UNIT I

Resolution and Composition of forces

1. Resolution of forces
2. Composition of forces
3. Components of forces
    i) Mutually perpendicular (Rectangular) components
    ii) Non-perpendicular/Oblique/ components along any two given lines
4. Determination of resultant for coplanar
    i) Concurrent force system
    ii) non-concurrent and non-parallel force system (General)
    iii) Parallel force system

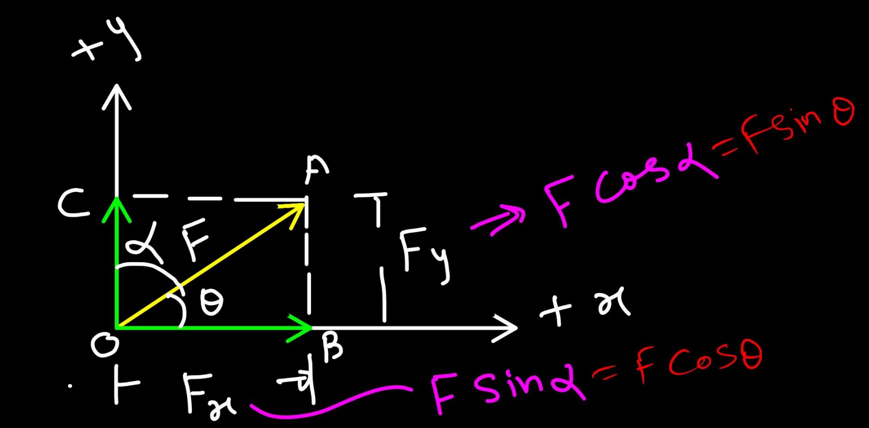
1. Resolution of a force :- It is the process of splitting of a force into its components (or replacing a force by several components) without chaning its effect is called "resolution of forces."

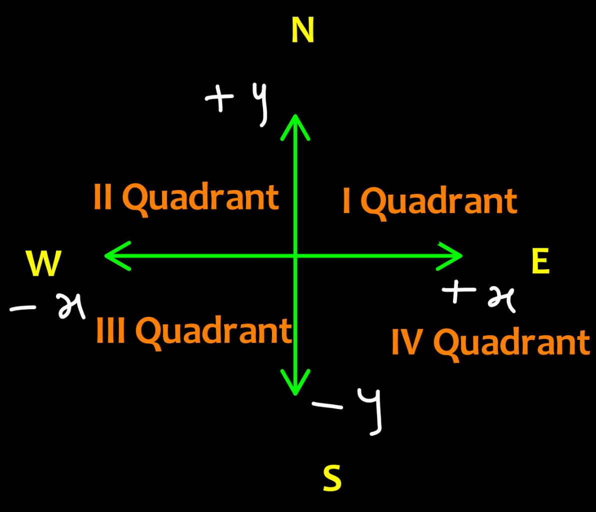
2. Composition of forces :- The reduction of a given system of forces to the simplest system that will be its equivalent is called composition of forces.

One single force obtained by the composition of several forcs is called the resultant force. (It is the process of determining the resultant of a number of forces acting on a body.)

