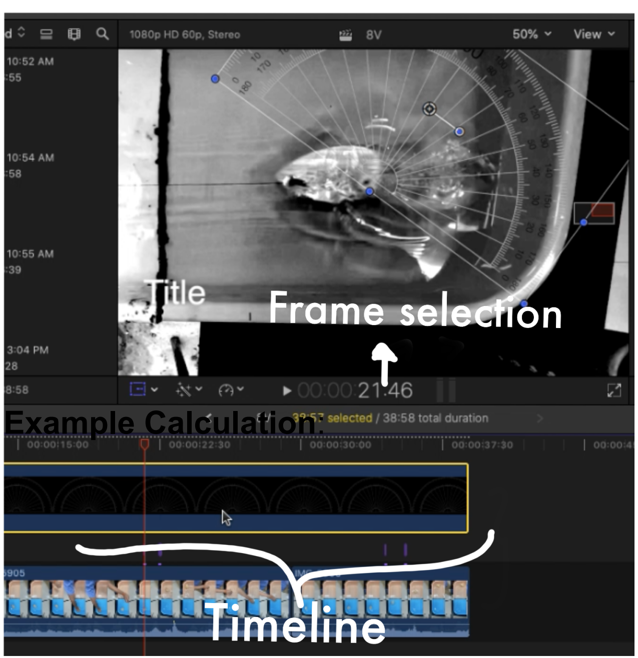
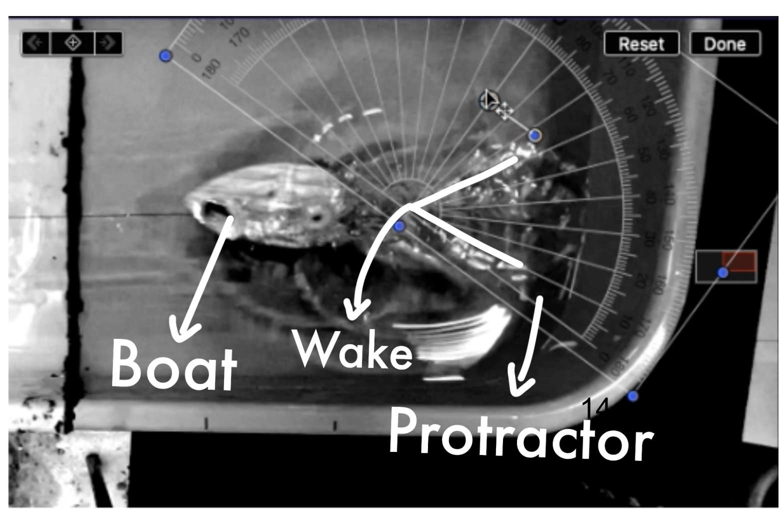


Figure 5.2 *(Measuring angle at the Frame)*



Example Calculation:

Data point 1:

Value = 109

Max Value = 110

Min Value = 108

$$Uncertainty = \frac{110 - 108}{2}$$

$$= \pm 1$$

On the other hand, the velocity of the boat was calculated by using the formula:

$$v = \frac{d}{t}$$ - Equation 3

Where the distance was marked on the tub (10 cm) and the time was recorded using the number of frames the boat took to travel:

Figure 6.1 *(Boat at the Initial Point of the marked distance)*

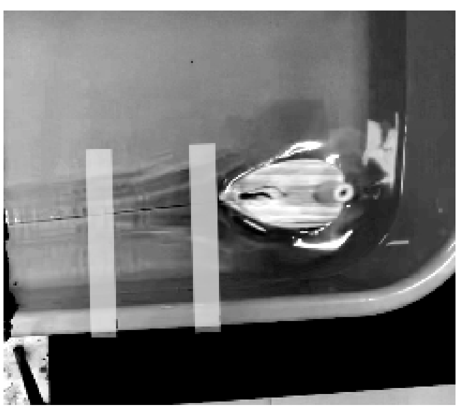
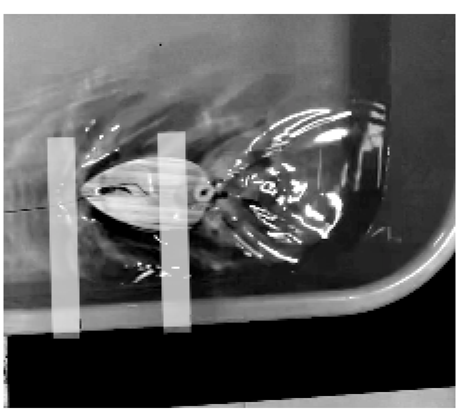


