Exercise 6

1. Consider the following null and alternative hypotheses:

$$H_0: \mu = 120 \text{ vs } H_1: \mu < 120$$

   A random sample of 81 observations taken from this population produced a sample mean of 116.5. The population standard deviation is known to be 15.

   a. If this test is made at a 10% significance level, would you reject the null hypothesis? Use the critical-value approach.

   $$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{116.5 - 120}{\frac{15}{\sqrt{81}}} = -2.1$$

   $z < -z_\alpha = -1.28$

   We have enough evidence to reject the null hypothesis.

   b. What is the probability of making a Type I error in part a?

   $P(\text{Type I error}) = \alpha = 0.1$

   c. Calculate the p-value for the test. Based on this p-value, would you reject the null hypothesis if $\alpha = 0.01$? What if $\alpha = 0.05$?

   p-value = $P(Z < z) = P(Z < -2.1) = P(Z > 2.1) = 0.01786$

   At $\alpha = 0.01$, we do not reject the null hypothesis as p-value > α.

   At $\alpha = 0.05$, we reject the null hypothesis as p-value < α.

2. Customers often complain about long waiting times at restaurants before the food is served. A restaurant claims that it serves food to its customers, on average, within 15 minutes after the order is placed. A local newspaper journalist wanted to check if the restaurant's claim is true. A sample of 36 customers showed that the mean time taken to serve food to them was 15.75 minutes with a standard deviation of 2.4 minutes. Using the sample mean, the journalist says that the restaurant's claim is false.

   a. Do you think the journalist's conclusion is fair to the restaurant? State the null and alternative hypotheses and use a 1% significance level to answer this question. You are free to use the p-value or critical value approaches.

   μ = average time taken to serve food in minutes

   $H_0: \mu = 15, \qquad H_1: \mu > 15$

$$z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{15.75 - 15}{\frac{2.4}{\sqrt{36}}} = 1.875$$

Reject H_0 if $z > z_{0.01} = 2.33$. In this case, $z = 1.875 < z_\alpha = 2.33$.

(Alternatively, p-value = $P(Z > 1.875) = 0.0301 > \alpha$)

Therefore, we do not have enough evidence to reject H_0.

There is not enough evidence to claim that the average time taken to serve food is more than 15 minutes.

The journalist's claim is not fair to the restaurant.

b. What are the Type I and Type II errors in part a?

Type I error: We reject the restaurant's claim, or conclude that the average time taken to serve food is more than 15 minutes, although in reality the restaurant's claim is true.

Type II error: We do not reject the restaurant's claim, or conclude that the average time taken to serve food is within 15 minutes, although in reality the restaurant's claim is false.

3. A consulting agency was asked by a large insurance company to investigate if business majors were better salespersons than those with other majors. A sample of 20 salespersons with a business degree showed that they sold an average of 11 insurance policies per week. Another sample of 25 salespersons with a degree other than business showed that they sold an average of 9 insurance policies per week. Assume that the two populations are approximately normally distributed with population standard deviations of 1.80 and 1.35 policies per week, respectively. Using a 1% significance level, can you conclude that people with a business degree are better salespersons than those who have a degree in another area? Use the critical value approach.

μ_1: average insurance policies sold by salespersons with business degree

μ_2: average insurance policies sold by salespersons with degrees other than business degree

$H_0: \mu_1 - \mu_2 = 0, \qquad H_1: \mu_1 - \mu_2 > 0$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(11 - 9) - 0}{\sqrt{\frac{1.80^2}{20} + \frac{1.35^2}{25}}} = 4.127$$

Reject H_0 if $z > z_\alpha = 2.33$. In this case, $z = 4.127 > z_\alpha$.

There is enough evidence to reject H_0.

There is enough evidence to conclude that the average insurance policies sold by people with business degree is greater than the average insurance policies sold by people without business degree.

4. A new type of sleeping pill is tested against an older, standard pill. Two thousand insomniacs are randomly divided into two equal groups. The first group is given the old pill, and the second group receives the new pill. The time required to fall asleep after the pill is administered is recorded for each person. The results of the experiment are given in the following table, where $\bar{x}$ and s represent the mean and standard deviation, respectively, for the times required to fall asleep for people in each group after the pill is taken.

	Group 1 (old pill)	Group 2 (new pill)
n	1000	1000
$\bar{x}$	15.4 minutes	15.0 minutes
s	3.5 minutes	3.0 minutes

a. Consider testing whether the new pill has a different effect when compared to the old pill. What would be the null and alternative hypotheses for this test?

μ_1: average time to fall asleep using old pill

μ_2: average time to fall asleep using new pill

$H_0: \mu_1 - \mu_2 = 0, \qquad H_1: \mu_1 - \mu_2 \neq 0$

b. Find the p-value for this test.

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}} = \frac{(15.4 - 15.0) - 0}{\sqrt{\frac{3.5^2}{1000} + \frac{3.0^2}{1000}}} = 2.74$$

p-value = $2 \times P(Z > 2.74) = 0.00614$

c. Does your answer to part a indicate that the result is statistically significant? Use $\alpha = 0.025$.

Result shows that the difference is statistically significant, as p-value $< \alpha$.

5. A clothing store chain is having a sale based on the use of a coupon. The company is interested in knowing whether the wording of the coupon affects the number of units of the

product purchased by customers. The company created four coupons for the same product, each with different wording. Four groups of 50 customers each were selected at random. Group 1 received the first version of the coupon; Group 2 received the second version; and so on. The units of the product purchased by each customer were recorded.

a. What would be the suitable null and alternative hypotheses that can be used to test whether the wording of the coupon affects the number of units of the product purchased by customers? What procedure is used to test the two hypotheses?

   H_0: The mean units sold by each of the groups are equal

   H_1: At least two groups have different means.

   The ANOVA procedure can be used here.

b. Suppose that the p-value is found to be 0.532. What conclusion can we draw from the test?

   p-value is large. In this case we do not have enough evidence to reject H_0. We can conclude that the wording of the coupon <u>does not affect</u> units purchased by customers.

c. Suppose that the p-value is found to be 0.00023. What conclusion can we draw from the test?

   p-value is very small. In this case we have enough evidence to reject H_0. We can conclude that the wording of the coupon <u>does affect</u> units purchased by customers.