3. Components of forces :-

i) Mutually perpendicular/Rectangular/Orthogonal components





In $\triangle OBA$

$$\cos\theta = \frac{OB}{OA} \Rightarrow OB = OA \cdot \cos\theta$$

$$\boxed{F_x = F\cos\theta}$$

→ x component F

$$\boxed{F_y = F\sin\theta}$$

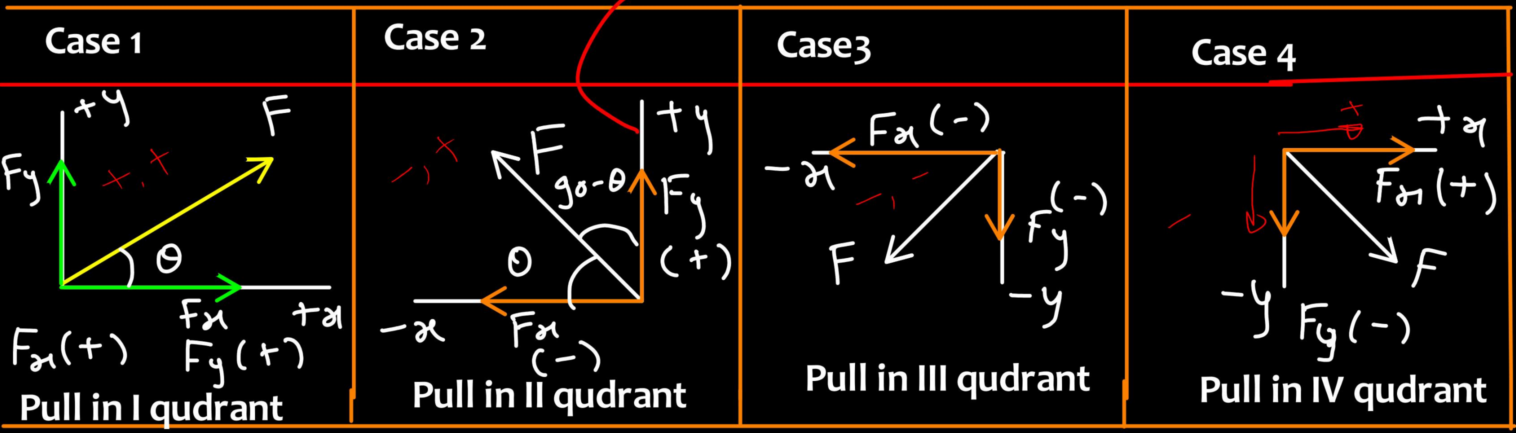
→ y component of F

$$\left\{ \sin\theta = \frac{AB}{OA}, \quad AB = OA\sin\theta \right\}$$

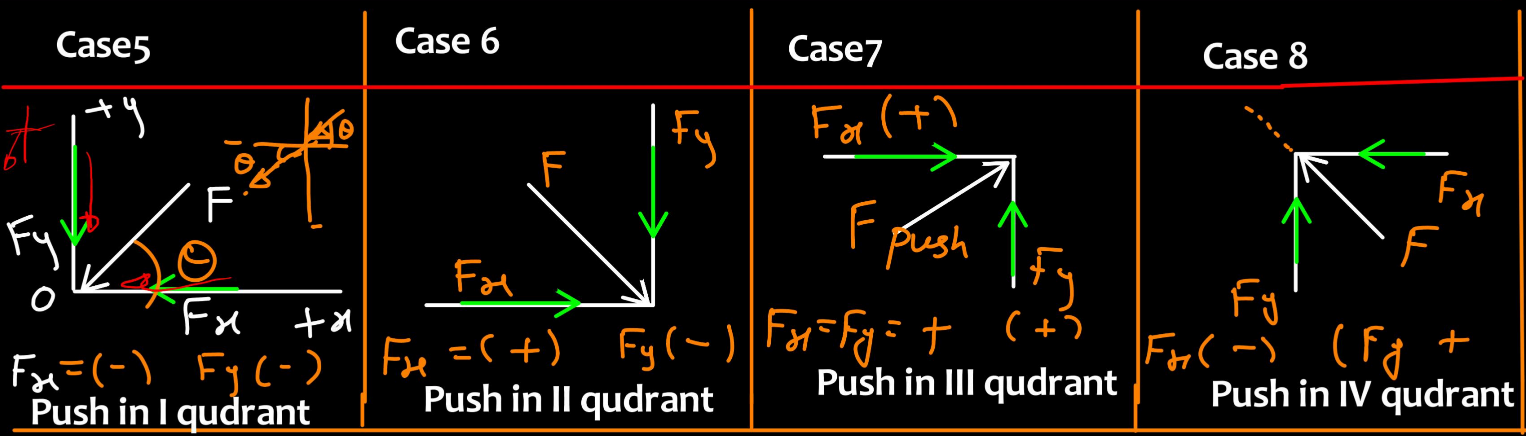
Different cass of Resolution of a Force

$F\cos(90-\theta) \Rightarrow F\sin\theta$

Part i :- Resolution when the force is pull in the respective quadrants :



Part ii :- Resolution when the force is push in the respective quadrants



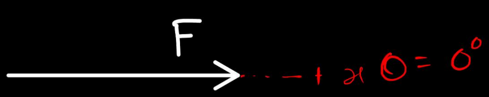
Conclusions for cases 1 to 8 :-

1. Components acting along horizontally rightwards(along + X axis) are taken as positive.
2. Components acting along horizontally leftwards(along - X axis) are taken as negative.
3. Components acting along vertically upwards(along + Y axis) are taken as positive.
4. Components acting along vertically downwards(along - Y axis) are taken as negative.

Note :- 1. If the force is pull, its components are also pull.
2. If the force is push its components are also push.

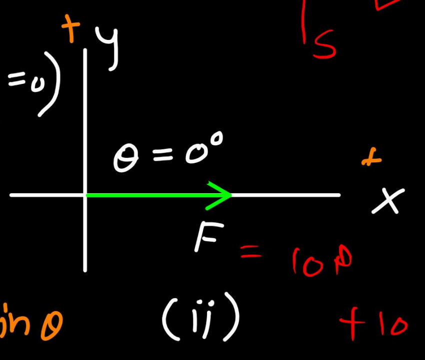
Part iii :- Resolution of a force when it lies on X and Y axes :

Case 9 : Force acting due East i.e. along + X axis $(F_x = F,\ F_y = 0)$



Force acting due East

(i)



$F_x = F\cos\theta$

$F_y = F\sin\theta$

(ii)

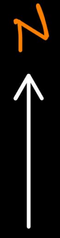
+10

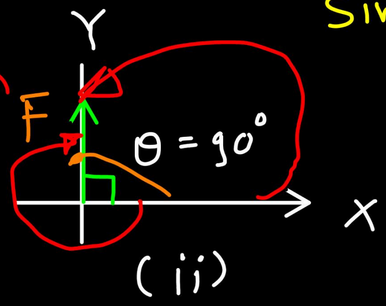
Case 10 : Force acting due North i.e. along + Y axis $(F_x = 0, F_y = F)$

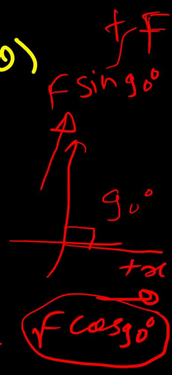
$F_x = F\cos(-270) = 0$

(i) $F_y = F\sin(-270°) = 1$

$\sin(-\theta)$





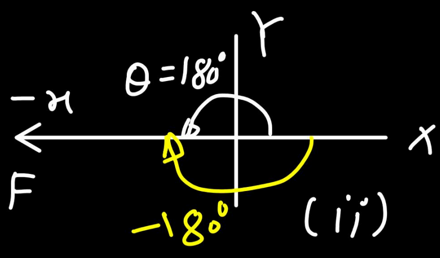


Force acting due North

$$\boxed{F_x = F\cos 90° \quad F_x = 0}$$

$$\boxed{F_y = F\sin 90° \Rightarrow F \times 1 = F}$$

Case 11 : Force acting due west i.e. along negative X axis $(F_x = -F, F_y = F)$



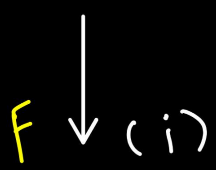
Force acting due West

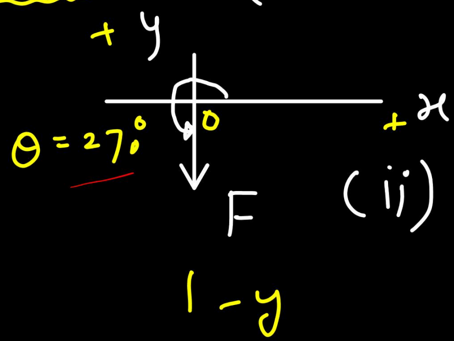
$$\boxed{F_x = F\cos 180° = F(-1) \quad F_x = -F}$$

$F_y = F\sin 180°$

$F_y = F \times 0 = 0$

Case 12 : Force acting due South i.e. along negative Y axis $(F_x = 0, F_y = -F)$





Force acting due South

$F_x = F\cos 270°$

$F_x = F(0)$

$$\boxed{F_x = 0}$$

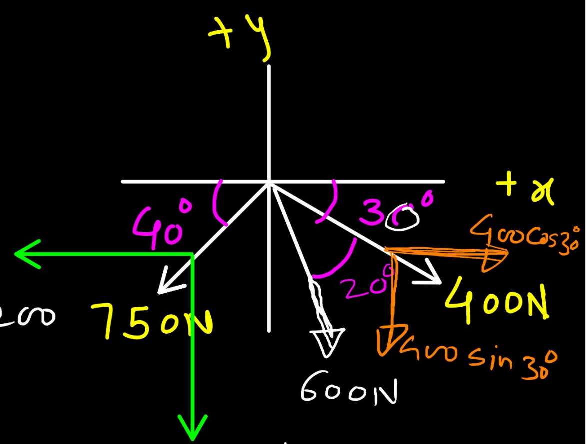
$F_y = F\sin 270°$

$F_y = F(-1)$

$$\boxed{F_y = -F}$$

Examples on rectangular components

1.Determine x and y components of the forces shown.



