http://www.topological-qp.jp/english/index.html

Topological aspects of Andreev bound state in superconductivity

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Argonne November 13 (2012)

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Contents of our talk

(1)Surface Andreev bound state up to now

(2)Majorana fermion

(3)Fabrication of Majorana Fermion at Nanowire and Interface

(4)Superconducting doped topological insulator

Bogoliubov-de Gennes equation

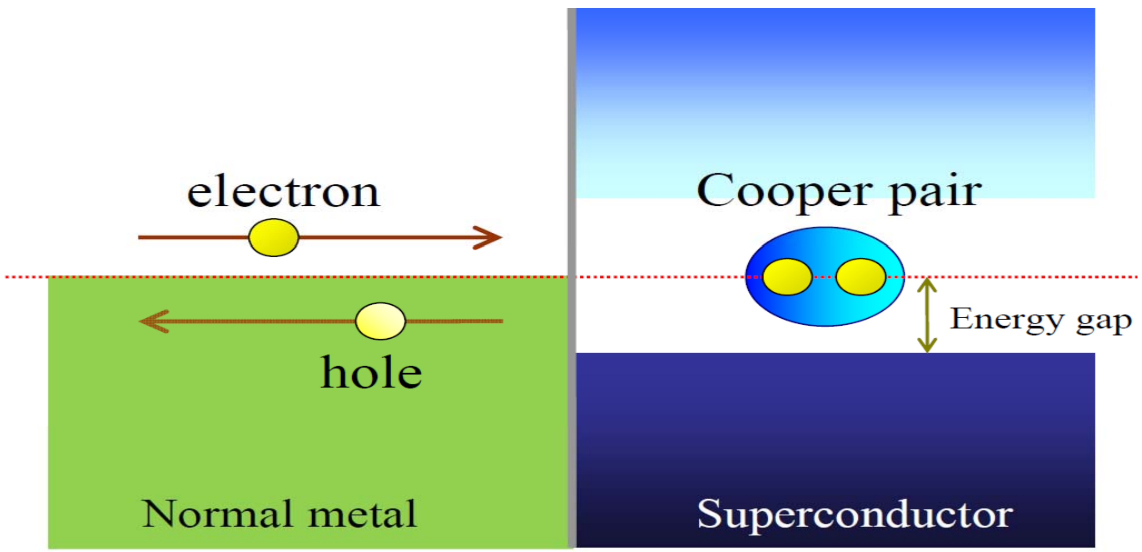
$$\left(-\frac{h^2}{2m}\frac{d^2}{dx^2}+U(x)-E_{\mathrm{F}}\right)u(x)+\Delta(x)v(x)=Eu(x),$$
$$-\left(-\frac{h^2}{2m}\frac{d^2}{dx^2}+U(x)-E_{\mathrm{F}}\right)v(x)+\Delta^*(x)u(x)=Ev(x).$$

$u(x), v(x)$: Wave functions

$\Delta(x)$: Pair potential

$U(x)$: Hartree Fock potential

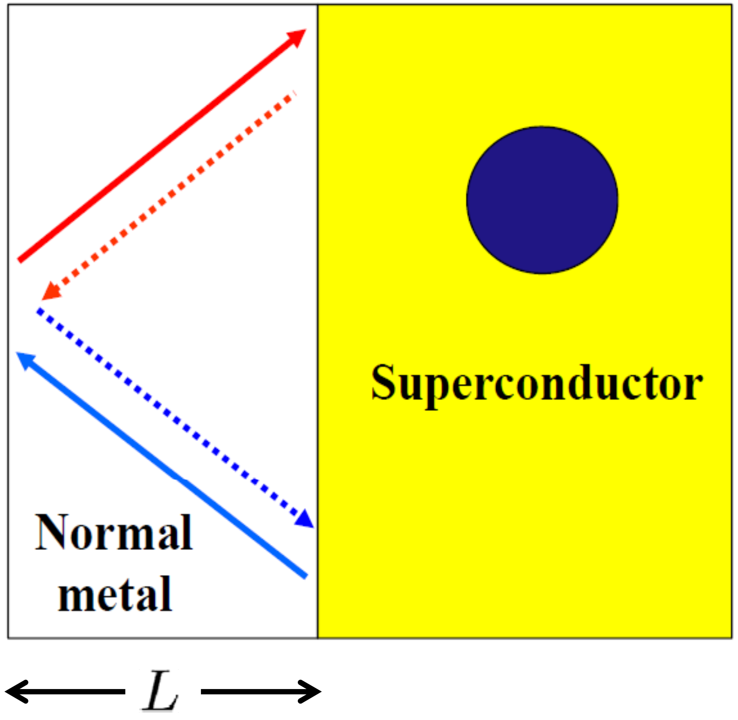
E_{F} : Fermi energy



Andreev reflection

Classical example of Andreev bound state

ABS with nonzero energy



ABS has been known since 1963.

P.G. de Gennes and D. Saint-James
Phys. Lett. 4 151 (1963).

McMillan(1966), Tomash(1965), Rowell(1966)

High transparent interface

Energy levels of Andreev bound state (nonzero energy)

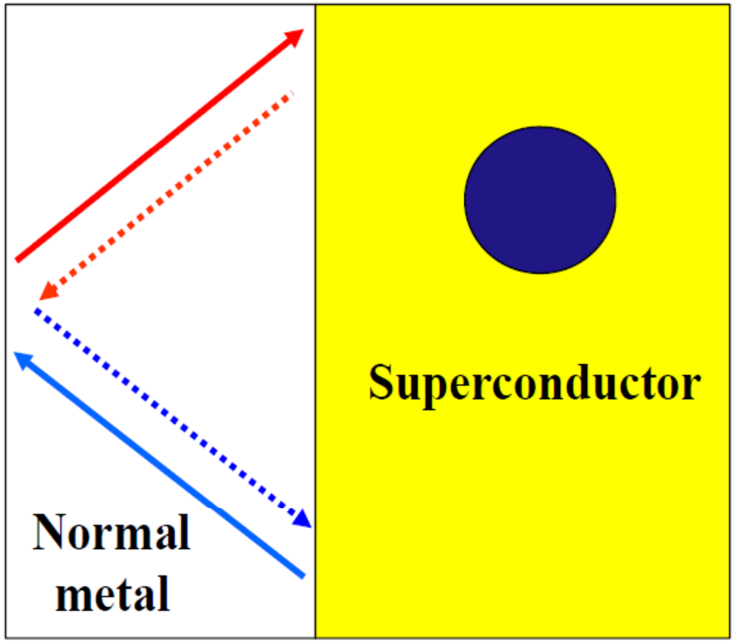
$$\varepsilon_n = \pm \frac{\pi v_{Fx}}{2L}(n + 1/2), \quad n = 0, 1, 2, ...$$

$\Delta_0 \gg |\varepsilon_n|$

Δ_0 magnitude of pair potential

v_{Fx} x component of Fermi velocity

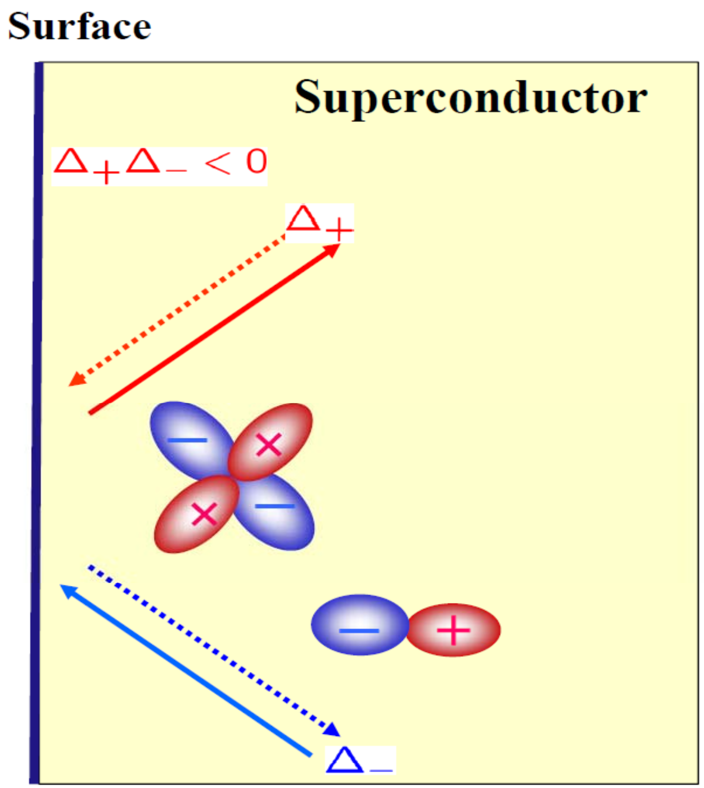
Andreev bound state (non-topological and topological)



Andreev bound state with non zero energy (de Gennes, Saint James)

Not edge state

Non topological



Mid gap (zero energy) Andreev bound state

Surface Andreev bound state

Edge state Topological

L. Buchholtz & G. Zwicknagl (81):, J. Hara & K. Nagai : Prog. Theor. Phys. 74 (86)
C.R. Hu : (94)

Surface Andreev bound state (ABS) up to now

(1) *d*-wave (cuprate)

(2) chiral *p*-wave (Sr_2RuO_4)

(3) helical (NCS superconductor)

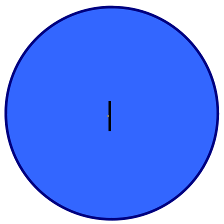
(4) 3d superconductor (superfluid ^3He)

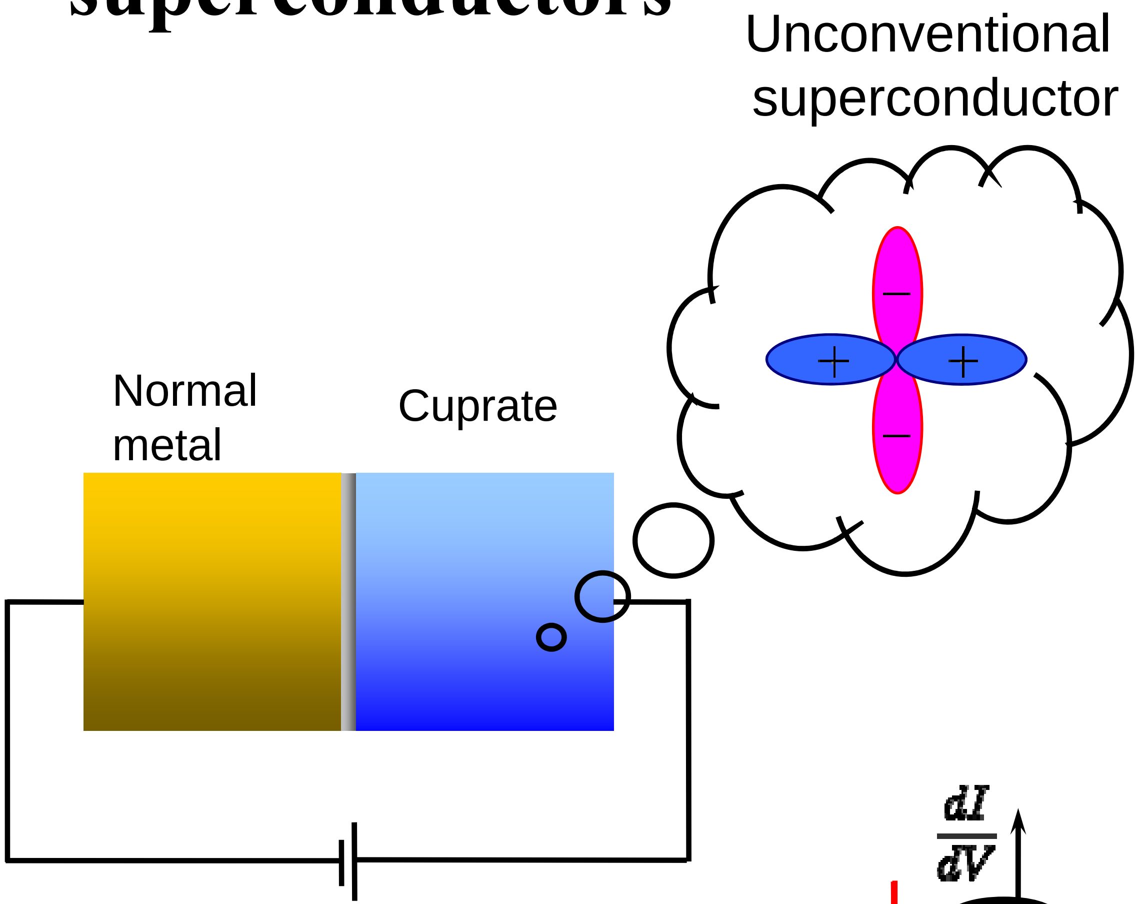
The presence of ABS is supported by the bulk topological invariant.

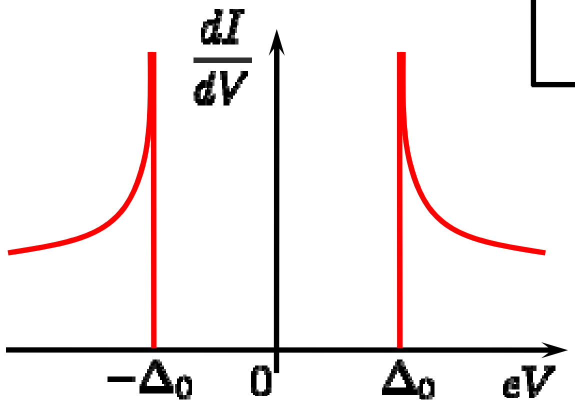
Y. Tanaka, M. Sato and N. Nagaosa, J. Phys. Soc. Jpn. 81 011013 (2012)

Tunneling effect in unconventional superconductors

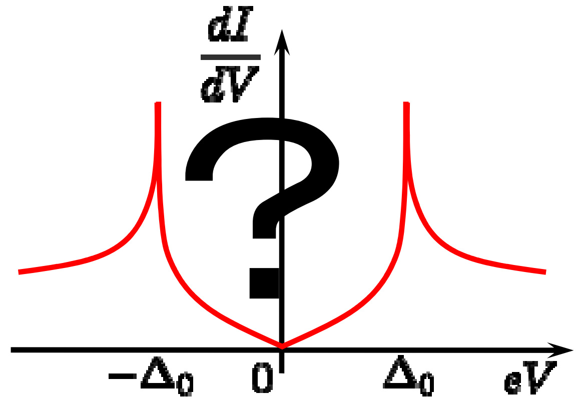
s-wave





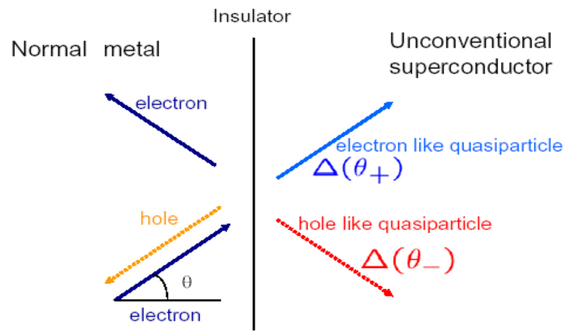


Important issue of cuprate in the 90s.



Conductance formula and Andreev bound state (ABS)

Tanaka and Kashiwaya PRL 74 3451



$$\sigma_T(E) = \frac{\int_{-\pi/2}^{\pi/2} d\theta \sigma_N(\theta)\sigma_R(E,\theta)\cos\theta}{\int_{-\pi/2}^{\pi/2} d\theta \sigma_N(\theta)\cos\theta}$$

$$\sigma_R(E,\theta) = \frac{1+\sigma_N(\theta) \mid \Gamma_+ \mid^2 + [\sigma_N(\theta)-1] \mid \Gamma_+\Gamma_- \mid^2}{\mid 1+[\sigma_N(\theta)-1]\Gamma_+\Gamma_- \exp[i(\phi_- - \phi_+)] \mid^2},$$

transparency

$$\exp(i\phi_+) = \frac{\Delta(\theta_+)}{|\Delta(\theta_+)|} = \frac{\Delta_+}{|\Delta_+|} \quad \exp(i\phi_-) = \frac{\Delta(\theta_-)}{|\Delta(\theta_-)|} = \frac{\Delta_-}{|\Delta_-|} \quad \Gamma_\pm = \frac{E-\sqrt{E^2-\Delta_\pm^2}}{|\Delta_\pm|} \quad \sigma_N(\theta) = \frac{\cos^2\theta}{\cos^2\theta + Z^2}$$

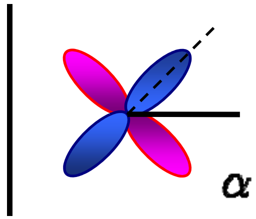
$$E = eV$$

Condition for ABS

$$\sigma_N(\theta) \to 0 \qquad \boxed{1 = \Gamma_+\Gamma_- \exp[i(\phi_- - \phi_+)]}$$

$$1 = \frac{-E+\sqrt{E^2-\Delta_0^2\sin^2 2\theta}}{E+\sqrt{E^2-\Delta_0^2\sin^2 2\theta}}$$

Flat zero energy band $\quad E = 0$

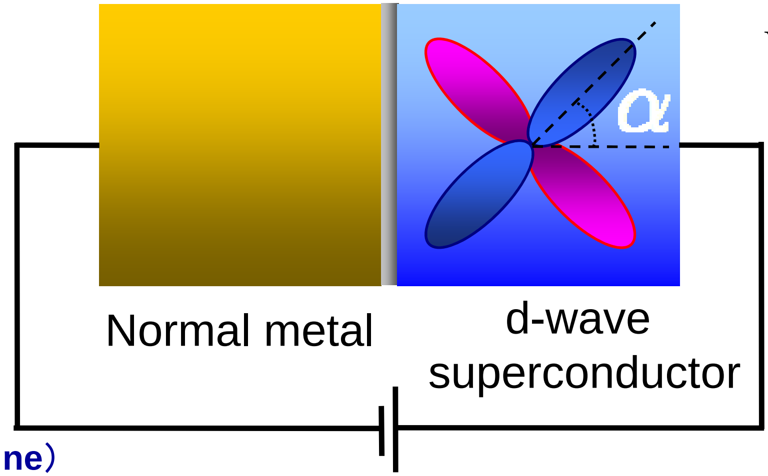


$\alpha = \frac{\pi}{4}$

surface

C.R. Hu : Phys. Rev. Lett. 72 (1994) 1526.

