MINING GRAPH DATA

STQD6414 PERLOMBONGAN DATA

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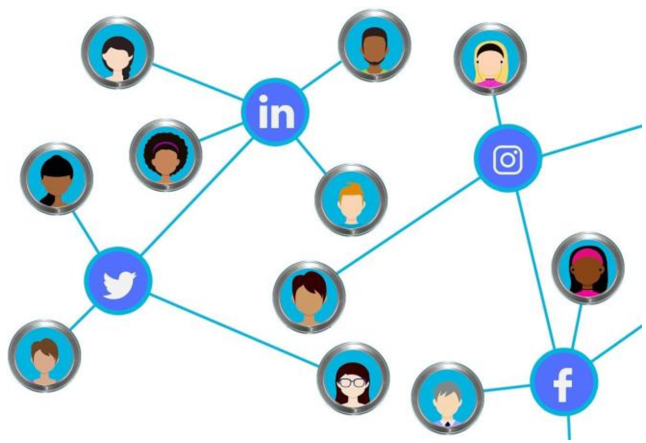
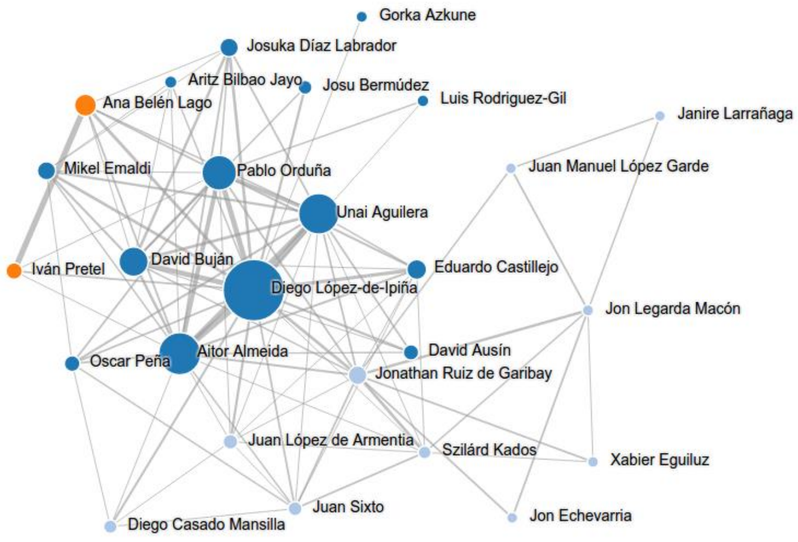
INTRODUCTION:

- A graph is a non-linear data structure that consist of nodes and edges.
- The objective of graph mining is to extract insightful knowledge from a data that is represented as a graph.
- Nowadays, graph-type data is everywhere, and available in many different fields.
- Examples: social network graphs, web graphs, cybersecurity networks, power grid networks, supply chain management, protein-protein interaction networks, and etc.

INTRODUCTION:

- Graphs often used to describe about links, relationships, or interconnections among some entities.
- Example: In the social science domain, the nodes in a graph are people and the links between them are friendship or professional collaboration, as can be seen from the platform of Facebook, LinkedIn, Instagram, twitter and etc.
- Through the analysis of graph data, we can gain a better understanding about the characteristics, behaviors or interaction trends among some particular entities.



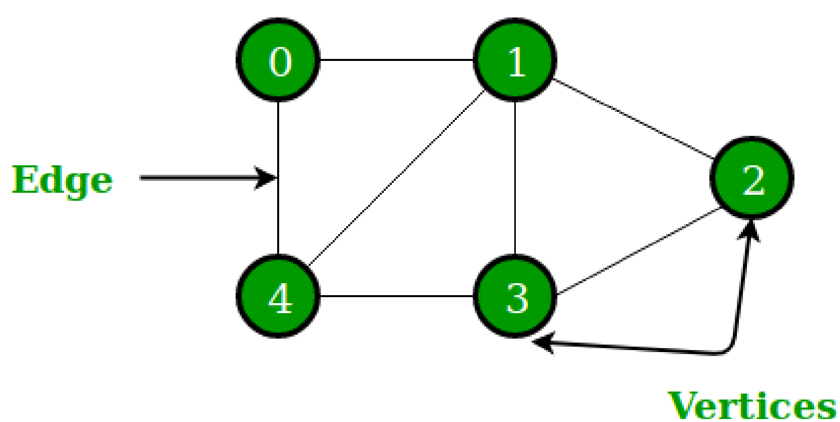


INTRODUCTION:

- To analyze the structure of a graph data, the knowledge of graph theory is required.
- Graph theory is a branch of mathematics that concerned with networks of points connected by lines.
- It provides a mathematical foundation used to model pairwise relations between objects.
- In general, graph represent structured data that contain vertices/nodes and edges:

i) Graph vertices represent information related to some entities.

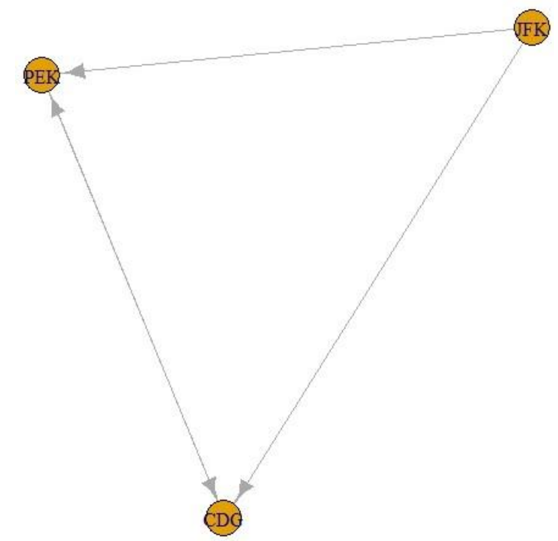
ii) The edges of the graph represent the relationship between the information and the entities.



