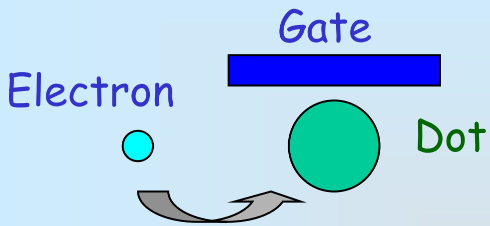
Building blocks for nanodevices

- Two-dimensional electron gas (2DEG)
- Quantum wires and quantum point contacts
- Electron phase coherence
- Single-Electron tunneling devices
  - Coulomb blockage
- Quantum dots (introduction)

Single Electron Tunneling Devices

Coulomb blockade

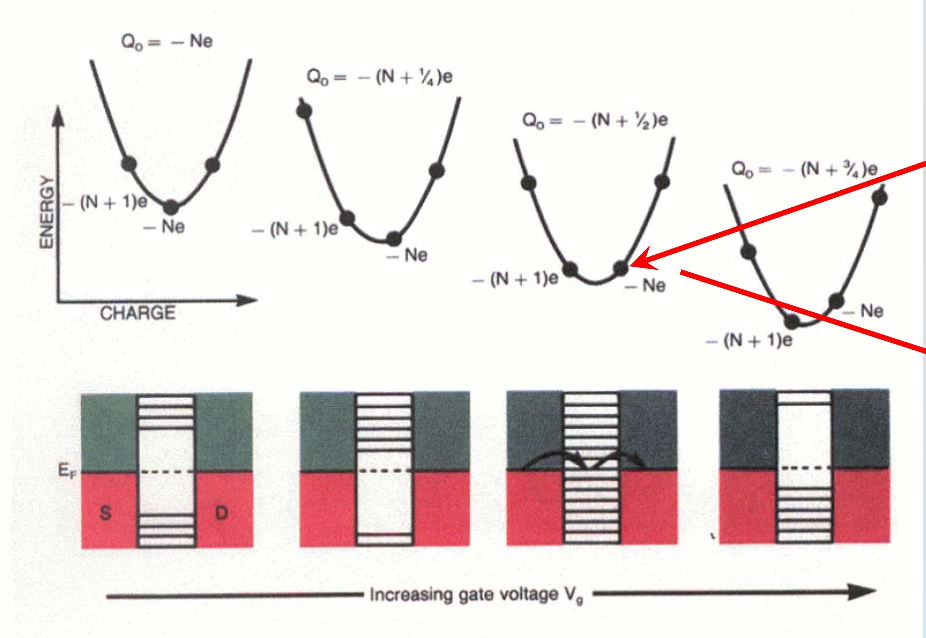


Repulsion at the dot

Cost $$E = QV_g + \frac{Q^2}{2C}$$

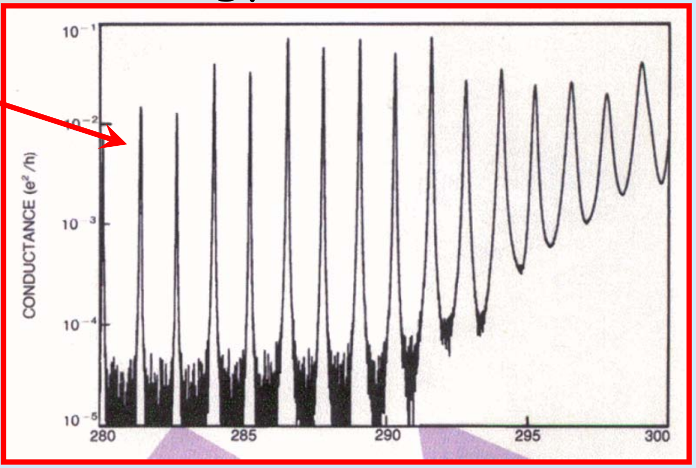
Attraction to the gate

$$Q = -Ne$$



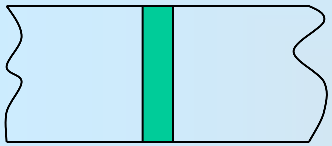
At $$V_g = -\left(N + \frac{1}{2}\right)\frac{e}{C}$$

the energy cost vanishes !



Single-electron transistor (SET)

Coulomb blockade in a tunnel barrier



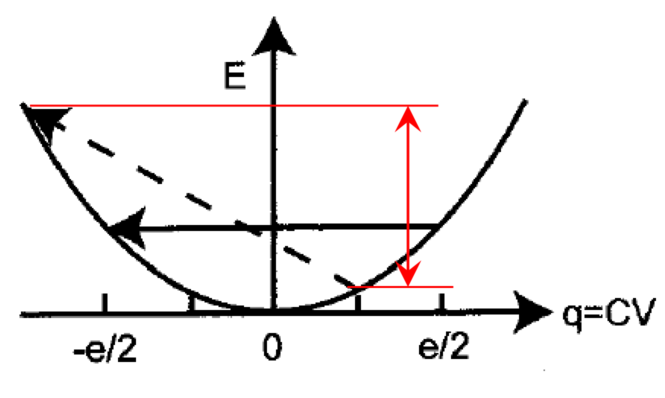
Why R matters?

time delay $\delta t = eR/V$

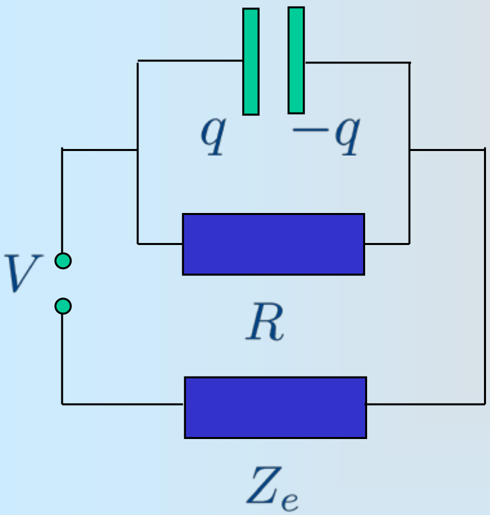
duration $\tau \sim \hbar/eV$

$\delta t \gg \tau \quad \rightarrow \quad R \gg \hbar/e^2$

Energy stored is $q^2/2C$

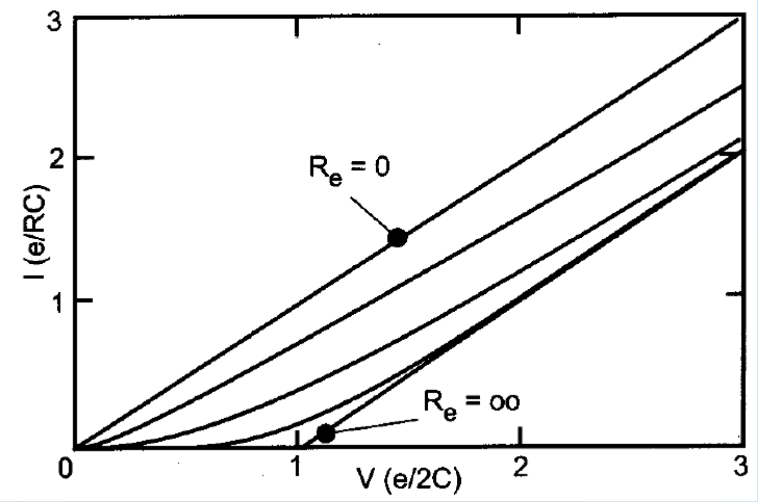


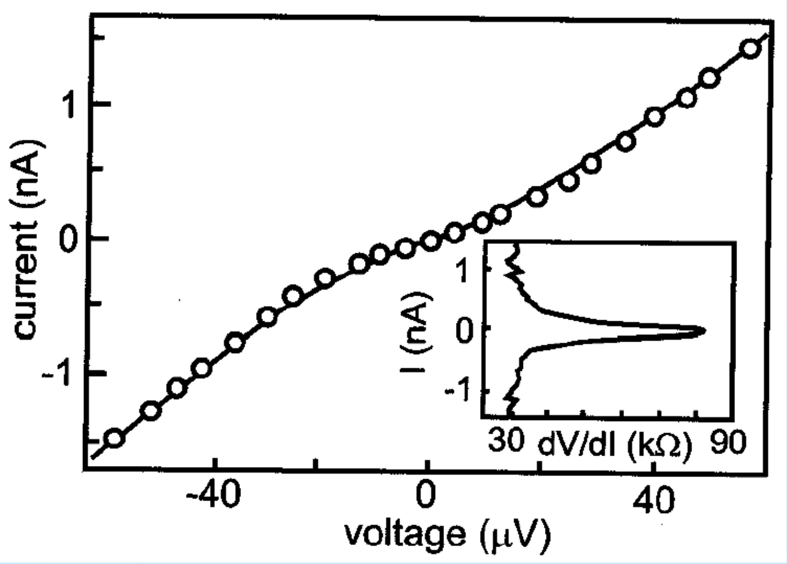
At $|q| < e/2$ the electron tunneling will increase the energy stored in the barrier – one has to pay for the tunneling by the bias voltage