x- component of 400 N $= 400\cos 30^\circ$

$= 346.4$ N

y component of 400N $= -400\sin 30^\circ = -200$

$\sqrt{(-200)^2+(346.4)^2} = 400$

x component of 750 $= -750\cos 40^\circ = -574.5$ N

y -II- -II- $= -750\sin 40^\circ = -482.0$ N

Fx (600N) $\Rightarrow$ $600\cos 50^\circ = 385.67$ KN

Fy (600 N) $\Rightarrow$ $-600\sin 50^\circ = -459.62$ N

.629

.63

x Compt of 750 N $= -700\cos 40^\circ$

$= -574.53$ N

y -II- -II- $= -700\sin 40^\circ = -482.09$ N

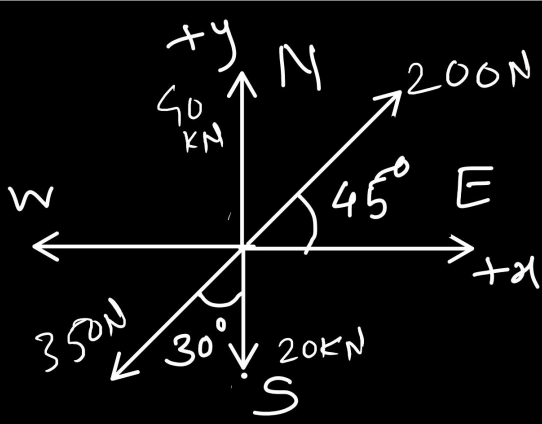
2. Find the components of each of the following forces.

i. 200 N at North-East

ii. 350 N at 30 degree west of south

iii. 20 kN acting due south

iv. 40 kN acting due North

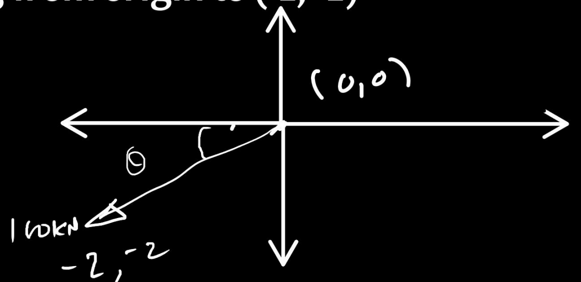


3. Resolve a force of 100 kN magnitude acting from origin to (-2, -2)

$\theta = 45^\circ$

$F_x = 100 \cos 45^\circ = -70.71$

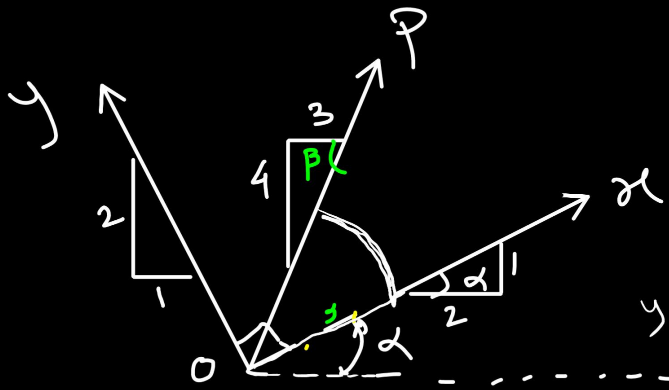
$F_y = 100 \sin 45^\circ = -70.71$



4 In the fig. the x- compnent of force P is 893 N. Determine the magnitude of P and it's y- component.

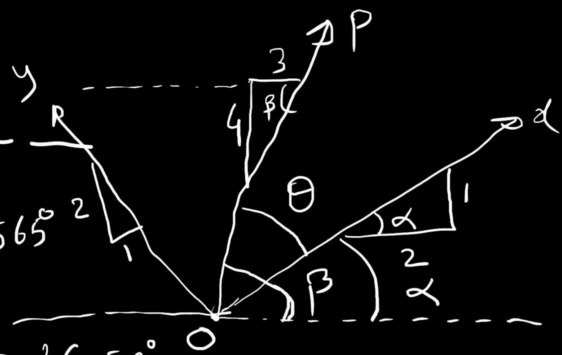
$P_x = 893\,N$

$P_y = ?$ $\quad P = ?$



$P = ?$

$P_y = ?$



$\theta = \beta - \alpha$

$\alpha = \tan^{-1}\left(\frac{1}{2}\right) = 26.565^\circ$

$\beta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$

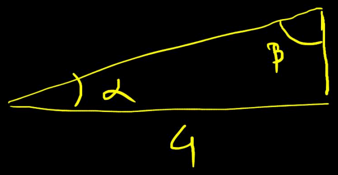
$\theta = \beta - \alpha = 53.13 - 26.565 = 26.52^\circ$

$P_x = P\cos\theta \Rightarrow 893 = P\cos 26.52^\circ \Rightarrow P = 998.01\,N$

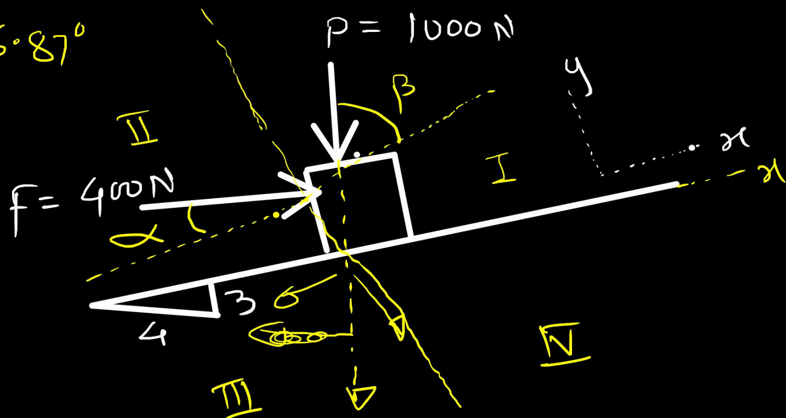
$P_y = P\sin\theta = 998.01 \sin 26.52^\circ \Rightarrow \underline{445.6^\circ}$

Q The body on the incline in fig. is subjected to the Vertical & horizontal forces shown. Find the components of each force along x & y axes oriented parallel & $\perp^{ar}$ to the plane