Figure 6.2 *(Boat at the Final Point of the marked distance)*



From this equation, the absolute uncertainty in the velocity can be calculated through:

$$\Delta v = v(\frac{\Delta d}{d} + \frac{\Delta t}{t})$$ - Equation 4

Example Calculation:

v = 21.4 cm/s

d = 10 cm

t = 0.467 s

Δd = 0.05 cm (Half of least count)

Δt = 0.017 s (1/60 due to the recording being 60 fps)

$$\Delta v = 21.4(\frac{0.05}{10} + \frac{0.017}{0.467})$$

$$\Delta v = \pm 0.87$$

To calculate the absolute uncertainty of the sine values, they must be calculated for each trial and then $\frac{max - min}{2}$ formula can be utilised:

$$Uncertainty = \frac{sin_{max}(\frac{\theta}{2}) - sin_{min}(\frac{\theta}{2})}{2}$$ - Equation 5

Example Calculation:

$$Uncertainty = \frac{sin(108/2) - sin(110/2)}{2}$$

$$Uncertainty = \pm 5.06 * 10^{-3}$$

Table 2: Velocity, Angle and Sine of Angle

Velocity (v ±)/(cm/s)		Angle (θ ± 1.0)/degrees		sin(Angle/2) $sin(\beta)$/(no units)*	
Value	Uncertainty	Average	Uncertainty	Value	Uncertainty
21.40	±0.87	109.0	±1.0	0.814116	$\pm 5.06 \times 10^{-3}$
35.30	±2.25	93.7	±1.5	0.729566	$\pm 8.97 \times 10^{-3}$
52.20	±4.80	77.7	±0.5	0.627284	$\pm 3.40 \times 10^{-3}$
75.00	±9.75	72.3	±2.0	0.589901	$\pm 1.41 \times 10^{-2}$
92.30	±14.66	64.7	±1.5	0.535090	$\pm 1.11 \times 10^{-2}$
120.00	±24.60	53.7	±2.5	0.451656	$\pm 1.95 \times 10^{-2}$

$*\theta = 2\beta$

Table 3: Velocity, Sine of Angle, Reciprocal of Sine of Angle

Velocity (v ± 11.86)/(cm/s)		sin(Angle/2) $sin(\beta)$/(no units)		1/sin(Angle/2) $\frac{1}{sin(\beta)}$/(no units)	
Value	Uncertainty	Value	Uncertainty	Value	Uncertainty
21.40	±0.87	0.814116	$\pm 5.06 \times 10^{-3}$	1.228	$\pm 7.63 \times 10^{-3}$
35.30	±2.25	0.729566	$\pm 8.97 \times 10^{-3}$	1.371	$\pm 1.68 \times 10^{-2}$
52.20	±4.80	0.627284	$\pm 3.40 \times 10^{-3}$	1.594	$\pm 8.64 \times 10^{-3}$
75.00	±9.75	0.589901	$\pm 1.41 \times 10^{-2}$	1.695	$\pm 4.05 \times 10^{-2}$
92.30	±14.66	0.535090	$\pm 1.11 \times 10^{-2}$	1.869	$\pm 3.88 \times 10^{-2}$
120.00	±24.60	0.451656	$\pm 1.95 \times 10^{-2}$	2.214	$\pm 9.56 \times 10^{-2}$

Example Calculation:

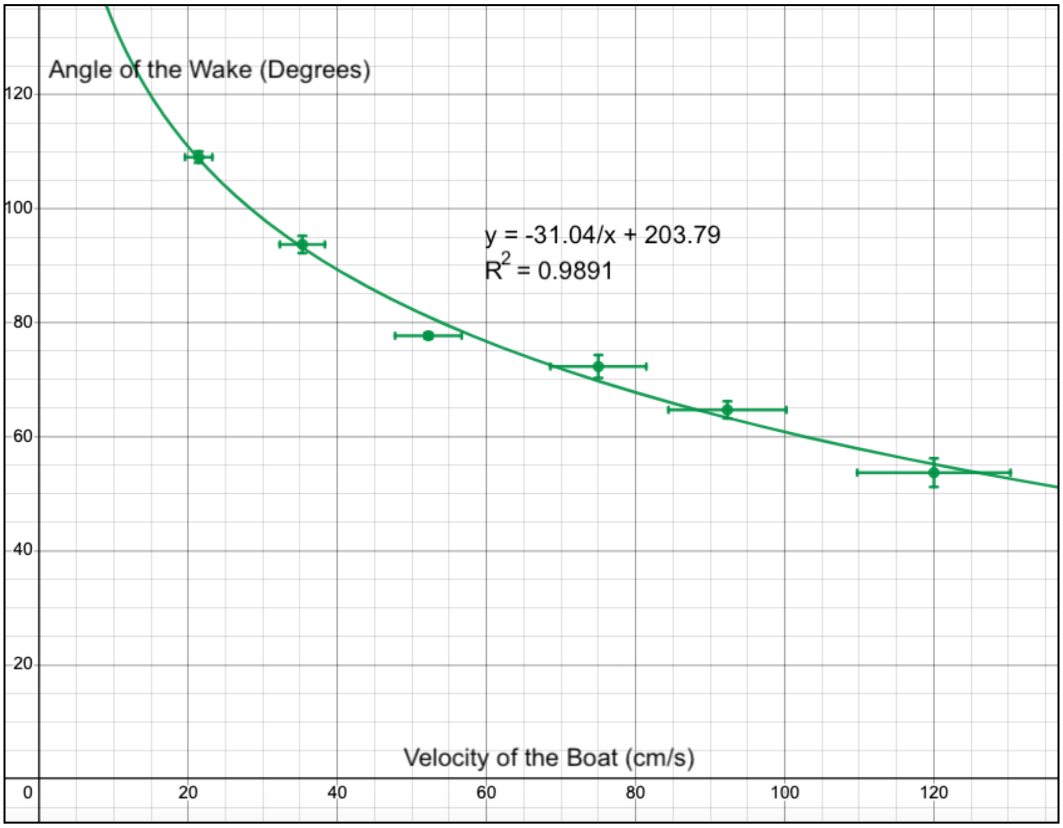
$$Uncertainty = 1/sin(\beta)(\frac{\Delta sin(\beta)}{sin(\beta)})$$ - Equation 6

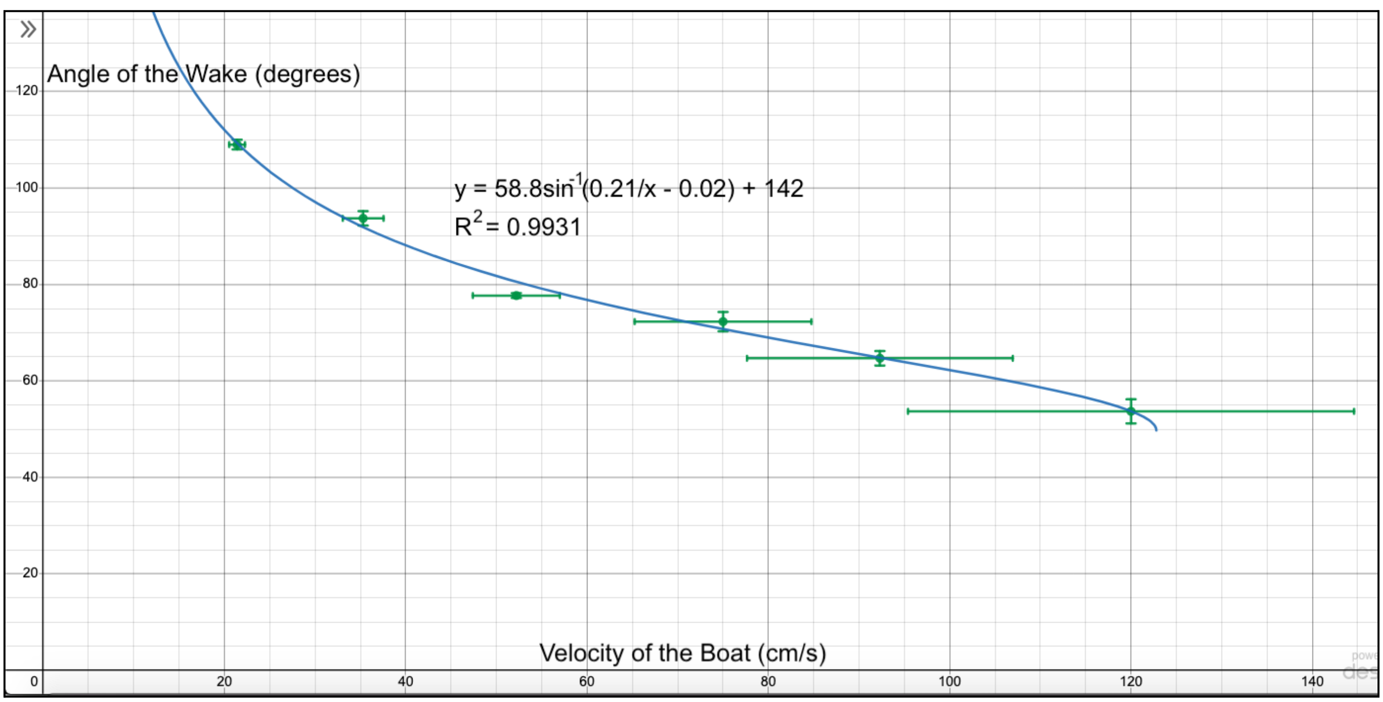
$$\Delta 1/sin(\beta) = 1.228(\frac{5.06(10^{-3})}{0.814116})$$