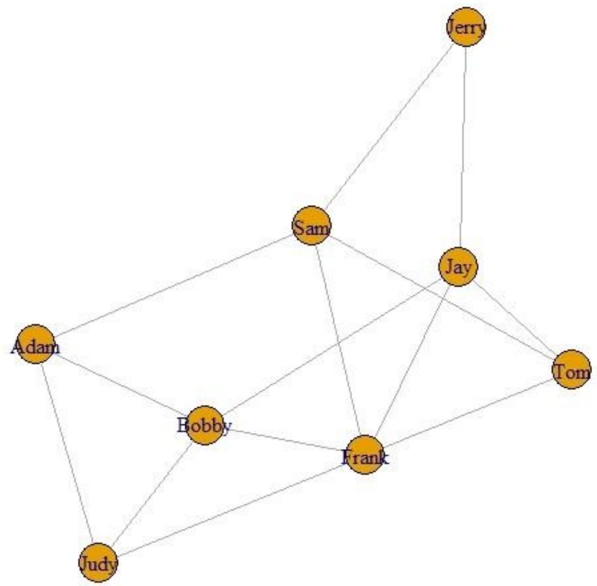
TYPES OF GRAPHS:

- There are various types of graph, among them are:

i) Undirected and Directed graphs:



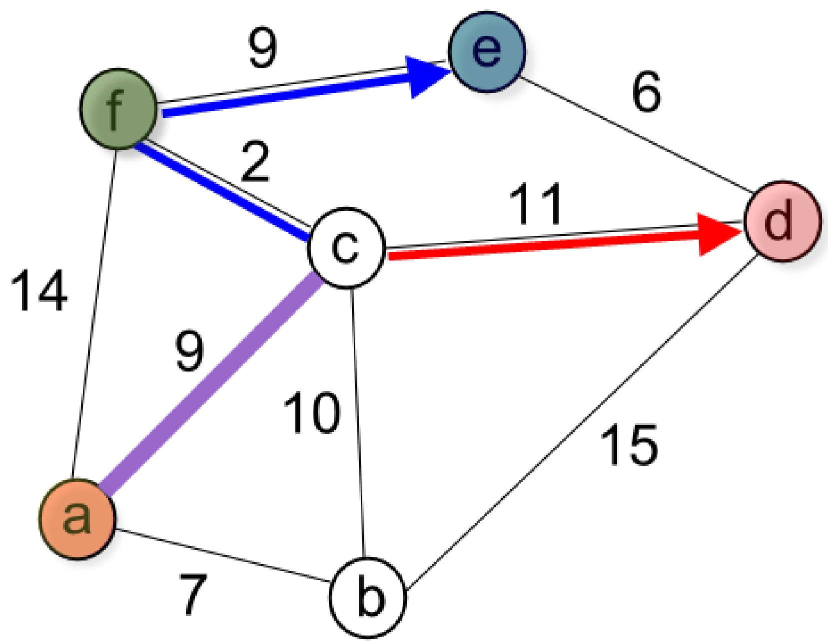
- A directed graph contains an ordered pair of vertices.
- Thus, directed graphs have edges with some specific directions.
- Undirected graph having unordered pair of vertices.
- Which implies that the edges for undirected graphs do not have a specific direction.



TYPES OF GRAPHS:

ii) Weighted and Unweighted graphs:

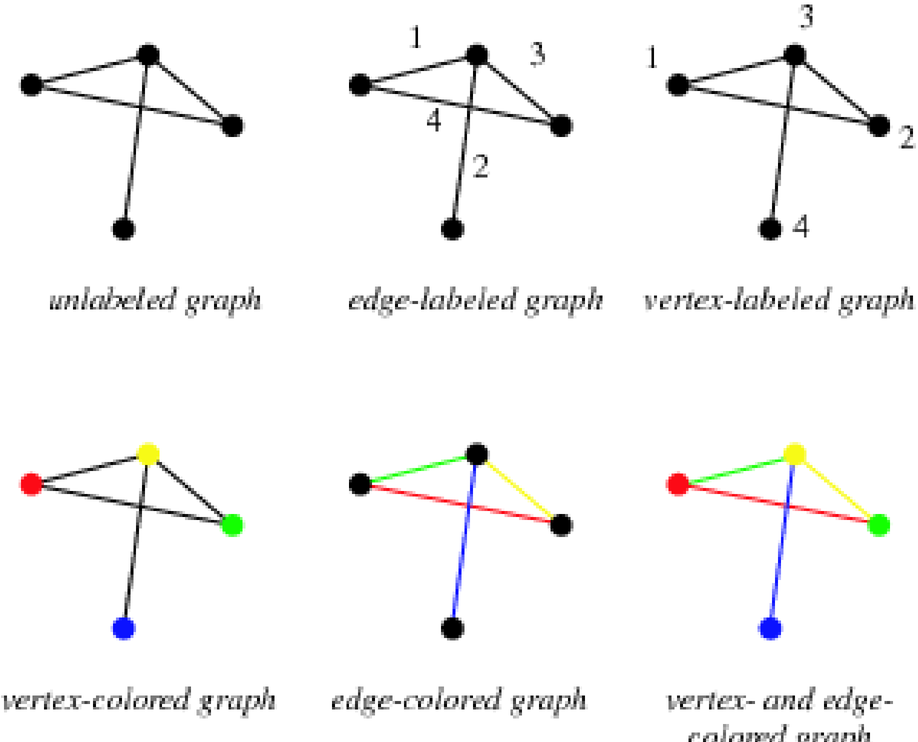
- A weight in graph represent a magnitude of relationships among the nodes and edges.
- A graphs that have weights is said to be a weighted graph, and vise versa.



TYPES OF GRAPHS:

iii) Labeled and Unlabeled graphs:

- An unlabeled graph is a graph whose nodes or edges do not have any indicator except through their relations.
- Whereas a labeled graph has several indicators in its nodes or edges.



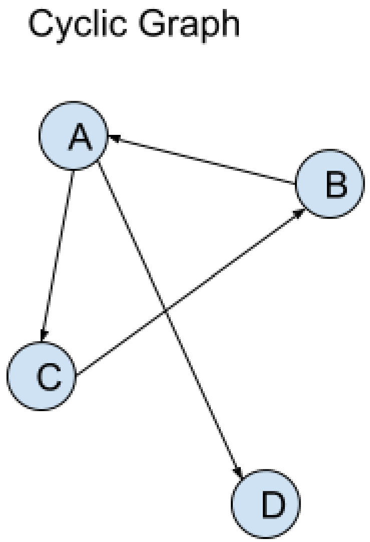
unlabeled graph *edge-labeled graph* *vertex-labeled graph*

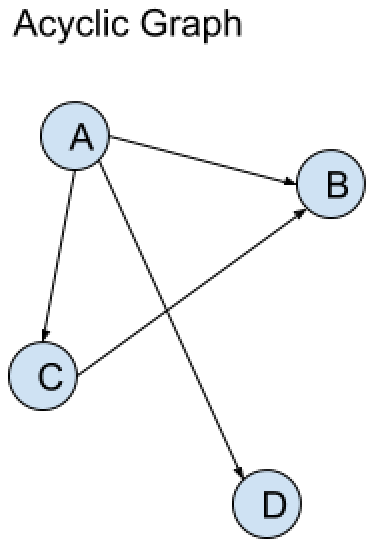
vertex-colored graph *edge-colored graph* *vertex- and edge-colored graph*

TYPES OF GRAPHS:

iv) Cyclic and acyclic graphs:

- A graph with at least one cycle is called a cyclic graph.
- A graph with no cycles is called an acyclic graph.