$\alpha = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$



$\beta = \tan^{-1}(4/3)$
$= 53.13^\circ$



$$\alpha = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$$

$$\beta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$

$$F_x = F\cos\alpha = 319.99 \simeq 320\ N$$

$$F_y = -F\sin\alpha = -240.00\ N$$

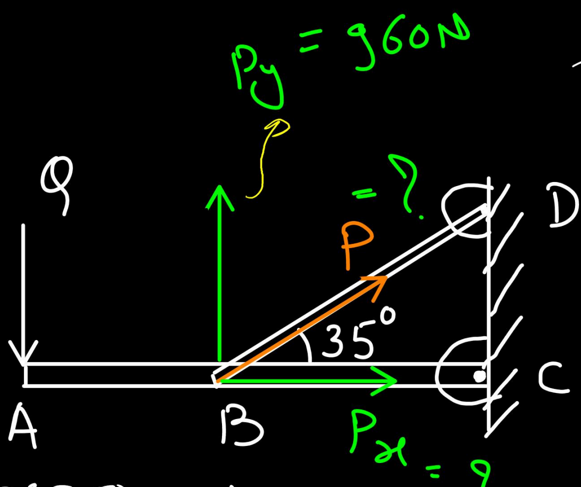
$$P_x = P\cos\beta = -600\ N$$

$$P_y = P\sin\beta = -800\ N$$

Q member DB exerts a force 'P' on the member ABC which is directed along the line BD. Knowing that P must have 960N vertical component, Determine

(i) magnitude of P

(ii) Horizontal component



$P_y = 960N \Rightarrow P\sin 35^\circ$

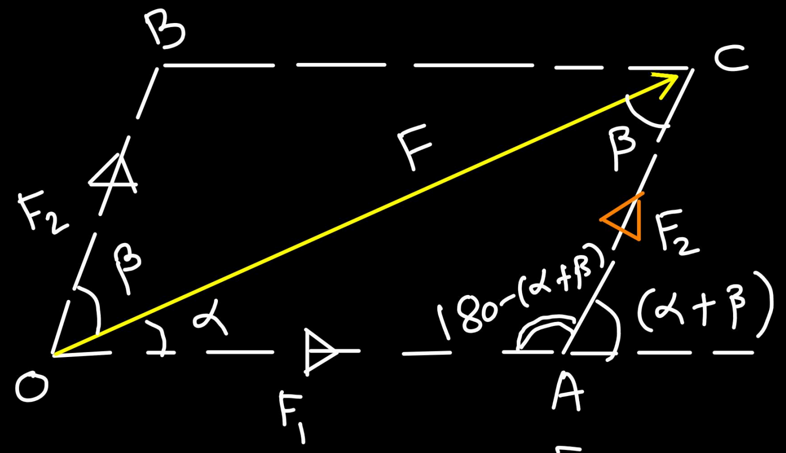
$P = \dfrac{960}{\sin 35} = 1673.70N$

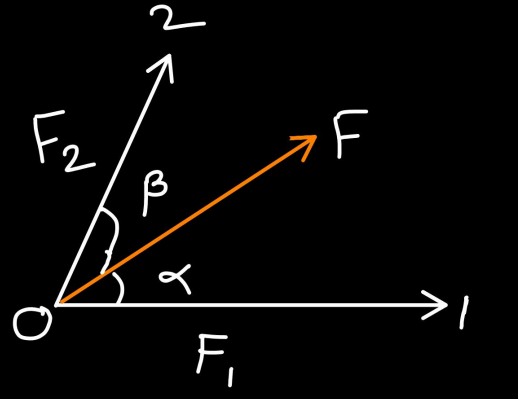
$P_x = P\cos 35^\circ \Rightarrow 1673.70 \cos 35^\circ = 1371 N$

ii)Non-perpendicular/Oblique/ components along any two given lines

F1 i.e component of F along the direction 1

F2 i.e component of F along the direction 2





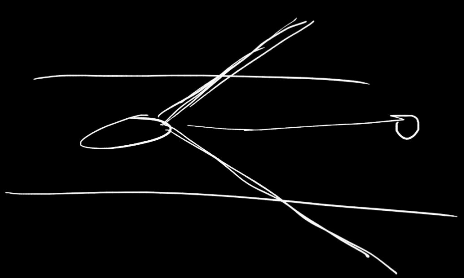
$$\frac{F_1}{\sin\beta} = \frac{F_2}{\sin\alpha} = \frac{F}{\sin\{180-(\alpha+\beta)\}}$$

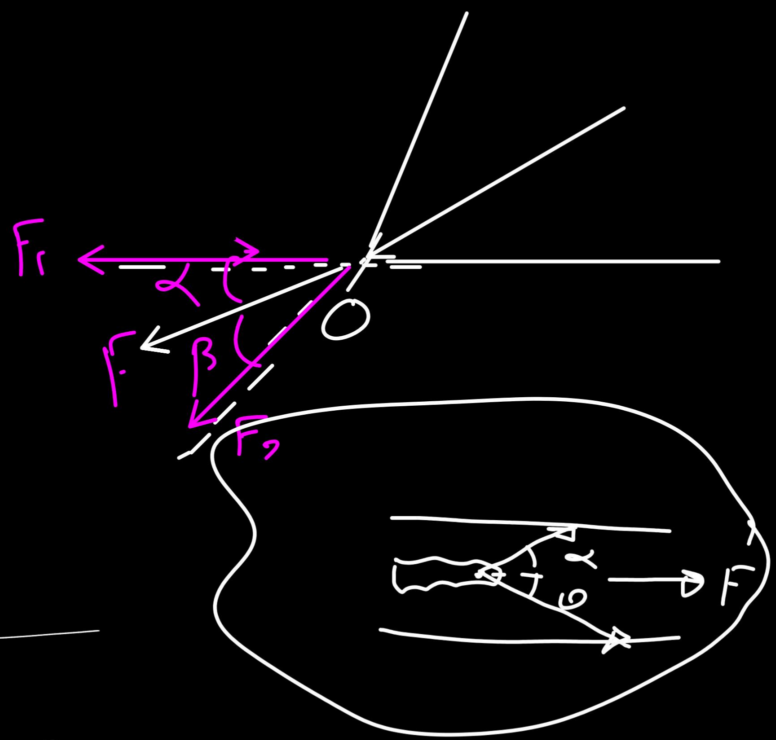
$\sin(\pi-\theta) \rightarrow \sin\theta$

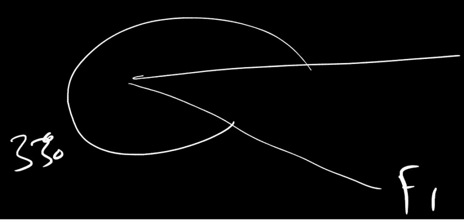
$$= \frac{F}{\sin(\alpha+\beta)}$$

$$\frac{F_1}{\sin\beta} = \frac{F}{\sin(\alpha+\beta)} \Rightarrow \boxed{F_1 = \frac{F\sin\beta}{\sin(\alpha+\beta)}}$$