$$\Delta 1/sin(\beta) = \pm 7.63 * 10^{-3}$$

3.7 Data Analysis

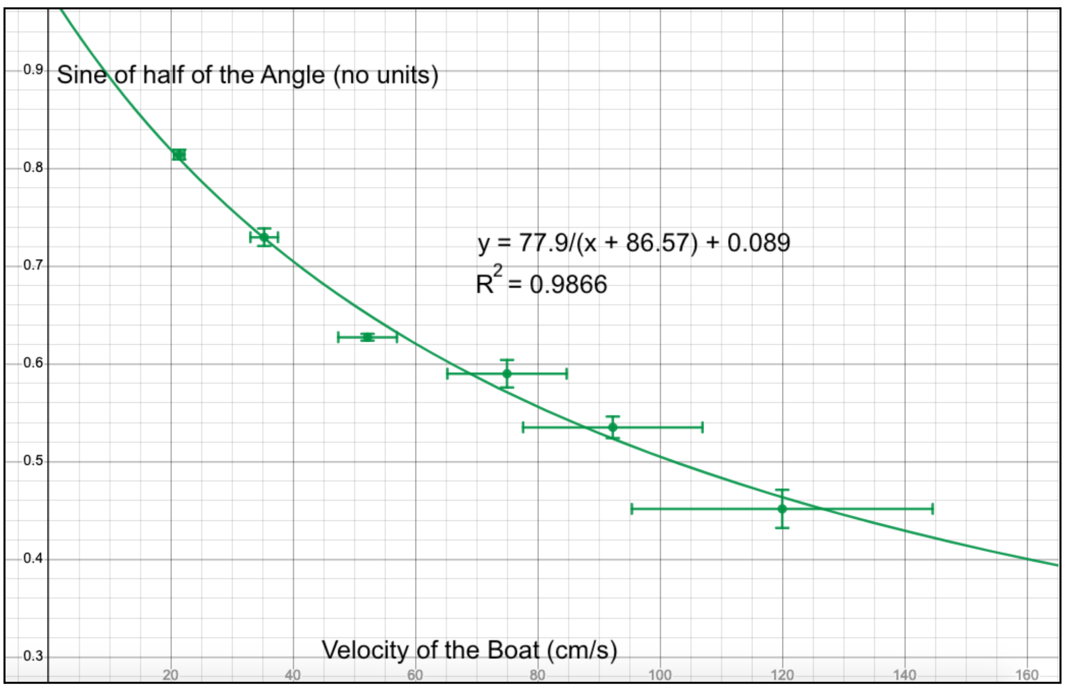
Velocity of the Boat Vs Angle of the Wake: (Graph 1.1 and 1.2 From Table 1)





From the two graphical representations of the data, it is visibly seen that the data correlates with the sine inverse function better. This function has a higher R^2 value and passes through most of the error bars. However, the equation of the curve of best fit also indicates that there is a systematic error in the experiment. This could be due to the reason that the angle measured is actually twice of the angle provided in the equation. Hence the graph with the sine of angle would provide a more accurate version of this data and confirm the equation's validity.

Graph 2: Velocity of the Boat Vs Sine of Angle: (Graph 2 From Table 2)



The expected graph was an inverse function as in the equation of the angle, sine of theta is inversely proportional to the velocity of the boat . This is confirmed by the shape of the graph. However, the graph also contains a significant systematic error. The coefficient of the fraction is also not equal to the critical speed, $(gd)^{½}$, as the equation suggests. The source of these differences will be discussed further in evaluation and conclusion.