Because of environment capacitances it is difficult to observe CB in single junctions

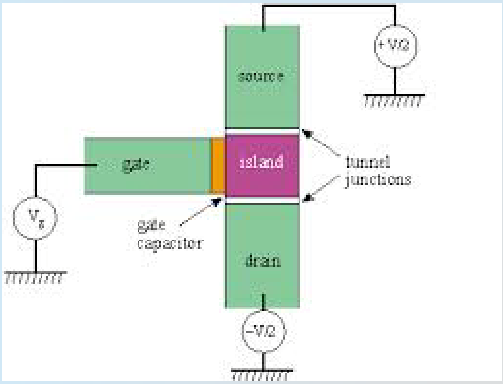
Resulting I-V curve

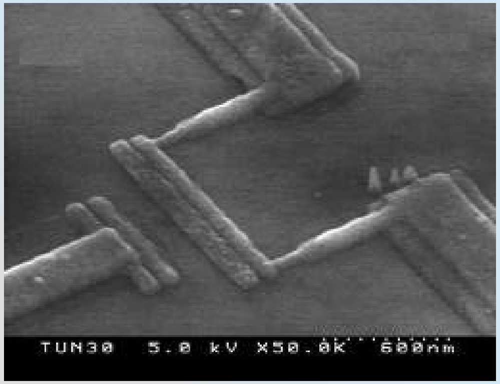


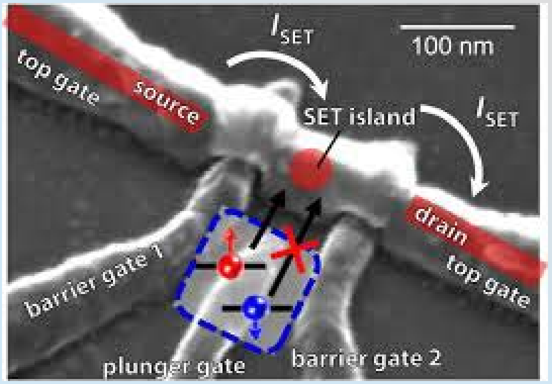


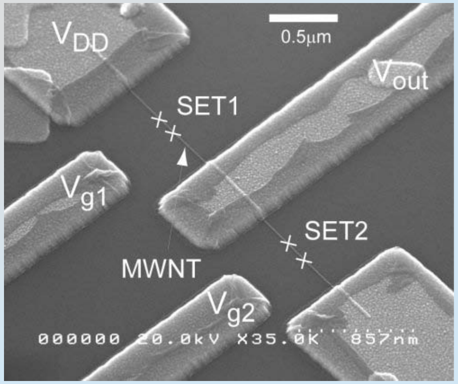
<u>Experiment:</u> $Al-Al_2O_3-Al$, 10 nm x10 nm (superconductivity was destroyed by magnetic field)

SET: Basic circuit and devices



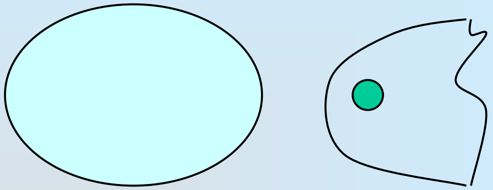






Basic tunneling circuits

Isolated island:

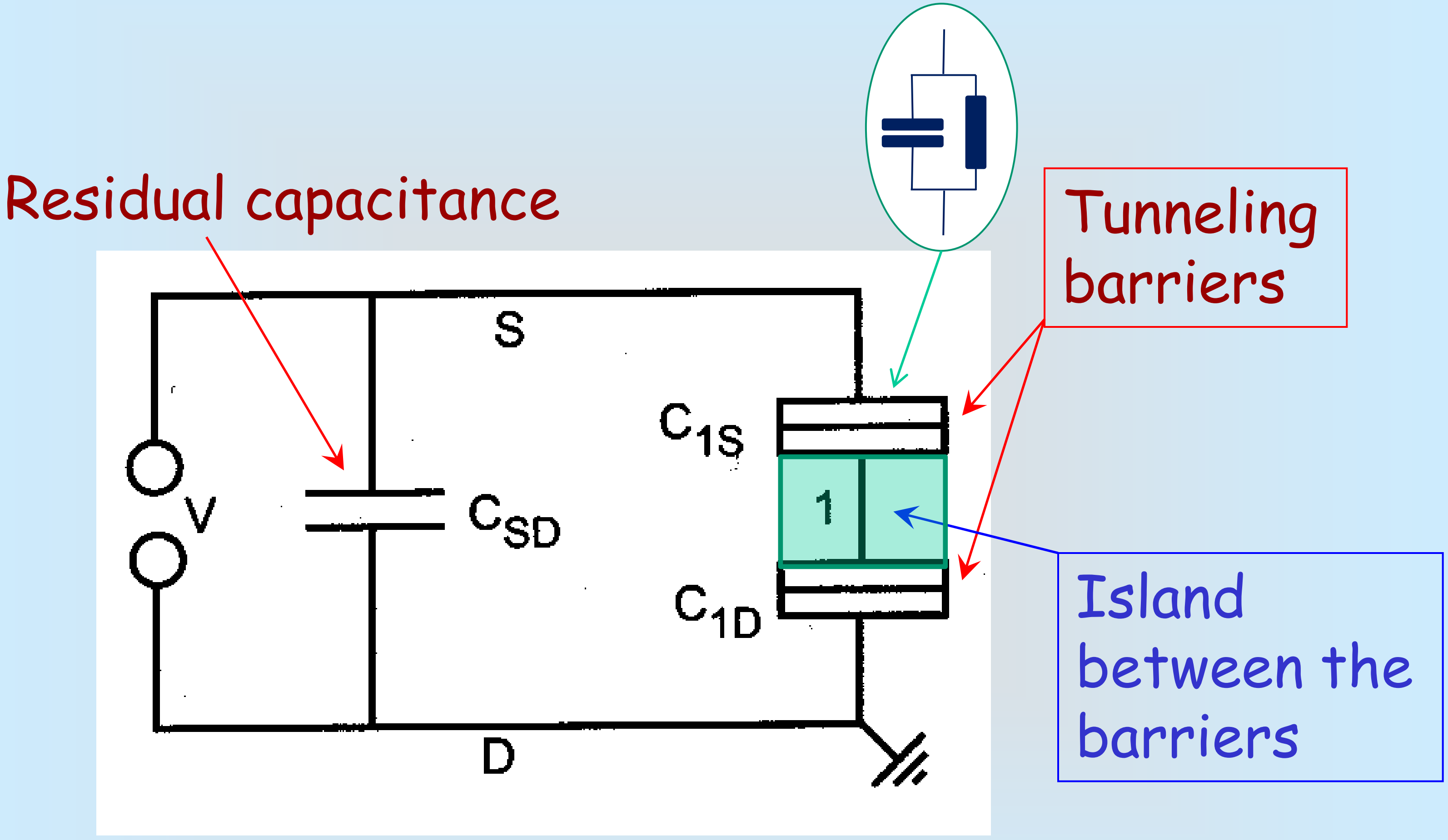


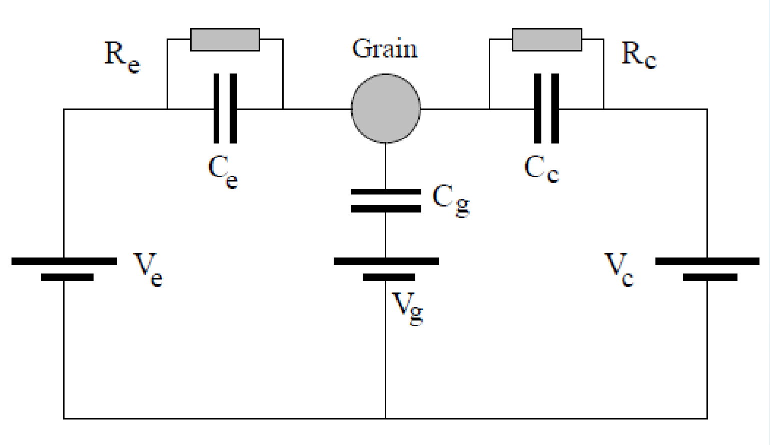
$$\Delta E = \frac{[(n+1)e+q_0]^2}{2C} - \frac{[ne+q_0]^2}{2C} = \frac{e}{C}\left[\left(n+\frac{1}{2}\right)+q_0\right]$$

Background charge

At $q_0 = -(n+1/2)e$ the energy cost vanishes!