Tunneling conductance in *d*-wave junction

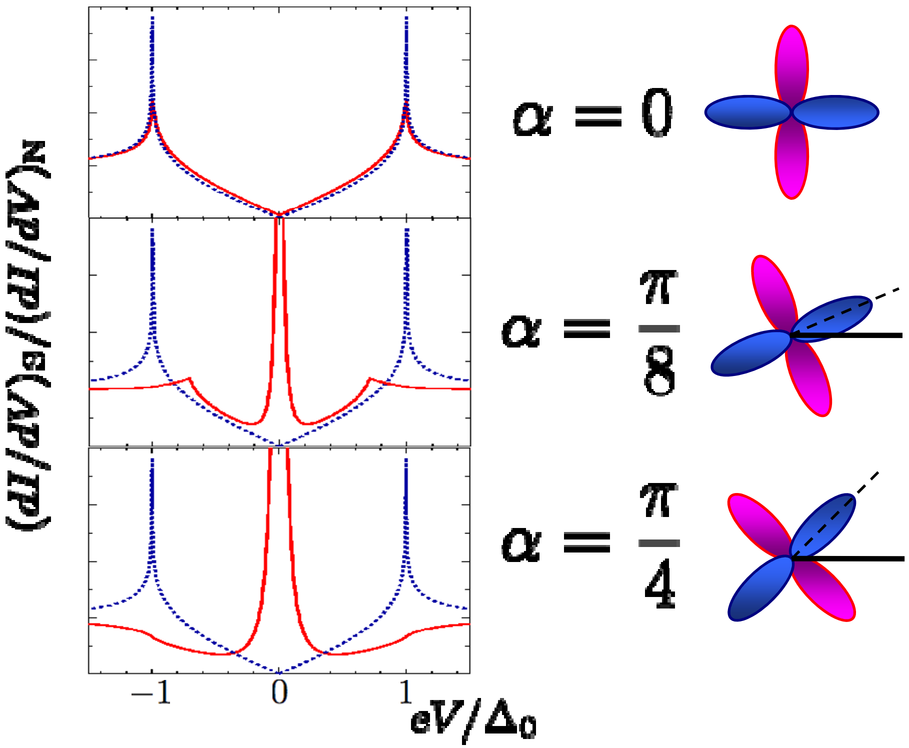


Y. Tanaka & S. Kashiwaya: Phys. Rev. Lett. **74** (1995) 3451.

$$\Delta_{\pm} = \Delta_0 \cos[2(\theta \mp \alpha)]$$

α angle between the normal to the interface and the lobe direction

Bulk ldos (blue line)



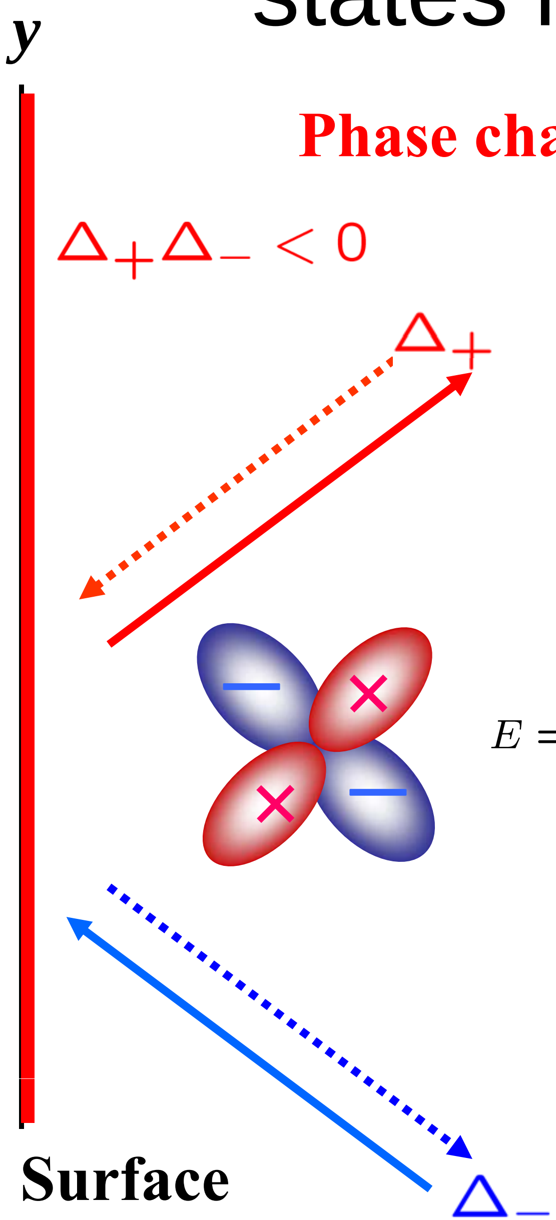
Zero bias conductance peak

Andreev bound state

Surface zero energy state

L. Buchholtz & G. Zwicknagl : Phys. Rev. B 23 (1981) 5788.
J. Hara & K. Nagai : Prog. Theor. Phys. 74 (1986) 1237.
C.R. Hu : Phys. Rev. Lett. 72 (1994) 1526.

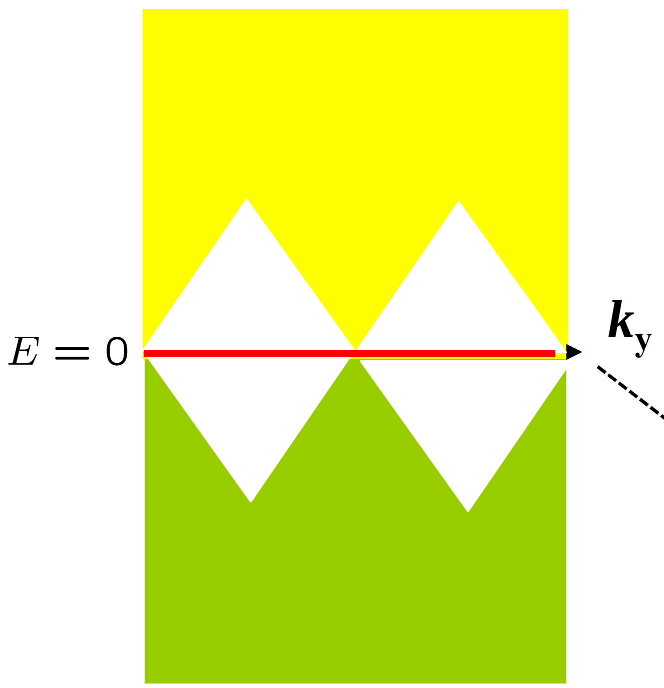
Well known example of Andreev bound states in *d*-wave superconductor

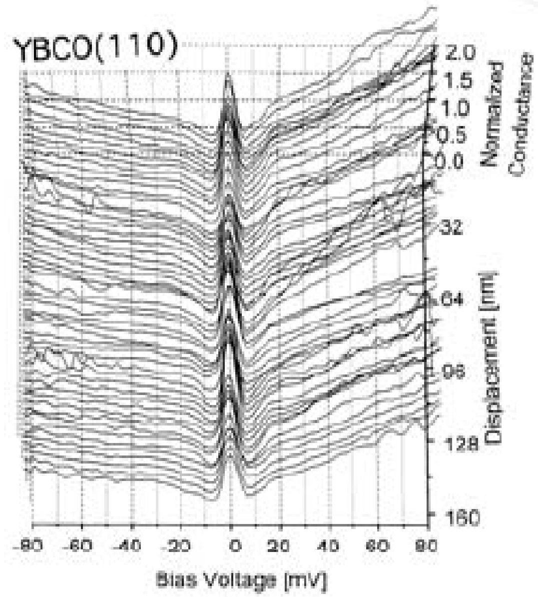


Phase change of pair potential is π

ABS in *d*-wave

(110)direction





Alff, Kashiwaya PRB (1998)

Flat dispersion!!

Zero energy

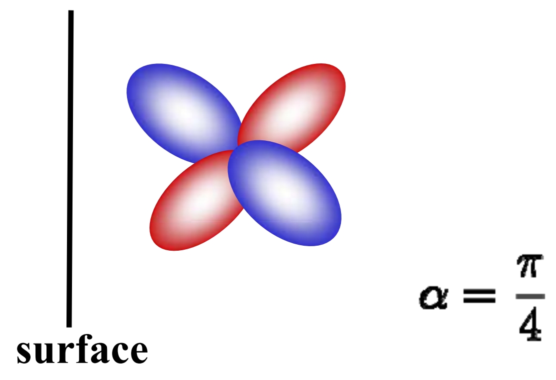
Tanaka Kashiwaya PRL 74 3451 (1995),
Kashiwaya, Tanaka, Rep. Prog. Phys. 63 1641 (2000)
Hu(1994) Matsumoto Shiba(1995)

Condition for ABS (without dispersion)

Spin-singlet d_{xy}-wave

$$1 = \frac{-E + \sqrt{E^2 - \Delta_0^2 \sin^2 2\theta}}{E + \sqrt{E^2 - \Delta_0^2 \sin^2 2\theta}} \qquad E = 0$$

Flat zero energy band



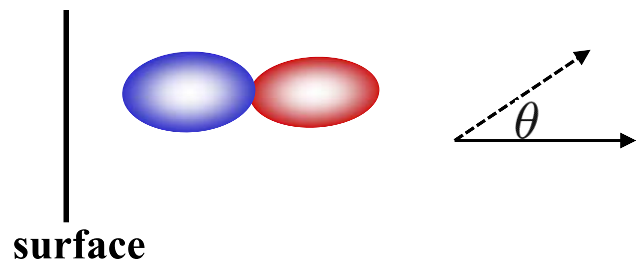
C.R. Hu : Phys. Rev. Lett. 72 (1994) 1526.

Spin-triplet p_x-wave

p_x $\quad \Delta = \Delta_0 \cos\theta$

$$1 = \frac{-E + \sqrt{E^2 - \Delta_0^2 \cos^2 \theta}}{E + \sqrt{E^2 - \Delta_0^2 \cos^2 \theta}} \Rightarrow E = 0$$

flat dispersion



Flat Andreev bound state and topology

Hamiltonian

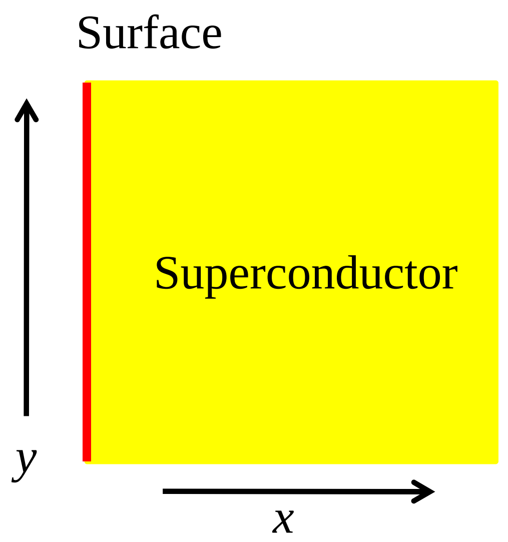
$$\mathcal{H} = \sum_{\boldsymbol{k}} \left(c^{\dagger}_{\boldsymbol{k}\uparrow}, c_{-\boldsymbol{k}\downarrow} \right) \mathcal{H}(\boldsymbol{k}) \begin{pmatrix} c_{\boldsymbol{k}\uparrow} \\ c^{\dagger}_{-\boldsymbol{k}\downarrow} \end{pmatrix} \quad \mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} \varepsilon(\boldsymbol{k}) & \Delta(\boldsymbol{k}) \\ \Delta(\boldsymbol{k}) & -\varepsilon(\boldsymbol{k}) \end{pmatrix},$$

$$\Delta(\boldsymbol{k}) = \begin{cases} \psi(\boldsymbol{k}) = \psi(\boldsymbol{k}) & \text{for spin-singlet} \\ d_z(\boldsymbol{k}) = -d_z(-\boldsymbol{k}) & \text{for spin-triplet} \end{cases} .$$

Winding number for fixed k_y

$$w_{1\mathrm{d}}(k_y) = \frac{1}{2\pi} \int_{-\pi}^{\pi} dk_x \partial_{k_x} \theta(\boldsymbol{k}).$$

$$\cos\theta(\boldsymbol{k}) = \frac{\varepsilon(\boldsymbol{k})}{\sqrt{\varepsilon(\boldsymbol{k})^2 + \Delta(\boldsymbol{k})^2}} \qquad \sin\theta(\boldsymbol{k}) = \frac{\Delta(\boldsymbol{k})}{\sqrt{\varepsilon(\boldsymbol{k})^2 + \Delta(\boldsymbol{k})^2}},$$



Sato, Tanaka, et al, PRB 83 224511 (2011)

Midgap Andreev bound state (Winding number)

$$w_{1\mathrm{d}}(k_y) = \frac{1}{2\pi}\int_{-\pi}^{\pi} dk_x \partial_{k_x}\theta(\boldsymbol{k})$$

$$= -\frac{1}{2}\sum_{k_x;\varepsilon(\boldsymbol{k})=0} \mathrm{sgn}[\Delta(\boldsymbol{k})]\cdot\mathrm{sgn}[\partial_{k_x}\varepsilon(\boldsymbol{k})]$$

$$\cos\theta(\boldsymbol{k}) = \frac{\varepsilon(\boldsymbol{k})}{\sqrt{\varepsilon(\boldsymbol{k})^2+\Delta(\boldsymbol{k})^2}}$$

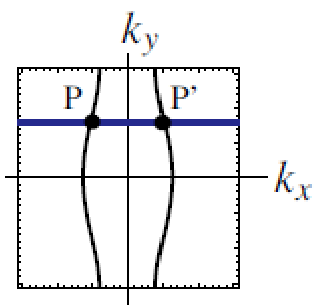
$$\sin\theta(\boldsymbol{k}) = \frac{\Delta(\boldsymbol{k})}{\sqrt{\varepsilon(\boldsymbol{k})^2+\Delta(\boldsymbol{k})^2}},$$

Topological invariant defined in bulk

If we consider simple Fermi surface

$$w_{1d} = -\frac{1}{2}\mathrm{sgn}[\partial_{k_x}\varepsilon(-k_x^0,k_y)]\left[\mathrm{sgn}[\Delta(-k_x^0,k_y)] - \mathrm{sgn}[\Delta(k_x^0,k_y)]\right].$$

$$w_{1d} \neq 0 \Longleftrightarrow \Delta(-k_x^0,k_y)\Delta(k_x^0,k_y) < 0$$



P: $(-k_x^0, k_y)$

P': (k_x^0, k_y)

Conventional condition

Sato, Tanaka, et al, PRB 83 224511 (2011)

Winding number & Index theorem (1)

From the <u>bulk-edge correspondence</u>, there exists the gapless states on the edge only when integer w_{1d} is nonzero.

BdG Hamiltonian has a symmetry (chiral symmetry)

$$\{\mathcal{H}(\boldsymbol{k}), \sigma_y\} = 0$$

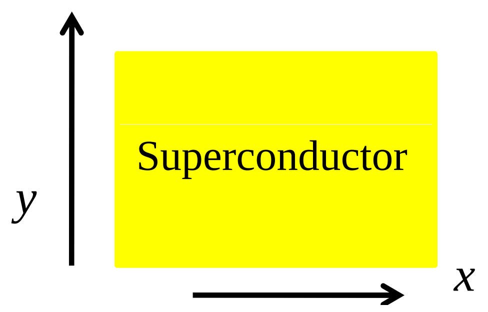
Zero energy ABS is an eigenstate of σ_y.

