Date of Report: 30/01/2024

Date of Experiment: 18/01/2024

Investigating the relation between Frequency of an Alternating Current Generator and Voltage across and Reactance of a Capacitor, Inductor and Resistor

Experiment Conducted by: Akella Neeraj Nandan

Lab Partner: Tobias Bhanji

Abstract

In this report, the voltage across and the reactance of a Capacitor or Inductor in an RC (Resistor-Capacitor) or RL (Resistor-Inductor) circuit driven by an AC generator (Alternating Current) was investigated. The experiment involved 4 parts to calibrate the setup and obtain the required data. The results have shown with a high accuracy that the voltage across a capacitor is inversely related to frequency of an AC generator and the voltage across an inductor is directly proportional to this frequency.

1 Introduction

This experiment aims to investigate the behaviour of a Capacitor in an RC-Circuit by measuring its voltage (V_c) and phase difference (ϕ) using an Oscilloscope across different frequencies of an AC generator. An AC generator or an AC power source supplies current in a sinusoidal fashion even alternating its direction.

On the other hand, a capacitor is a circuit component which can hold charge and as a result, store electrical energy. A capacitor can be described as two conducting plates separated by a dielectric medium in between. When a voltage is applied to a capacitor, each plate accumulates a charge (plate connected to the positive terminal accumulates positive charge as electrons leave the plate and vice versa). As a result, an electric field is developed between the two plates which store energy (Capacitance). This process is known as charging. When a charged capacitor is disconnected from a power source it starts discharging and dissipates all the energy it has stored to the circuit it has been connected to.

An AC driven RC-circuit has unique characteristics, mainly the Capacitor's Reactance. As the direction of the current is flipped continuously, the capacitor's electric field's direction switches as well. However, the circuit is met with some resistance when flipping direction as the Capacitor is charged by an amount which the circuit is opposing to. This opposition to the change in direction is known as 'Capacitor Reactance' (X_c). Due to this there is a phase difference observed in the Voltage across the capacitor and the voltage across the entire circuit. This phase difference is represented by a 'phasor diagram' which shows the sinusoidal motion of an AC circuit as circular motion. The voltages across various components are shown as vectors traveling along this circular direction at the frequency of the AC generator:

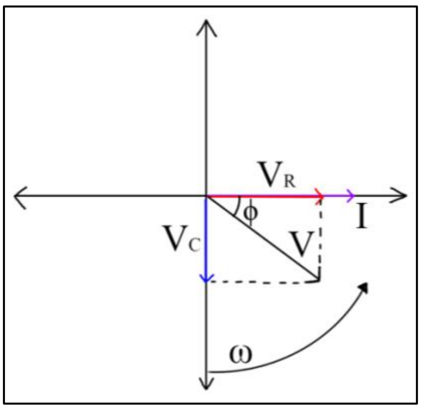

Figure 1: An example of a phasor diagram

In this diagram V_R is the voltage across the resistor which is in phase with the current while V_c is the voltage across the capacitor which is 90 degrees behind. The combination of this vectors V represents the voltage across the entire circuit. V_c is given by the formula (Urone and Hinrichs, 2022):

$$V_c = \frac{I}{2\pi f C} \tag{1.1}$$

Where I is the current in the circuit, f is the frequency of the AC generator and C is the capacitance of the capacitor. Using this equation, the reactance of a Capacitor can be calculated:

$$X_c = \frac{1}{2\pi f C} \tag{1.2}$$

It is important to also note that the current in an AC driven RC-circuit is dependent on the frequency (Saini, 2022):

$$I = \frac{V}{R_T}$$

$$I = \frac{V}{\sqrt{R^2 + X_C{}^2}} \tag{1.3}$$

For this investigation, an Oscilloscope is the primary measuring device necessary to outline the characteristics of RC-circuit. An Oscilloscope is an instrument which can graphically show an electrical signal; hence, it can show a sinusoidal signal explicitly and provide detail on phase difference, time period and amplitude of the wave unlike a regular multi-meter.

Along with the RC-circuit, an Inductor will also be tested with the AC generator. An Inductor is a passive electric component which stores energy in magnetic fields (Inductance), which similarly resists change in the direction of current, also known as ‘Inductor Reactance’. A brief test on the resistor will also be conducted.