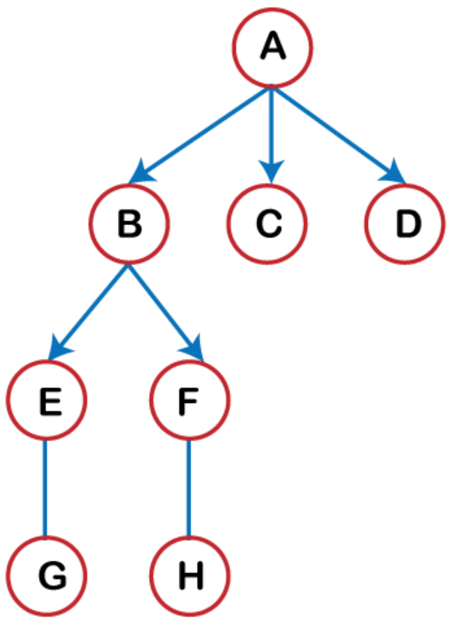


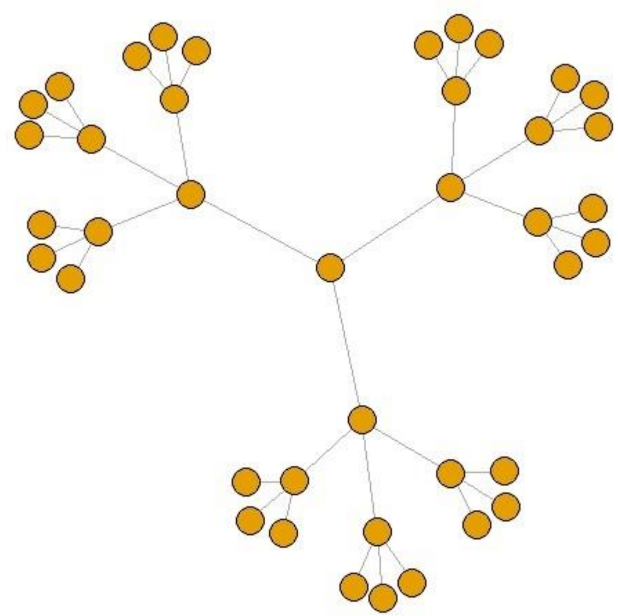


TYPES OF GRAPHS:

v) Trees Graph:

- This is undirected graph with any two vertices are connected by exactly one path.
- There is no cycles in this graph.
- Its also known as a connected acyclic undirected graph

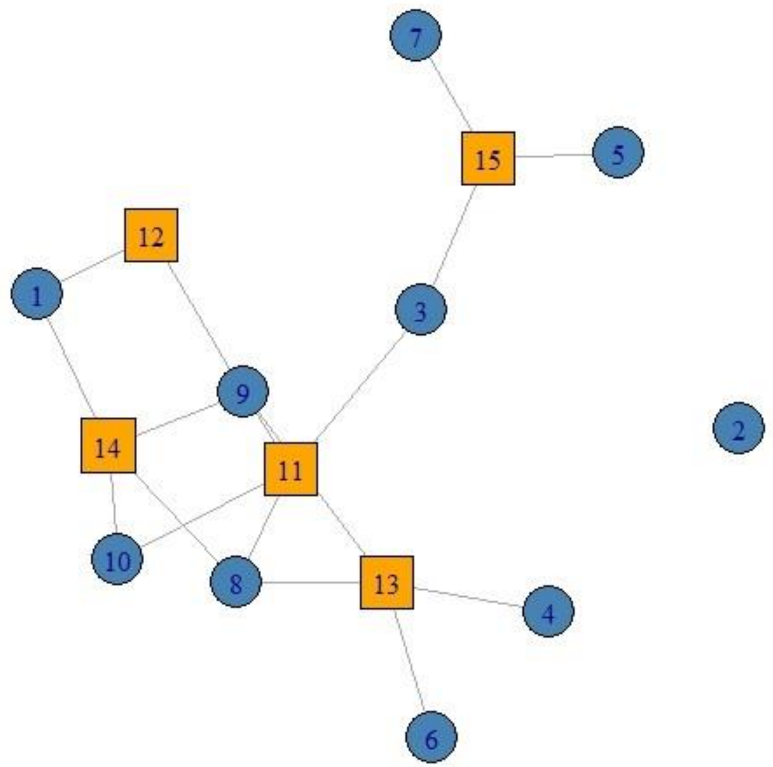




TYPES OF GRAPHS:

vi) Bipartite graph:

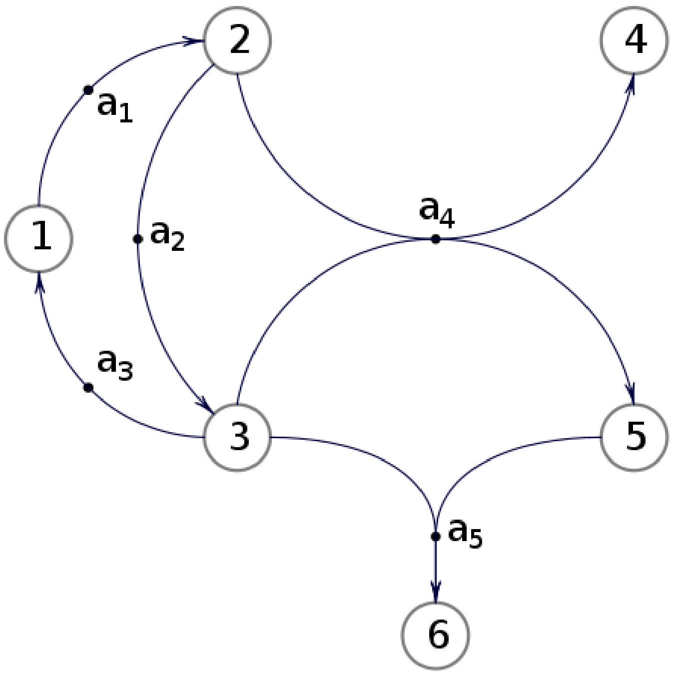
- A Bipartite Graph is a graph whose vertices can be divided into two independent sets (U and V)
- Every edge (u,v) either connects a vertex from U to V or a vertex from V to U.
- There is no edge that connects vertices of same set.
- The concept of bipartite graph can be generalized into multipartite graph.

