**Exercise 3**

1. A random sample of 250 adults was taken, and they were asked whether they prefer watching sports or opera on television. The following table gives the two-way classification of these adults.

| | Prefer Watching Sports | Prefer Watching Opera | Total |
|---|---|---|---|
| Male | 96 | 24 | 120 |
| Female | 45 | 85 | 130 |
| Total | 141 | 109 | 250 |

a. If one adult is selected at random from this group, find the probability that this adult

i. prefers watching opera

$$P(Opera) = \frac{n(Opera)}{n(Total)} = \frac{109}{250} = 0.436$$

ii. prefers watching sports given that the adult is a female

$$P(Sports|Female) = \frac{n(Sports\ and\ Female)}{n(Female)} = \frac{45}{130} = 0.3462$$

iii. is a female and prefers watching opera

$$P(Female\ and\ Opera) = \frac{n(Female\ and\ Opera)}{n(Total)} = \frac{85}{250} = 0.34$$

iv. prefers watching sports or is a male

$$P(Sports\ or\ Male) = P(Sport) + P(Male) - P(Sports\ and\ Male)$$
$$= \frac{141}{250} + \frac{120}{250} - \frac{96}{250} = 0.66$$

Or,

$$P(Sports\ or\ Male) = \frac{96 + 24 + 45}{250} = 0.66$$

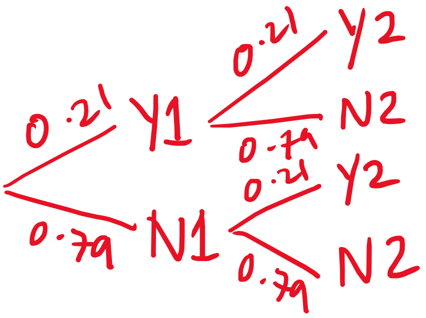
b. Are the events female and prefers watching sports independent? Are they mutually exclusive? Explain why or why not.

The events female and prefers watching sports are not independent. This is because $P(Female\ and\ Sports) \neq P(Female) \times P(Sports)$. You can calculate this.

The events female and prefers watching sports are not mutually exclusive because both can happen together. $n(Female\ and\ Sports) \neq 0$.

2. In a Gallup Annual Economy and Personal Finance poll, conducted April 3–6, 2014, 21% of adults aged 18 to 29 said that college costs and loans were the biggest financial problem their families were dealing with. Suppose two adults aged 18 to 29 are selected. Find the following probabilities.

   a. Draw a tree diagram for this event.




   b. Both adults will say that college costs and loans are the biggest financial problem their families are dealing with.

   These are independent events.

   $P(both\ said\ so) = P(Y1\ and\ Y2) = 0.21 \times 0.21 = 0.0441$

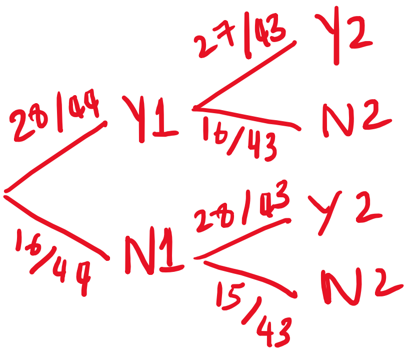
   c. Exactly one adult will say that college costs and loans are the biggest financial problem their families are dealing with.

   $P(exactly\ one\ said\ so) = P(Y1\ and\ N2) + P(N1\ and\ Y2)$

   $= 0.21 \times 0.79 + 0.79 \times 0.21 = 0.3318$

3. A car rental agency currently has 44 cars available, 28 of which have a GPS navigating system. Two cars are selected at random from these 44 cars.

   a. Draw a tree diagram for this event.




   b. Find the probability that both of these cars have GPS navigation systems.