2 Methods

This investigation is divided into 4 parts: Calibrating the AC generator, Investigation with Capacitor, Inductor and Resistor.

The first part of the experiment was to Calibrate the AC generator and the Oscilloscope, essentially verifying that the instruments are working correctly. For this, the terminals of the generator were connected to one of the channels of the Oscilloscope. The generator was turned on and its amplitude was set to 10 Volts. Its frequency was varied to the values: 100, 500, 1000, 1500, 2000, 2500, 3000, 3500, 4000, 4500, 5000 Hz and the frequency was calculated on the oscilloscope by measuring the time between two crests.

In the second part of the experiment an RC-circuit was constructed with a Capacitor (C) of Capacitance 0.2 µF and a Resistor (R) of Resistance 5kΩ. This circuit was connected to the Oscilloscope as shown below:

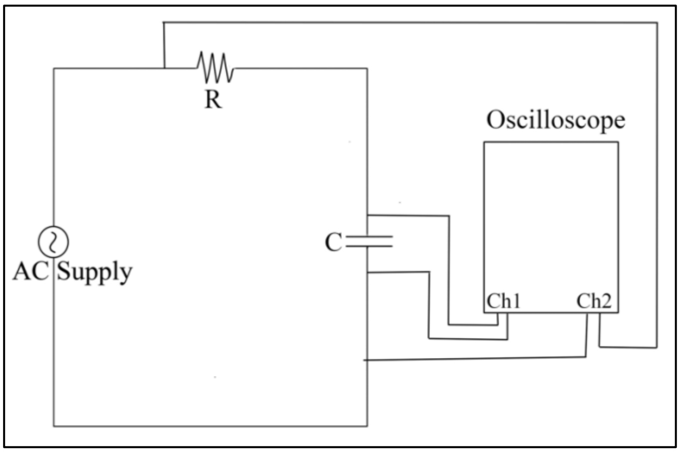

Figure 2: Diagram of the RC Circuit setup

Using this setup, V_c (amplitude of Voltage) was measured for the above calibrated frequencies and ϕ was measured for frequencies 100, 1000, 2500 and 5000 Hz.

For the third part, the circuit from Figure 2 was altered by replacing the Capacitor with an Inductor (L) of an Inductance 100 mH and the same measurements were recorded.

For the final part of the experiment, the circuit was once again altered by replacing the inductor with a 5kΩ Resistor. This investigation focused on how the frequency would affect the amplitude of V_R.

3 Results

The results of each part of the investigation are shown below in the form of tables. Table 1 is the data obtained from the Calibration:

Frequency of the AC Generator/ f (Hz)	**Measured Frequency (Hz ± 10%)**
100.000	100
500.000	500
1000.000	1000
1500.000	1490
2000.000	2000
2500.000	2500
3000.000	2990
3500.000	3510
4000.000	4000
4500.000	4460
5000.000	4950

Table 1: Results from the Calibration of the AC generator and the Oscilloscope

As the variation between the frequencies is mild, the frequency values displayed on the generator were used for the rest of the investigation. The cause for these variations is mainly due to the least count on the Oscilloscope as certain frequencies' time period cannot be accurately measured (as it would require infinite precision).

The frequency (f) was calculated using:

$$f = \frac{1}{T} \tag{2.1}$$

Where T was the measured time period. From equation (2.1) the error of the frequency was calculated:

$$\Delta f = f\left(\frac{\Delta T}{T}\right) \tag{2.2}$$

As previously mentioned, $\frac{\Delta T}{T}$ is equal to 10%, hence the percentage error in f was 10% as well.

Table 2 represents the results obtained from the second part of the investigation; The voltage across the capacitor V_c and the phase difference ϕ in terms of time difference of the peaks:

Frequency/f (Hz)	**Voltage/V_c (V ± 0.01) (V ± 0.001)**	**Time Period/T (ms ± 0.1)**	**Phase Difference/ϕ (μs ± 1)**
100	8.24	10.0	1000
500	2.76	-	-
1000	1.52	1.0	220
1500	1.03	-	-
2000	0.805	-	-
2500	0.651	0.5	120
3000	0.566	-	-
3500	0.471	-	-
4000	0.408	-	-
4500	0.371	-	-
5000	0.331	0.2	50

Key:

□ : V ± 0.01

■ : V ± 0.001

Table 2: Results of V_c and ϕ from changing the frequency of AC generator in an RC-circuit

The uncertainty for each value was obtained from their least count on the oscilloscope.