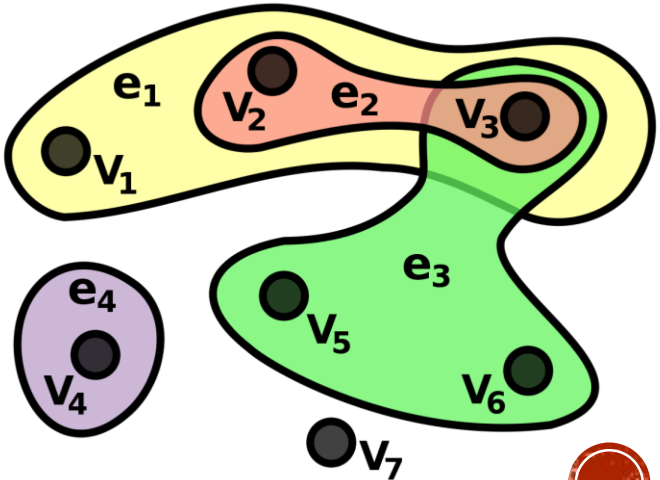


TYPES OF GRAPHS:

vii) Hypergraph:

- A hypergraph is a generalization of a graph where an edge can join any number of nodes.
- Hyper-edges (generalized edges) can connect to a subset of nodes compared to a non-hyper graph that only connects to 2 nodes on one edge.
- A k-hypergraph has all such hyper-edges connecting exactly k nodes.
- A common hyper graph is a 2-hypergraph (one edge connects 2 nodes).

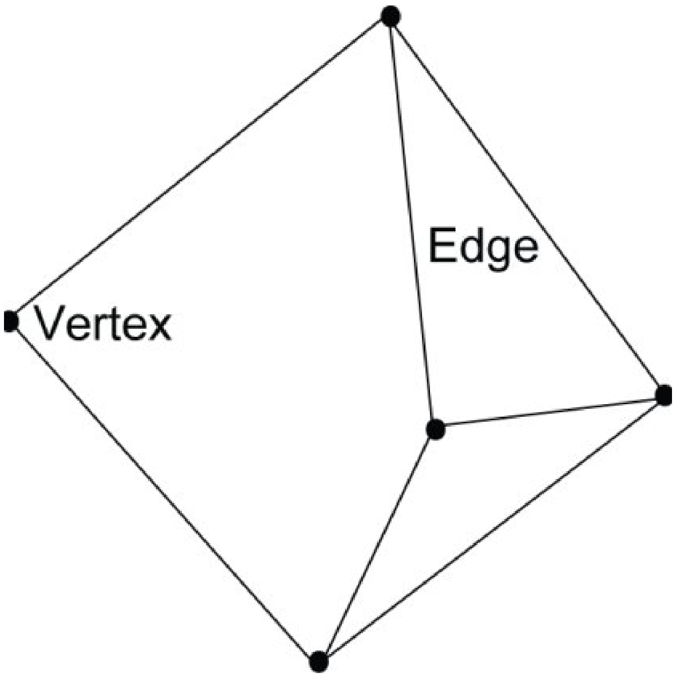




BASIC OF GRAPH THEORY:

- Some important definitions:

i) Graph: A graph G is composed of two sets: a set of vertices, denoted V(G), and a set of edges, denoted E(G).



ii) Edge: An edge in a graph G is an unordered pair of two vertices (v_1,v_2) such that $v_1 \in V(G)$ and $v_2 \in V(G)$.

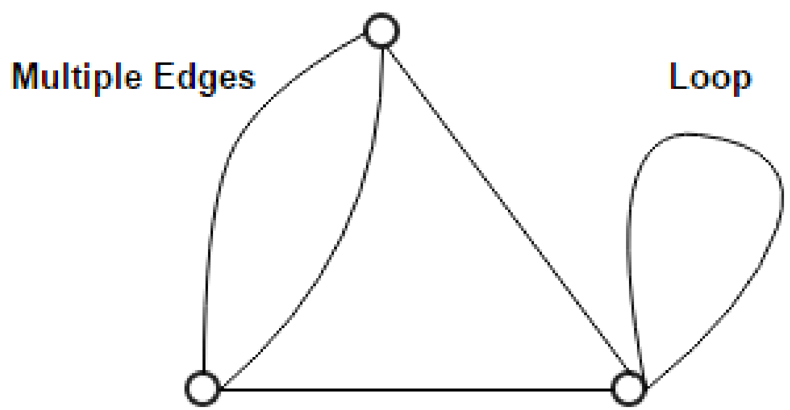
iii) Degree: degree(v), is the number of times vertex v occurs as an endpoint for the edges E(G).

BASIC OF GRAPH THEORY:

iv) Loop: A loop is an edge that joins a vertex to itself.

v) Multiple Edge: An edge is a multiple edge if there is another edge in E(G) which joins the same pair of vertices.

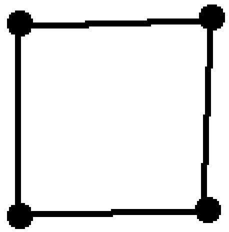
vi) Simple Graph: A graph with no loops or multiple edges.



BASIC OF GRAPH THEORY:

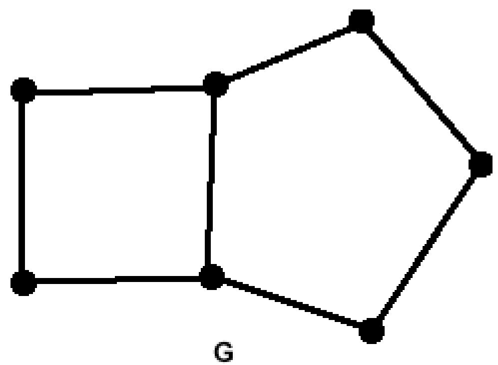
vii) Subgraph:

- A subgraph S of a graph G is a graph whose vertex set V(S) is a subset of the vertex set V(G), that is (V(S)⊆V(G)).
- While, an edge set E(S) is a subset of the edge set E(G), that is (E(S)⊆E(G)).



