Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with  = 45 minutes and  = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

A. 0.3875

B. 0.2676

C. 0.5

D. 0.6987

Ans: stats.norm.cdf (50,45,8) = 0.26 So the answer is B. The probability that the service manager cannot meet his commitment is 26%.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean  = 38 and Standard deviation  =6. For each statement below, please specify True/False. If false, briefly explain why.
   1. More employees at the processing center are older than 44 than between 38 and 44.
   2. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:

1. False, most of the data lies around the mean. If the most of the employees are above 44 the mean would be around 44.
2. TRUE

Z= xi-µ/𝜎

Z= 30-38/6= -1.333

Z= 1.333

Stats.norm.cdf(30,38,6) =0.0912

0.0912 x 400= 38.48

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: Both are independent random variables, X1 + X2 is normal with N(µ1+µ2,𝜎12 + 𝜎2²). 2X1 will show normal distribution by 2 times.

As both are independent normal random variables, X1 + X2 is normal with N(µ1+µ2,σ12+σ22). And 2X1 will just scale the normal distribution by 2 times

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

A. 90.5, 105.9

B. 80.2, 119.8

C. 22, 78

D. 48.5, 151.5

E. 90.1, 109.9

Ans: stats.norm.ppf(0.995,100,20) = 151.51 Stats.norm.ppf(0.005,100,20) = 48.5 So the answer is D

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
   1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
   2. Specify the 5th percentile of profit (in Rupees) for the company
   3. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:

1. stats.norm.interval(0.95, 225, 3) [219.120 - 230.879] Stats.norm.interval(0.95,315,4) [307.160 - 322.839]
2. stats.norm.ppf(0.05, 225, 3) stats.norm.ppf(0.05, 313, 4) 220.06, 306.42= 526.48
3. The second division