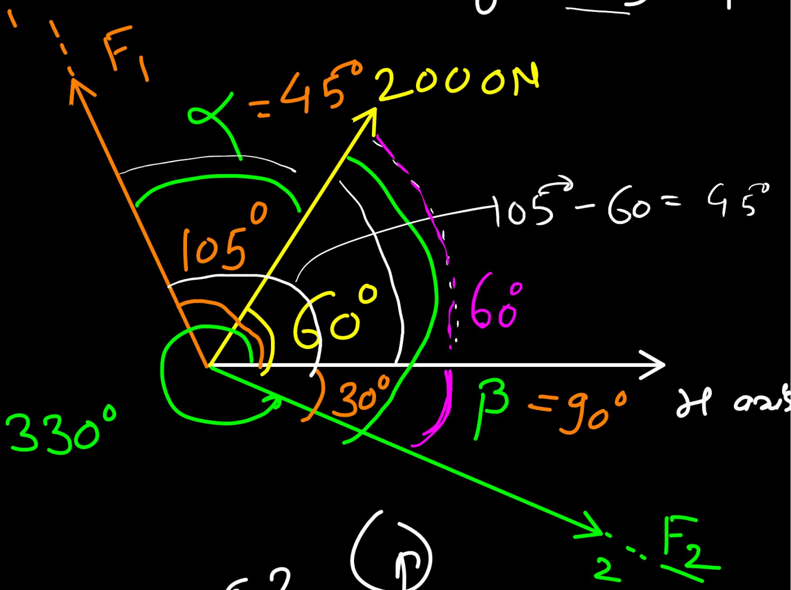
$$\frac{F_2}{\sin\alpha} = \frac{F}{\sin(\alpha+\beta)} \Rightarrow \boxed{F_2 = \frac{F\sin\alpha}{\sin(\alpha+\beta)}}$$







A Force of 2000 N acts at an angle of 60° with x-axis. Find its components along 105° & 330° with x-axis



$\Rightarrow$

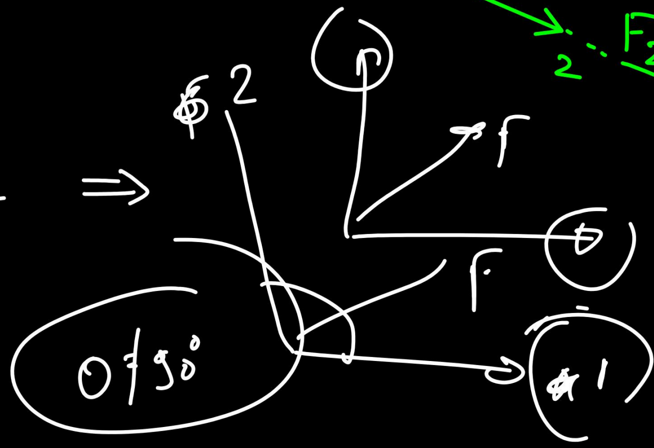
$$F_1 = \frac{F \sin\beta}{\sin(\alpha+\beta)}$$

$$= \frac{2000 \sin 90^\circ}{\sin(45^\circ + 90^\circ)}$$

$$F_1 = 2828.43 \text{ N}$$

$$F_2 = \frac{2000 \sin 45^\circ}{\sin(45^\circ + 90^\circ)}$$

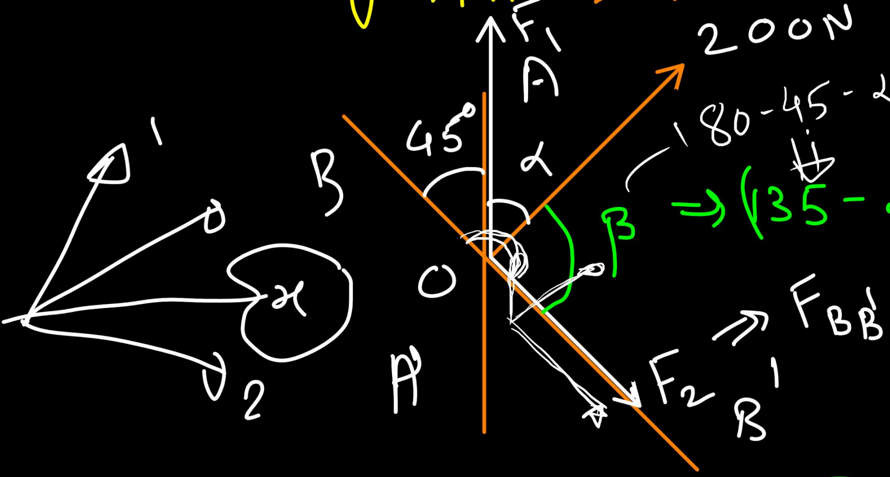
$$= 2000.3 \text{ N}$$

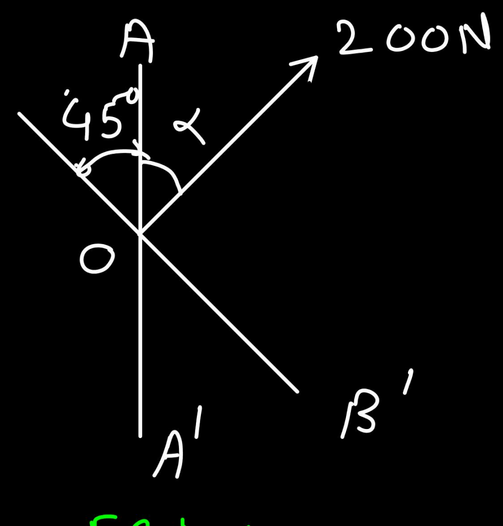


② The 200 N force is to be resolved into components along AA' & BB'. Determine

(i) angle 'α' knowing that components along BB' is to be 120 N

(ii) What is corr. value of the comp. along AA' $= F_{AA'}$





$$F_{AA'} = F_1 = \frac{F \sin\beta}{\sin(\alpha+\beta)} \Rightarrow$$

$$F_2 = F_{BB'} = \frac{F \sin\alpha}{\sin(\alpha+\beta)}$$

$$120 = \frac{200\sin\alpha}{\sin\{\cancel{\alpha} + (135-\cancel{\alpha})\}} = \frac{200\sin\alpha}{\sin 135^\circ}$$