Double barrier structure: Circuitry





Circuitry

The charge conservation requires that

$$
\begin{aligned}
-ne &= Q_e + Q_c + Q_g \\
&= C_e(V_e - U) + C_c(V_c - U) + C_g(V_g - U)\,,
\end{aligned}
$$

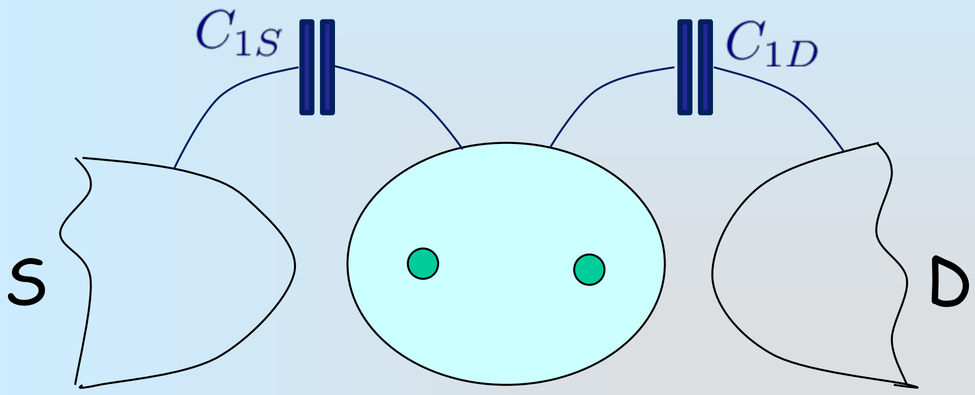
where U is the potential of the grain. The effective charge of the grain is hence

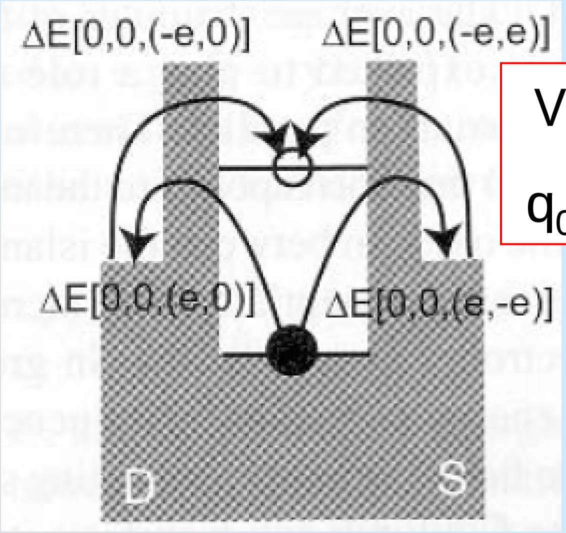
$$Q = CU = ne + \sum_{i=e,c,g} C_i V_i\,, \quad C \equiv \sum_i C_i\,.$$

This charge consists of 4 contributions, the charge of excess electrons and the charges induced by the electrodes. Thus, the electrostatic energy of the grain is

$$E_n = \frac{Q^2}{2C} = \frac{(ne)^2}{2C} + \frac{ne}{C}\sum_i C_i V_i + \frac{1}{2C}\left(\sum_i C_i V_i\right)^2\,.$$

Electrostatics: 4 different charge transfer events are relevant





V=0

q_0=0

$$\Delta E = \frac{e}{C_{1S}+C_{1D}}\left[\frac{e}{2} \pm (ne - q_0 + C_{1D}V)\right]$$

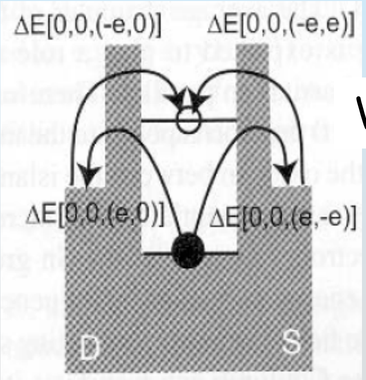
$$\Delta E = \frac{e}{C_{1S}+C_{1D}}\left[\frac{e}{2} \pm (ne - q_0 - C_{1S}V)\right]$$

induced charge

Symmetric system: $C_{1S} = C_{1D} \equiv C/2$

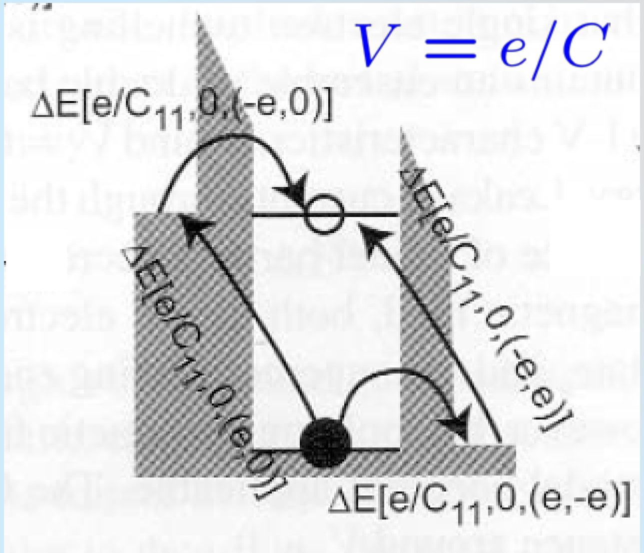
For *n=0* and *q_0=0*, all transfers are suppressed until

$$-\frac{e}{C} \leq V \leq \frac{e}{C}$$



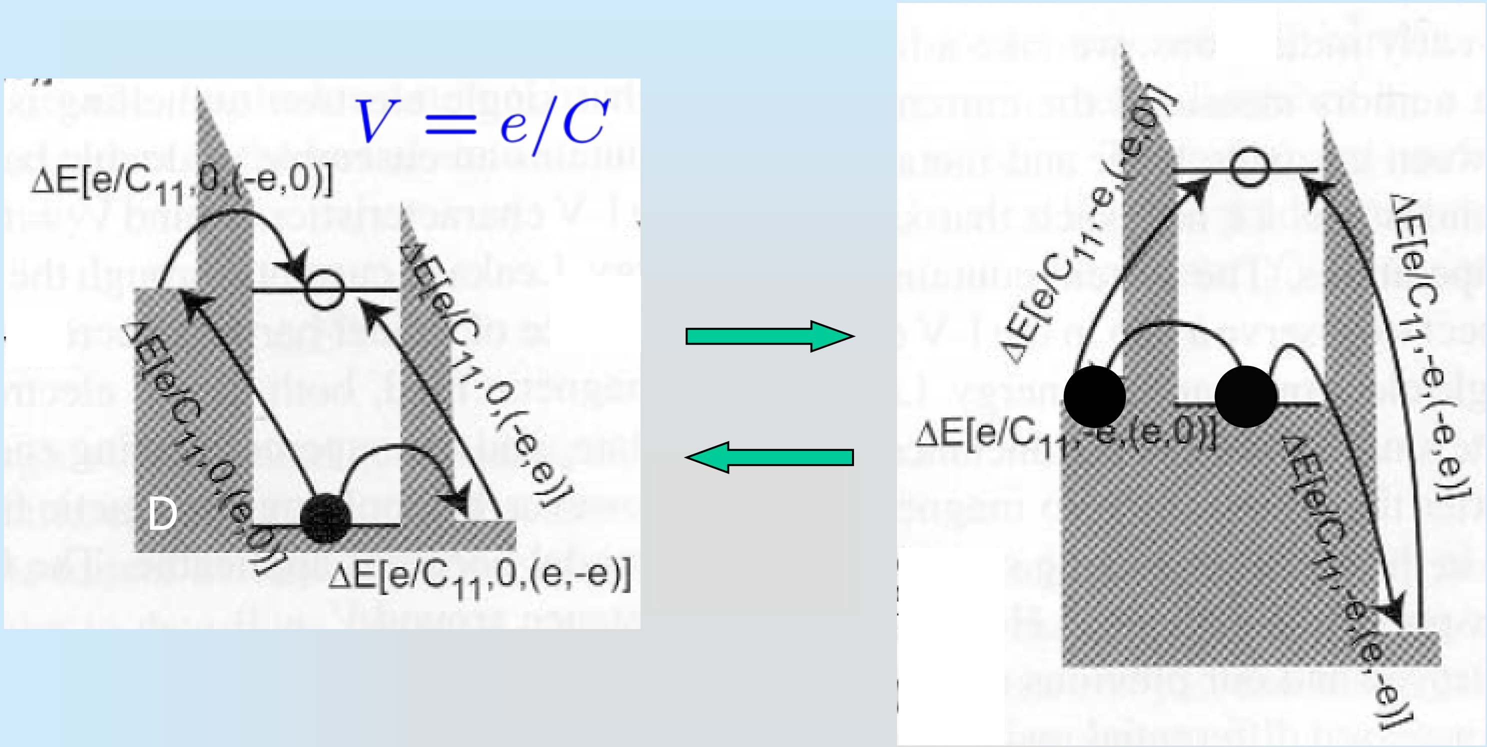
V=0

Coulomb blockade of transport



Electrons can tunnel both from drain onto island and from island to source

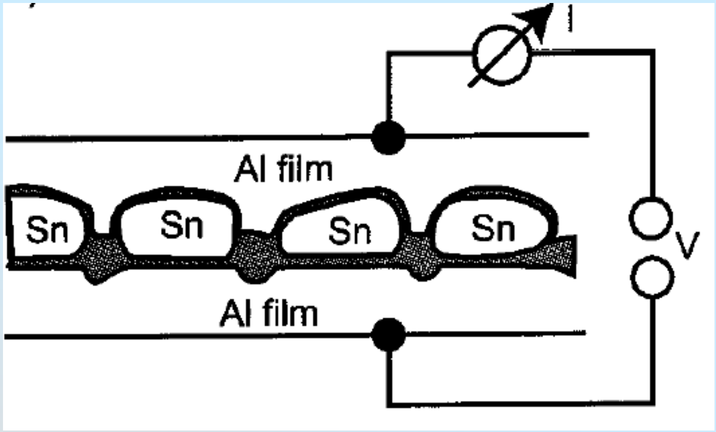
What happens when an electron reaches the island?