Table 3 represents the data obtained from the third part of the experiment with the inductor, the Voltage across the Inductor (V_L) and its phase difference (ϕ_L):

Frequency/f (Hz)	**Voltage/V_L (V ± 0.01) (V ± 0.001)**	**Time Period/T (ms ± 0.1)**	**Phase Difference/ϕ_L (μs ± 1)**
100	0.424	10.0	500
500	0.720	-	-
1000	1.248	1.0	180
1500	1.860	-	-
2000	2.38	-	-
2500	3.00	0.5	72
3000	3.48	-	-
3500	4.00	-	-
4000	4.48	-	-
4500	4.92	-	-
5000	5.32	0.2	30

Key:

: V ± 0.01

: V ± 0.001

Table 3: Results of V_L and ϕ_L from changing the frequency of AC generator in an RC-circuit

Similar to Table 2, the errors were obtained from the least count of the Oscilloscope.

Table 4 is a summary of all the results obtained including the reactance of the of the capacitor (using Equation (1.2)) and the inductor (X_L). The phase difference was converted to radians:

Frequency/f (Hz)	Voltage (V ± 0.01) (V ± 0.001)		Reactance (kΩ)		Phase Difference (radians)	
	V_c	V_L	X_C	X_L	ϕ	ϕ_L
100	8.24	0.424	7.958	0.063	0.628 ± 0.001	0.314 ± 0.001
500	2.76	0.720	1.592	0.314	-	-
1000	1.52	1.248	0.796	0.628	1.382 ± 0.006	1.131 ± 0.006
1500	1.03	1.860	0.530	0.942	-	-
2000	0.805	2.38	0.398	1.257	-	-
2500	0.651	3.00	0.318	1.571	1.885 ± 0.016	1.131 ± 0.016
3000	0.566	3.48	0.265	1.885	-	-
3500	0.471	4.00	0.227	2.199	-	-
4000	0.408	4.48	0.199	2.513	-	-
4500	0.371	4.92	0.177	2.827	-	-
5000	0.331	5.32	0.159	3.142	1.571 ± 0.031	0.942 ± 0.031

Key:

☐ : V ± 0.01

☐ : V ± 0.001

Table 4: Voltage, Reactance and Phase Difference summarized for both Capacitor and Inductor

The Reactance for the Inductor was calculated using the following equation:

$$X_L = 2\pi f L \tag{3.1}$$

Where L is the inductance of the inductor. This equation also highlights the relation between V_L and frequency:

$$V_L = IX_L \tag{3.2}$$

The phase difference and its error were calculated using equation (4.1) and (4.2) (Ku, 1966):

$$\phi = 2\pi f(\phi_m) \tag{4.1}$$

$$\Delta\phi = \sqrt{\left(\frac{d\phi_m}{d\phi}\right)^2 (\Delta\phi_m)^2}$$

$$\Delta\phi = 2\pi f(\Delta\phi_m) \tag{4.2}$$

Where ϕ_m is the time difference measured between the peaks of the waves.

The Calibration of the experiment was plotted using the least squares method to see the accuracy of the claim that the measured frequency can be approximated to be the frequency displayed on the AC generator:

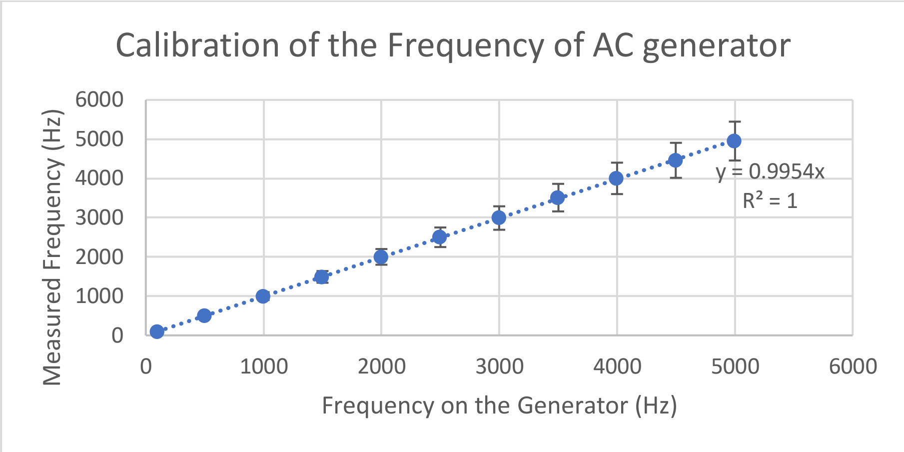

Figure 3: Results of Table 1 plotted to visually represent the calibration of the experiment

The slope obtained from this plot is 0.9954 ± 0.0016 (obtained through least squares method). This shows that it is a fair simplification (as the expected slope of 1.0 lies within 2 standard deviations) to use the frequency of the AC generator for the rest of the experiments.

The results of the second experiment were plotted (V_c and X_C against Frequency) to see the relation between Voltage across the capacitor and Frequency:

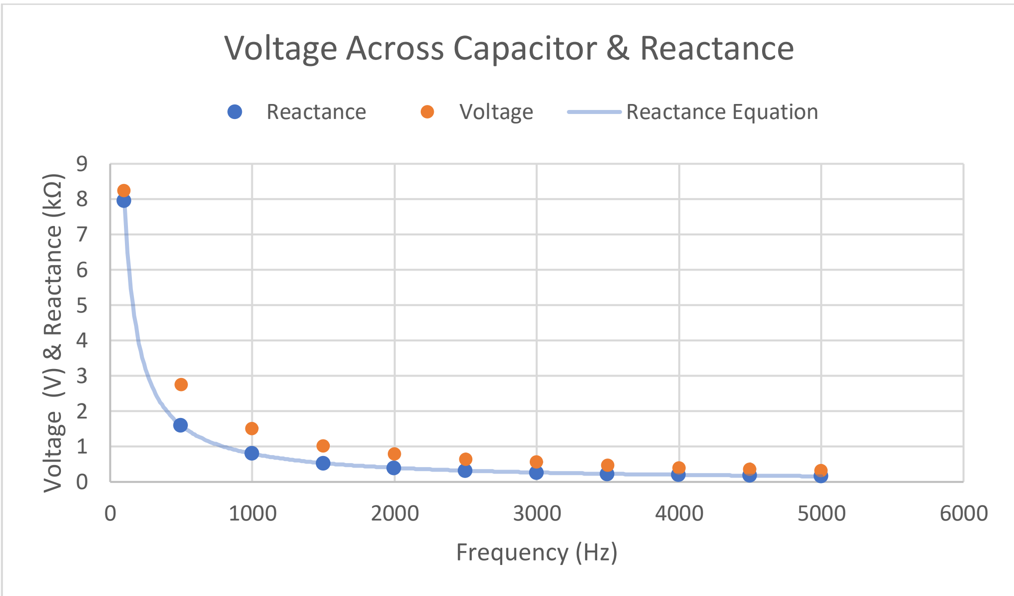

Figure 4.1: Voltage and Reactance of Capacitor against Frequency (Error bars were plotted but not visible)