$n_0^{(+)}$ **<u>Number of ZES where the eigenvalue of σ_y is 1</u>**

$n_0^{(-)}$ **<u>Number of ZES where the eigenvalue of σ_y is −1</u>**

Index Theorem

$$w_{1d} = (n_0^{(+)} - n_0^{(-)})$$



Winding number & Index theorem (2)

Two-dimensional spin-singlet d_{xy}-wave superconductor

$$\varepsilon(\boldsymbol{k}) = \frac{\boldsymbol{k}^2}{2m} - E_F, \qquad \Delta(\boldsymbol{k}) = \Delta_0 \frac{k_x k_y}{\boldsymbol{k}^2}.$$

$$w_{1\mathrm{d}}(k_y) = \begin{cases} 1, & \text{for } 0 < k_y < k_\mathrm{F} \\ -1, & \text{for } 0 > k_y > -k_\mathrm{F} \\ 0, & \text{for } |k_y| > k_\mathrm{F} \end{cases}, \qquad k_F = \sqrt{2mE_F}, \quad k_x = \sqrt{k_F^2 - k_y^2}$$

$$|u_0(x)\rangle = C \begin{pmatrix} 1 \\ -i\,\mathrm{sgn} k_y \end{pmatrix} e^{ik_y y} \sin(k_x x) e^{-x/\xi}$$

superconductor on x>0

Winding number & Index theorem (3)

(a) d_{xy}-wave superconductor on $x > 0$

k_y	$n_0^{(+)}$	$n_0^{(-)}$	$n_0^{(+)} - n_0^{(-)}$	$w_{1\mathrm{d}}(k_y)$
$0 < k_y < k_\mathrm{F}$	0	1	-1	-1
$0 > k_y > -k_\mathrm{F}$	1	0	1	1
$\lvert k_y \rvert > k_\mathrm{F}$	0	0	0	0

(b) d_{xy}-wave superconductor on $x < 0$

k_y	$n_0^{(+)}$	$n_0^{(-)}$	$n_0^{(+)} - n_0^{(-)}$	$w_{1\mathrm{d}}(k_y)$
$0 < k_y < k_\mathrm{F}$	1	0	1	-1
$0 > k_y > -k_\mathrm{F}$	0	1	-1	1
$\lvert k_y \rvert > k_\mathrm{F}$	0	0	0	0

(a) $w_{1d}(k_y) = (n_0^{(+)} - n_0^{(-)})$ Superconductor $x > 0$

(b) $w_{1d}(k_y) = -(n_0^{(+)} - n_0^{(-)})$ Superconductor $x < 0$

Surface Andreev bound state (ABS) up to now

(1)*d*-wave (cuprate)

(2)chiral *p*-wave (Sr_2RuO_4)

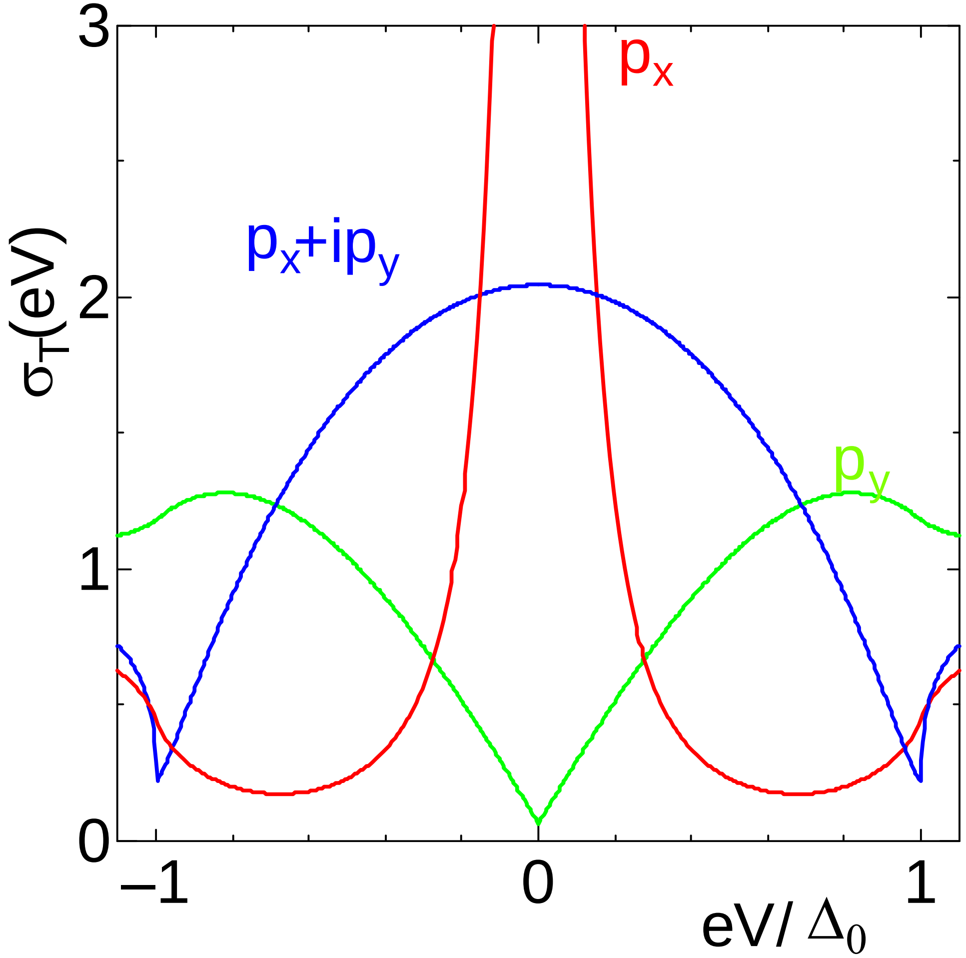
(3)helical (NCS superconductor)

(4)3d superconductor (superfluid ^3He)

The presence of ABS is supported by the bulk topological invariant.

Y. Tanaka, M. Sato and N. Nagaosa, J. Phys. Soc. Jpn. 81 011013 (2012)

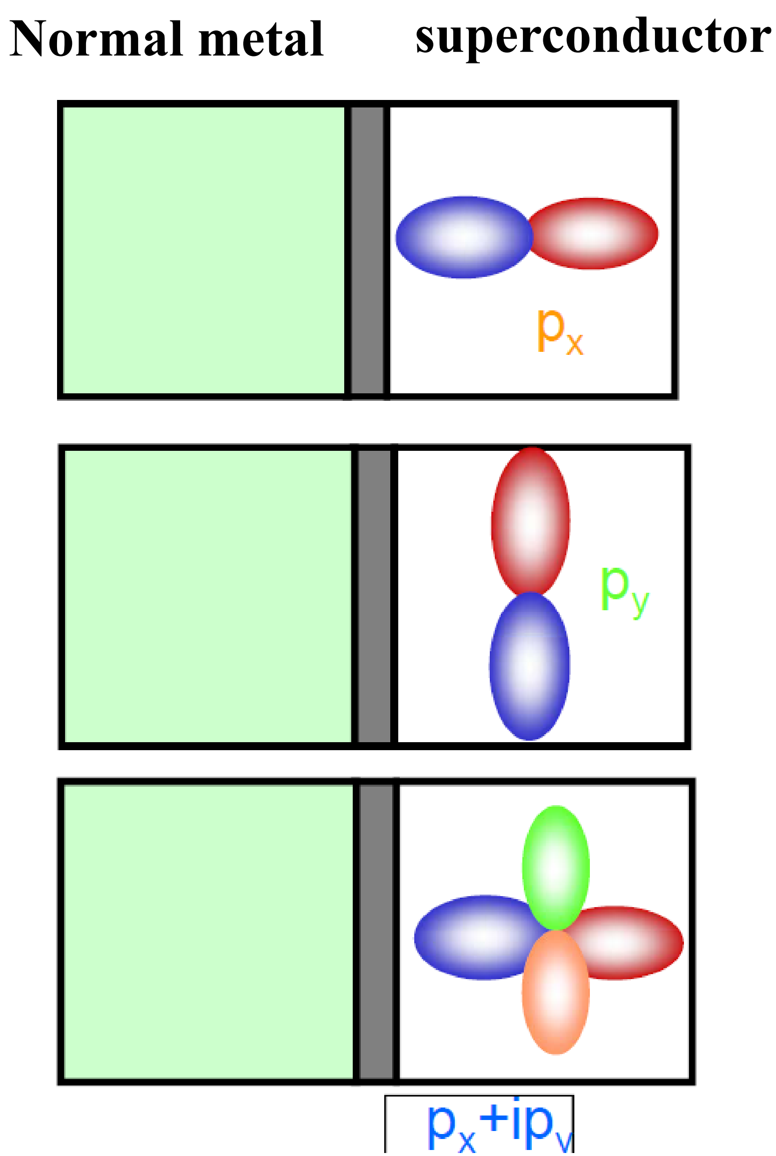
Extension to spin-triplet superconductors



Phys. Rev. B. 56, 7847 (1997)

J. Phys. Soc. Jpn. 67, 3224 (1998)

L. Buchholtz & G. Zwicknagl : Phys. Rev. B 23 (1981) 5788.
J. Hara & K. Nagai : Prog. Theor. Phys. 74 (1986) 1237



Condition for ABS

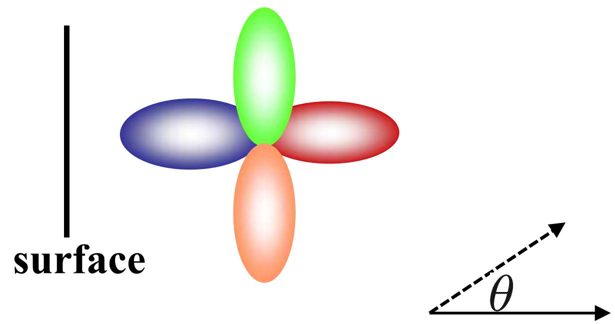
$$\Delta = \Delta_0 \exp(i\theta)$$

chiral p

$$1 = \frac{-E+\sqrt{E^2-\Delta_0^2}}{E+\sqrt{E^2-\Delta_0^2}} \exp(-2i\theta)$$

$$\Rightarrow \quad E = \Delta_0 \sin\theta \sim k_y/k_F$$

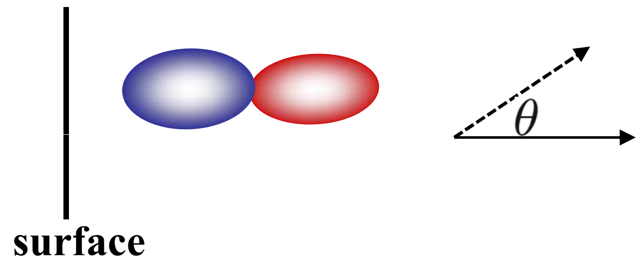
linear dispersion



p_x $\quad \Delta = \Delta_0 \cos\theta$

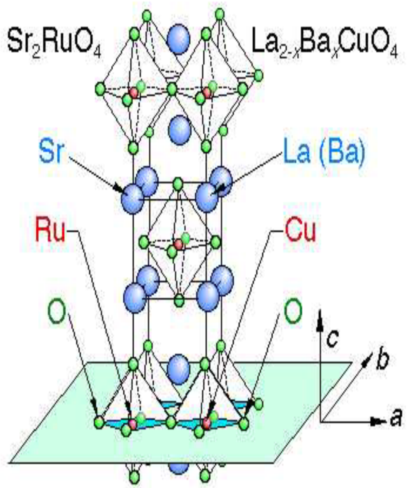
$$1 = \frac{-E+\sqrt{E^2-\Delta_0^2\cos^2\theta}}{E+\sqrt{E^2-\Delta_0^2\cos^2\theta}} \Rightarrow \quad E = 0$$

flat dispersion



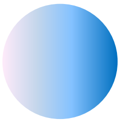
Chiral superconductor Sr_2RuO_4

Edge surface current



Similar structure to cuprate

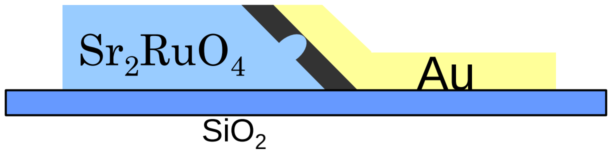
Maeno (1994)

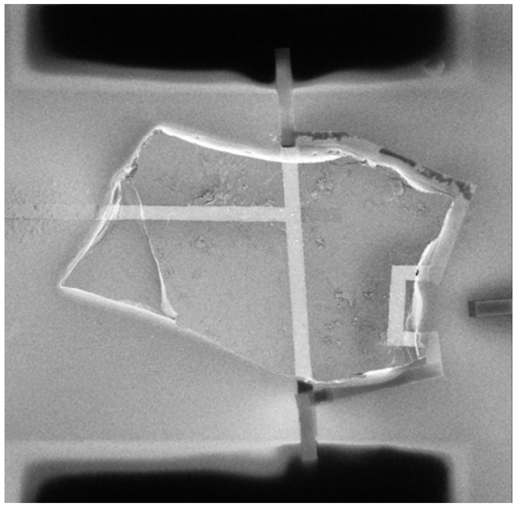


$$p_x + ip_y$$

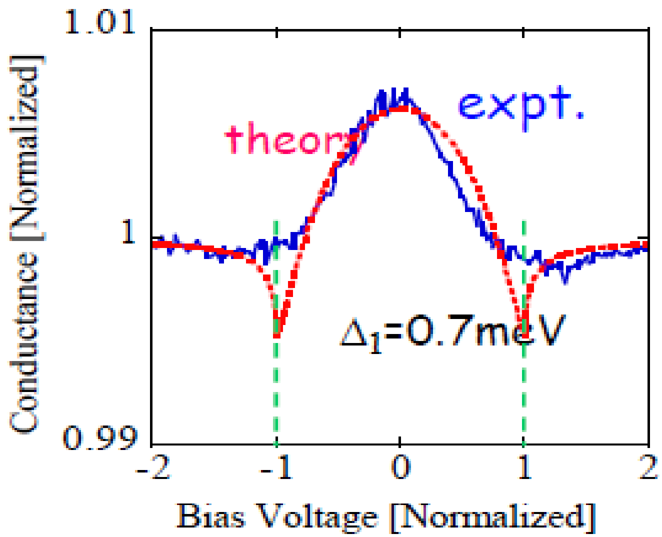
Recent experiment of Sr_2RuO_4

S/I/N





Experiment

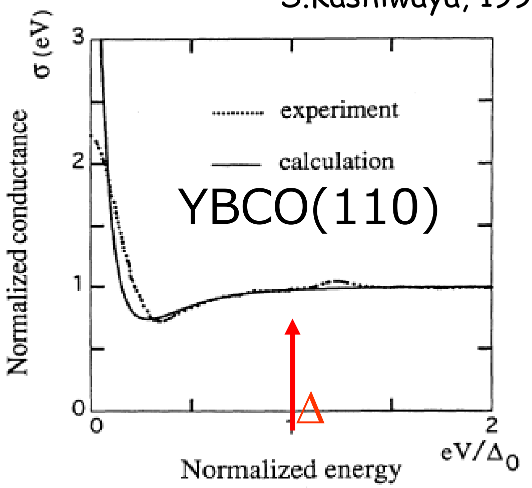


It is possible to fit experimental data taking into account of anisotropy of pair potential.

S. Kashiwaya, *et al*, Phys. Rev. Lett. **107**, 077003 (2011)

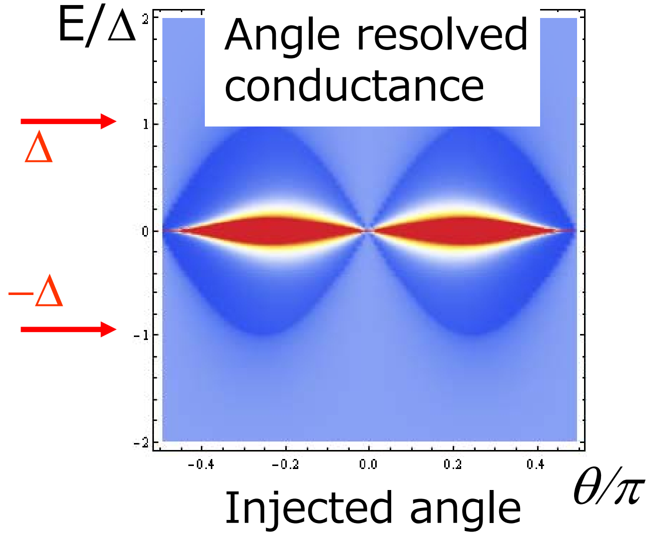
Tunneling spectrum in two-dimensional topological superconductors

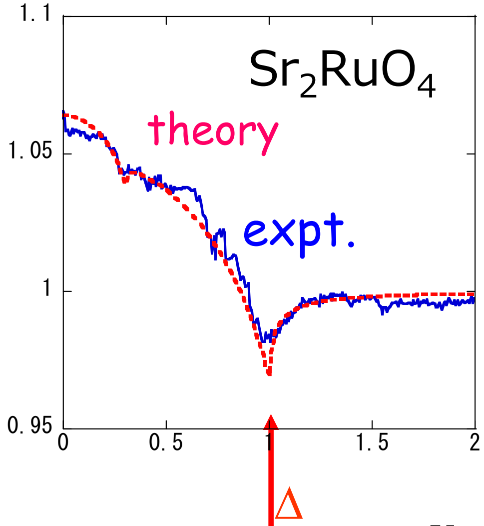
S.Kashiwaya, 1995



d_{x2-y2}-wave
nodal gap

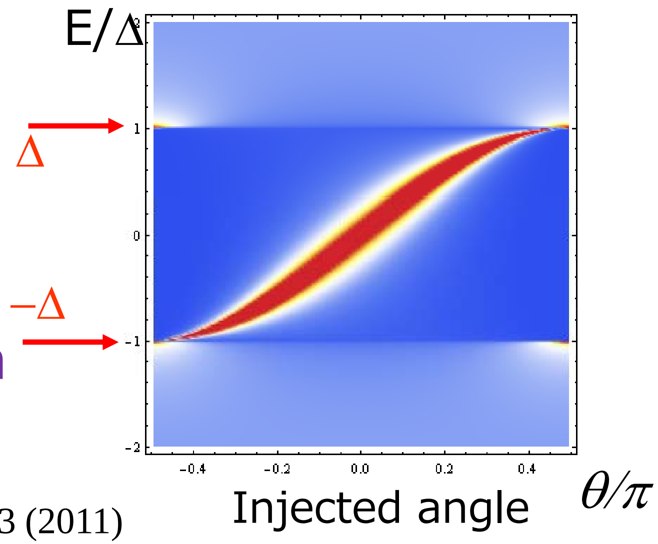
zero energy flat
band of surface ABS





chiral *p*-wave
full gap
chiral edge state (ABS)

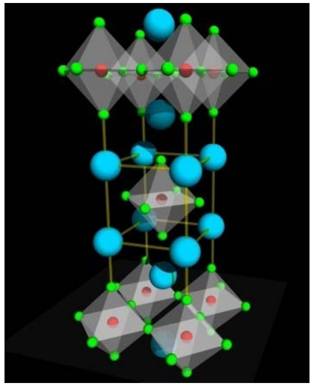
broad zero-bias peak
due to linear dispersion



Kashiwaya *et al*, Phys. Rev. Lett. **107**, 077003 (2011)

Chiral superconductor

Topological invariant defined in bulk



Sr_2RuO_4

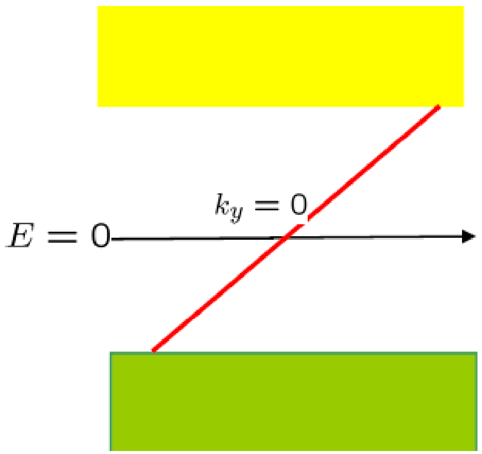
Volovik(88), Furusaki(01)

$$w_{2\mathrm{d}} = -\frac{1}{8\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} dk_x dk_y \epsilon^{ij}\epsilon^{abc} m_a(\boldsymbol{k})\partial_{k_i} m_b(\boldsymbol{k})\partial_{k_j} m_c(\boldsymbol{k}),$$

$$m_1(\boldsymbol{k}) = \frac{\mathrm{Re}\Delta(\boldsymbol{k})}{\sqrt{\varepsilon(\boldsymbol{k})^2 + |\Delta(\boldsymbol{k})|^2}},\quad m_2(\boldsymbol{k}) = \frac{\mathrm{Im}\Delta(\boldsymbol{k})}{\sqrt{\varepsilon(\boldsymbol{k})^2 + |\Delta(\boldsymbol{k})|^2}},\quad m_3(\boldsymbol{k}) = \frac{\varepsilon(\boldsymbol{k})}{\sqrt{\varepsilon(\boldsymbol{k})^2 + |\Delta(\boldsymbol{k})|^2}}.$$

$\Delta(\boldsymbol{k})$ Energy gap function

$\varepsilon(\boldsymbol{k})$ Quasiparticle energy
Measured from Fermi surface



$w_{2D} = 1$

Edge sate is possible for nonzero w_{2D}

Broken time reversal symmetry

Andreev bound state (topological edge state) and topological invariant

Andreev bound state	Topological invariant	Time reversal symmetry	Materials	Theory of tunneling	Insulator (semi-metal)
Flat	1d winding number	○	Cuprate p_x-wave	PRL (1995) JPSJ(1998)	**Graphene (zigzag edge)**
Chiral	2d winding number	×	Sr_2RuO_4 ^3He A	PRB (1997)	**QHS**
Helical	Z_2	○	***s*+*p*-wave** (NCS)	PRB (2007)	**QSHS** **(2D Topological insulator)**
Cone	3d winding number	○	^3He B	PRB (2003)	**Topological insulator**

M. Sato, et al Kotai Butsuri in Japan

Surface Andreev bound state (ABS) up to now

(1)*d*-wave (cuprate)

(2)chiral *p*-wave (Sr_2RuO_4)

(3)helical (NCS superconductor)

(4)3d superconductor (superfluid ^3He)

The presence of ABS is supported by the bulk topological invariant.