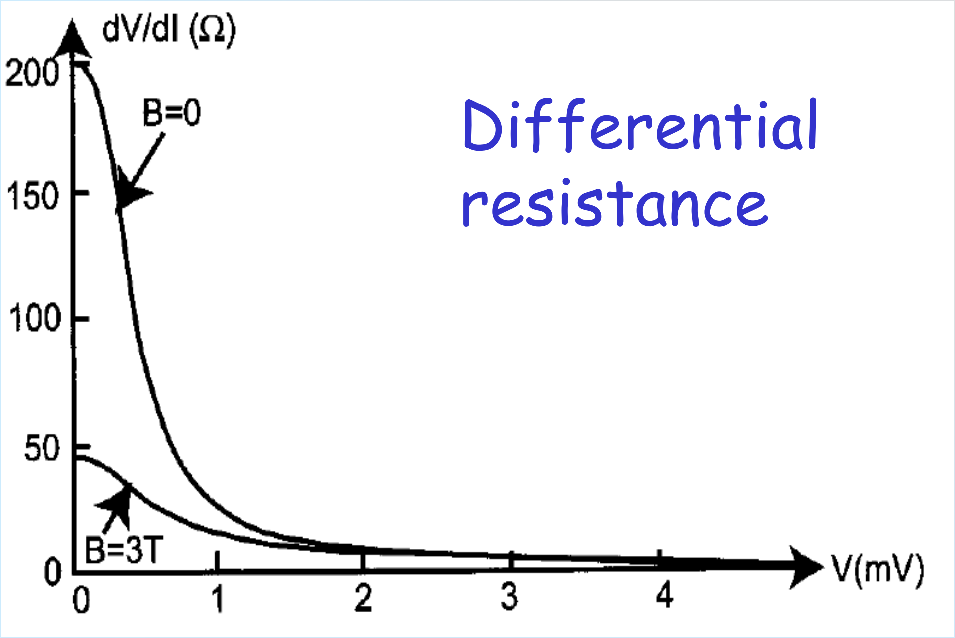


During each cycle a single electron is transferred!

First observation:

Giæver and Zeller, 1968 – granular Sn film, superconductivity was suppressed by magnetic filed

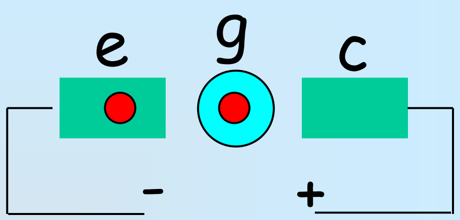




Coulomb gap manifests itself as increased low-bias differential resistance

The background charges, q_0, influence Coulomb blockade and can even lift it.

I-V curves: Coulomb staircase



How one can calculate I-V curve?

For simplicity, we will do it only for a stationary case.

The current through the emitter-grain transition we get

$$I = e \sum_n p_n \left[\Gamma_{e\to g} - \Gamma_{g\to e}\right] .$$

Here p_n is the stationary probability to find n excess electrons at the grain. It can be determined from the balance equation,

$$\boxed{p_{n-1}\Gamma^n_{n-1} + p_{n+1}\Gamma^n_{n+1} - \left(\Gamma^{n-1}_n + \Gamma^{n+1}_n\right) p_n = 0 .}$$

Here

Master equation

$$\begin{aligned}
\Gamma^n_{n-1} &= \Gamma_{e\to g}(n-1) + \Gamma_{c\to g}(n-1) ; \\
\Gamma^n_{n+1} &= \Gamma_{g\to e}(n+1) + \Gamma_{g\to c}(n+1) .
\end{aligned}$$

The partial probabilities, Γ, can be calculated from the Fermi Golden Rule: the probability is

$$W_{i\to f} = \frac{2\pi}{\hbar}\left|\langle i|\widehat{\mathcal{H}}|f\rangle\right|^2 \delta(E_f - E_i - \Delta E)$$

To get the rate we have to multiply the probability by

$$g_i g_f F(i)[1 - F(f)]$$

Fermi function

and then sum over initial and final states.

Since only the vicinity of the Fermi level matters we can take the densities of states and matrix elements at the Fermi level and express the results through tunneling conductance, G, of the junctions.

We use the tunneling Hamiltonian $\mathcal{H}_{e\leftrightarrow g} = \sum_{\mathbf{k},\mathbf{q},\sigma} T_{\mathbf{kq}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{q}\sigma} + \mathrm{h.c.}$

that gives the following expression for the tunneling conductance

$$G_e = (4\pi e^2/\hbar)\, \underbrace{g_e(\epsilon_F) g_g(\epsilon_F)}_{\text{DOS}}\, \underbrace{\mathcal{V}_e \mathcal{V}_g}_{\text{Volume}}\, \langle |T_{\mathbf{kq}}|^2 \rangle$$

As a result, for, e. g. , for the transition between the emitter and the grain

$$\Gamma_{e\to g} = \frac{G_e}{e^2} \int d\epsilon_k \int d\epsilon_q\, F_e(\epsilon_k)[1 - F_g(\epsilon_q)] \times \delta(\epsilon_q - \epsilon_k + E_{n+1} - E_n - eV_e)$$

Finally,

$$\Gamma_{e\to g}(n, V_e) \; = \; \Gamma_{g\to e}(-n, -V_e) = \frac{2G_e}{e^2}\mathcal{F}(\Delta_{+,e})\,;$$

$$\Gamma_{g\to c}(n, V_c) \; = \; \Gamma_{c\to g}(-n, -V_c) = \frac{2G_c}{e^2}\mathcal{F}(\Delta_{-,c})\,.$$

Temperature

Here

$$\mathcal{F}(\epsilon) = \frac{\epsilon}{1+\exp(-\epsilon/kT)} \to \epsilon\,\Theta(\epsilon) \text{ at } \Theta \to 0\,,$$

Heaviside

while

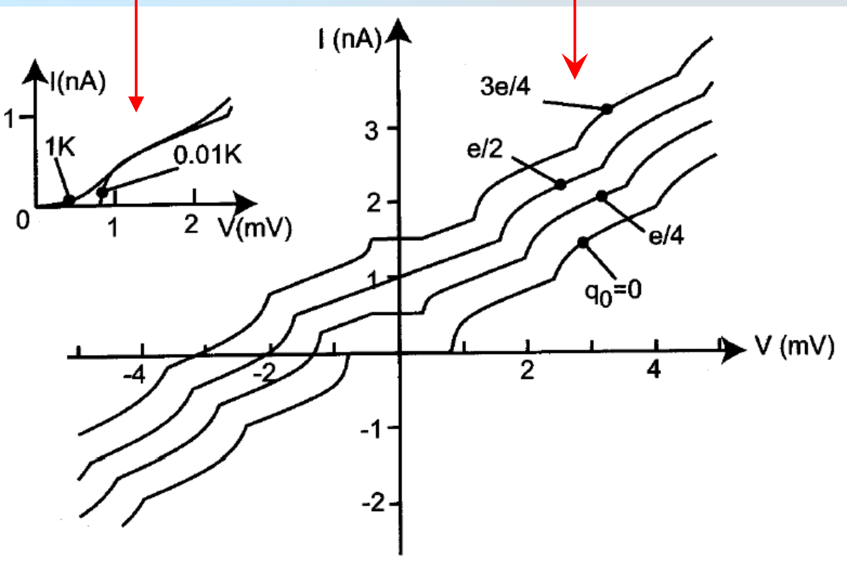
$$\Delta_{\pm,\mu}(n) = E_n - E_{n\pm 1} \pm eV_\mu = \frac{1}{C}\left[\frac{e^2}{2} \mp en \qquad\qquad\right] \pm eV_\mu$$

is the $\boxed{\text{energy cost}}$ of transition. The temperature-dependent factor arise from the Fermi occupation factor for the initial and final states, physically they describe thermal activation over Coulomb barrier.