S

(Subgraph of G)

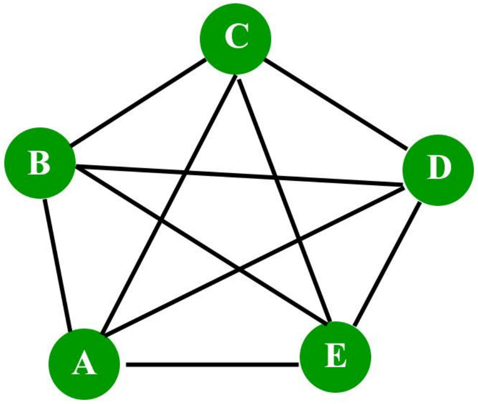


G

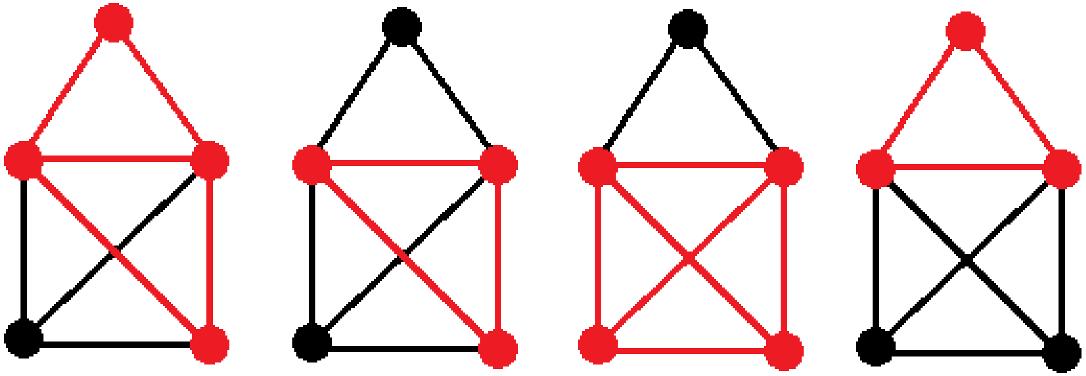
BASIC OF GRAPH THEORY:

viii) Clique:

- A subset A⊆V is a complete graph if all vertex pairs in A are connected by an edge.
- A graph G = (V,E) is complete if the vertex set V is complete.
- A clique is a maximal complete subset, if a complete subset is not contained in a larger complete subset.
- The set of cliques of a graph G is denoted by C(G).



Complete Graph

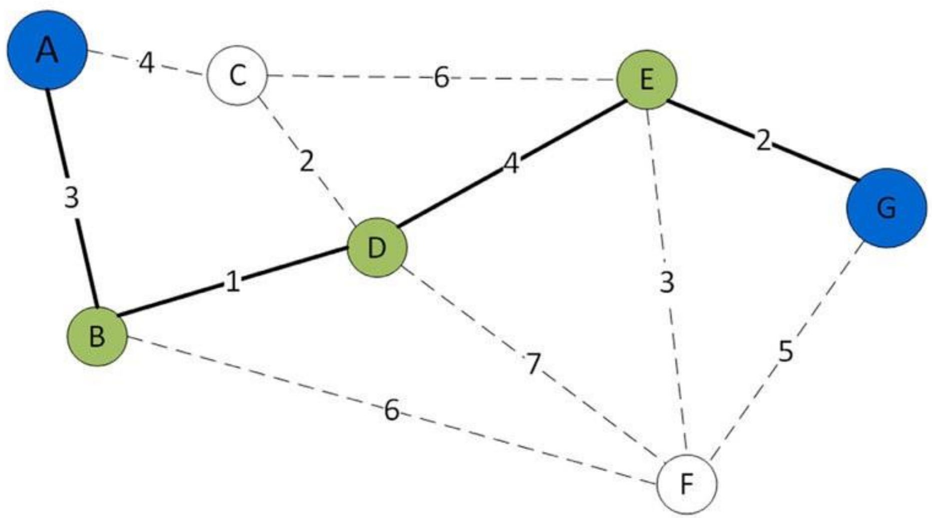


not a clique　　non-maximal clique　　maximal clique　　maximal clique

BASIC OF GRAPH THEORY:

ix) Path and circle:

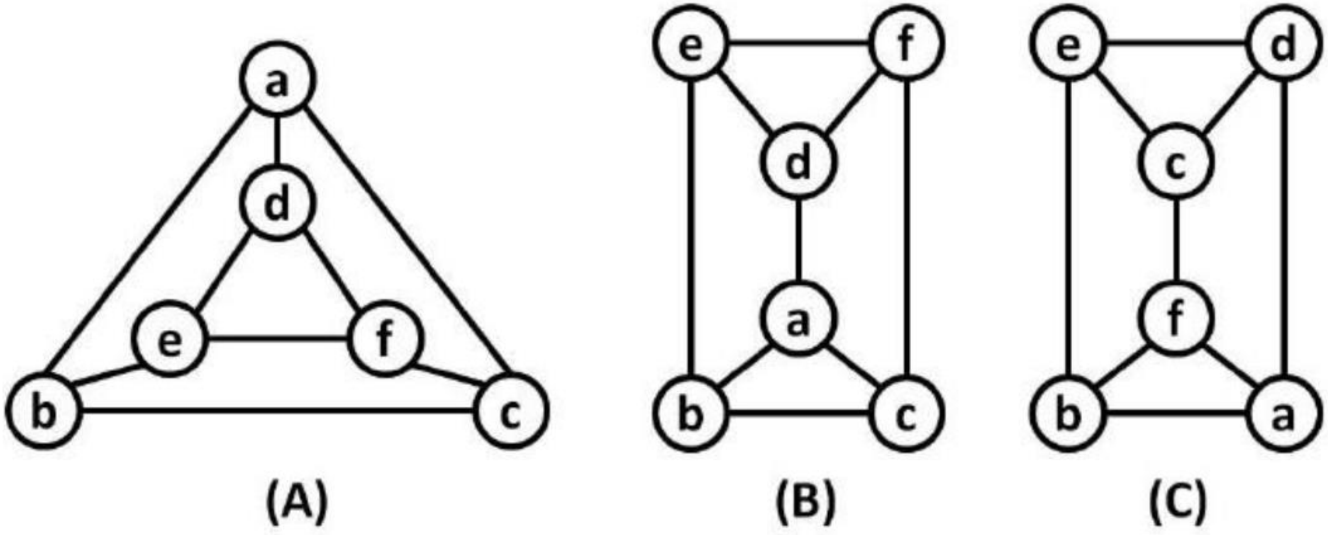
- A path (of length n) between α and β in an undirected graph is a set of vertices, such that $\alpha = \alpha_0, \alpha_1, \ldots, \alpha_n = \beta$.
- If a path $\alpha=\alpha_0,\alpha_1,\ldots,\alpha_n=\beta$ has $\alpha=\beta$ then the path is said to be a cycle of length n.



BASIC OF GRAPH THEORY:

x) Isomorphic graphs: A graph that can exist in different forms but having the same number of vertices, edges, and also having same edge connectivity.

xi) Automorphic Graphs: A graphs that have the same structure, but they have different labels. Thus, they are not exactly the same.