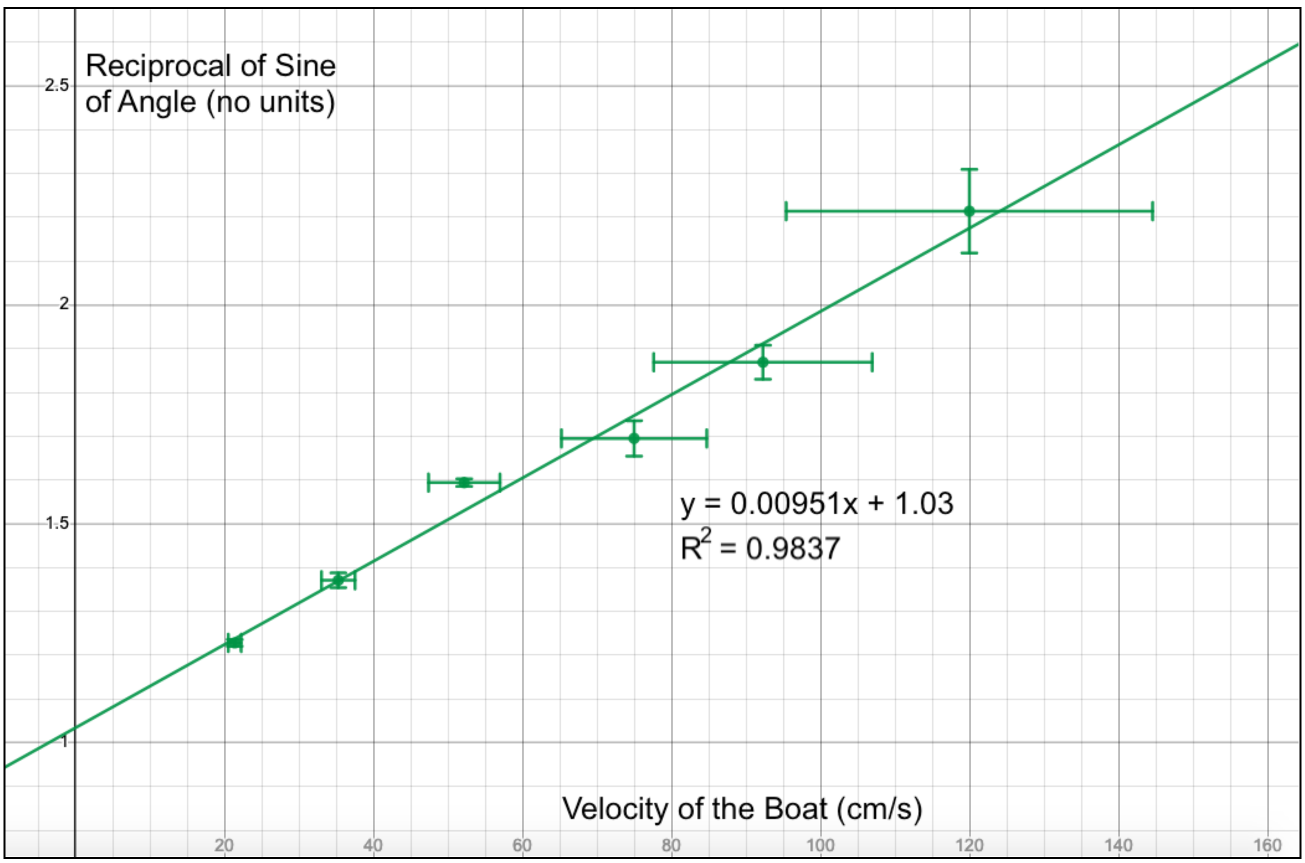
Final Relationship:

$$sin(\beta) = \frac{77.9}{V + 86.6} + 0.089$$ - Equation 7

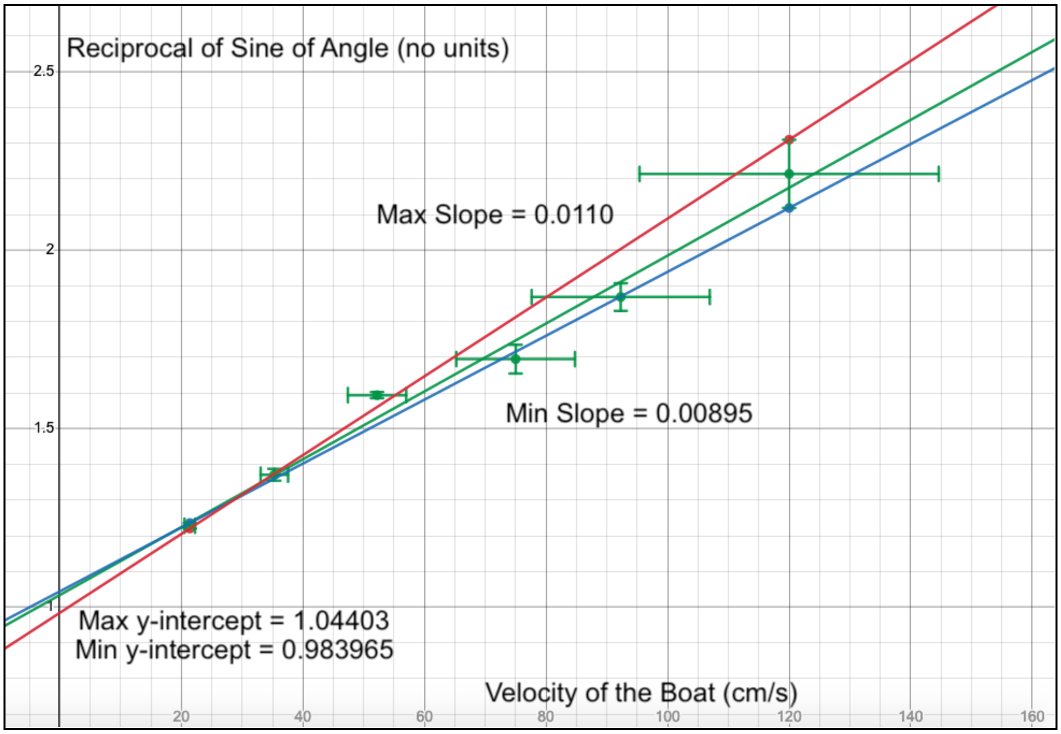
Where β is half of the angle of the wake and