$$120 = \frac{200\sin\alpha}{\sin 135^\circ} \rightarrow \alpha = 25.11^\circ$$

in (1) $\alpha = \sin^{-1}\left\{\frac{120\sin 135^\circ}{200}\right\}$

$$F_{AA'} = F_1 = \frac{200\sin\{135-\alpha\}}{\sin 135^\circ}$$

$$= 265.79\ N$$

$$265.97\ N$$

→ $F = 1500N$ < a-a, b-b

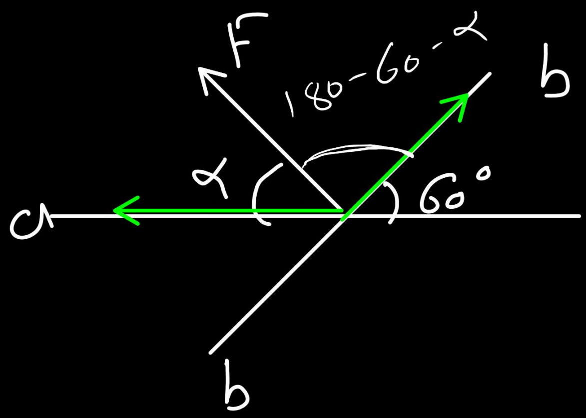
$\alpha = ?$ when $F_{a-a} = 1200N$

$F_{b-b} = ?$

$$F_1 = \frac{F\sin\beta}{\sin(\alpha+\beta)}$$

1200

$\alpha =$



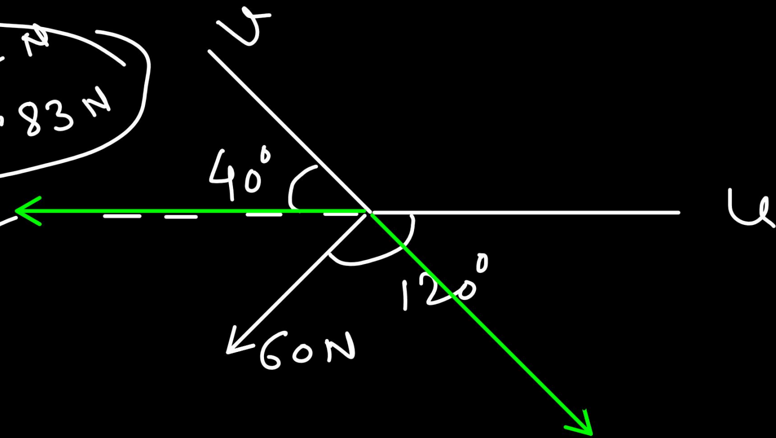
$\alpha = 76.1^\circ$

$F_{b-b} = 1681.6\ N$

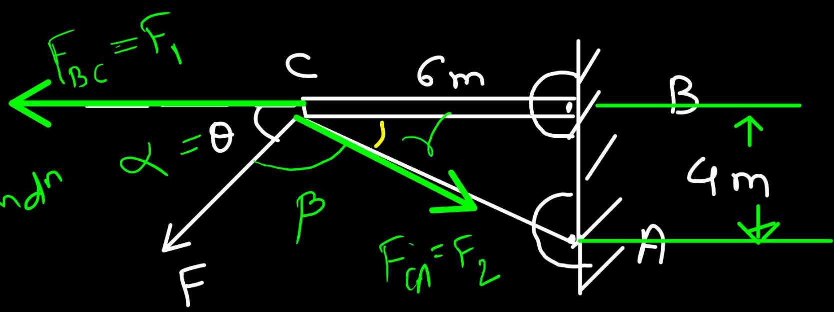
Resolve 60N Force into Components acting along u & v axes.

$F_u = ?$ ⇒ 91.92 N

$F_v = ?$ ⇒ 80.83 N



' At what angle 'θ' should the force 'F' be directed so, that the mag. of it's components along 'CA' does not exceeds 80% of mag. of its component along BC



$\Rightarrow F_{CA} = \left(\frac{80}{100}\right) F_{BC}$

max condn

$\gamma = \tan^{-1}(4/6) = 33.69°$

$\alpha + \beta + \gamma = 180° \Rightarrow \alpha + \beta = 180 - \gamma = 180 - 33.69°$

$\alpha + \beta = 146.31 \Rightarrow \beta = 146.31 - \alpha \Rightarrow 146.31 - \theta$

$F_2 = 0.8 F_1 \Rightarrow F_1 = \frac{F \sin\beta}{\sin(\alpha+\beta)}, \; F_2 = \frac{F \sin\alpha}{\sin(\alpha+\beta)}$

$\frac{\cancel{F}\sin\alpha}{\cancel{\sin(\alpha+\beta)}} = 0.8 \frac{\cancel{F}\sin\beta}{\cancel{\sin(\alpha+\beta)}} \Rightarrow \sin\alpha = 0.8 \sin\beta$

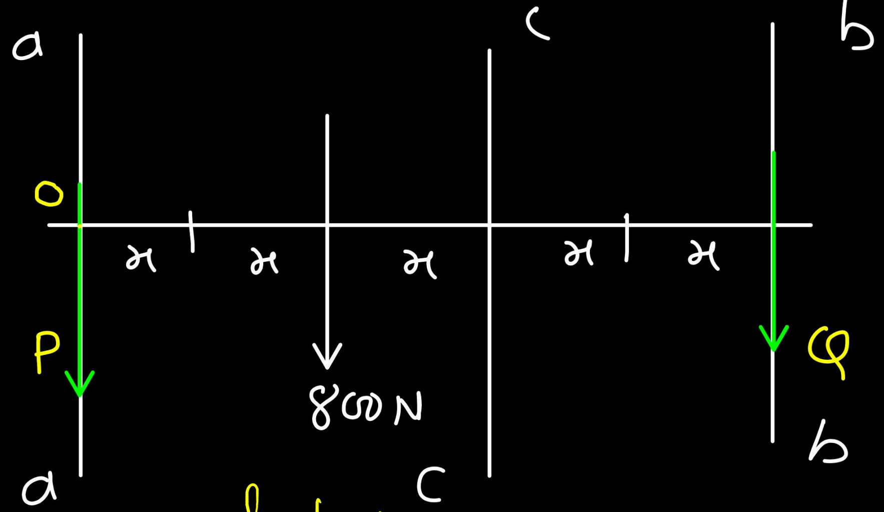
$\sin\theta = 0.8 \sin(146.31 - \theta)$

$\theta = 52.78 \Rightarrow 53°$

A Force of 800N is acting as shown in fig resolve this force into ||el compo P & Q acting along the linges