With induced charges, SET

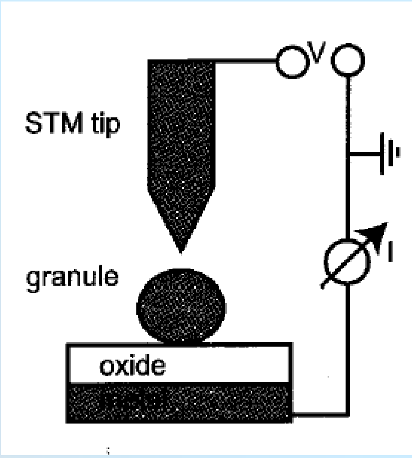
Thermal smearing

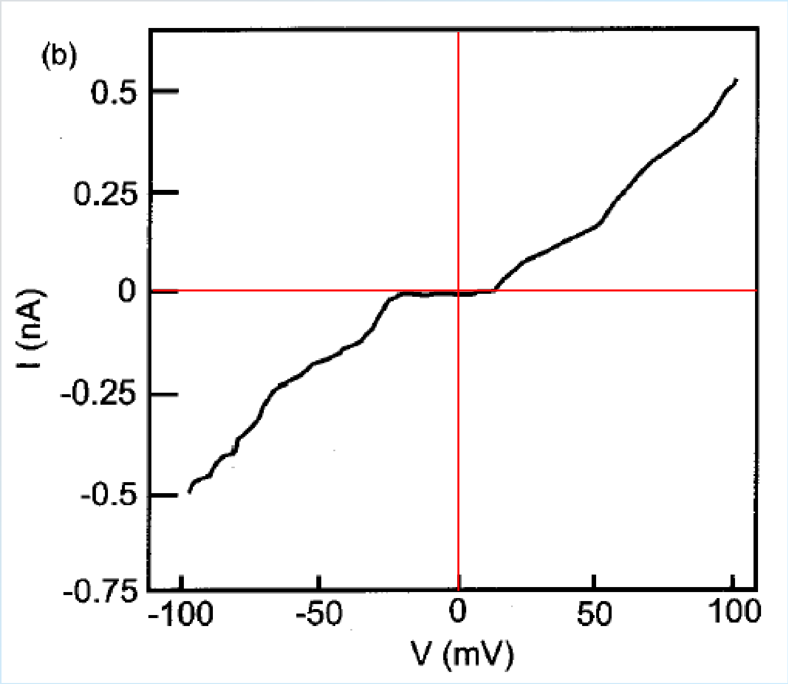
Coulomb staircase



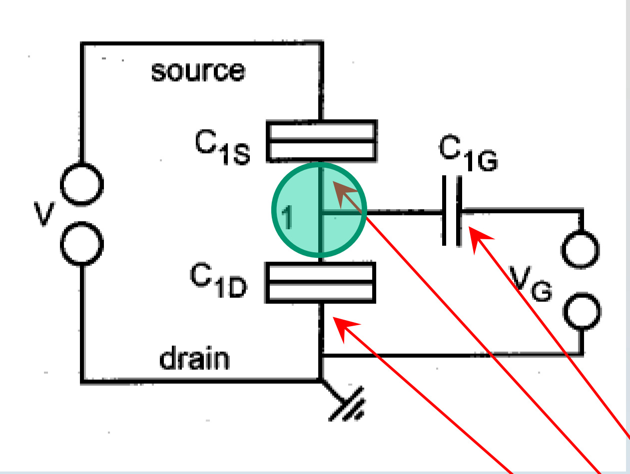
Calculations for different background charges

Experiment: STM of surface clusters





The SET transistor



Fulton & Dolan, 1987

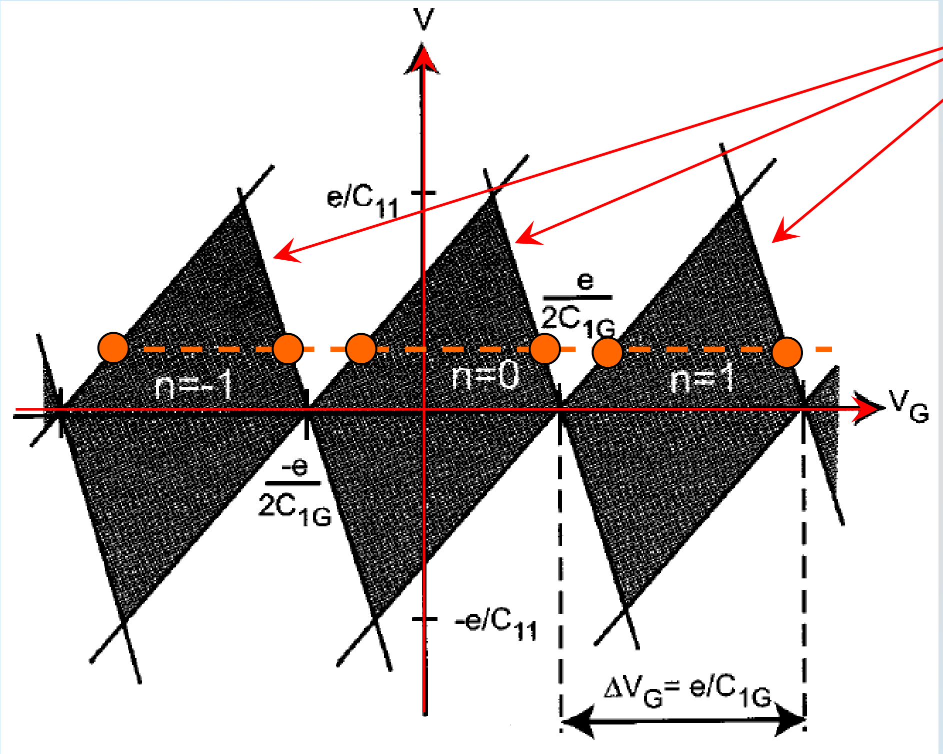
An extra electrode (gate) defined in a way to have very large resistance between it and the island.

That allows to tune induced charges by the gate voltage

The so-called "orthodox" theory discussed before is valid; we have just to remember that the energy cost is

$$\frac{1}{C}\left[\frac{e^2}{2} \mp en \mp e\sum_i C_i V_i\right] \pm eV_\mu$$

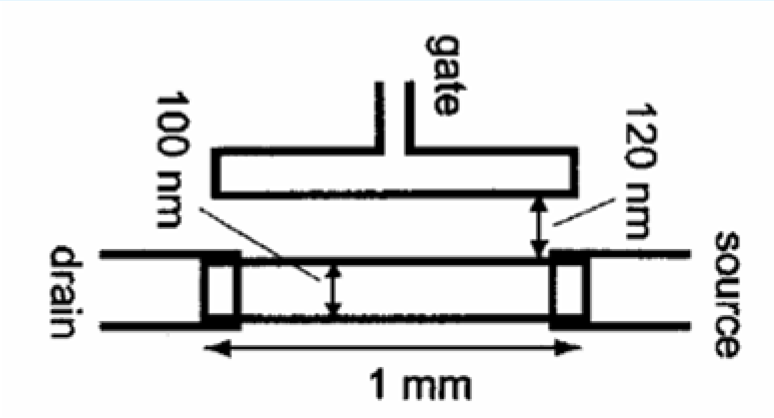
In this way we arrive at the so-called stability diagram of Single Electron Transistor (SET)

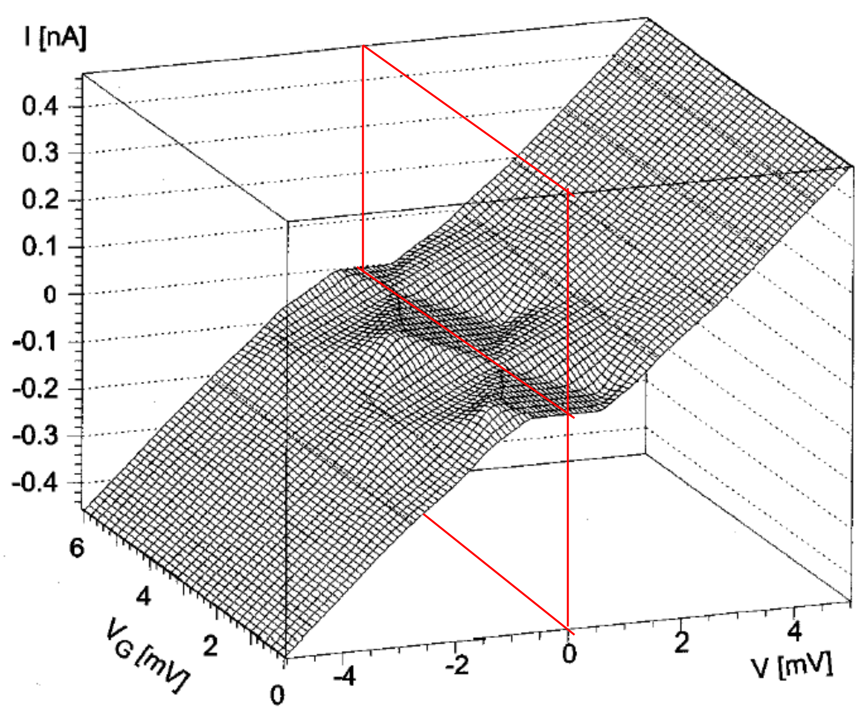


Coulomb diamonds: all transfer energies inside are positive

Conductance oscillates as a function of gate voltage – Coulomb blockade oscillations