   These are dependent events. The second outcome depends on the first outcome.

$$P(both\ has\ GPS) = P(Y1\ and\ Y2) = P(Y1) \times P(Y2|Y1) = \frac{28}{44} \times \frac{27}{43} = 0.3996$$

c. Find the probability that only one of these cars has GPS navigation systems.

$$P(only\ one\ has\ GPS) = P(Y1\ and\ N2) + P(N1\ and\ Y2) = \frac{28}{44} \times \frac{16}{43} + \frac{16}{44} \times \frac{28}{43}$$
$$= 0.4736$$

4. A certain state's auto license plates have three letters of the alphabet followed by a three-digit number.

a. How many different license plates are possible if all three-letter sequences are permitted and any number from 000 to 999 is allowed?

<u>26</u> <u>26</u> <u>26</u> <u>10</u> <u>10</u> <u>10</u>

$$Total\ way = 26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17576000$$

b. Arnold witnessed a hit-and-run accident. He knows that the first letter on the license plate of the offender's car was a B, that the second letter was an O or a Q, and that the last number was a 5. How many of this state's license plates fit this description?

First letter is known. Second letter is either of two letters. Last number is known

<u>1</u> <u>2</u> <u>26</u> <u>10</u> <u>10</u> <u>1</u>

$$Total\ way = 1 \times 2 \times 26 \times 10 \times 10 \times 1 = 5200$$

5. A box contains 10 red marbles and 10 green marbles.

a. Sampling at random from this box five times with replacement, you have drawn a red marble all five times. What is the probability of drawing a red marble the sixth time?

$$\frac{10}{20} = \frac{1}{2}$$

b. Sampling at random from this box five times without replacement, you have drawn a red marble all five times. Without replacing any of the marbles, what is the probability of drawing a red marble the sixth time?

$$\frac{5}{15} = \frac{1}{3}$$

c. You have tossed a fair coin five times and have obtained heads all five times. A friend argues that according to the law of averages, a tail is due to occur and, hence, the probability of obtaining a head on the sixth toss is less than .50. Is he right? Is coin tossing mathematically equivalent to the procedure mentioned in part a or the procedure mentioned in part b above? Explain.

He is not right. The coin tosses follow procedure in part (a) where they are independent. (This is also known as gambler's fallacy where gamblers believe they are more likely to win after losing consecutively)

6. A thief has stolen Roger's automatic teller machine (ATM) card. The card has a four-digit personal identification number (PIN). The thief knows that the first two digits are 3 and 5, but he does not know the last two digits. Thus, the PIN could be any number from 3500 to 3599. To protect the customer, the automatic teller machine will not allow more than three unsuccessful attempts to enter the PIN. After the third wrong PIN, the machine keeps the card and allows no further attempts.

   a. What is the probability that the thief will find the correct PIN within three tries? (Assume that the thief will not try the same wrong PIN twice.)

   There are a total of $10 \times 10 = 100$ possible numbers.

$$\begin{aligned} P(correct\ within\ 3\ tries) &= P(correct\ after\ 1\ try) + P(correct\ after\ 2\ tries) \\ &\quad + P(correct\ after\ 3\ tries) \\ &= \frac{1}{100} + \frac{99}{100} \times \frac{1}{99} + \frac{99}{100} \times \frac{98}{99} \times \frac{1}{98} = 0.03 \end{aligned}$$

   Alternatively,

$$P(correct\ within\ 3\ tries) = 1 - P(incorrect\ 3\ times) = 1 - \frac{99}{100} \times \frac{98}{99} \times \frac{97}{98} = 0.03$$

   b. If the thief knew that the first two digits were 3 and 5 and that the third digit was either 1 or 7, what is the probability of the thief guessing the correct PIN in three attempts?

   There are $2 \times 10 = 20$ possible numbers.

$$\begin{aligned} P(correct\ within\ 3\ tries) &= P(correct\ after\ 1\ try) + P(correct\ after\ 2\ tries) \\ &\quad + P(correct\ after\ 3\ tries) \\ &= \frac{1}{20} + \frac{19}{20} \times \frac{1}{19} + \frac{19}{20} \times \frac{18}{19} \times \frac{1}{18} = \frac{3}{20} = 0.15 \end{aligned}$$