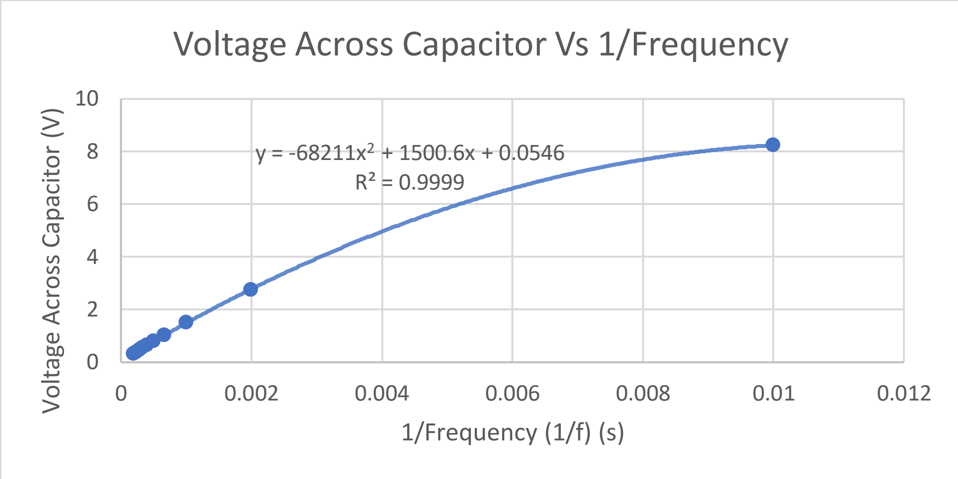

Figure 4.2: Graph of Table 2 adjusted to provide a polynomial fit (Error bars were plotted but not visible)

Figures 4.1 and 4.2 explicitly show that V_c is inversely 'related' to the frequency of the AC generator as predicted by equation (1.1). Figure 4.2 indicates it is not inverse proportionality as according to equation (1.1) and (1.3), V_c is dependent on current, which itself varies with frequency. However, there are a few errors as indicated by the y-intercept in the curve of best fit equation. The equation for the curve of best fit was obtained using the least squares method.

The results of the third experiment were plotted (V_L and X_L against Frequency) showcasing the relation between Voltage across the Inductor and Frequency of the AC generator:

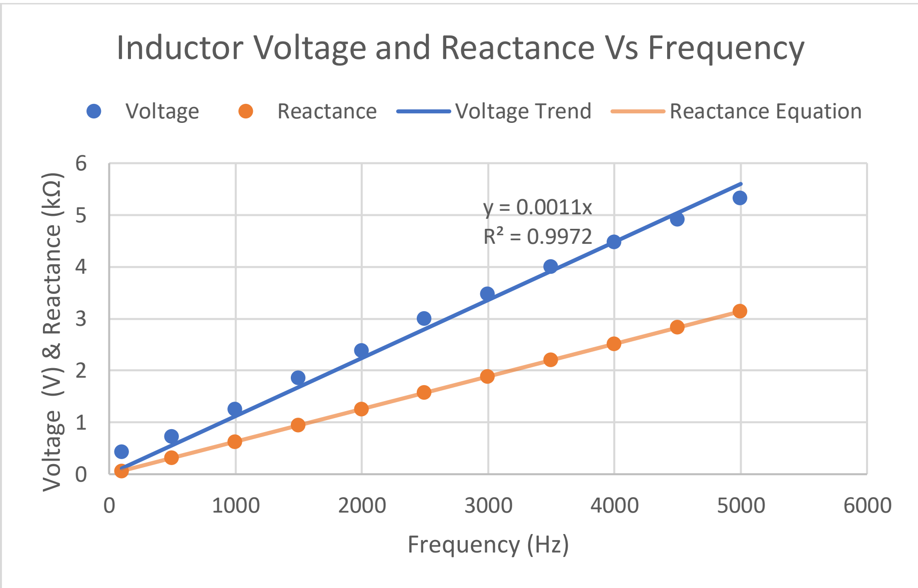

Figure 5: Voltage across Inductor, Inductor Reactance against Frequency (Errors bars plotted but not visible)

Figure 5 clearly shows that the voltage across the Inductor is directly proportional to the Frequency hence validating equation (3.2). This figure also highlights the accuracy as indicated by the high value of coefficient of determination. A caveat here is that the Voltage is once again related to the current in the circuit, however unlike in the previous figure, the effect of current is negligible. Similar to the previous plot, the line of best fit was calculated using the least squares method.