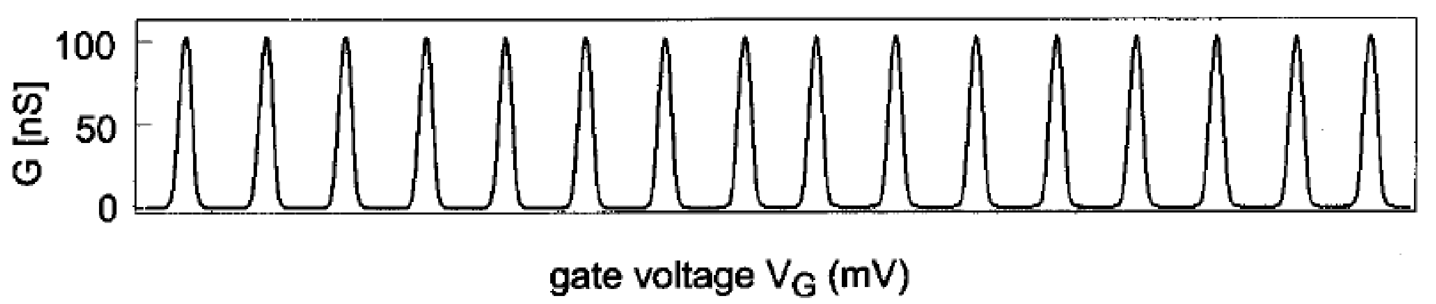
Experimental test: Al-Al_2O_3 SET, temperature 30 mK



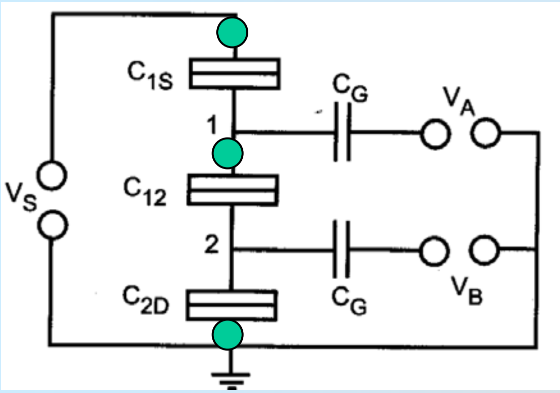


Coulomb blockade oscillations

V=10 μV



The single electron pump



Two islands – each one can be tuned by a nearby gate electrode.

The structure is symmetric.

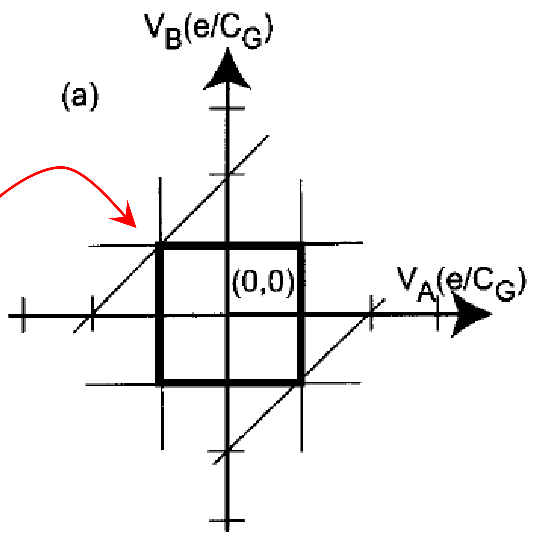
Six electron transfers are important.

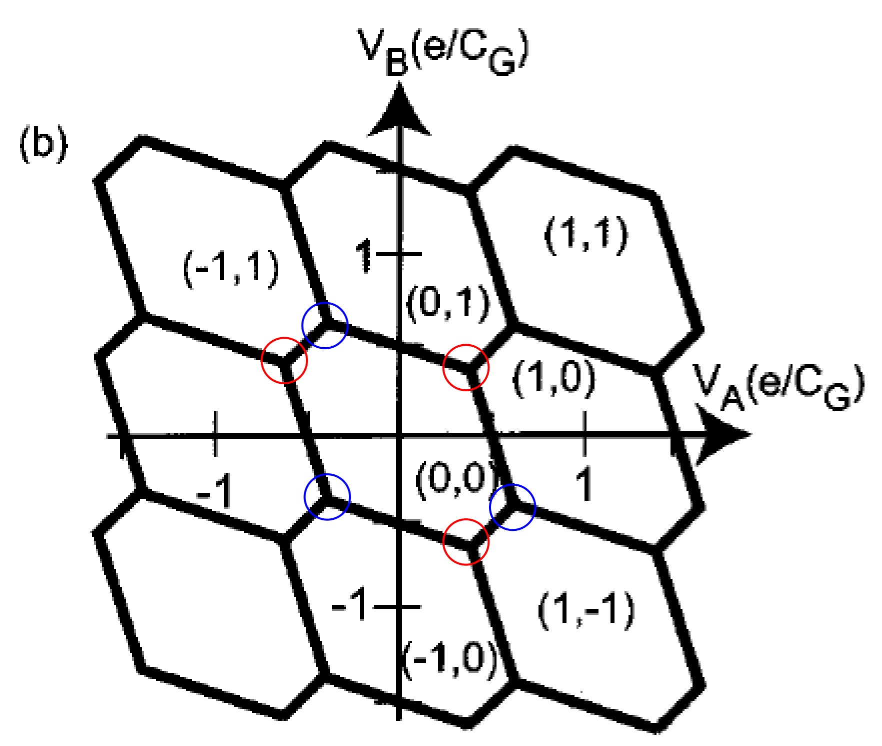
Equalities between direct and reverse processes define lines at the stability diagram.

It define the regions of stable configurations characterized specific charges of the island.

The inter-island capacitance C_{12} is responsible for the interplay of the contributions from V_A and V_B.

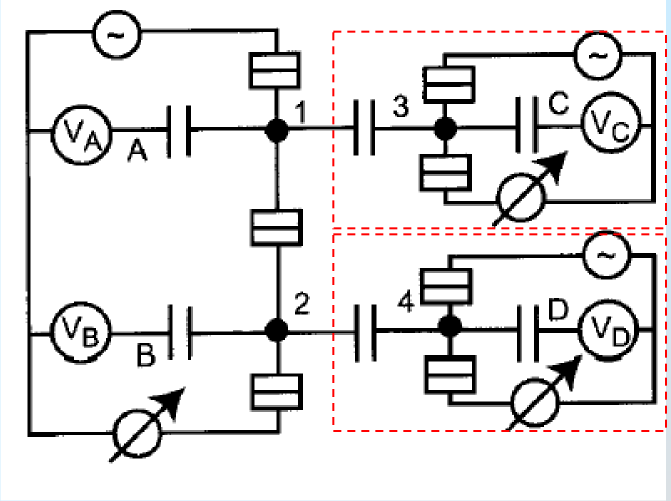
At $C_{12}=0$, the diagram is a set of squares, unaffected by the transitions of 3[d] type – the corresponding lines just touch corners





In the general situation there are "triple points", where and electron can transfer the whole system for free.

We will show that it allows one to "pump" electrons one by one.