(i) a-a & b-b

(ii) b-b & c-c



(i) along line a-a & b-b

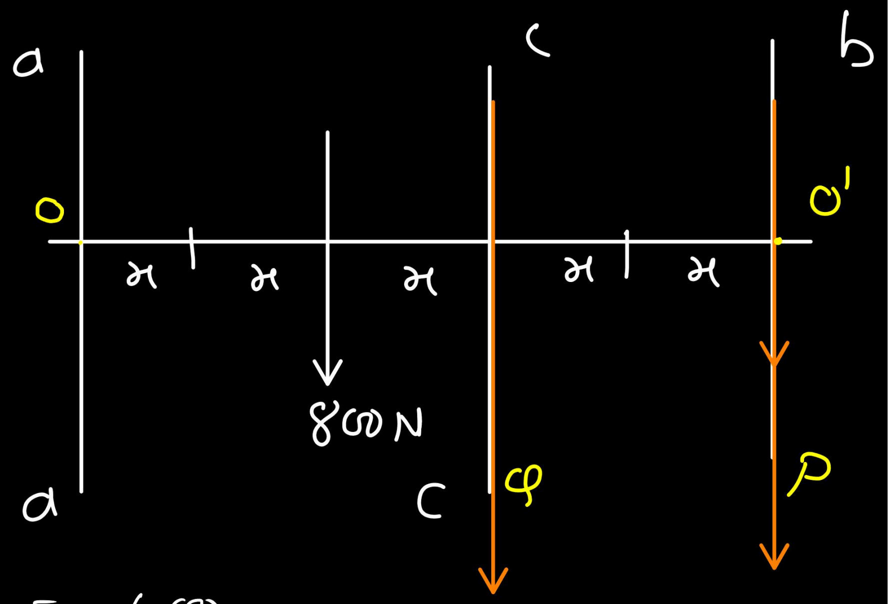
$P + Q = 800$ —(1) $-P - Q = -800$

Taking mmt @ pt 'O' & using V. T.

$0 + Q(5x) = 800(2x) \Rightarrow Q = 320 \downarrow$

$P = 800 - 320 = 480 N \downarrow$

(ii) along b-b & c-c



$P + Q = 800$

mmt @ O' & using V. T.

$-(2x) \times Q = -\{800(3x)\}$

$Q = 1200 N \downarrow$

from eqn (1)

$P = 800 - 1200 = -400$

$P = 400 N \uparrow$

Moment of force -

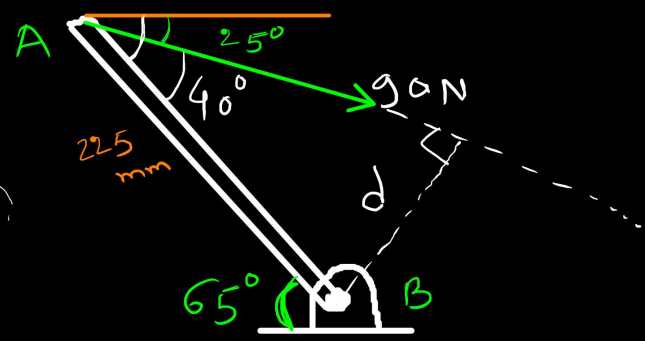
A 90 N force is applied to controll rod AB as shown in figure. Length of the rod is 225mm. Determine the moment of force about point B.

$d = 225 \sin 40^\circ =$

mmt @ B $= 90 \times (225 \sin 40^\circ)$

$\boxed{= 13.016 \text{ Nm (CW)}}$

~~13.06 ACW~~



In figure the tension 'T' in the cable supplies a moment of 72 kNm about point O.Determine the magnitude of T.

$\boxed{-T \times d = -72}$

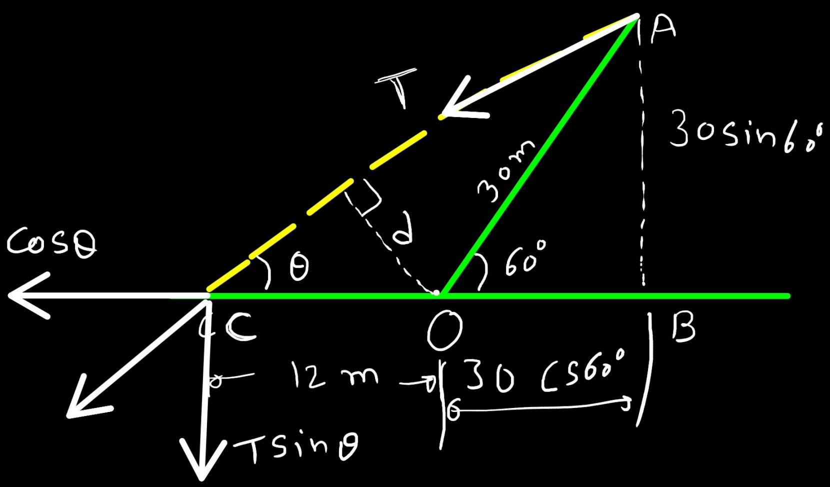
$-(T \sin\theta \times 12) = -72$

$\boxed{T\sin\theta = 6}$ — (1)

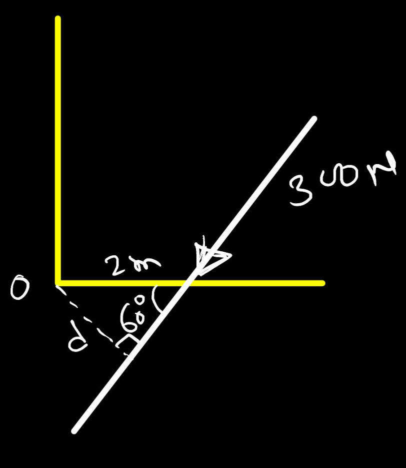
$\tan\theta = \frac{AB}{CB} = \frac{30 \sin 60^\circ}{12 + 30\cos 60^\circ}$

$\theta = 43.89^\circ$

$T = 8.65$

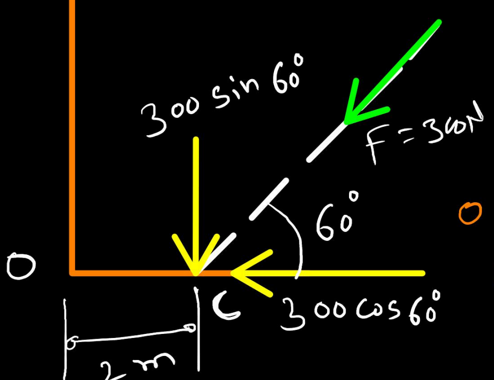


Find the moment of 300 N force about the point 'O'.