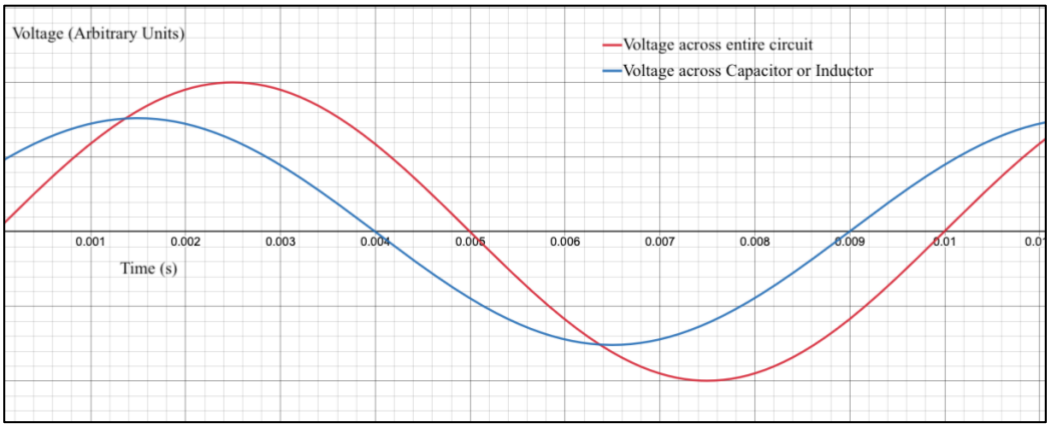

Figure 6: Visual Recreation of the Phase Difference observed on the Oscilloscope

The above figure is a simple graph to show the phase difference caused by the reactance of the capacitor and inductor.

The results of the fourth and final experiment essentially showed that the amplitude of voltage across the resistor, V_R, remains constant even when the frequency was changed. As a resistor is in phase with current, no reactance was expected, therefore no phase difference was observed.

4 Discussion

There are few sources of error for all the experiments causing minor deviations from the theoretical values. In the first experiment the main source of error was the least count of the device. The time measurements made with the Oscilloscope had an error of 10% and due to that the values of frequencies obtained were not as precise leading to slight deviations. The time measurements also required infinite precision to obtain the exact frequency displayed (due to recurring decimals). Considering this source of error, the frequency displayed on the generator was used instead. However, a possible improvement is that lower frequencies could have been used to obtain better readings for the time period.

In the second and third experiment, an issue found when recording the amplitude of the voltage was that there was noise in the electrical signal. This could have skewed the results as the amplitude was harder to record accurately (due to a fuzzier area rather than a crisp line). This issue becomes more problematic with larger frequencies hence the discrepancy with the best fit line in both Figure 4.2 and Figure 5 can be seen at larger frequency values (For Figure 4.2 a y-intercept is observed while in Figure 5 the values are further from the line). Once again, this error could have been rectified by testing with lower frequencies as this would have reduced the noise in the signal.

Therefore, future investigations should focus on selecting the appropriate frequencies (possibly in the range of 10 to 500 Hz) to acquire more accurate data with minimal noise in the signal and obtain, with more precision, the measurement of time period.

5 Conclusions

This experiment allows to understand the properties of a Capacitor and an Inductor when in the presence of an AC generator. With a simple setup and procedure, it helps highlight the Voltage across and Reactance of a Capacitor or an Inductor and helped validate theoretical formulas with high accuracy.

It has also been highlighted why the results vary slightly with the theory and it is also been discussed how these sources of errors can be rectified with few changes to the procedure. Future investigations can focus more on the phase shift of the voltage across the Capacitor or Inductor caused by the reactance. By measuring the current in the circuit, the theoretical phase shift and the voltage across the RC-circuit can be acquired and compared with the measured values.

In summary, this experiment successfully shows how an AC driven RC or RL circuit behaves and highlights the reactance property of Capacitors and Inductors.

References

Urone, P. P., and Hinrichs, R. (2022, 13 07). *"Reactance, Inductive and Capacitive"*. OpenStax, College Physics 2e, Chp 23 Sec 11.

Saini, M. K. (2022, 21 07). *"Difference between RL and RC Circuit"*. Retrieved 25 01, 2024, from www.tutorialspoint.com.

Ku, H.H. (1966). *"Notes on the Use of Propagation of Error Formulas"*. J Research of National Bureau of Standards-C. Engineering and Instrumentation, No 4 Vol 70C pg. 263-273.