REPRESENTATIONS FOR GRAPHS:

- Generally, the graph will be stored in four basic formats:

i) Adjacency lists:

- An adjacency list is a collection of unordered lists.
- Each unordered list describes the set of neighbors of a specific vertex in the graph within an adjacency list.

ii) Edge lists:

- An edge list is a two-column table to list all the node pairs in the graph.

```
$Adam
+ 3/8 vertices, named, from d339868:
[1] Judy  Bobby Sam

$Judy
+ 3/8 vertices, named, from d339868:
[1] Adam  Bobby Frank

$Bobby
+ 4/8 vertices, named, from d339868:
[1] Adam  Judy  Frank Jay

$Sam
+ 4/8 vertices, named, from d339868:
[1] Adam  Frank Tom   Jerry

$Frank
+ 5/8 vertices, named, from d339868:
[1] Judy  Bobby Sam   Jay   Tom
```

	V1	V2
1	Adam	Judy
2	Adam	Bobby
3	Adam	Sam
4	Judy	Bobby
5	Judy	Frank
6	Bobby	Frank
7	Bobby	Jay
8	Sam	Frank
9	Sam	Tom

REPRESENTATIONS FOR GRAPHS:

iii) Adjacency matrix:

- This matrix shows whether two vertices in the graph are connected or not.
- If there is a link between two nodes “i and j,” the row-column indices (i, j) will be marked as 1, otherwise 0.

8 x 8 sparse Matrix of class "dgCMatrix"

	Adam	Judy	Bobby	Sam	Frank	Jay	Tom	Jerry
Adam	.	1	1	1	.	.	.	.
Judy	1	.	1	.	1	.	.	.
Bobby	1	1	.	.	1	1	.	.
Sam	1	.	.	.	1	.	1	1
Frank	.	1	1	1	.	1	1	.
Jay	.	.	1	.	1	.	1	1
Tom	.	.	.	1	1	1	.	.
Jerry	.	.	.	1	.	1	.	.

iv) Incidence Matrix:

- This is a logical matrix that shows the incidence relation between an vertex.
- The entry in row x and column y is 1 if x and y are related and 0 if they are not.

	Acciaiuoli	Albizzi	Barbadori	Bischeri	Castellani	Ginori	Guadagni
Acciaiuoli	0	0	0	0	0	0	0
Albizzi	0	0	0	0	0	1	1
Barbadori	0	0	0	0	1	0	0
Bischeri	0	0	0	0	0	0	1
Castellani	0	0	1	0	0	0	0
Ginori	0	1	0	0	0	0	0
Guadagni	0	1	0	1	0	0	0
Lamberteschi	0	0	0	0	0	0	1
Medici	1	1	1	0	0	0	0
Pazzi	0	0	0	0	0	0	0

GRAPH MANIPULATION:

- Among the important techniques of graph manipulation are:

i) remove specific nodes/vertices.

ii) generate subgraph.

iii) join graphs.

iv) modify the nodes data.

v) modify the edge data.

LINK AND NETWORK ANALYSIS:

- Link refer to a relationship between two entities.
- Network refer to a collection of entities and links between them.
- Graph mining provide a basis for link and network analysis.

- Example:

i) Graph mining can be used to interpret networks by determining clustering of nodes.

ii) Graph mining useful in determining how densely nodes connected in network data.

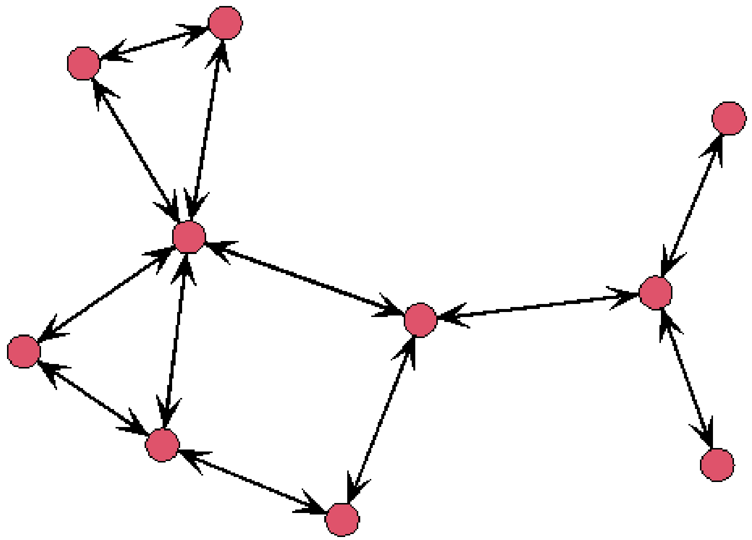
iii) Graph mining is useful in identifying the layout structure on network data.

NODE PROMINENCE ANALYSIS:

- Networks data are interesting because of their specific structural patterns.
- The structures will affect the characteristics of nodes/members in the network.
- Example: A person who is connected to many other members of a network is likely to view entire network in a different context than somebody who is relatively isolated from the other members.
- Thus, by examining the location of individual network members, we can assess the prominence of those nodes.
- A node is prominent if their ties make that node visible to the other members in the network.

NODE PROMINENCE ANALYSIS:

- Among the measures that can be used to measure prominence node :



i) Degree Centrality:

- Based on this measure, nodes that have more direct ties are more prominent.

ii) Closeness Centrality:

- Based on this measure, nodes are more prominent if they are more closer to all other nodes in the network.

NODE PROMINENCE ANALYSIS:

iii) Betweenness Centrality:

- Nodes are more prominent if their location sits ‘between’ pairs of other nodes in the network.
- A paths between the other nodes has to go through prominent node

iv) Eigenvector Centrality Scores:

- Measures the transitive influence of nodes.
- A high eigenvector score means that a node is connected to many nodes who themselves have high scores.

v) Information Centrality Scores:

- Nodes with higher information centrality have a greater control over the flow of information within a network.
- Its implies the existence of a large number of short paths within the network structure.