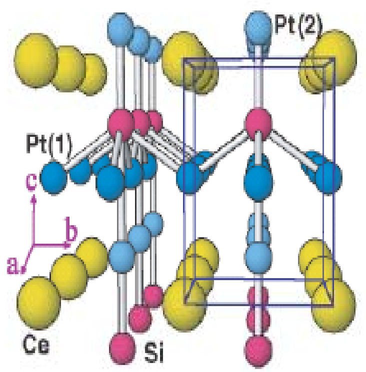
Y. Tanaka, M. Sato and N. Nagaosa, J. Phys. Soc. Jpn. 81 011013 (2012)

Andreev bound state in the presence of spin-orbit coupling

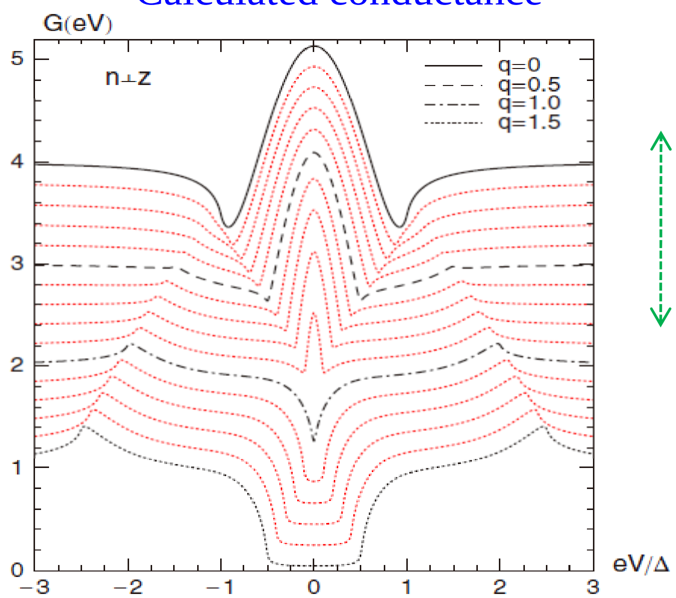
Spin-singlet (*s*-wave) Δ_s spin-triplet(*p*-wave) Δ_p

$\Delta_p > \Delta_s$	**Andreev bound state**	**Bulk energy gap**
$\Delta_p = \Delta_s$	**No Andreev bound state**	**Gap closes**
$\Delta_s > \Delta_p$	**No Andreev bound state**	**Bulk energy gap**

$CePt_3Si$



Calculated conductance



$$q = \Delta_s / \Delta_p$$

$q < 1$ Helical superconductor

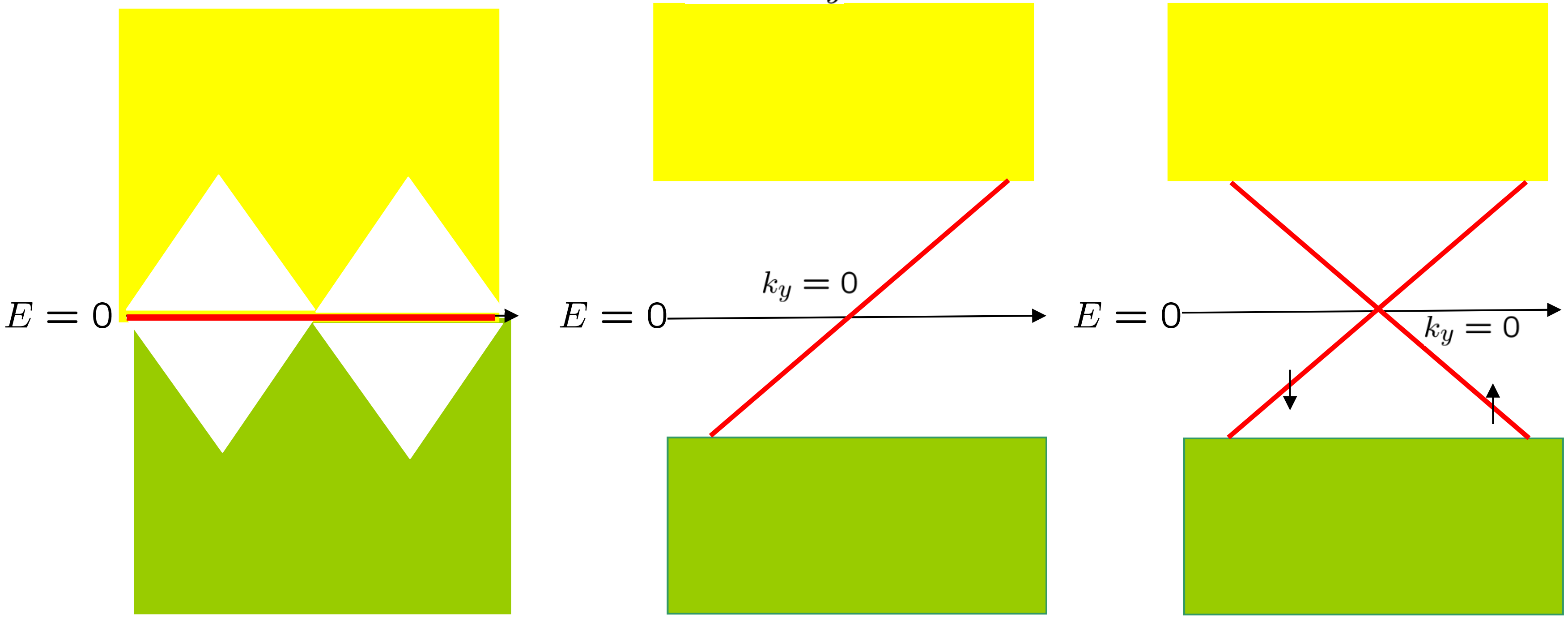
Zero bias conductance peak by Andreev bound state

Iniotakis, Tanaka et al,
Phys. Rev. B 76, 012501 (2007)

(Topological) Andreev bound states (1)

Non-centrosymmetric superconductor (NCS)

d_{xy}-wave	Chiral *p*-wave $p_x + ip_y$ -wave	NCS (Helical) *p*+*s* -wave



Hu(94) Tanaka Kashiwaya (95)	Tanaka Kashiwaya (97) Sigrist Honerkamp (98)	Iniotakis (07) Eschrig(08) Tanaka (09)
Flat	**Chiral**	**Helical**

Superconducting Materials where zero bias conductance peak by ABS is observed

$YBa_2CuO_{7-\delta}$ (Geerk, Kashiwaya, Iguchi, Greene, Yeh,Wei..)

$Bi_2Sr_2CaCu_2O_y$ (Ng, Suzuki, Greene….)

$La_{2-x}Sr_xCuO_4$ (Iguchi)

$La_{2-x}Ce_xCuO_4$ (Cheska)

$Pr_{2-x}Ce_xCuO_4$ (R.L.Greene)

Sr_2RuO_4 (Mao, Maeno, Laube,Kashiwaya)

κ–(BEDT-TTF)$_2$X, X=Cu[N(CN)$_2$]Br (Ichimura)

UBe_{13} (Ott)

$CeCoIn_5$ (Wei Greene)

$PrOs_4Sb_{12}$ (Wei)

$PuCoGa_5$ (Daghero)

Superfluid ^{3}He (Okuda, Nomura, Higashitani, Nagai)

Surface Andreev bound state (ABS) up to now

(1)*d*-wave (cuprate)

(2)chiral *p*-wave (Sr_2RuO_4)

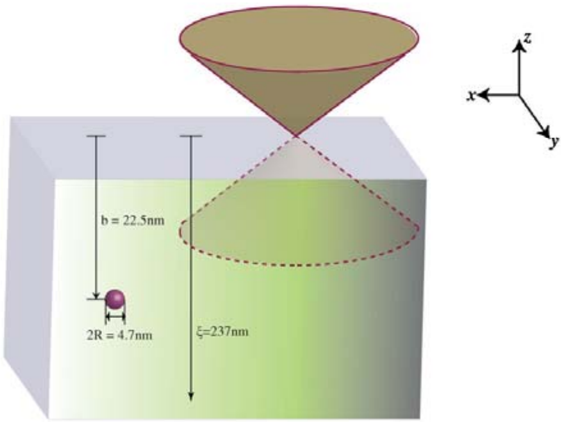
(3)helical (NCS superconductor)

(4)3d superconductor (superfluid ^3He)

The presence of ABS is supported by the bulk topological invariant.

Y. Tanaka, M. Sato and N. Nagaosa, J. Phys. Soc. Jpn. 81 011013 (2012)

ABS in B-phase of superfluid ^{3}He

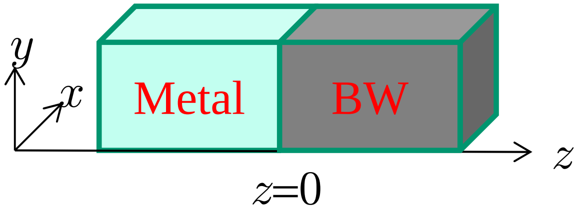


Cone type ABS

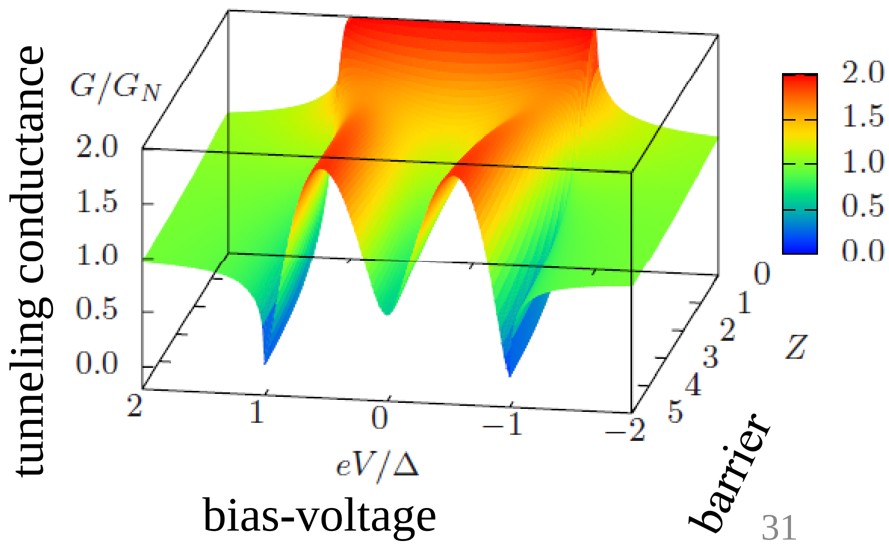
Salomaa Volovik (1988)
Schnyder (2008)
Roy (2008) Nagai (2009)
Qi (2009)
Kitaev(2009)
Chung, S.C. Zhang (2009)
Volovik (2009)

perpendicular injection ZES: Buchholtz and Zwicknagle (1981)

BW state (B-phase in ^{3}He)
full gap superconductor



no zero-bias peak
due to linear dispersion
of surface ABS

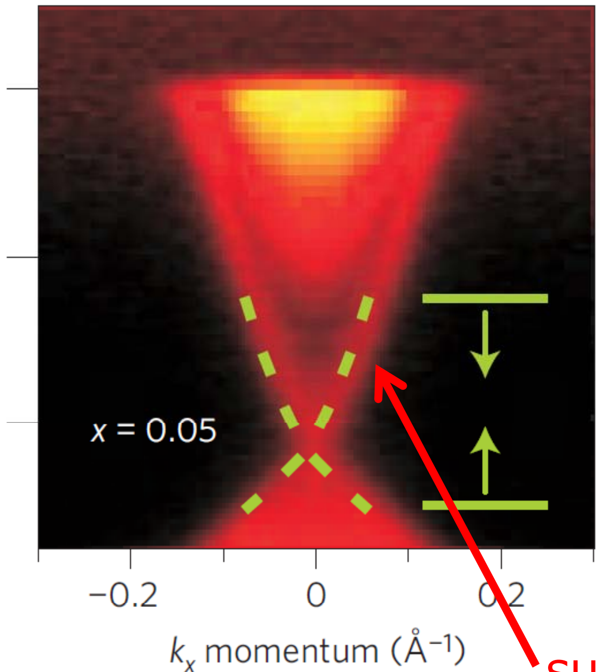


Y. Asano *et al*, PRB '03

Superconductivity in doped topological insulator

topological insulator
······metallic surface states

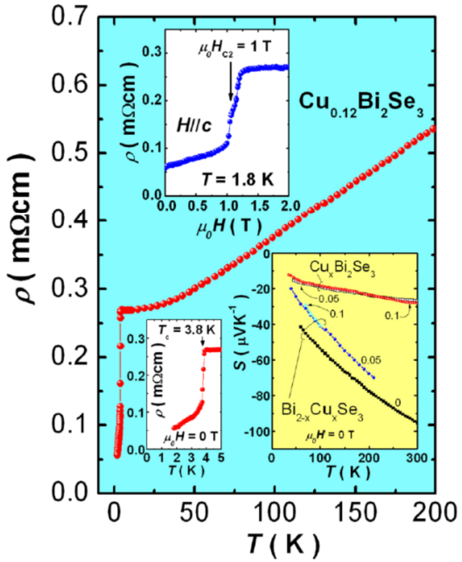
superconducting topological insulator
$Cu_xBi_2Se_3$

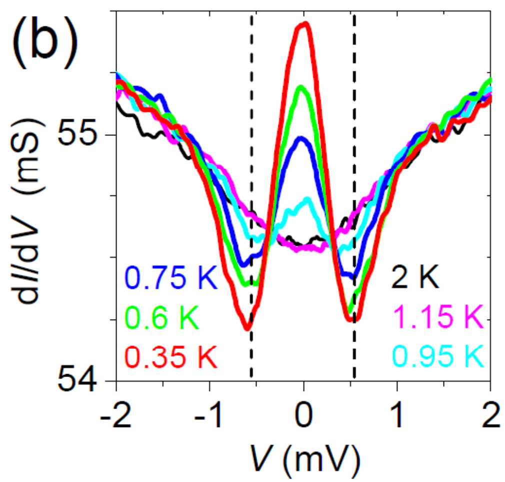


surface states

L. A. Wray *et al*, Nature Phys. 10

tunneling conductance
(point contact)



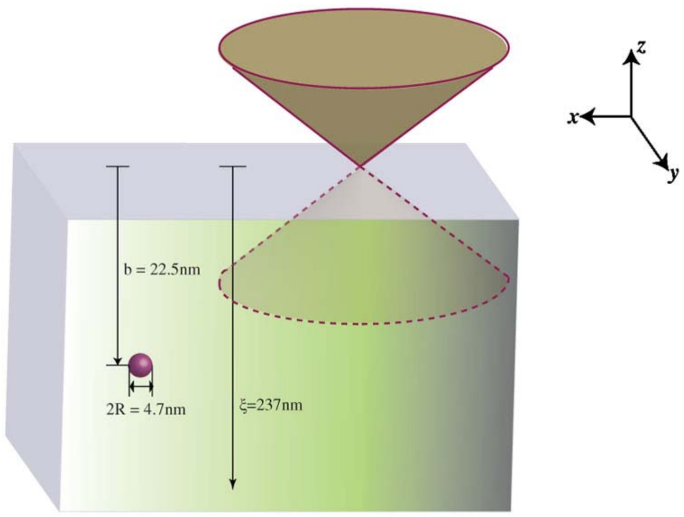


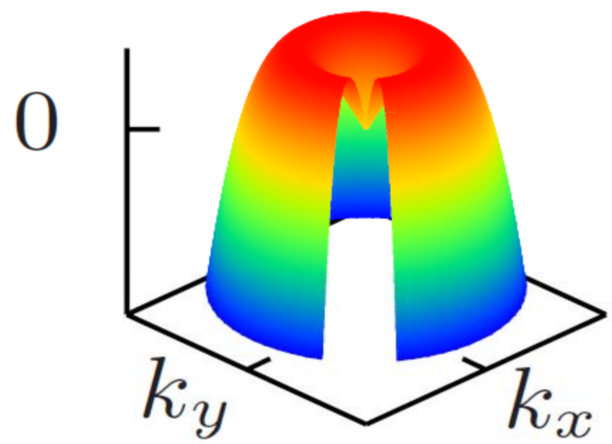
Y. S. Hor *et al*, PRL '10   S. Sasaki *et al*, PRL '11

zero-bias peak⇒surface states ABS

new type of three-dimensional
topological superconductor

(Topological) Andreev bound states (2)





Cone

Superfluid ^{3}HeB

Okuda Nomura (Review) (12)

Caldera

Doped topological insulator

Yamakage Yada Sato Tanaka (12)

Summary (1)

(1)Surface Andreev bound state can be interpreted as a topological edge state.