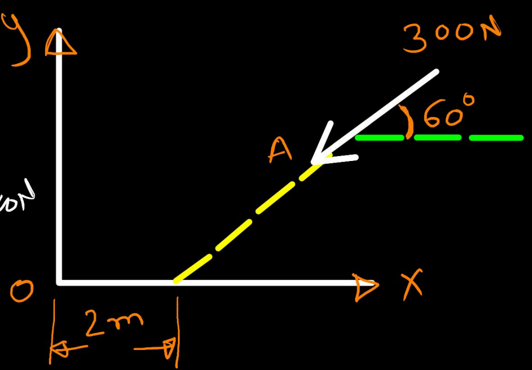


①

or



②



$d = 2\sin 60°$

(1)

mmt of 300N @O =

$300 \times (2\sin 60°)$

$= 519.6$ N

mmt of F @ O = sum of mmts of its components @ O

(2)

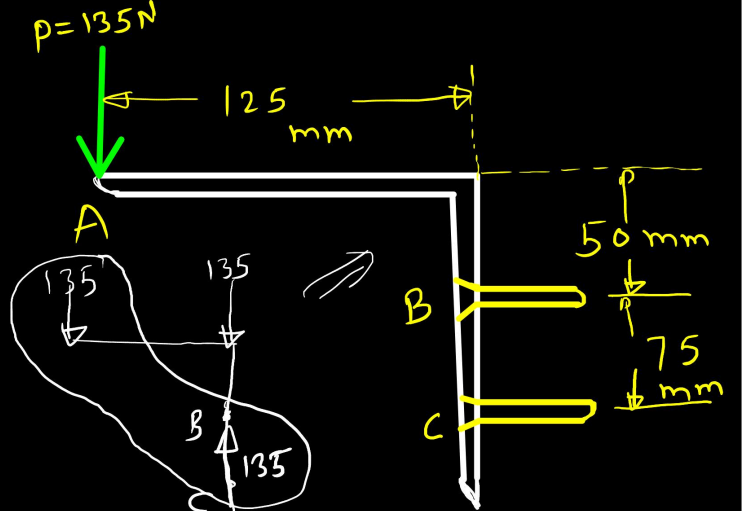
$= (300\cos 60° \times 0) + (300\sin 60°) \times 2$

$= 519.6$ N

135 N vertical force 'P' is applied at 'A' to the bracked (shown in fig), which is held by screws at B and C.

1)Replace 'P' with an equivalent force couple system at 'B'.

2) Find the two horizontal forces at B and C that are equivalent to the couple obtained in part 1.

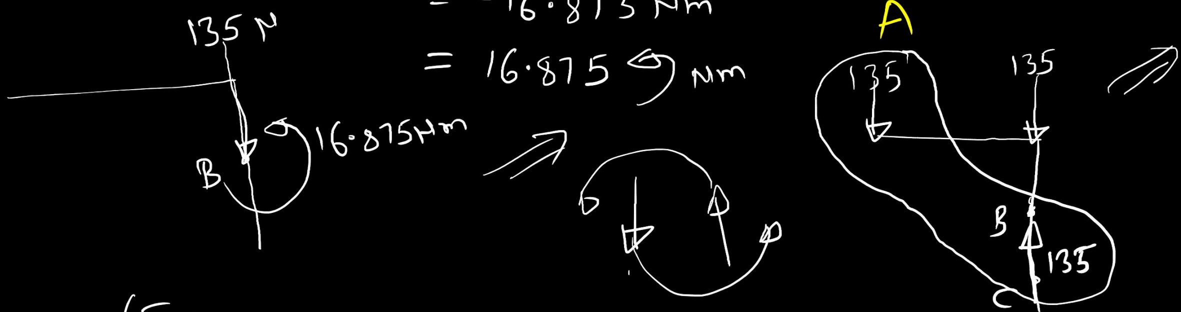


Soln ⇒

(1) $M_B = -P \times 125 = -135 \times 125$

$= -16.875$ Nm

$= 16.875$ Nm

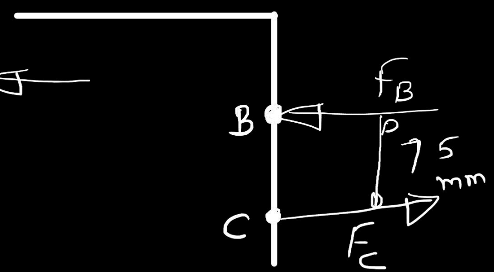


or $-(F_B$

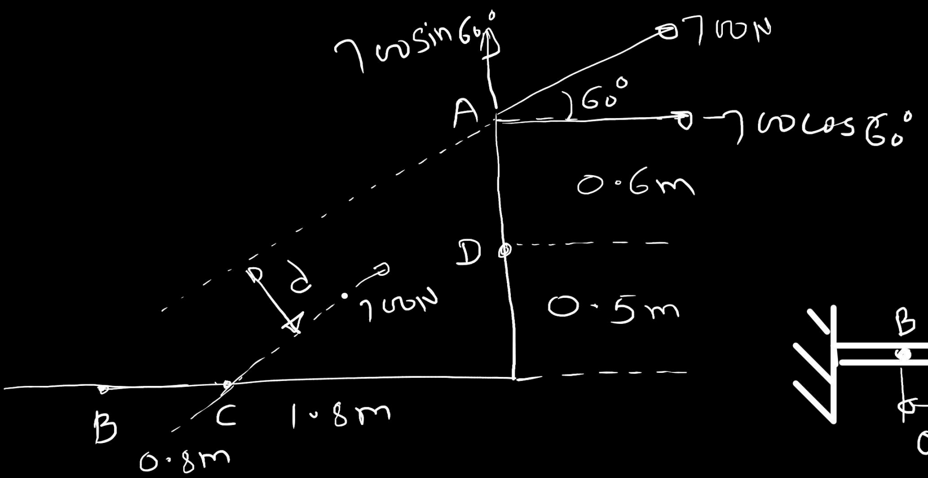
$-(F_B \times 0.075) = -16.875$

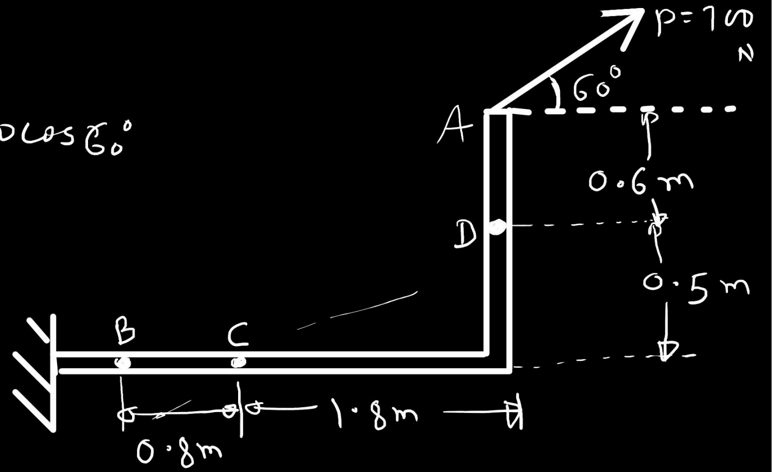
$F_B = \frac{16.875}{0.075} = 225\text{N}$

$F_C = 225$ N



A 700 N force 'P' is applied at point 'A' of the bracket as shown in fig.Replace the force 'P' with an equivalent force couple system at point 'C'.





$$M_c = (700\cos 60^\circ \times 1.1) - (700\sin 60^\circ \times 1.8)$$
$$= -706.19$$
$$M_c = 706.19 \text{ Nm}$$