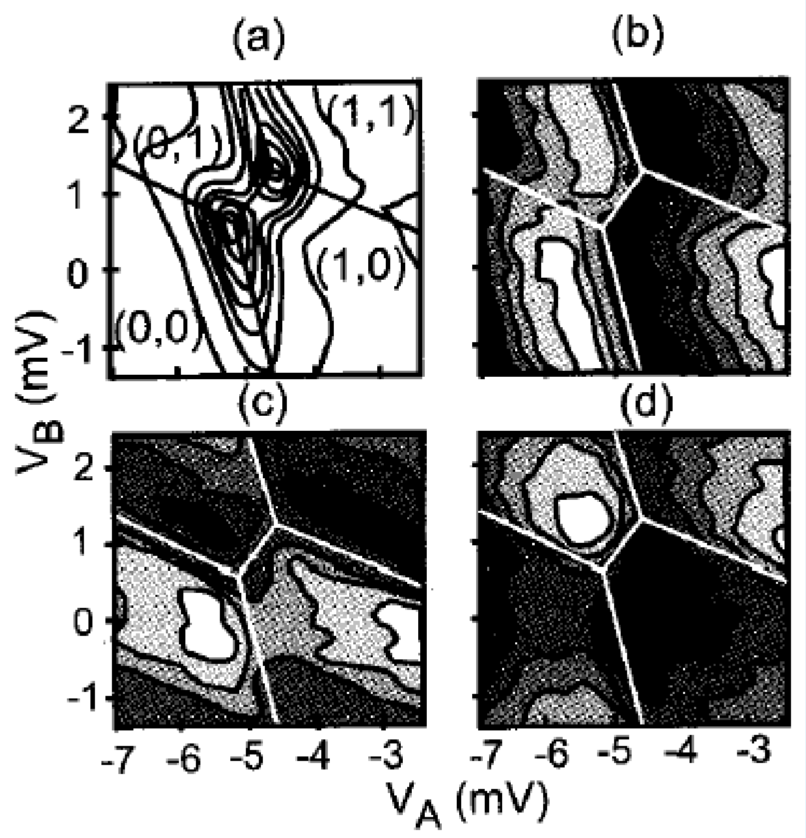


Experiment:

SETs 3 and 4 work as electrometers to measure charges at the islands 1 and 2

Conductance of the double island

Conductance of el. 4



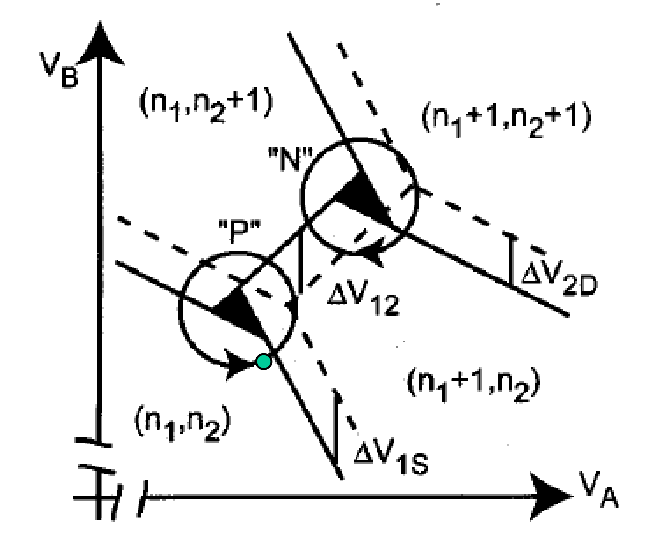
Conductance of el. 3

Difference signal

How one can pump the electrons?

Bias voltage moves lines at the stability diagram.

It creates triangles where the energy is relatively large, and CB is impossible.



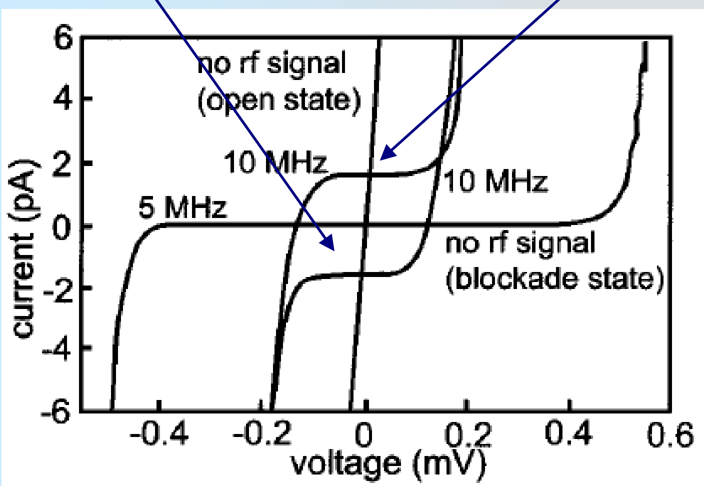
Let us adjust the gate voltages to start within a triangle, and then apply to the gates AC voltages shifted in phase. Then the path in the phase space is

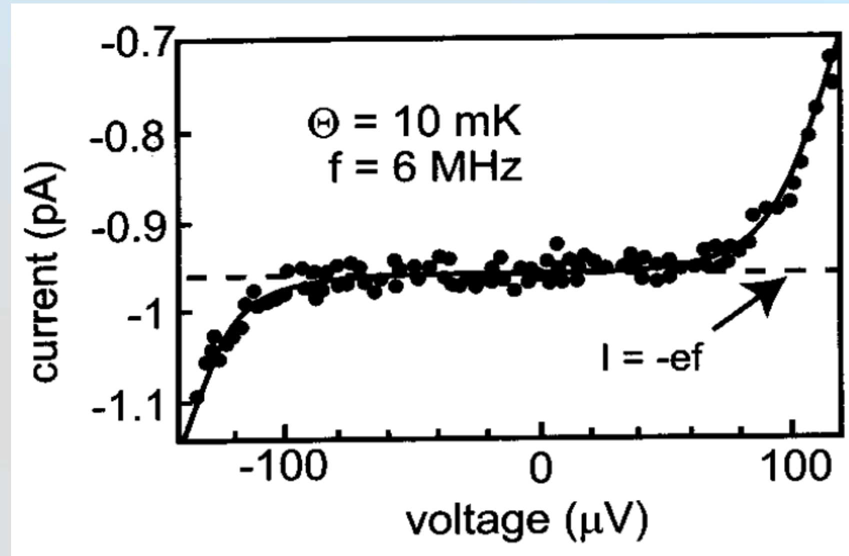
$$(n_1, n_2) \to (n_1+1, n_2) \to (n_1, n_2+1) \to (n_1, n_1)$$

Exactly one electron has passed through the device!