(2)Topological classification corresponding to bulk topological invariant

Y. Tanaka, M. Sato and N. Nagaosa, J. Phys. Soc. Jpn. 81 011013 (2012)

Periodic table of the topological materials

T: time-reversal
C: particle-hole, S: “chiral”

Cartan label	T	C	S
A (unitary)	0	0	0
AI (orthogonal)	+1	0	0
AII (symplectic)	−1	0	0
AIII (ch. unit.)	0	0	1
BDI (ch. orth.)	+1	+1	1
CII (ch. sympl.)	−1	−1	1
D (BdG)	0	+1	0
C (BdG)	0	−1	0
DIII (BdG)	−1	+1	1
CI (BdG)	+1	−1	1

Cartan	dimension 1	2	3
Complex case:			
A	0	$\mathbb{Z}$	0
AIII	$\mathbb{Z}$	0	$\mathbb{Z}$
Real case:			
AI	0	0	0
BDI	$\mathbb{Z}$	0	0
D	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	$2\mathbb{Z}$	0
CI	0	0	$2\mathbb{Z}$

IQHE

chiral-*p* (Sr_2RuO_4, ^{3}He-A)

^{3}He-B

Z_2 TI

NCS (helical) Rashba

QSHE

chiral-*d*

d(*p*)-wave fixed k_y, flat

Ryu *et al.*, NJP **12** (2010) 065005.

Schnyder, Ryu, Furusaki, Ludwig, Phys. Rev. B 78, 195125 (2008)

Contents of our talk

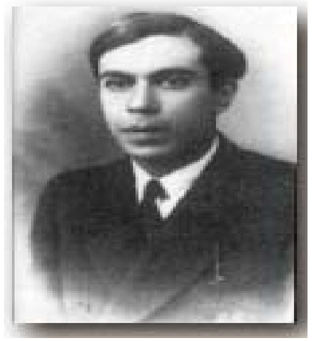
(1)Surface Andreev bound state up to now

(2)Majorana fermion (mode)

(3)Fabrication of Majorana Fermion at Nanowire and Interface

(4)Superconducting doped topological insulator

Majorana relation



Majorana (1933)

(1)Spinless fermion

Fully polarized, spin degree of freedom is half

$$\gamma_k = (u_k c_k + v_k c^{\dagger}_{-k}) \Rightarrow \boxed{\gamma_k = \gamma^{\dagger}_{-k}} \qquad u_k = v^{*}_{-k}$$

(2)Spinfull fermion (Equal spin pairing)

$$\gamma_{k,\nu} = (u_{k,\nu} c_{k,\nu} + v_{k,\nu} c^{\dagger}_{-k,\nu}) \Rightarrow \boxed{\gamma_{k,\nu} = \gamma^{\dagger}_{-k,\nu}} \qquad u_{k,\nu} = v^{*}_{-k,\nu}$$

$$\nu = \uparrow, \downarrow$$

(3)Spinfull fermion (Opposite spin pairing)

$$\gamma_{k,\nu} = (u_{k,\nu} c_{k,\nu} + v_{k,-\nu} c^{\dagger}_{-k,-\nu}) \Rightarrow \boxed{\gamma_{k,\nu} = \gamma^{\dagger}_{-k,-\nu}} \qquad u_{k,\nu} = v^{*}_{-k,\nu}$$

$$\nu = \uparrow, \downarrow$$

(3) It is not called Majorana Fermion for spin-singlet pairing.

(Topological) Surface Andreev Bound State

Type of dispersion	TRS	Examples	Type of Majorana mode
Flat	○	Cuprate	**Not Majorana**
Chiral	×	Sr_2RuO_4 (without orbital effect) ^{3}He A phase	**double chiral Majorana** **double chiral Majorana**
Helical	○	***s*+*p* –wave** (NCS superconductor) $\Delta_p > \Delta_s$	**helical Majorana**
Cone **Caldera**	○	^{3}He B phase $Cu_xBi_2Se_3$	2D **helical Majorana**

Sato、Kashiwaya、Maeno(固体物理) Tanaka, Sato, Nagaosa (JPSJ Review)

Summary (2)

(1)Majorana fermion (mode) is a special type of Andreev bound state.

(2)Chiral Majorana (TRS broken)

Helical Majorana (TRS is not broken)

⬇

Future application for quantum computation

X.L. Qi and S.C. Zhang, Rev. Mod. Phys. 83 1057 (2011)
Y. Tanaka, M. Sato and N. Nagaosa, J. Phys. Soc. Jpn. 81 011013 (2012)

Contents of our talk

(1)Surface Andreev bound state up to now

(2)Majorana fermion (mode)

(3)Fabrication of Majorana Fermion in Nanowire and Interface

(4)Majorana fermion in the nanowire

Condition of the formation of Andreev bound state (Majorana fermion)

Sign Change of the Pair potential on the Fermi surface (orbital degree of freedom)

Is it possible to generate Andreev bound state (Majorana fermions) based on the conventional spin-singlet s-wave superconductor?

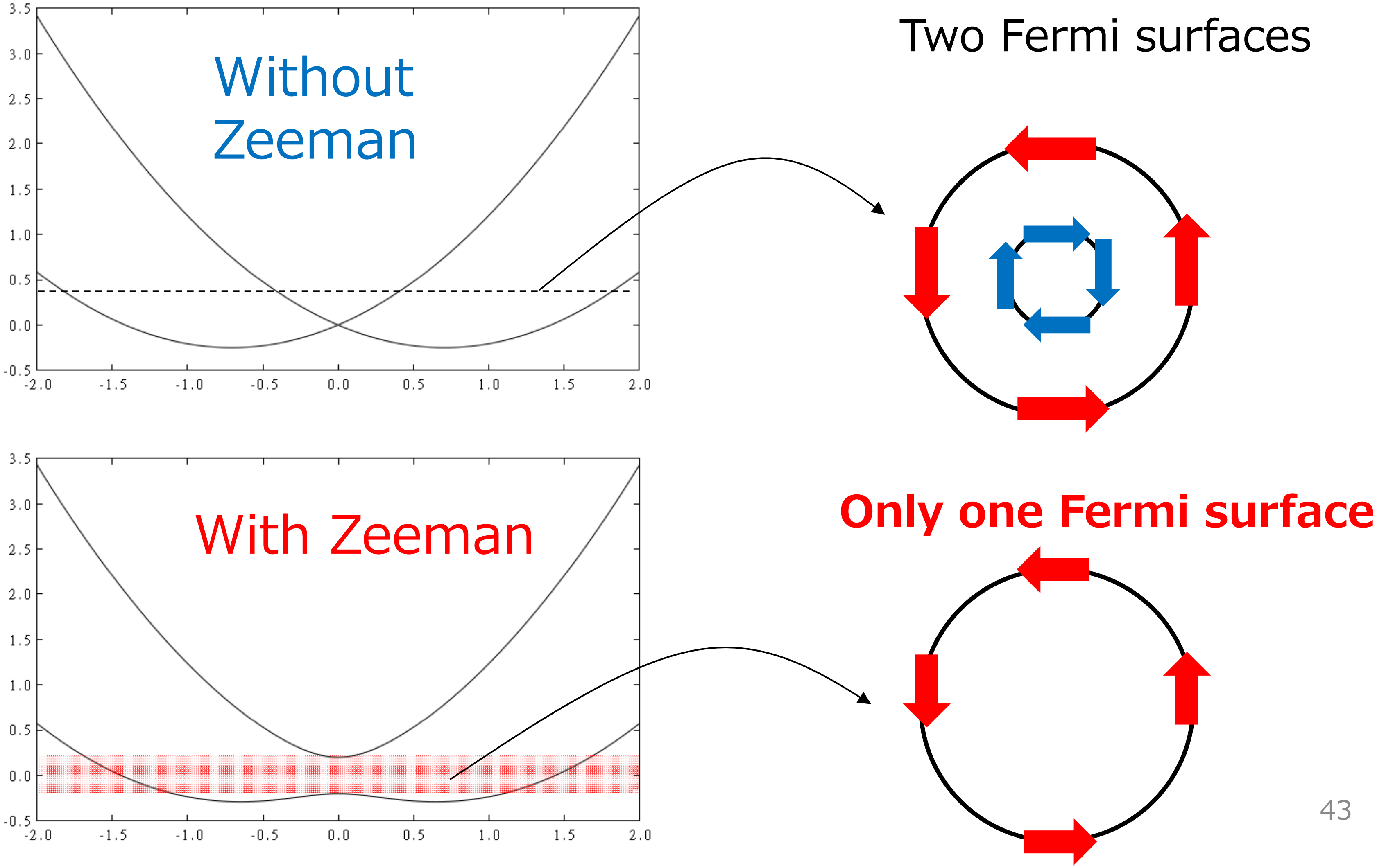
↓

Yes

Spin-orbit interaction is a key ingredient

Role of spin-orbit(SO) coupling

direction of spin is quenched



Creation of Majorana fermion (spinless) based on the Nano-structures

- Nano-wire, Interface of oxides, surface
- Mainly proximity coupling to conventional spin singlet s-wave pairing

- **Spin-orbit coupling**
- **Reduction of the electron's spin degree of freedom**

(Exchange field, hybridization of bands）

Chiral Majorana Fermion (mode) (1)

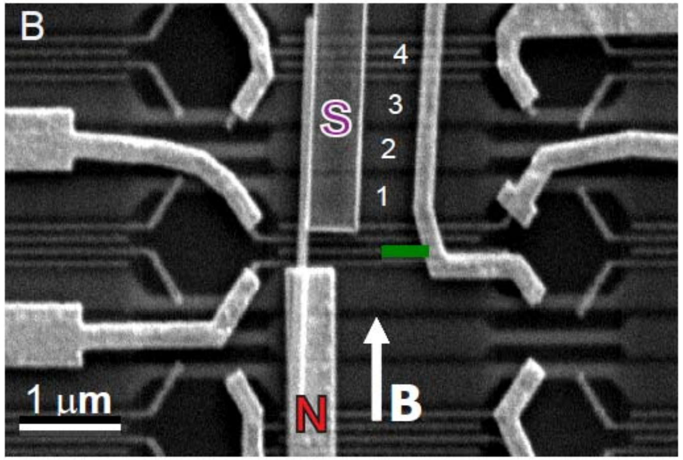
Spinless, TRS broken

zero-dimensional Majorana

s-wave superconductor
+ InSb Nanowire

Magnetic field

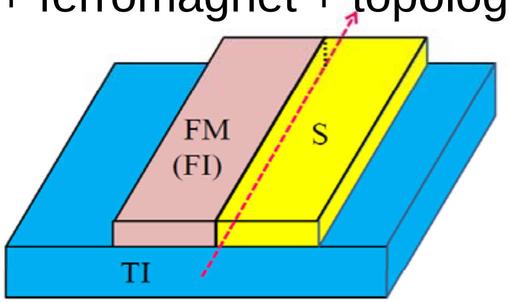
Kitaev(01); Lutchyn ,Sau, Das Sarma(10), Oreg(10)
Beenakker(11)



Kouwenhoven (12)

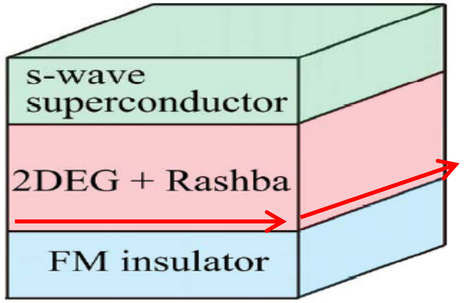
one-dimensional Majorana (Chiral)

s-wave superconductor
+ ferromagnet + topological insulator (TI)



Fu Kane(08), Beenakker (09)
Tanaka & Nagaosa(09), Law Lee (09)

s-wave superconductor
+ ferromagnet + 2D electron gas



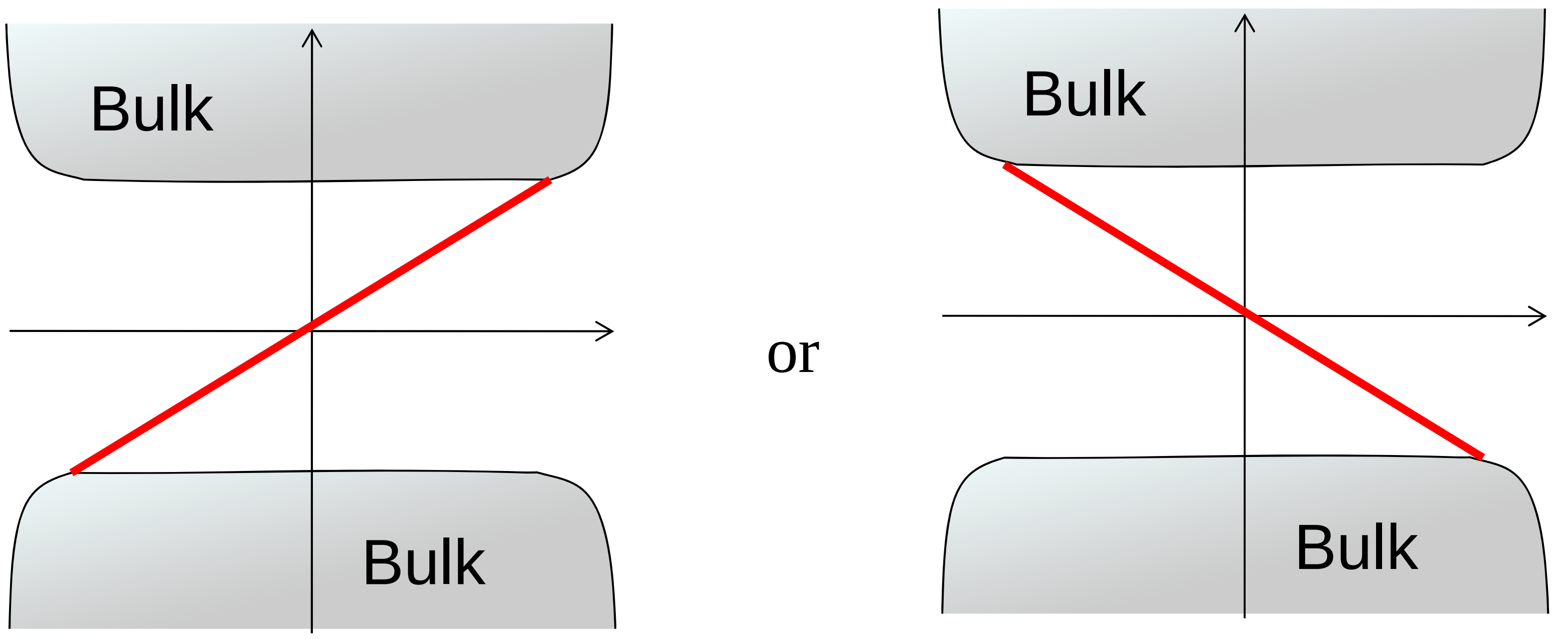
Rashba superconductor

Sato 09; Sau 10; Alicea 10, Lutchyn 10 Yamakage 11

Chiral Majorana Fermion
(Special type of Andreev bound state)

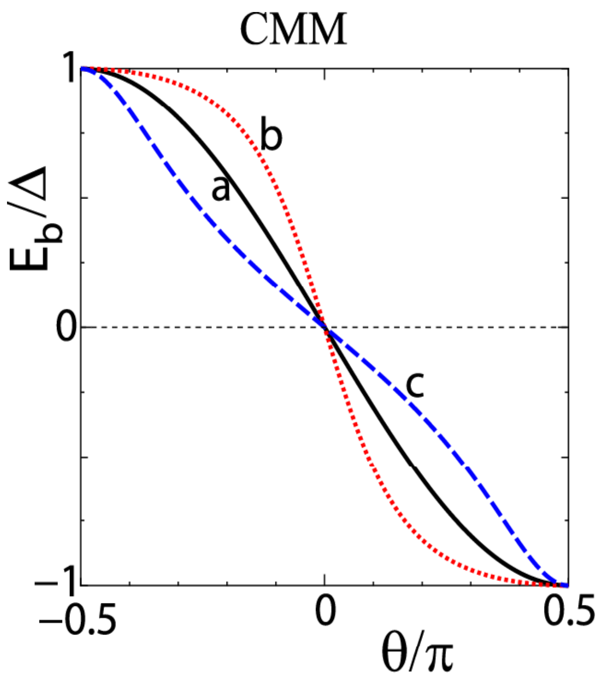
Broken Time reversal symmetry!!