V is the velocity of the boat measured in cm/s

Velocity of the boat Vs Reciprocal of Sine of Angle: (Graph 3 From Table 3)



Uncertainty in Slope:



$$Uncertainty = \frac{Max - Min}{2}$$

$$Uncertainty = \frac{0.01104 - 0.00895}{2}$$

$$Uncertainty = \pm 1.05 * 10^{-3}$$

$$m = 9.51 * 10^{-3} \pm 1.05 * 10^{-3}$$

$$Uncertainty(\%) = \frac{1.05 * 10^{-3}}{9.51 * 10^{-3}} \times 100$$

$$Uncertainty(\%) = 11.0$$

Therefore, the percentage uncertainty in slope is 11%.

Uncertainty in y-intercept:

$$Uncertainty = \frac{Max - Min}{2}$$

$$Uncertainty = \frac{1.04403 - 0.983965}{2}$$

$$Uncertainty = \pm 3.0 * 10^{-2}$$

$$c = 1.03 \pm 3.0 * 10^{-2}$$

$$Uncertainty(\%) = \frac{3.0 * 10^{-2}}{1.03} \times 100$$

$$Uncertainty(\%) = 2.9$$

Therefore, the graph shows a systematic error of 1.03 (represented by the y-intercept in the function) with a percentage error of 2.9%.

Theoretical Slope Vs Experimental Slope:

By rearranging equation 2, the theoretical slope can be represented in terms of constants:

$$\frac{1}{sin(\beta)} = \frac{V}{(gd)^{\frac{1}{2}}}$$

Therefore, the theoretical slope of the graph is $\frac{1}{(gd)^{\frac{1}{2}}}$ where g is the acceleration due to gravity and d is the depth of water used in the experiment.

g = 981 cms^{-2} (9.81 ms^{-2} x 100)

d = 10 cm

$$m(slope) = \frac{1}{9810^{\frac{1}{2}}}$$

$$m = 0.0101$$

However, the actual slope from the regression is 0.00951. This indicates that there is a source of random error. The graph also has a systematic error due to the noticeable y-intercept. The sources for these two errors will be discussed further in conclusion and evaluation.

$$Theoretical(m) = 0.0101(cms^{-1})^{-1}$$

$$Experimental(m) = 0.00951(cms^{-1})^{-1}$$

Efficiency of the System:

To calculate and identify the efficiency, the power input of the circuit and the power output in the boat can be calculated. Power input would simply equal Voltage times Current in the circuit. Power output however, is more complicated. To find the power output the formula Fv can be used. As the boat will be at constant velocity, the force would equal to the resistive force of the water and the velocity of the boat is already measured which is dependent on the voltage and resistive force.

Hence Efficiency equals:

$$E(\%) = \frac{Fv}{VI} * 100$$

To create a rough understanding about the efficiency of the system, the resistive force can be approximated based on the angle of the wake. Here an assumption should be made, which is that the resistive force is proportional to the weight of the water displaced. Next it can be assumed that the boat displaces around 0.5 cm depth of the water when travelling to simplify calculations. Therefore, the resistive force becomes the surface area of the wake (which can be calculated using the angle and distance travelled by the boat) and the depth of water.

The surface area of the wake can be approximated as a triangle.

$A = 2(\frac{height * distance}{2})$ as the height is only for half of the wake.

