Monte Carlo Simulation of Option Pricing with Jump-Diffusion Model

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Introduction

- Options are financial contracts granting the holder the right, but not the obligation, to buy (call) or sell (put) an underlying asset at a predetermined price within a specified time frame.
- Underlying asset prices can experience sudden, unpredictable jumps, which can greatly impact option values.
- Jump diffusion model can capture real-world market dynamics by incorporating these jumps (modeled as a Poisson process) in addition to continuous random price movements (modeled by Geometric Brownian Motion/GBM).
- Monte Carlo method is a numerical method that uses random sampling to simulate multiple future price paths in the jump diffusion model for option pricing.

Equations used in the Jump Diffusion Model

- The Jump Diffusion Stochastic Differential Equation is:

$$dS_t = \underbrace{\mu S_t dt + \sigma S_t dW_t}_{DiffusionComponent} + \underbrace{(J-1)dN(t)}_{JumpComponent}$$

- The corresponding Jump Diffusion Equation is:

$$\ln(S_T) = \ln(S) + \int_0^t \left(\mu - \frac{\sigma^2}{2}\right) dt + \int_0^t \sigma dW(t) + \sum_{j=1}^{N(t)} Q_j$$

- The jump component is represented using a compound poisson process where: N(t) represents a process with probability of k jumps by it's PMF:

$$P(N(t) = k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}$$

- The Diffusion Component is Geometric Brownian Motion which describes the continuous, stochastic evolution of the underlying asset's price, S_t, is defined by:

$$dS_t = rS_t dt + \sigma S_t dW_t$$

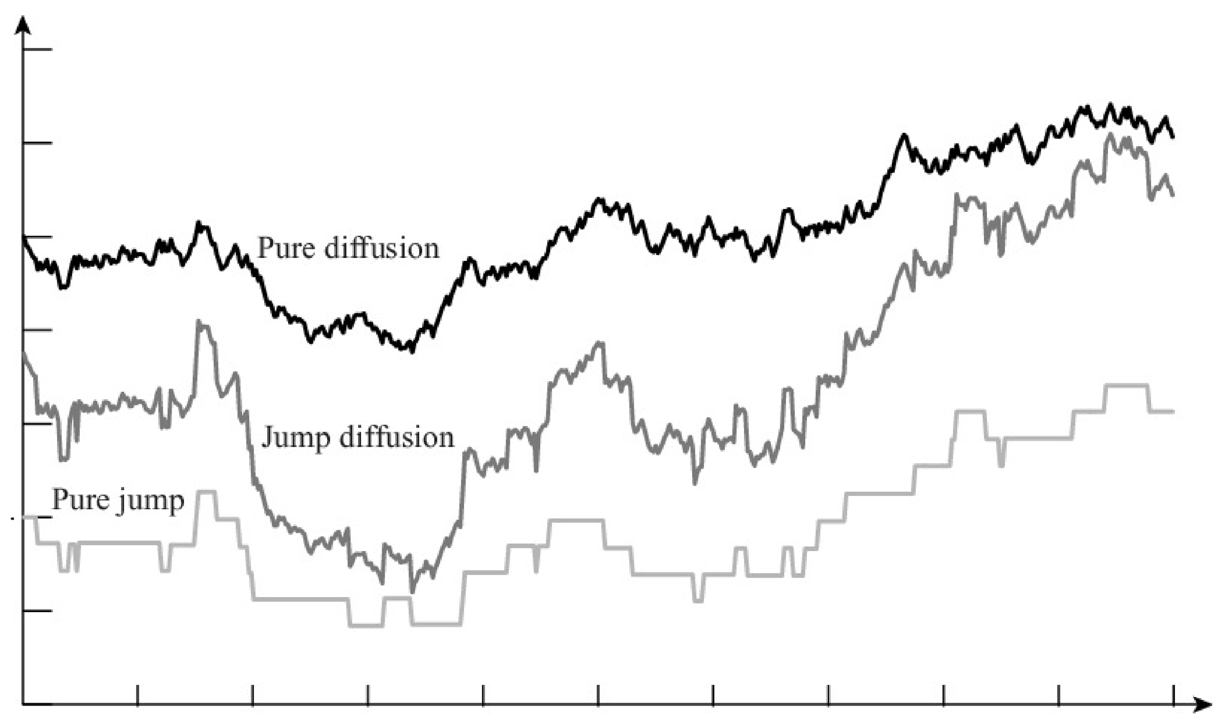


Figure 1: Comparison of jump and diffusion processes

Figure 2: Scan Me! :)