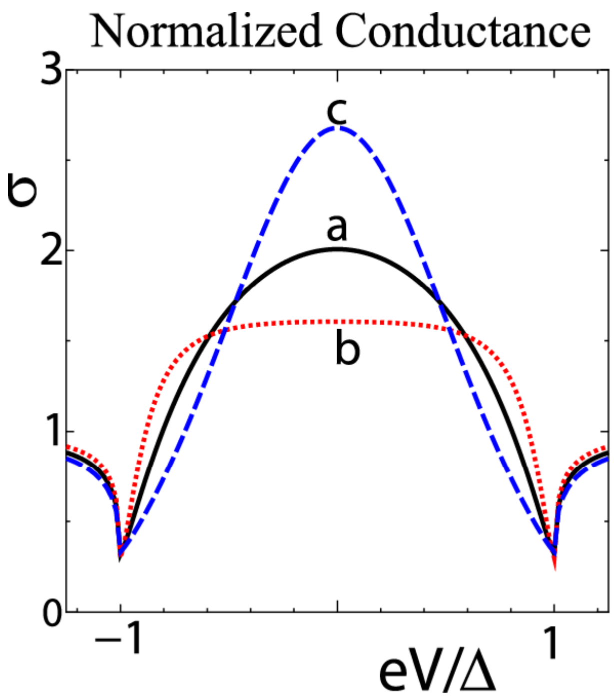
$$\gamma = \gamma^\dagger$$

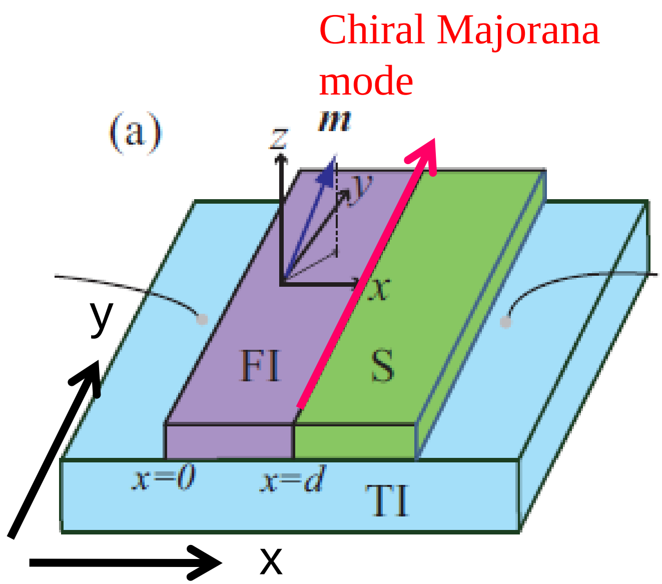


1d Chiral Majorana fermion

Chiral Majorana Fermion (mode) on TI







$m_z d/v_F = 1$ $m_y/m_z = 0$ a $\mu/m_z = 1$ b $\mu/m_z = 0.5$ c $\mu/m_z = 2$

1 Chiral Majorana mode (CMM) is tunable by material parameters of TI.

2 Normalized conductance has a peak at zero voltage similar to chiral *p*-wave case.

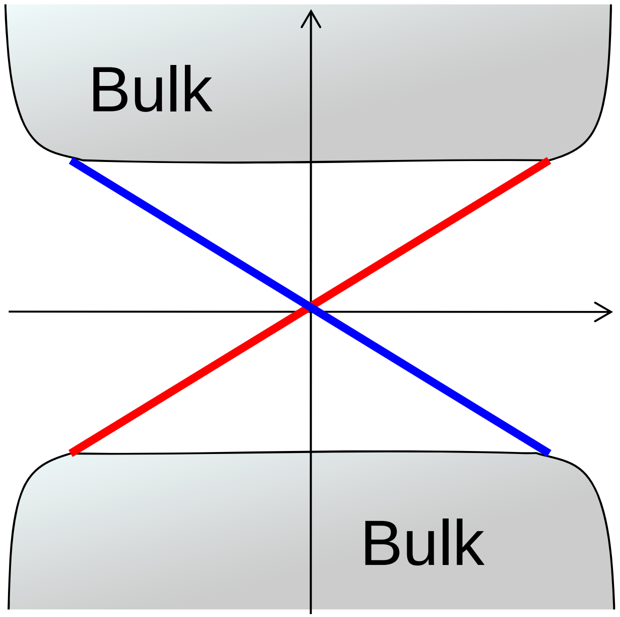
Tanaka, Yokoyama, Nagaosa, PRL 103, 107002 (2009)

Helical Majorana Fermion

(Special type of Andreev bound state)

Time reversal symmetry

$$\gamma_\nu = \gamma_\nu^\dagger$$



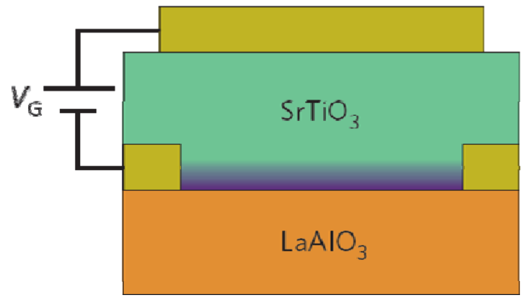
1d helical Majorana fermion

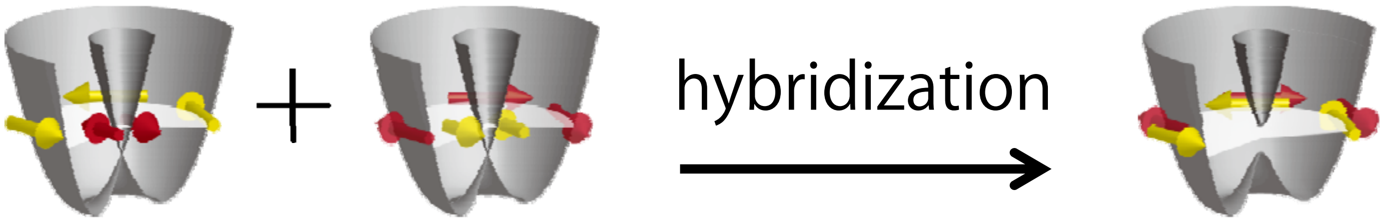
Helical Majorana fermion

Using Interface superconductivity

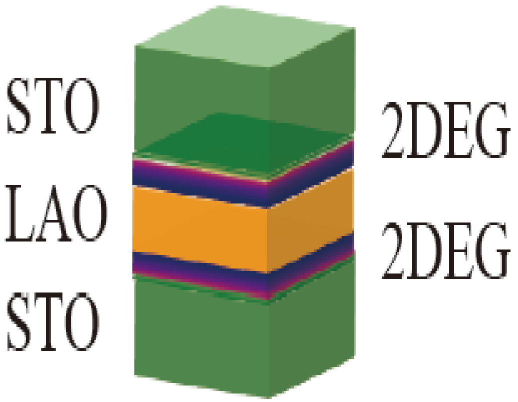
interface of transition metal oxides

- 2d electron gas — Ohtomo & Hwang Nature 2004
- superconductivity — Reyren *et al.* Science 2007
- tunable Rashba SOI — Caviglia *et al.* PRL 2010





One-dimensional Majorana (Helical)



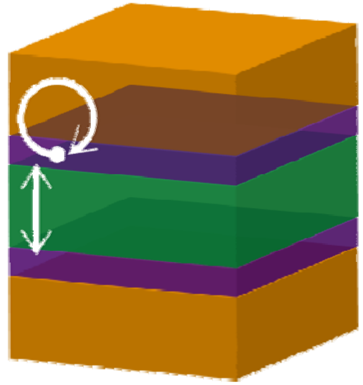
$$\Delta_1 = -\Delta_2$$

Intra-layer pairing with different sign

Nakosai, Tanaka Nagaosa, PRL(2012)

Model construction

kinetic Hamiltonian



$$\mathcal{H}_0(\boldsymbol{k}) = \frac{k^2}{2m} \boxed{-\varepsilon\sigma_x} + \boxed{\alpha\left(k_x s_y - k_y s_x\right)\sigma_z}$$

hybridize : transfer   SOI : Rashba SOI

$\boldsymbol{s}$: spin $\boldsymbol{\sigma}$: layer

electron density-density interaction

$$\mathcal{H}_{\text{int}}(\boldsymbol{x}) = \boxed{-U\left(n_1^2(\boldsymbol{x}) + n_2^2(\boldsymbol{x})\right)} \boxed{-2Vn_1(\boldsymbol{x})\, n_2(\boldsymbol{x})}$$

intra-layer   inter-layer

Bogoliubov de-Gennes Hamiltonian

$$\mathcal{H}_{\text{BdG}} = \begin{pmatrix} \mathcal{H}_0 - \mu & \Delta \\ \Delta & -\mathcal{H}_0 + \mu \end{pmatrix}$$

cf. Fu and Berg PRL 2010

S. Nakosai , Y . Tanaka and N. Nagaosa PRL(2012)

Pairing potentials

determination of pairing potentials (= mean-field)

- spin and layer degrees of freedom
- short range int. $\longrightarrow$ k-indep. pairing amplitudes
- lattice symmetry D_{4h}

parity under an inversion operation

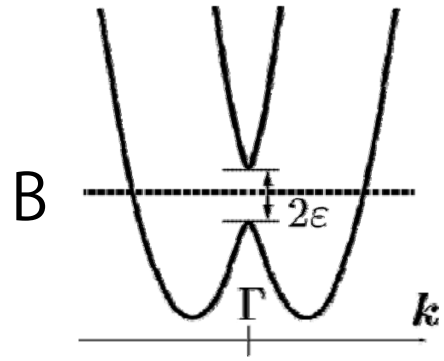
	irreps	matrix	symmetry	I	node	spin
$\hat{\Delta}_1$	A_{1g}	I σ_x	$\langle c_{1\uparrow}c_{1\downarrow}\rangle = \langle c_{2\uparrow}c_{2\downarrow}\rangle = \Delta_1/2$ $\langle c_{1\uparrow}c_{2\downarrow}\rangle = -\langle c_{1\downarrow}c_{2\uparrow}\rangle = \Delta_1'/2$	+	full	singlet
$\hat{\Delta}_2$	A_{1u}	$s_z\sigma_y$	$\langle c_{1\uparrow}c_{2\downarrow}\rangle = \langle c_{1\downarrow}c_{2\uparrow}\rangle = \Delta_2/2$	−	full	triplet
$\hat{\Delta}_3$	A_{2u}	σ_z	$\langle c_{1\uparrow}c_{1\downarrow}\rangle = -\langle c_{2\uparrow}c_{2\downarrow}\rangle = \Delta_3/2$	−	full	singlet
$\hat{\Delta}_4$	E_u	$\begin{pmatrix} s_x\sigma_y \\ s_y\sigma_y \end{pmatrix}$	$\langle c_{1\uparrow}c_{2\uparrow}\rangle = \langle c_{1\downarrow}c_{2\downarrow}\rangle = \Delta_4/2$ $\langle c_{1\uparrow}c_{2\uparrow}\rangle = -\langle c_{1\downarrow}c_{2\downarrow}\rangle = \Delta_4/2$	−	point node	triplet

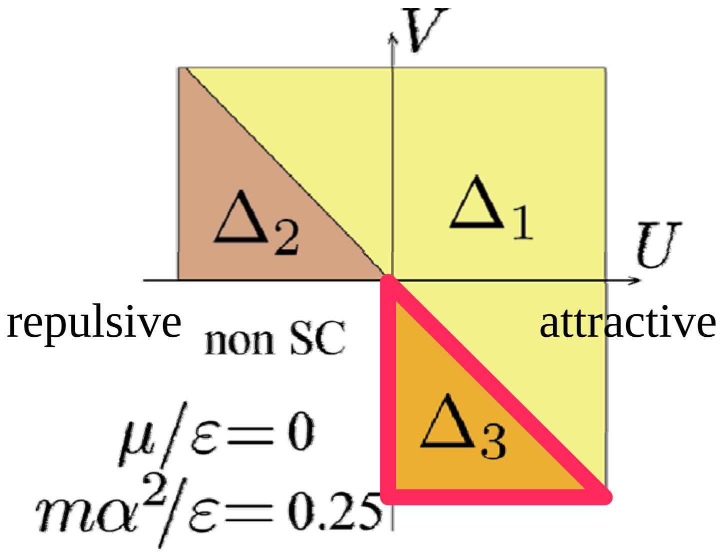
S. Nakosai , Y . Tanaka and N. Nagaosa PRL(2012)

Topological SC ?

We set the Fermi energy
within the hybridization gap.

1. [Fermi level] OK





2. [odd parity pairing potential] OK

NOTE:
Pairing amplitudes for Δ_2 and Δ_3 are proportional to α .

SOI-induced SC phases

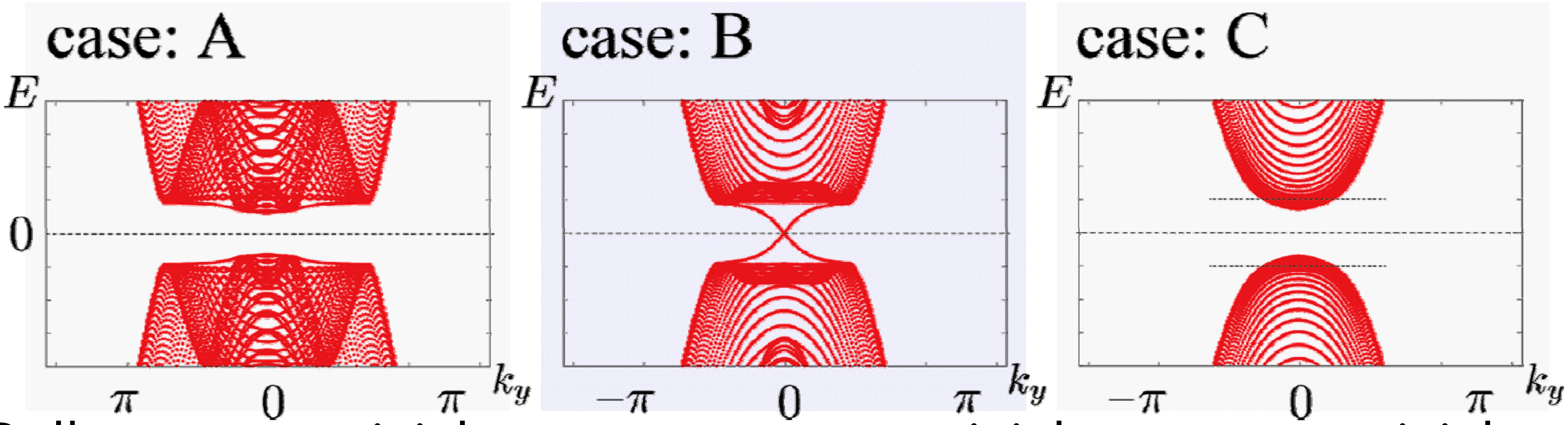
Unconventional SC phase appears
in a feasible parameter region.

intra-layer : attractive (phonon mechanism)

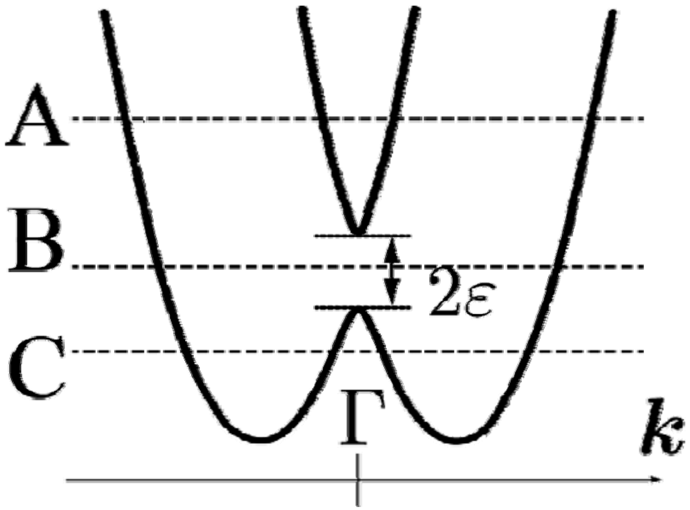
inter-layer : repulsive (Coulomb interaction)

Spectrum and Edge states Δ_3

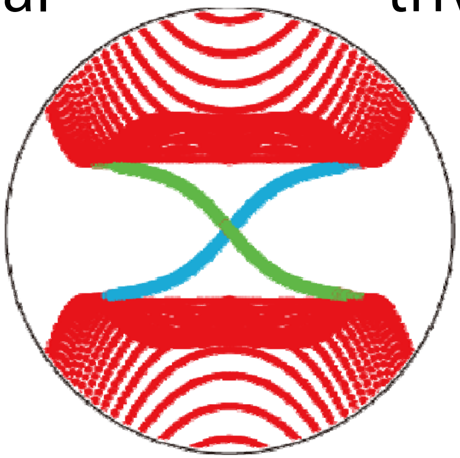
For each Fermi level



Bulk state trivial non-trivial trivial



Fermi energy



helical Majorana edge states

S. Nakosai , Y . Tanaka and N. Nagaosa PRL(2012)

Summary (3)

(1) There are many interesting systems where spinless Majorana fermion can be generated from conventional s-wave pairing.

(2) Fabrication of Majorana Fermion in nanowire, surface and interface is a new direction of condensed matter physics.

J. Alicea, Rep. Prog. Rep. Prog. Phys. 75, 076501 (2012)

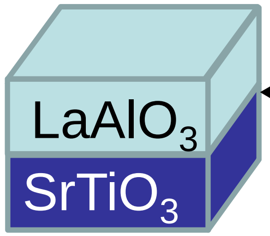
X.L. Qi and S.C. Zhang, Rev. Mod. Phys. 83 1057 (2011)

Majorana Fermion with flat dispersion

Unconventional (anisotropic) pairing at the interface

One-dimensional Majorana (Flat)

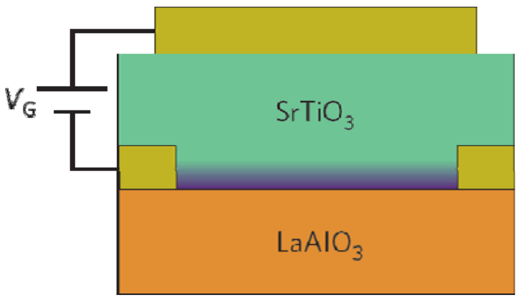
Time reversal symmetry

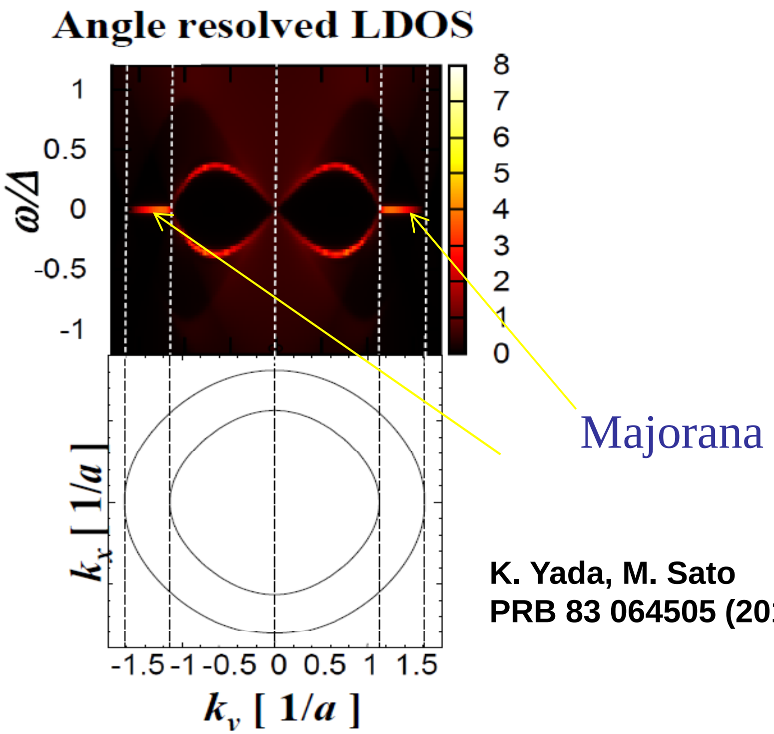


edge （surface）

Interface superconductivity **Reyren et al (2007)**

Possible d+p-wave pairing **Yada Onari (2009)**





K. Yada, M. Sato
PRB 83 064505 (2011)

Contents of our talk

(1)Surface Andreev bound state up to now

(2)Majorana fermion

(3)Fabrication of Majorana Fermion at Nanowire and Interface

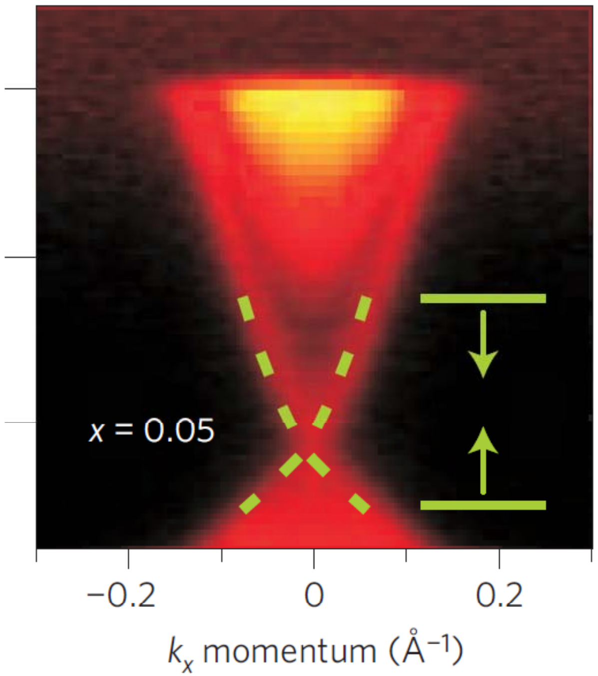
(4)Superconducting doped topological insulator