$height = distance(tan(\beta))$.

Meanwhile, the depth can be taken as a separate variable and this would give the volume of water in terms of cubic centimetres.

$$Volume = stan(\beta)(s)d$$

Here another assumption must be made. It must be assumed that the density of water in any conditions would be the same. Thus the volume equation can be converted to grams as the density of water is 1 cm^3 = 1 g. Hence the resistive force or the weight of the water displaced becomes:

$$F = stan(\beta)(s)gd$$

$$F = 490.5tan(sin^{-1}(\frac{77.9}{v+86.6} + 0.089))s^2$$

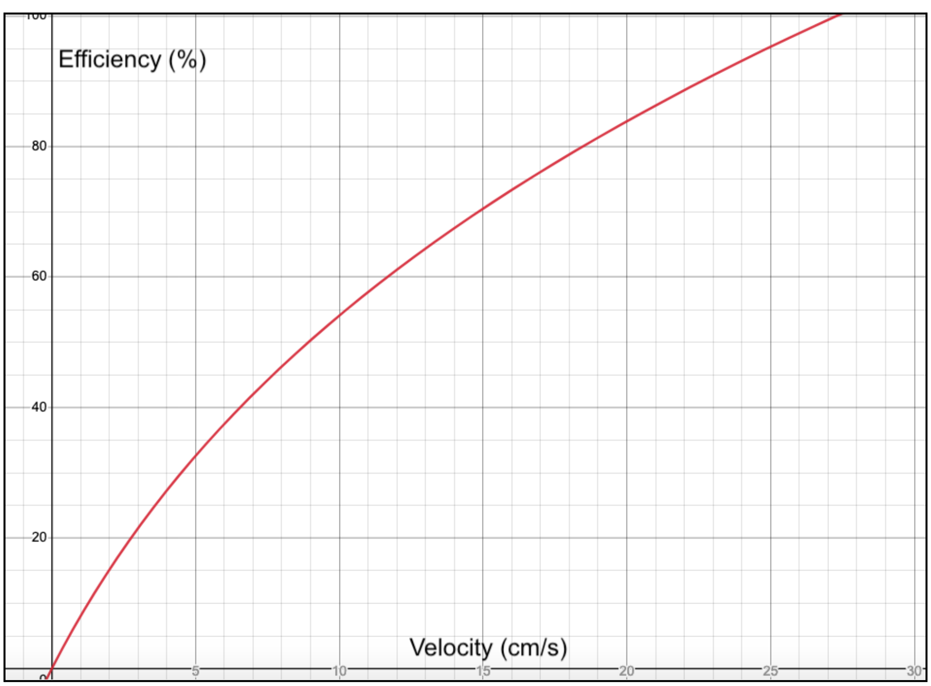
As g = 981 cms^{-2}

Therefore efficiency becomes:

$$E(\%) = \frac{490.5tan(sin^{-1}(\frac{77.9}{v+86.6} + 0.089))vs^2}{VI} * 100$$

For the input power, the power rating of the motor can be taken, which is 15 Watts (150000 when converted to the cm scale). Using 10 cm for the distance travelled the relation between velocity and Efficiency becomes:

$$E(\%) = \frac{49050tan(sin^{-1}(\frac{77.9}{v+86.6} + 0.089))v}{150000} * 100$$ - Equation 8



This graph clearly shows that Efficiency and velocity share a relation similar to a root function, that is, the efficiency has a large increase with velocity at start but the increase in efficiency becomes smaller with increase in velocity at larger values. One caveat here is that the force, distance and velocity are not in SI units but they can be converted which would still maintain this relationship between velocity of the boat and efficiency.

4 Evaluation and Conclusion

4.1 Conclusion

From the above analysis, it can be concluded that the velocity of the boat is inversely proportional to the sine of angle of half the wake. This supports the equation of the wake and validates the hypothesis of the experiment. The data shows a noticeable decrease in the angle with increase in velocity, and when graphed according to the equation derived, the data fits well with a relatively low amount of random error. This is evident in Graph 3 where the data has a clear linear trend and passes through most of the error bars indicating a precise data set. However, it is important to note that the uncertainty in the slope is significant and there is also a slight disagreement in the theoretical value and the experimental value of the slope. The graph also leans towards the minimum line which could be because of the lower 4th and 5th data points. Either systematic and/or random error is the cause for this making the experiment less accurate. Graph 3 also has a y-intercept which indicates systematic error in the experiment. This same error was also observed in Graph 1.2 and Graph 2. The source of this systematic error will be discussed further in evaluation.