Applying Monte Carlo on Option Pricing

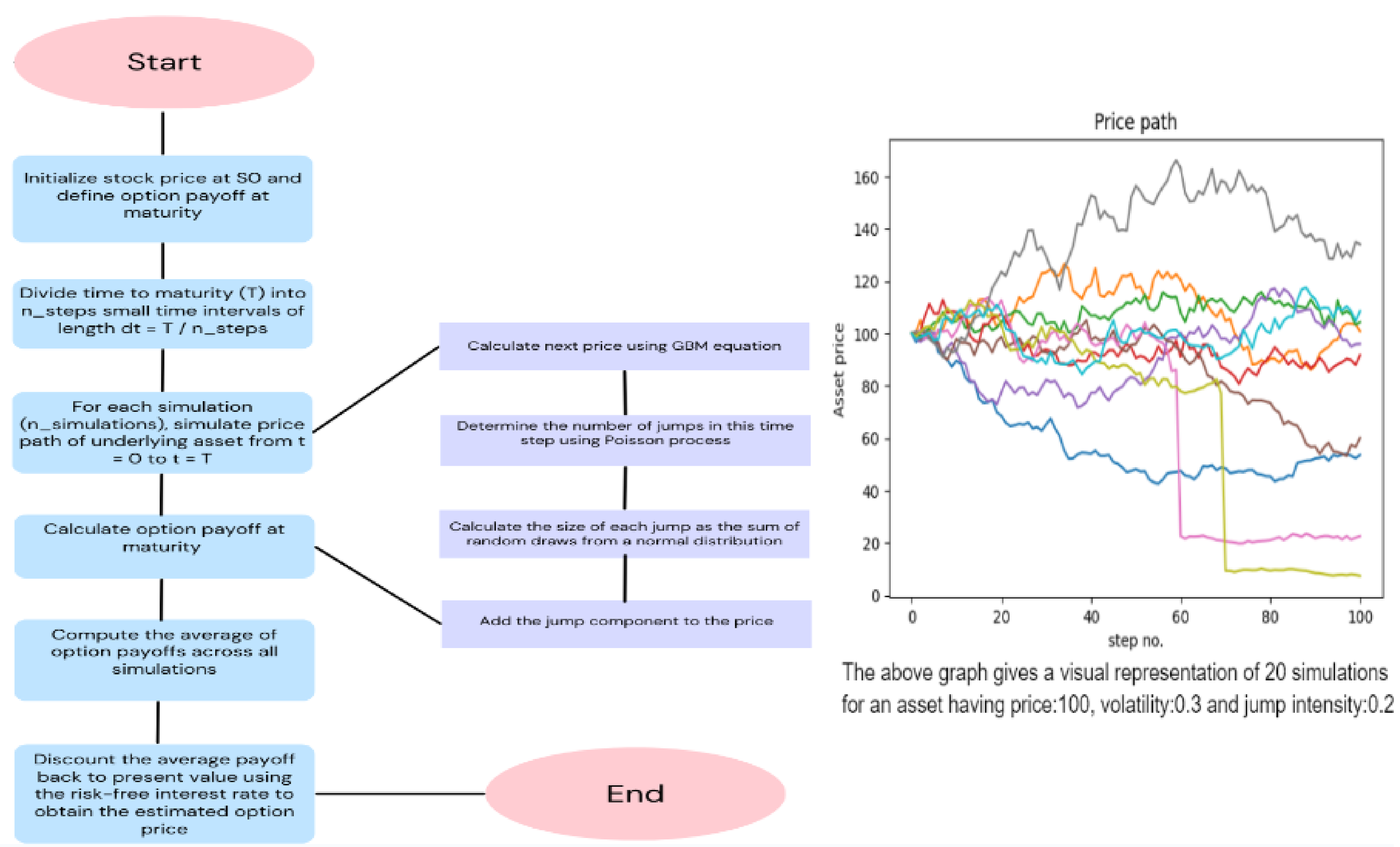
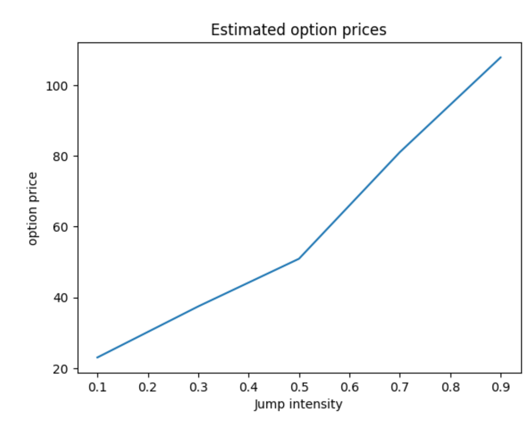
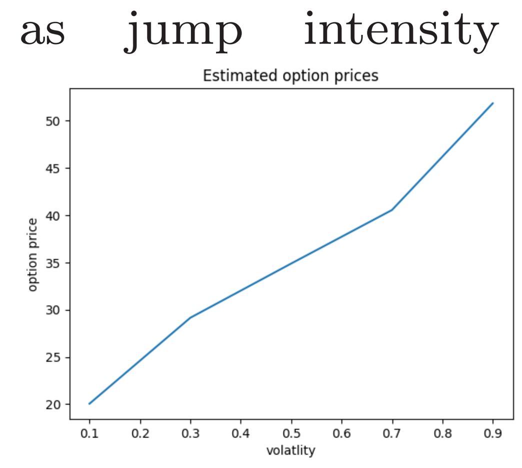


Figure 3: Flowchart of the process

Results



Estimated option price increases as jump intensity increases



Estimated option price increases as volatility increases