Doped topological insulator

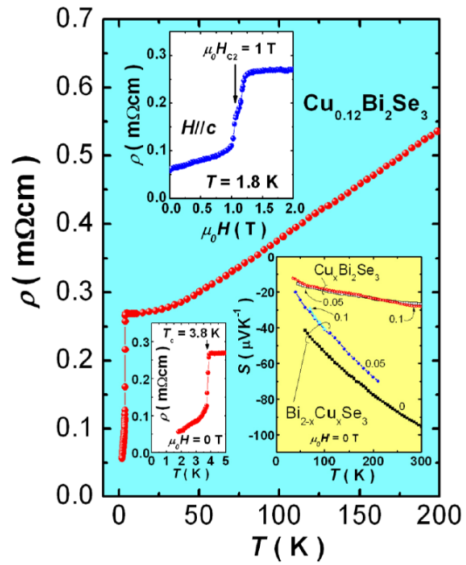
Topological insulator
$Cu_xBi_2Se_3$

⇒

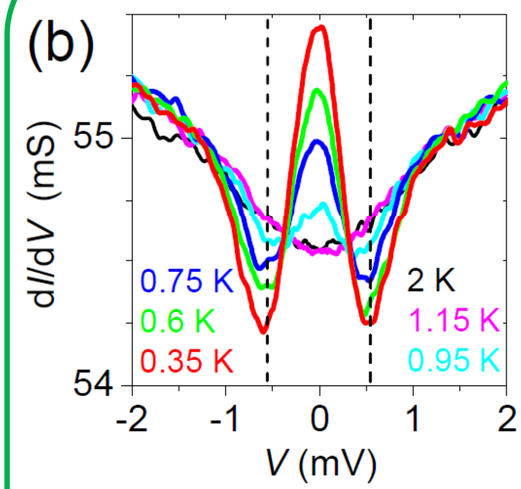
Superconducting topological insulator $Cu_xBi_2Se_3$



L. A. Wray *et al*, Nature Phys. 10



Y. S. Hor *et al*, PRL '10

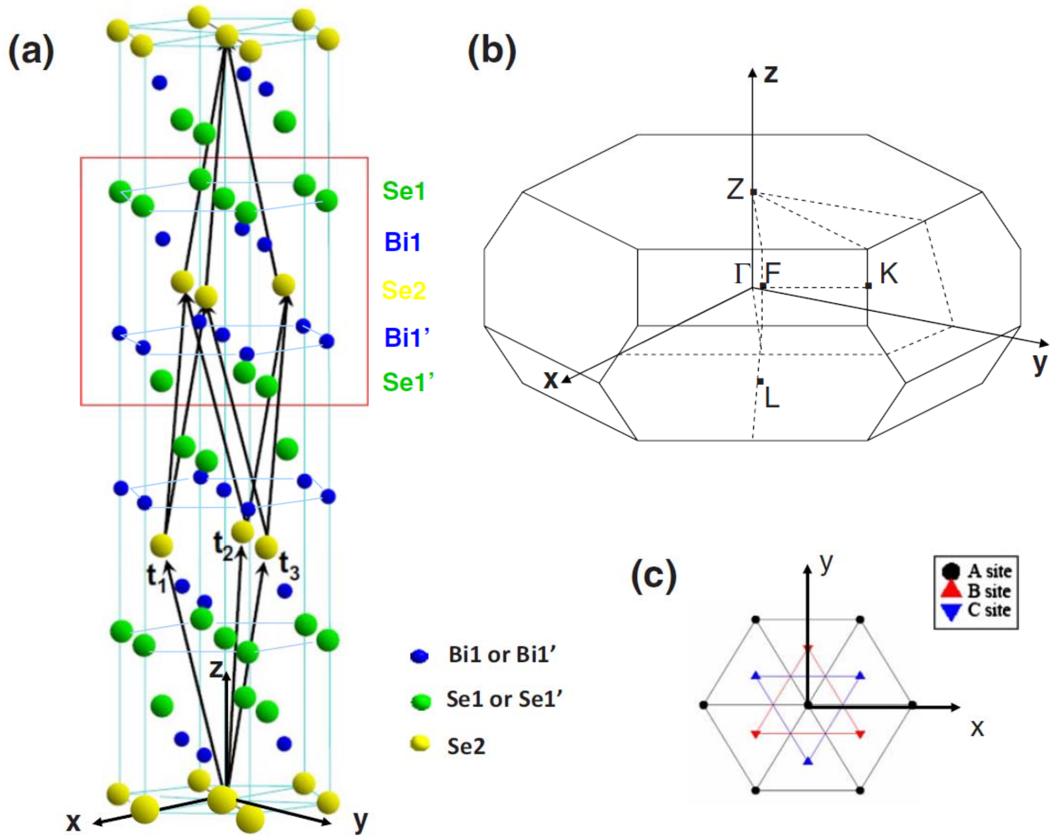


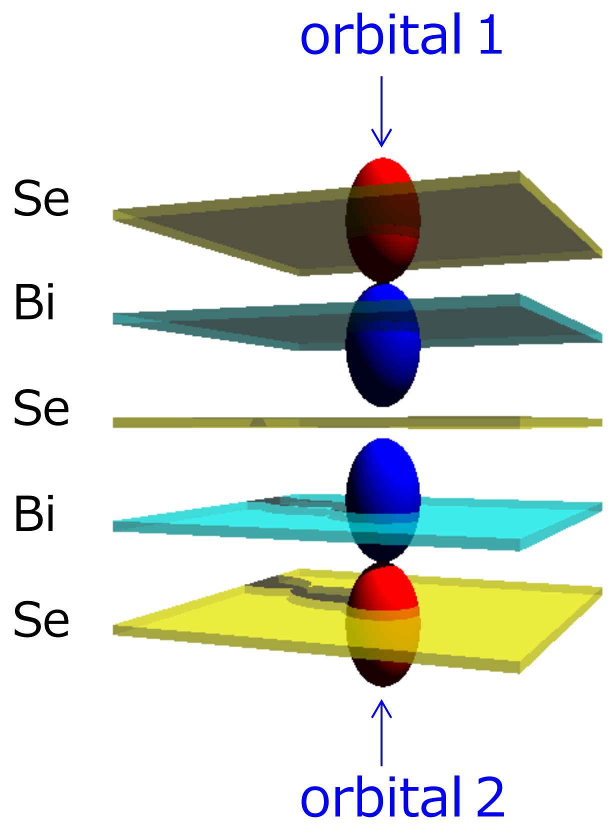
S. Sasaki *et al*, PRL '11

⇒ Andreev bound state

3 d topological superconductor

Crystal structure of topological insulator





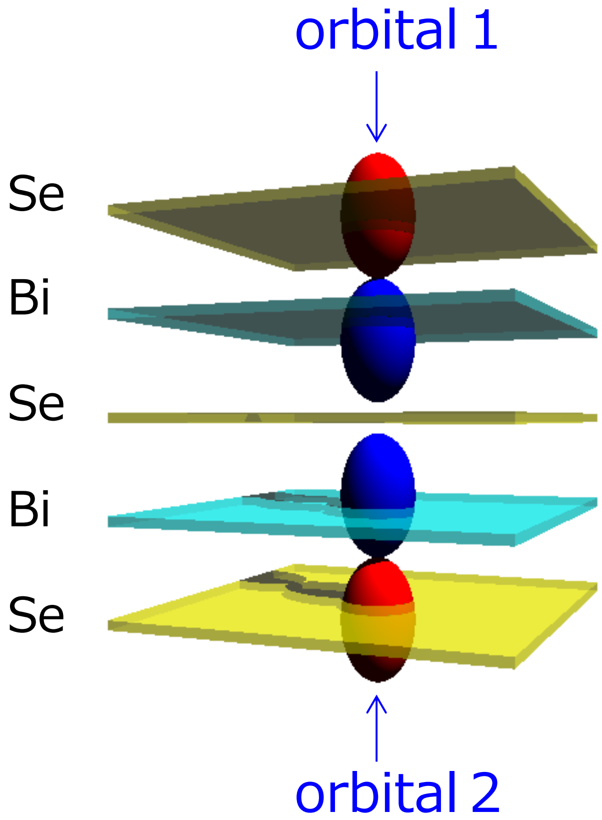
Zhang, *et al.*, 09

Effective Hamiltonian of topological insulator

$$H_{\mathrm{TI}}(\boldsymbol{k}) = m(\boldsymbol{k})\sigma_x + v_z k_z \sigma_y + v\sigma_z(k_x s_y - k_y s_x)$$

orbital

spin



Zhang, *et al.*, 09

Model of superconducting topological insulator

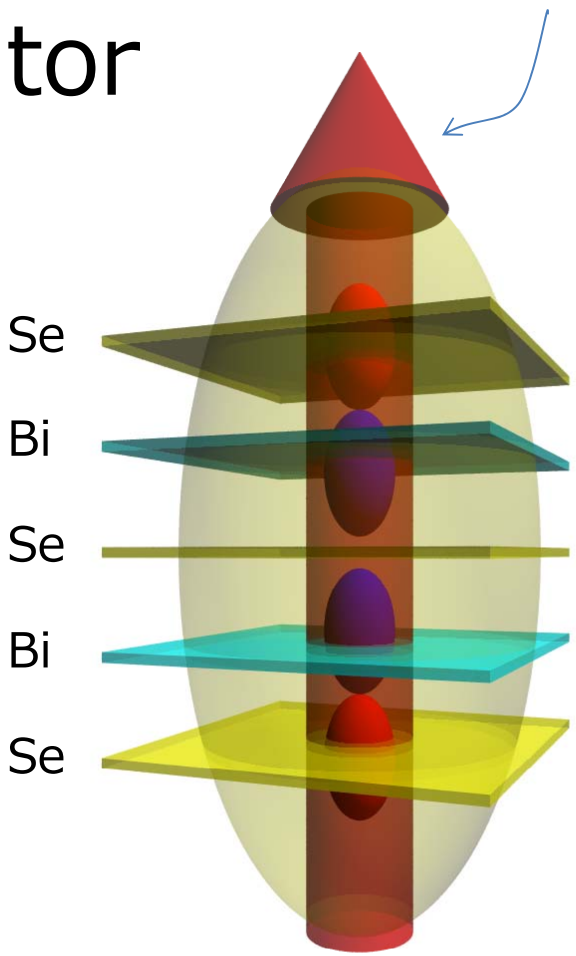
$$H_{\mathrm{TI}}(\boldsymbol{k}) = m(\boldsymbol{k})\sigma_x + v_z k_z \sigma_y + v\sigma_z(k_x s_y - k_y s_x)$$

orbital spin

$$H_{\mathrm{STI}}(\boldsymbol{k}) = \begin{pmatrix} H_{\mathrm{TI}}(\boldsymbol{k}) - \mu & \Delta_{\mathrm{STI}}\sigma_y s_z \\ \Delta_{\mathrm{STI}}\sigma_y s_z & -H_{\mathrm{TI}}(\boldsymbol{k}) + \mu \end{pmatrix}$$

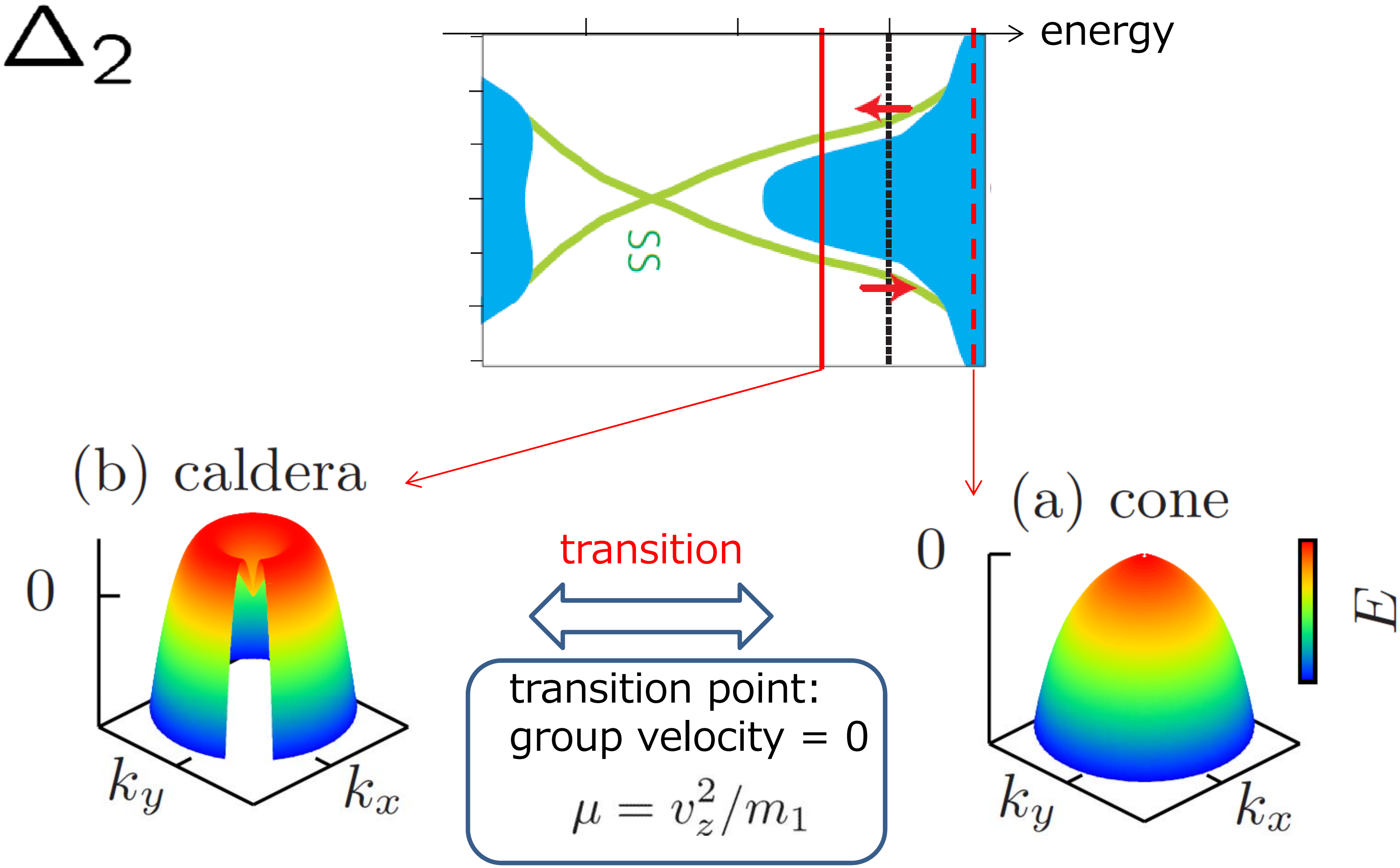
$$k_i \to \sin k_i,$$
$$k_i^2 \to 2(1 - \cos k_i).$$