The current is then, $I=-ef$

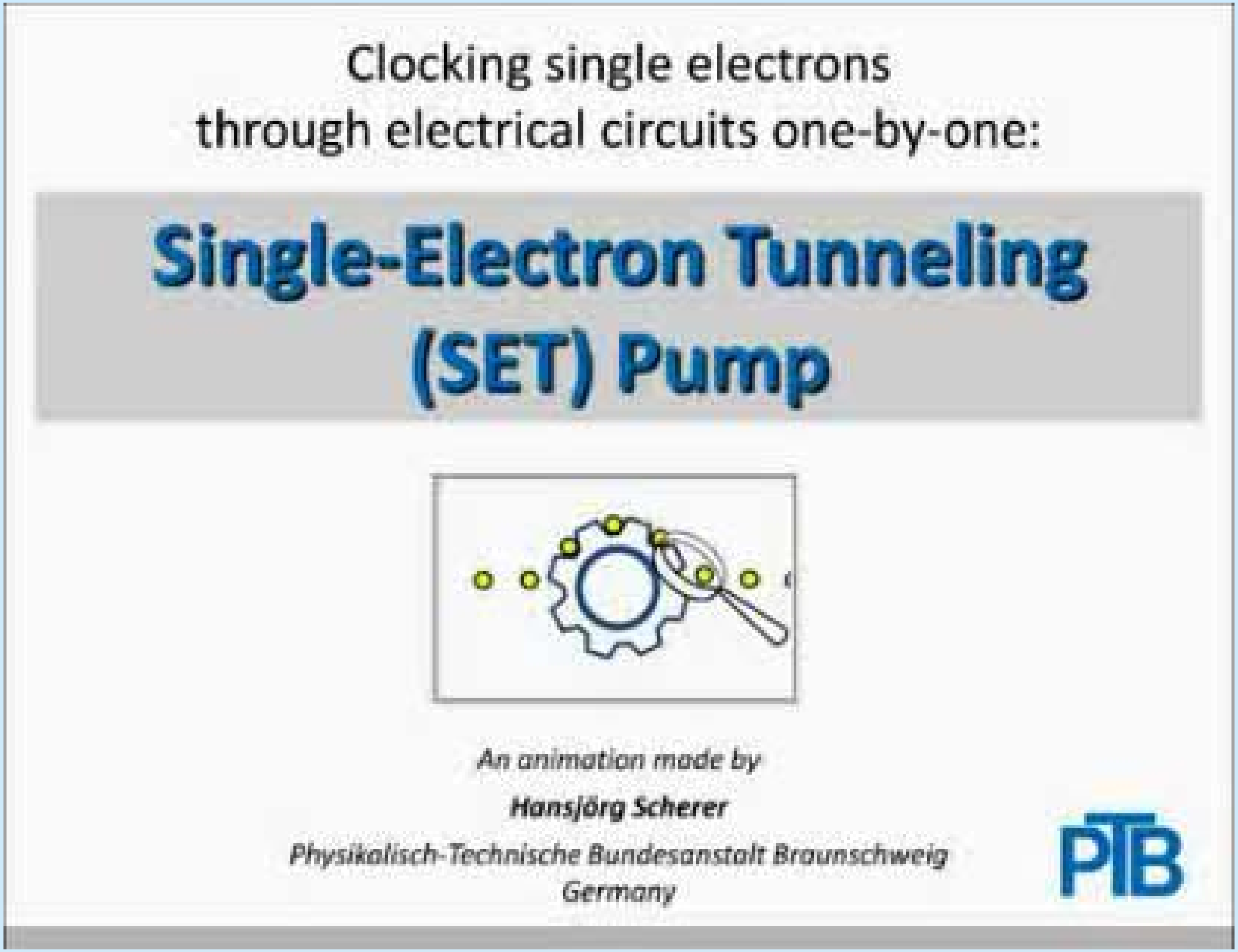
Opposite phase shifts



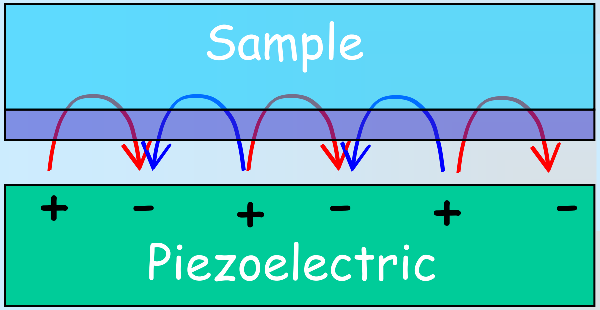


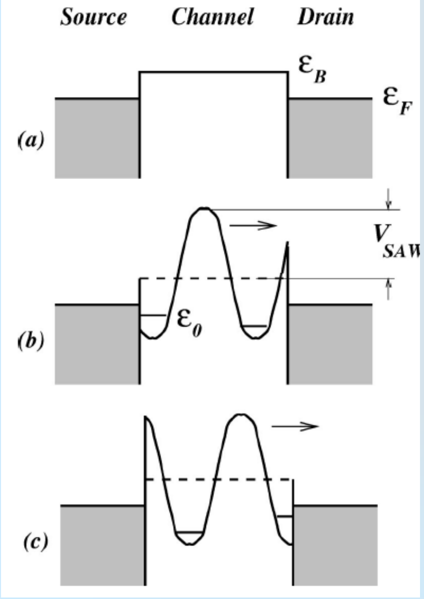
Accuracy check: 10^{-6}

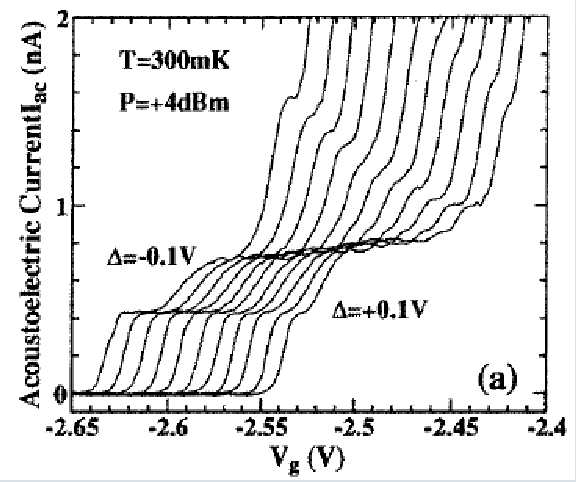
This is an excellent current standard!

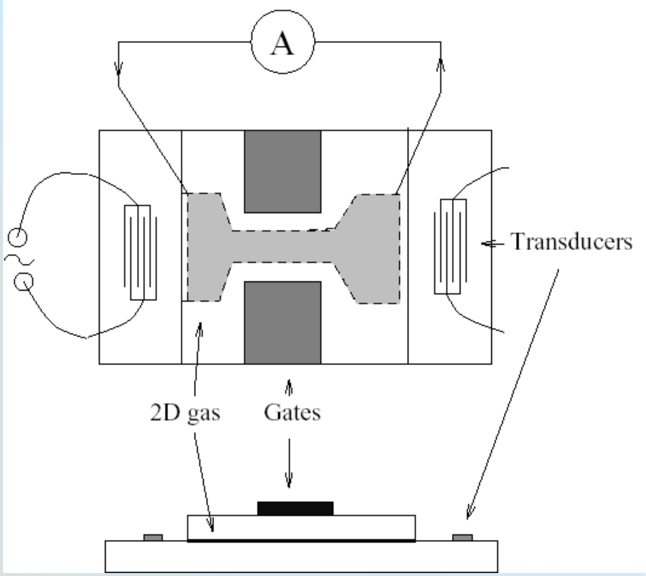

Clocking single electrons
through electrical circuits one-by-one:
Single-Electron Tunneling
(SET) Pump
An animation made by
Hansjörg Scherer
Physikalisch-Technische Bundesanstalt Braunschweig
Germany
PTB

Another current standard: Quantized electron drag in quantum wires



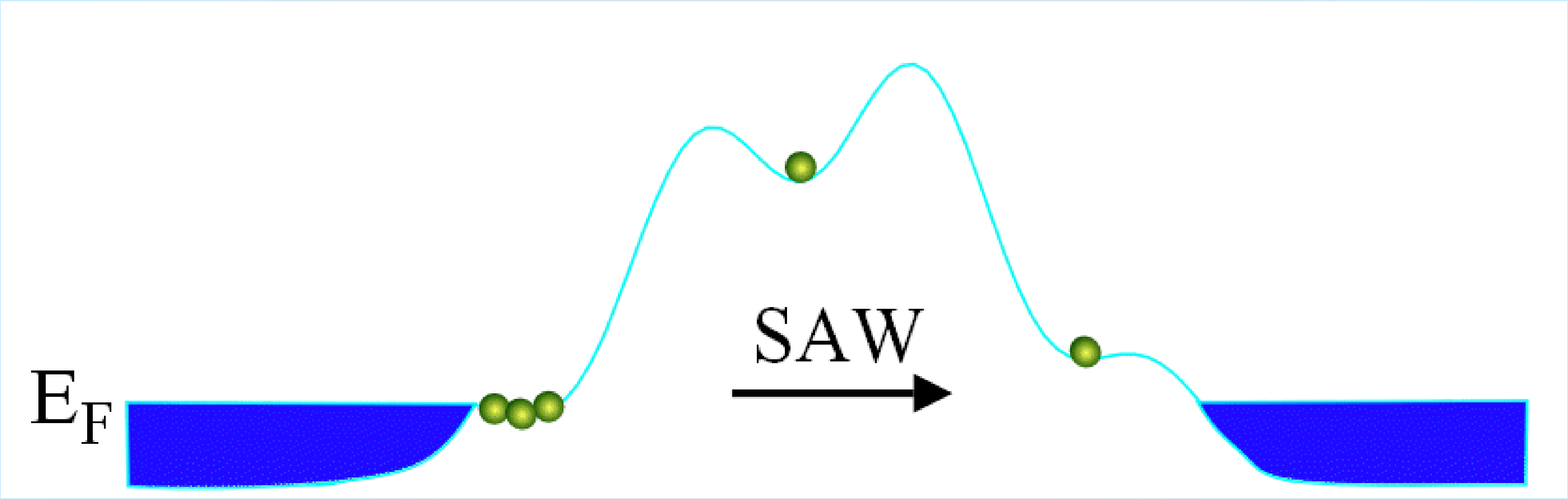






Only integer number of electrons can be trapped in the potential wells – drag current is quantized in units of ***ef,*** depending on the gate voltage.

Talyanskii *et al*., 1996


SAW
E_F

How accurate are the Coulomb-blockade devices? Are there principle limitations?

We discussed only sequential tunneling through a grain, which is exponentially suppressed by the Coulomb blockade.

In addition, there is a coherent transfer. Consider the initial and final states in different leads. Then the transition rate in the second order of the perturbation theory is

$$W_{i\to f} = \frac{2\pi}{\hbar}\left|\sum_{\psi}\frac{\langle i|\mathcal{H}_{\rm int}|\psi\rangle\langle\psi|\mathcal{H}_{\rm int}|i\rangle}{E_\psi - E_i}\right|^2\delta(E_i - E_f)$$

The grain states are involved only in virtual transitions!

This process is called the quantum co-tunneling. Its rate is

$$\Gamma_{\rm cot} = \frac{\hbar G_e G_c}{2\pi e^4}\int_e d\epsilon_k \int_g d\epsilon_q \int_g d\epsilon_{q'} \int_c d\epsilon_{k'}\, f(\epsilon_k)[1-f(\epsilon_q)]f(\epsilon_{q'})[1-f(\epsilon_{k'})] \times \left[\frac{1}{\Delta_{-,e}(n+1)} + \frac{1}{\Delta_{+,c}(n-1)}\right]^2 \delta(eV+\epsilon_k-\epsilon_q+\epsilon_{q'}-\epsilon_{k'})\,.$$

We pay by additional small tunneling transparency (one more factor containing conductance).

However, the energy costs enter as powers rather than exponents.

Due to its importance, the quantum co-tunneling has been thoroughly studied. It can lead to the contributions to the current proportional to V^3 and V.

Open questions

In the previous lecture we discussed electrons in terms of waves. However, in this lecture we spoke about particles, their charge, etc.

Are we running two horses at the same time?

How single-electron effects interplay with quantum interference?

These problems are solved to some extent and we will discuss them later.