Using approximations and assumptions, the relation between velocity and efficiency of the boat was also graphed and the importance of wake angle is clearly understood in identifying the ideal velocity of a boat in travel. Essentially, the angle of the wake indicates the resistive force that the body travelling in water is experiencing. A wider wake angle indicates that the body in the water is interfering with more water and thus using most of its input power to overcome the resistive force rather than propelling forward. This is why it is important to reduce the angle of the wake by increasing velocity. This way, the body would interact with less water and would experience less resistive force. Obviously, this is not a foolproof solution as a higher velocity would also require a higher power input which would again reduce the efficiency. This is what the experiment showed as well as the efficiency is affected less by the change in velocity with larger values of velocity. However, this relation was not predicted in the hypothesis as prior to the experiment, the velocity and efficiency of the boat were assumed to be directly proportional at first but then efficiency would start to decrease. This was not the exact case as the rate at which efficiency increases with change in velocity has lowered but has not become negative like the hypothesis predicted it would.

Due to the relationship between efficiency and velocity, it would make sense to run a boat at its maximum velocity possible. Though, this is not a practical solution as the boat's maximum speed could cause heavy turbulence for any of the passengers. Therefore, a better solution is to use less power as this would decrease the input power and increase efficiency. With further experimentation of velocity, efficiency and wake angle, the ideal velocity for a ship can be identified. This type of solution can allow engineers to design engines and propellers specific to the boat's purpose and try to maximise the efficiency by observing the angle of the wake. The results of this experiment proves that wake physics can have a larger application in modelling and designing efficient ships.

However, even though the data analysis answers the research question, the data analysis shows that there are few errors in the graphs (specially 1.2, 2 and 3). The primary cause for this is the caveat behind the formula. As mentioned when deriving, the formula only applies when the velocity of the boat is higher than the critical speed. After calculating the velocities at different voltages, it is seen that the boat does not cross critical speed for most of the data. This is why there is a systematic error as all of the data shows the relation between velocity and wake angle below the critical speed. This error does not make the data obsolete either as the efficiency formula and the equation derived do apply for cases where the ships are not moving with a large velocity. Most ships have a cruising speed lesser than the critical speed of the water they travel in, making this data still valuable in a real life scenario.

4.2 Evaluation

Systematic Error:

There is one source for systematic error in this experiment which could be improved

1. Critical Speed of the Water: The boat does not cross the critical speed of the water in the first few data points which leads to an observable systematic error in the final graph.

Method of Correction: This can be avoided by reducing the resistance of the circuit to allow the boat to reach a higher velocity. However, this could lead to a smaller dataset as the smaller time taken for the boat to travel the fixed distance would increase the error in the reading by a significant percentage. To avoid this, a larger water tub would be required. Therefore, to avoid this systematic error, a larger distance for the boat to travel and a smaller resistance in the motor circuit are required.

Random Error:

There are multiple sources of random error in this experiment which could be improved

1. The Replacement of String: During the experiment, the string which was used to pull the boat had to be replaced multiple times. The reason behind this was that the string was not able to withstand the tension for a long time and started to coil up and tangle which led to a slower acceleration of the boat.

   Method of Correction: To avoid this, a stronger material for the string could have been used which could have withstood the tension force of the motor boat for a longer time.

2. Deflection of the Boat: In certain trials, the boat has deflected to the side. The reason behind this was that the immediate change from rest to a high RPM caused the motor to move which made it not parallel to the side of the tub. This led to minor errors in the velocity calculations but was mostly compensated by measuring the angle deflection and dividing the velocity into horizontal and vertical components.

Method of Correction: To reduce this source of random error, a clamp system could have been used to hold the motor to prevent it from deflecting heavily.

3. Frictional Forces: Due to the nature of the setup, friction is expected to occur. When the string wraps around the motor axle, it could cause friction. This friction could have reduced the rpm. This would be more significant at higher speeds as the string wraps around the motor much faster creating more friction around the axle. However, the frictional force against the rotation is not constant at each trial as the string does not always wrap in the same way. Due to this, tiny differences in the velocity of the boat and thus the angle of the wake are created.

   Method of Correction: This could have been avoided with even more trials (5 to 8) to get accurate data on the velocity of the boat and the angle of the wake.

4.3 Extensions and Further Areas of Research

The experiment can be extended further by measuring all the factors which affect the angle of the wake. A possible extension of this investigation could be investigating the relation between the depth of the water and the angle of the wake where velocity of the boat is kept constant. The equation suggests that the sine of the angle would be directly proportional to the root of the depth. However, this can be investigated further and validated properly and can also help understand the efficiency of a boat at different depths of water.

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