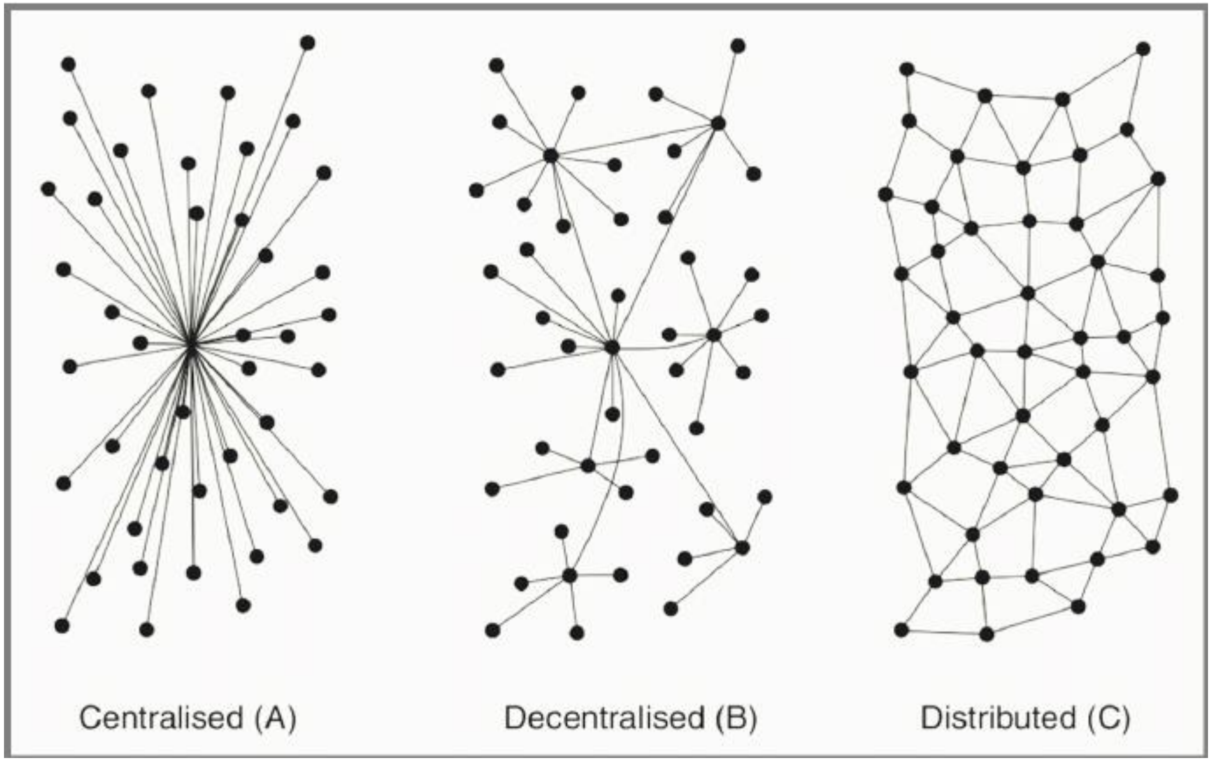
vi) Flow Betweenness Scores:

- Measures the total maximum flow of a particular nodes.

NODE PROMINENCE ANALYSIS:

vi) Centralization:

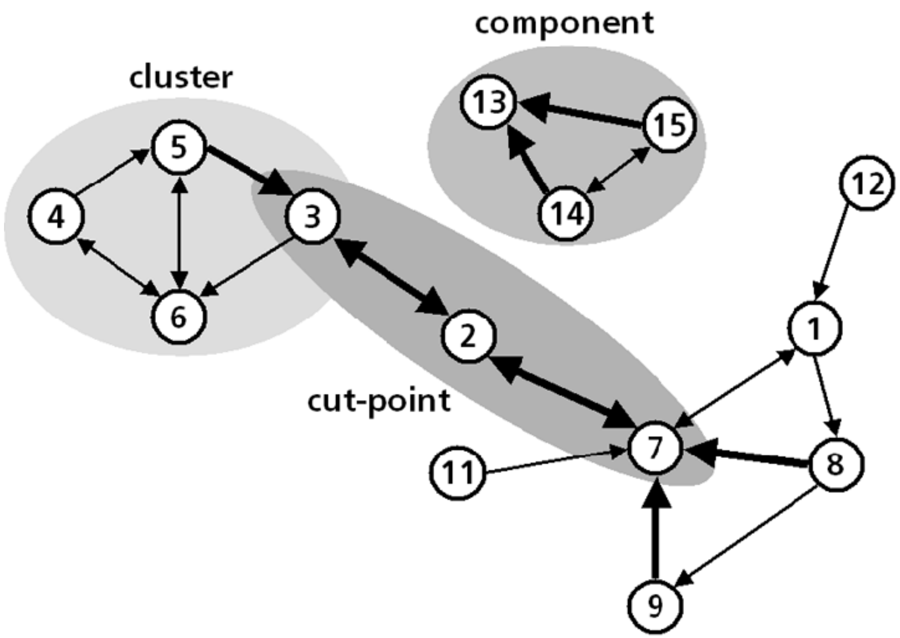
- Based on the given nodes measure, we can analyze the centralization behaviors of a network.
- Centralization provides a measure of the variability of the network centrality.



NODE PROMINENCE ANALYSIS:

vii) Cutpoints:

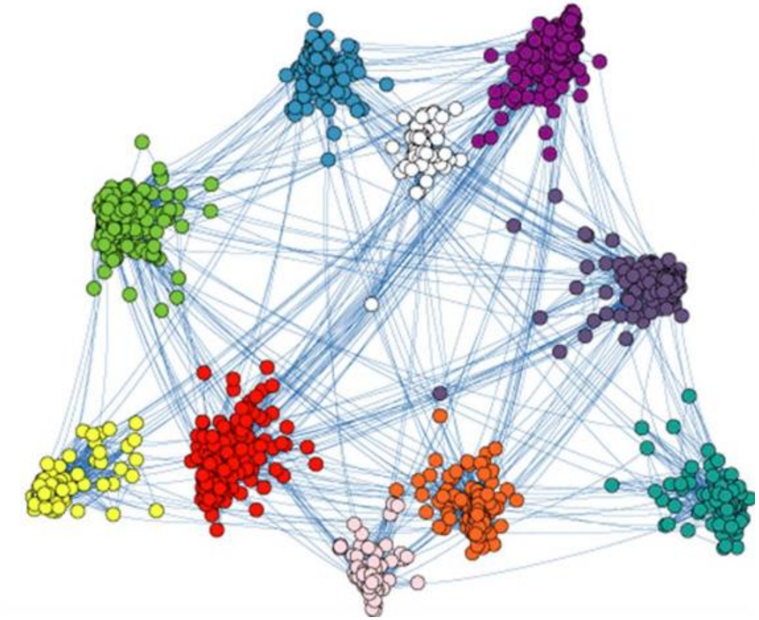
- A cutpoint refer to a node that, if we delete it, the number of components in the network will be increase.
- Cutpoints is a node with an important position that connects different parts of the network.
- If a cutpoint node is removed, it will result in two subsets of nodes that will not be able to communicate with each other.



SUBGROUPS ANALYSIS:

- Network data can be form by several densely connected subgroups that are themselves only connected via less common ties.

- Example: Friendship subnetworks can be found between acquaintances.
- These subgroups contain different information between each other.
- Thus, for a large network data, it is important to be able to define and identify such subgroups for further analysis.