mapping to lattice model



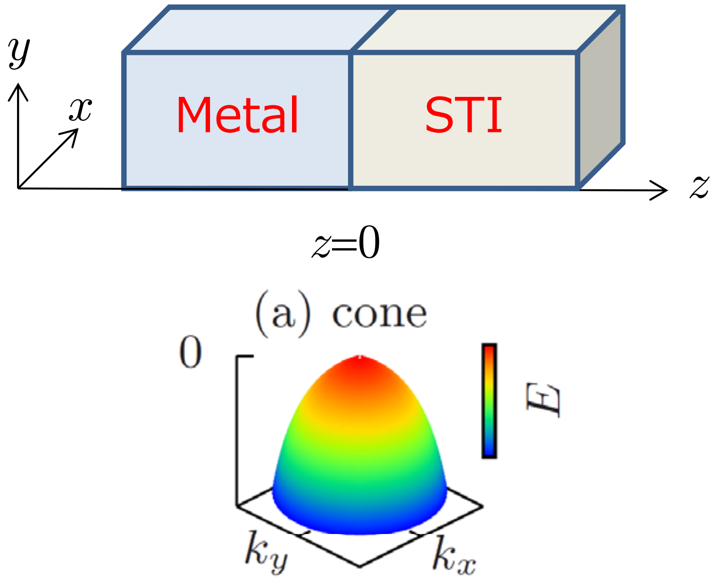
Fu and Berg, 10

Emergence of new type of Andreev Bound State

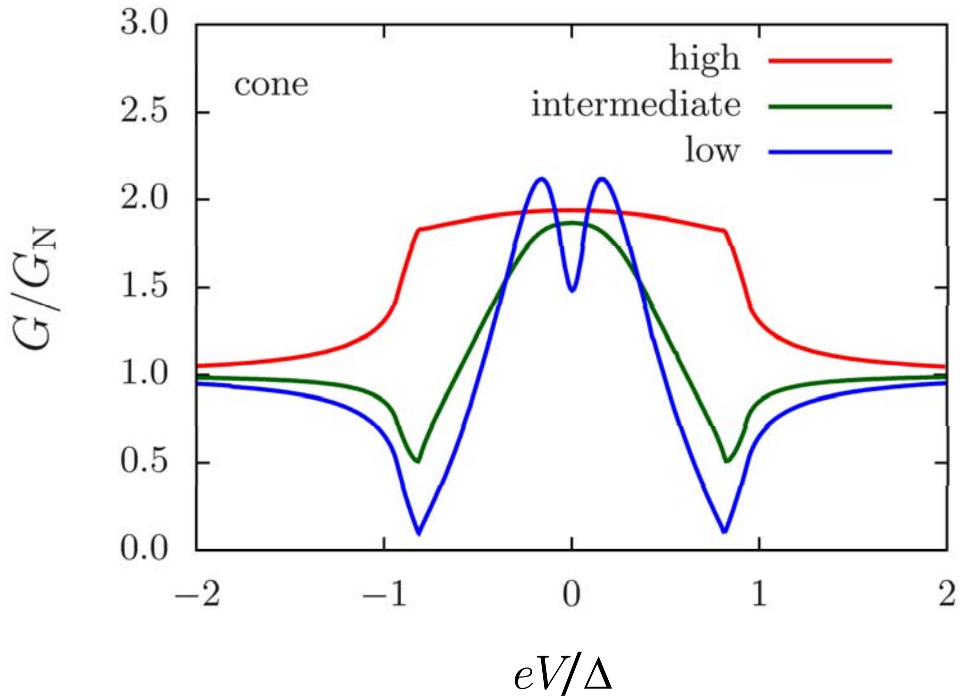


L. Hao and T. K. Lee, PRB '11 T. H. Hsieh and L. Fu, PRL '12
A. Yamakage, K. Yada, M. Sato, and Y. Tanaka, PRB 2012

Tunneling conductance in superconductor realized in doped topological insulator



full-gap case

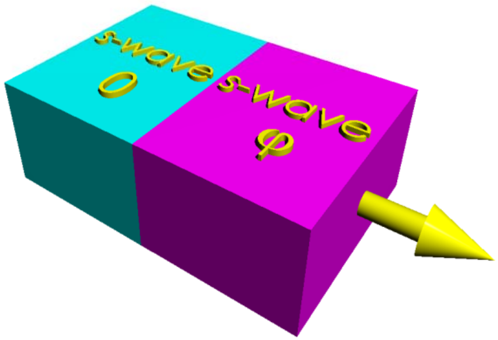


structural transition
-> group velocity ~ zero
-> large surface DOS

zero-bias peak even in the full gap case

A. Yamakage , K. Yada, M. Sato, and Y. Tanaka, PRB2012

Current phase relation in Josephson junctions



φ

Spin structure | Current
singlet／singlet : $J(\varphi) \sim \sin\varphi$

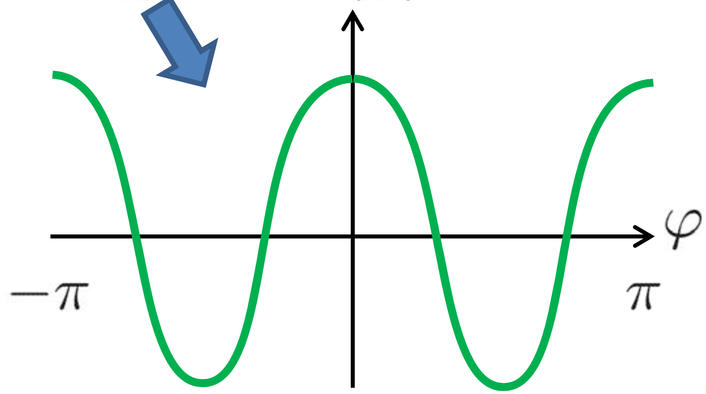
singlet／triplet : $J(\varphi) \sim \sin 2\varphi$

J. A. Pals, 76

Free energy

$$F(\varphi) = \frac{\hbar}{2e}\int^{\varphi} d\varphi' J(\varphi')$$

$J(\varphi) \sim \sin 2\varphi$



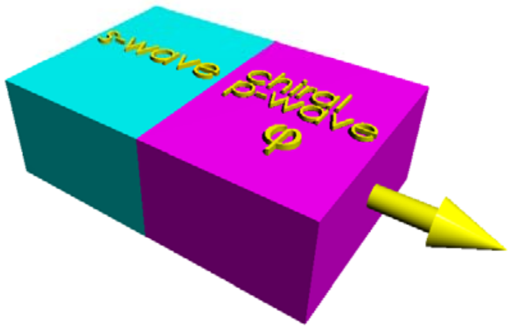
sin2φ →quantum two level system

Anomalous current phase relation
sin 2φ

$J(\varphi) \sim \sin 2\varphi$

Pals, 76

Yamashiro, 98



spin-singlet／spin-triplet

d-wave／s-wave

Tanaka, 94

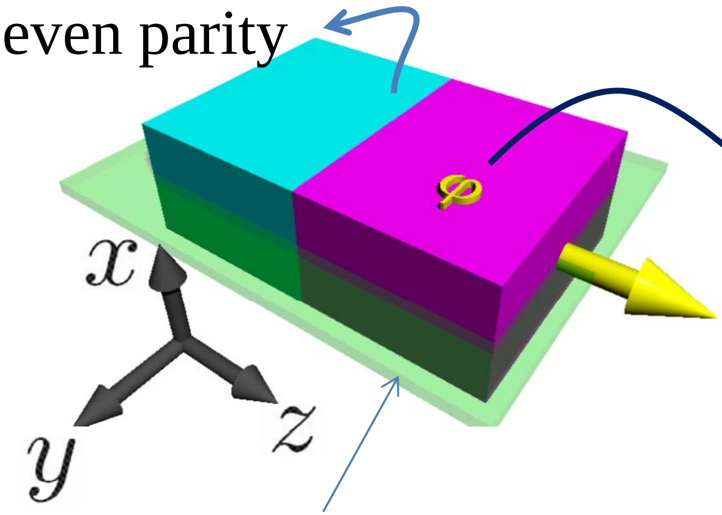
The effect of spin-orbit coupling

$J(\varphi) \sim \sin 2\varphi \rightarrow \cos\varphi$ Asano, *et al*., PRB, 03

Robust in superconducting topological insulator $J(\varphi) \sim \sin 2\varphi$

Yamakage, *et al*, arXiv1208.5306

Mirror symmetry and current phase relation



yzplane : mirror

mirror inversion: $(x,y,z) \rightarrow (-x,y,z)$

If one of the pair potential has an odd parity as mirror inversion operation $[\varphi \rightarrow \varphi + \pi]$

$$J(\varphi) = J(\varphi + \pi) \Rightarrow J(\varphi) \sim \sin 2\varphi$$

If spin-orbit coupling has an odd-parity by mirror inversion

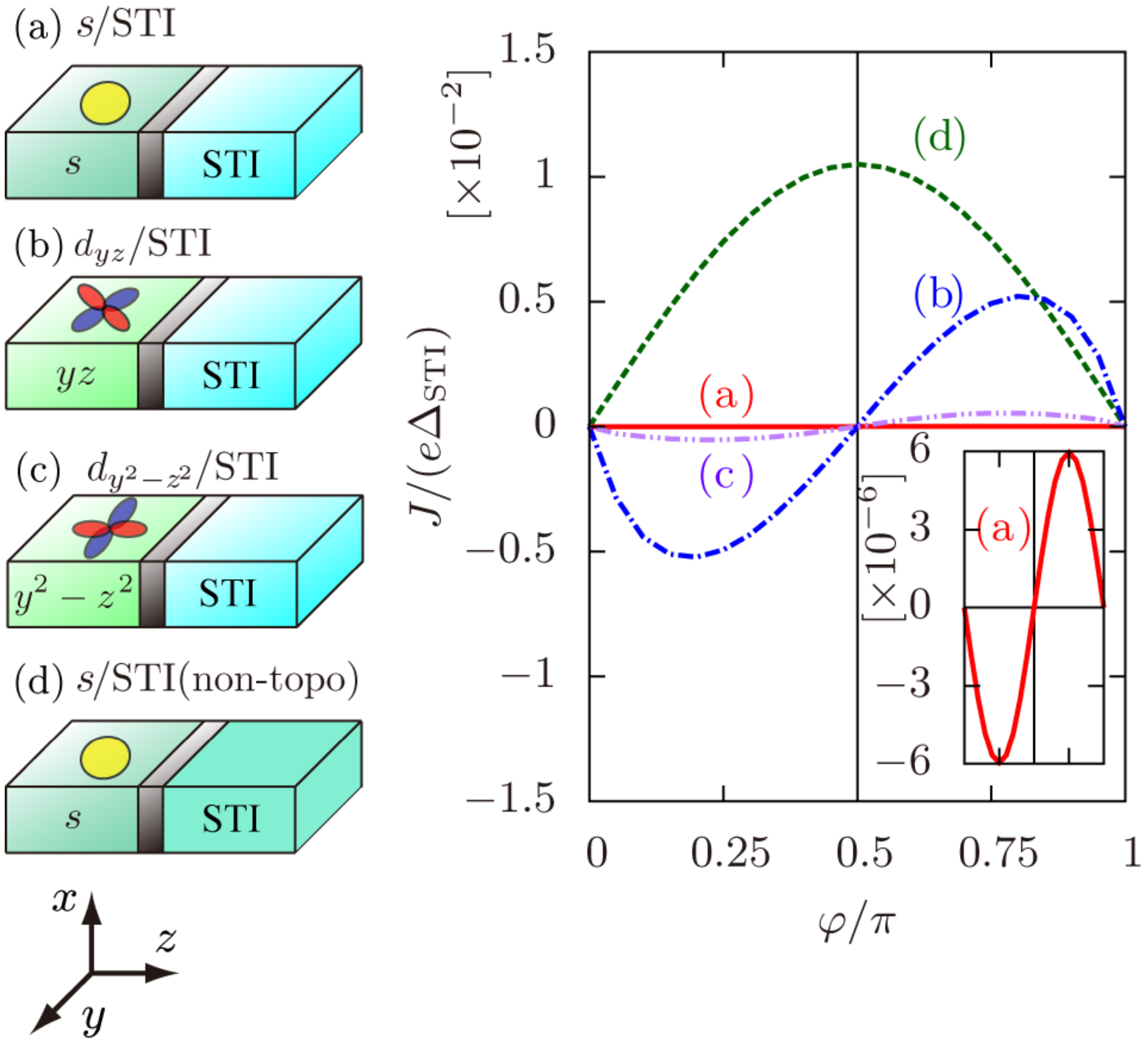
$J(\varphi) \sim \sin 2\varphi$ Yip, *et al*., PRB, 90

Fu and Berg, 10
Hsieh and Fu, 12
Yamakage, *et al*., 12

Pair potential in superconducting Topological insulator has an odd-parity by mirror inversion

$\Rightarrow J(\varphi) \sim \sin 2\varphi$ robust feature

Current phase relation in superconducting topolgical insulator



Yamakage, *et al*, arXiv1208.5306

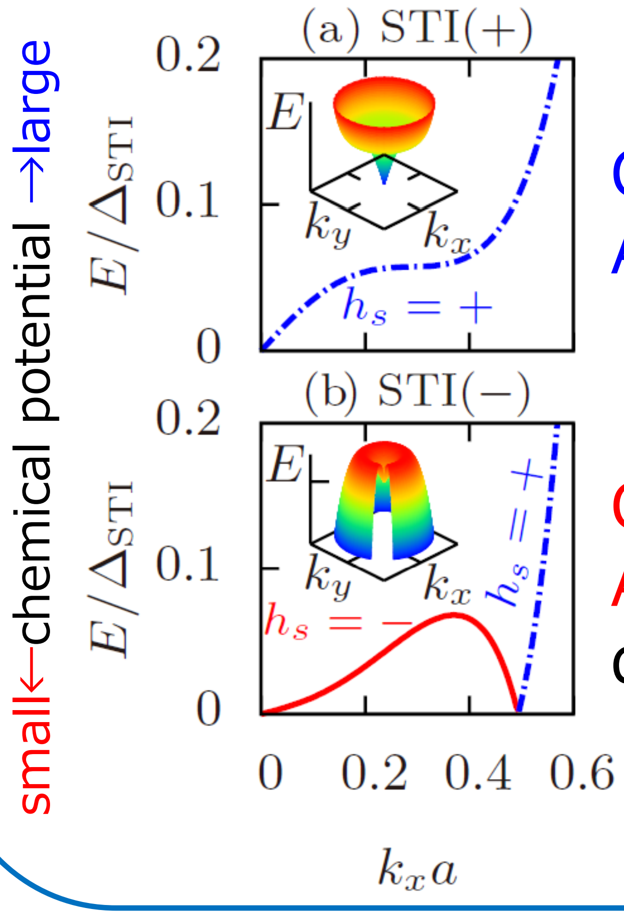
$$J(\varphi) \sim \sin 2\varphi$$

Josephson current depending on spin-helicity

Hao and Lee, 11
Hsieh and Fu, 12
Yamakage, *et al*, 12

Yamakage, *et al*, arXiv1208.5306

Surface Andreev bound state of superconducting topological insulator



Cone-type ABS

Caldera-type ABS

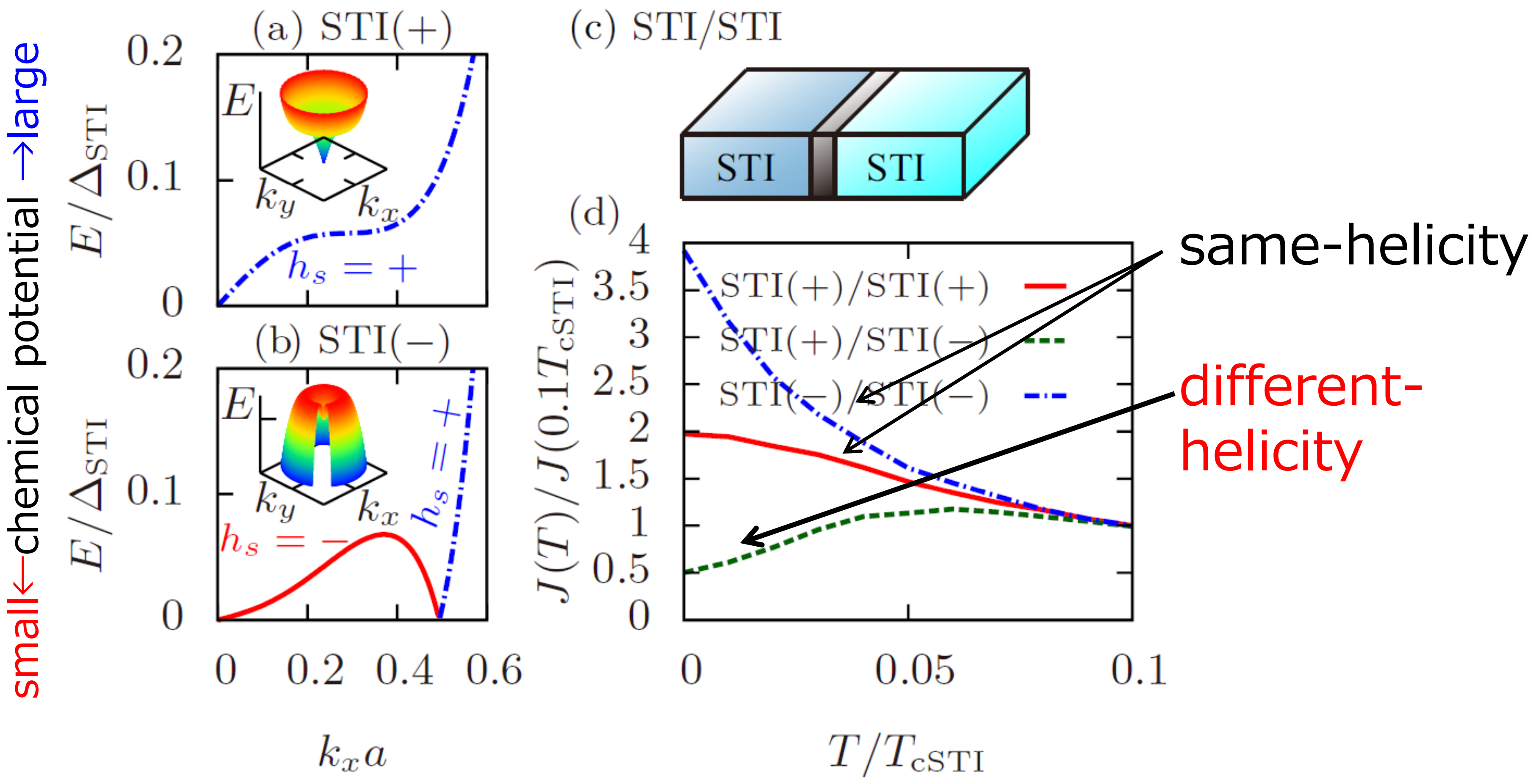
different helicity

$$H_{\mathrm{surf}}(\boldsymbol{k}_{\|}) = h_{\mathrm{s}} v_{\mathrm{surf}} (\boldsymbol{k} \times \boldsymbol{s})_z$$

Spin-helicity dependent Josephson current

Hao and Lee, 11
Hsieh and Fu, 12
Yamakage, *et al*, 12

Yamakage, *et al*, arXiv1208.5306



Summary (4)

(1)New type of Andreev bound state (surface state) is expected in superconducting doped topological insulator.

(2) Josephson current has an anomalous current phase relation.

(3)Anomalous temperature dependence due to spin helicity

Yamakage, *et al*, arXiv1208.5306

Yamashiro, 98

Summary

Josephson current in superconducting topological insulator

- symmetry protected $J(\varphi) \sim \sin 2\varphi$
- Josephson current depending on the spin-helicity of Andreev bound state(Majorana fermion)