SUBGROUPS ANALYSIS:

- In a real-application, for a large networks, the existence of subgroup structure usually difficult to be detected clearly.
- Thus, a systematic analysis need to done to determine the existence of subgroup structure.
- Subgroup structure can be detected based on the concept of social cohesion.
- Cohesive subgroups refer to a sets of nodes that are tied together through frequent, strong, and direct ties.

- Two most important types of cohesive subgroups are:

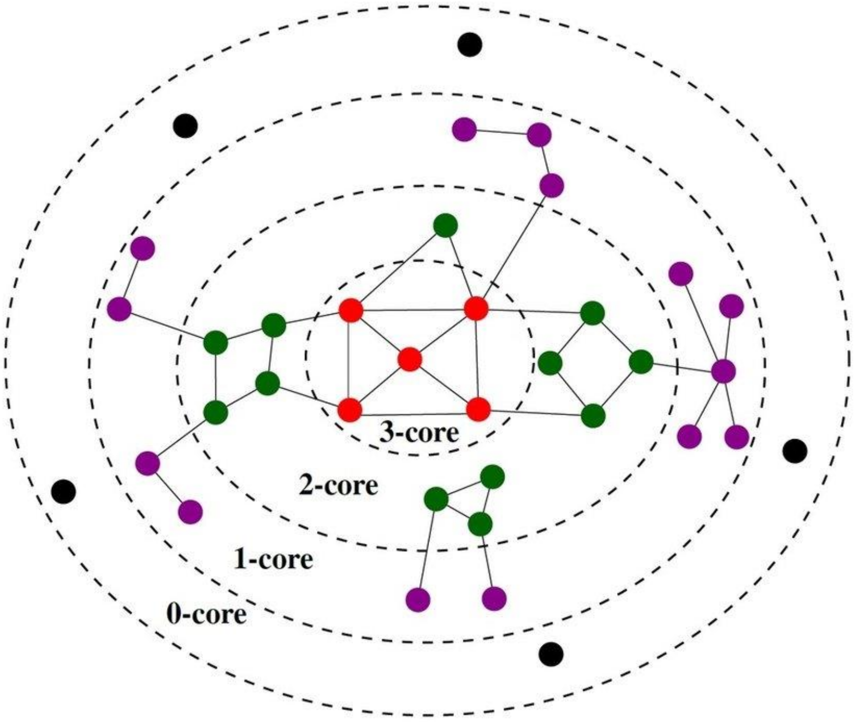
i) Cliques:

- A clique is a maximally complete subgraph.
- It is a subset of nodes that have all possible ties among them.

SUBGROUPS ANALYSIS:

ii) k-Cores:

- Cliques sometimes difficult to be determine because it require a condition of maximally complete subgraph.
- k-core is modification of cliques which refer to a maximal subgraph where each vertex is connected to at least k other vertices in the subgraph.

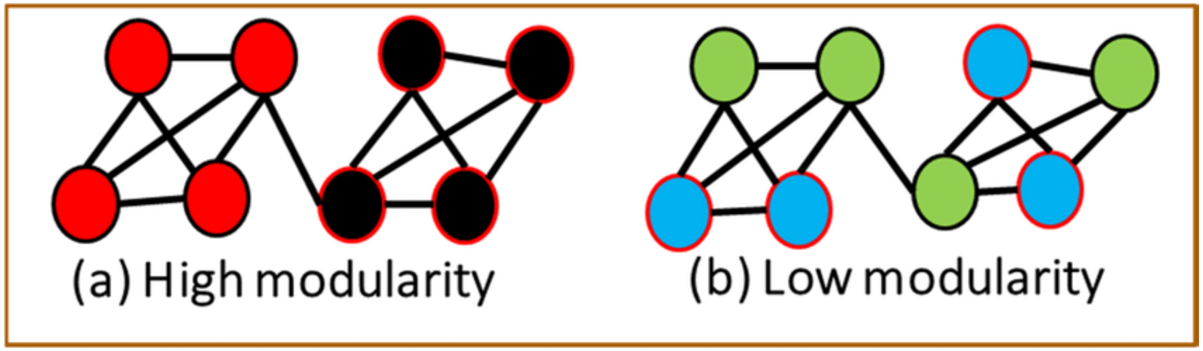


SUBGROUPS ANALYSIS:

Other approach to investigate a subgroup structure is based on community detection which include the techniques of:

i) Modularity:

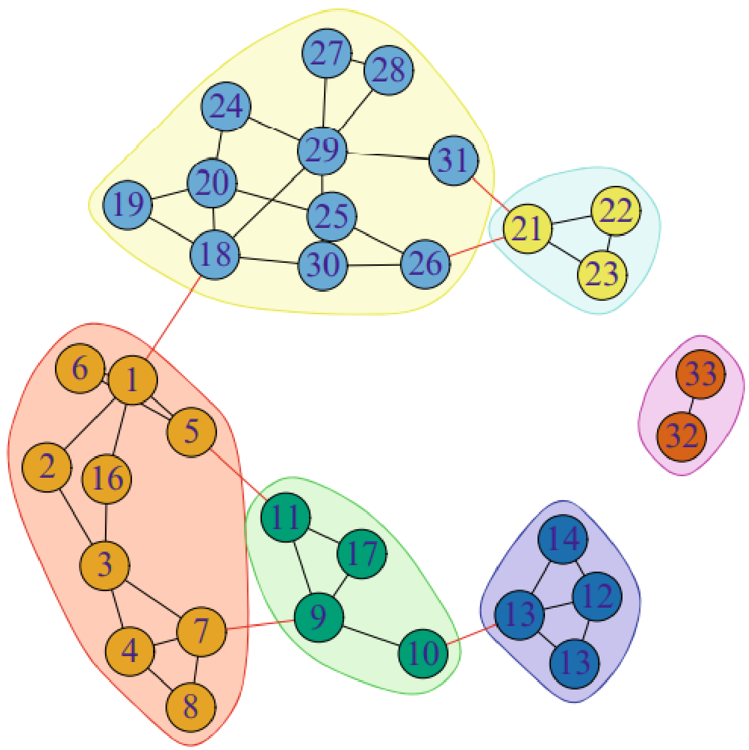
- This is a measure of the structure of the network, in which nodes exhibit clustering if there exist greater density within the clusters or less density between them.
- Value close to 1 indicates strong community structure. While, value equal to 0 indicates the community division is just a random.



SUBGROUPS ANALYSIS:

ii) Community Detection:

- A community in graph refer to a subset of nodes that are densely connected to each other and loosely connected to the nodes in the other communities.



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NEXT TOPIC:

Mining Web Data