Mechanical vibration - System identification and modal analysis of 3-DOF linear system

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1 Dynamical system

1.1 The linear model

The chosen model is a linear plant consisting of 3 masses, 3 springs between them, and 3 dampers between each mass and the ground. The model is shown in Figure 1.

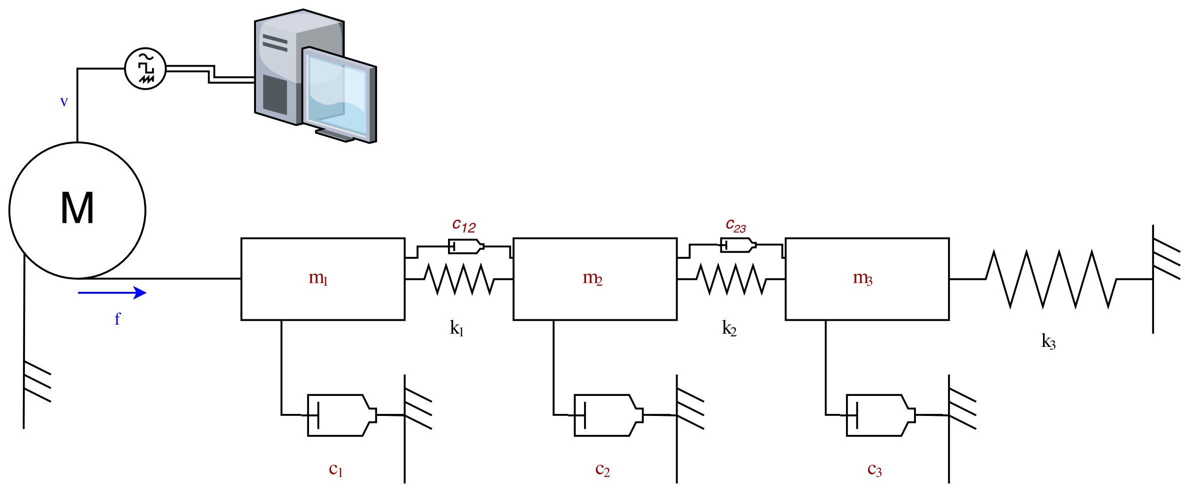

Figure 1: The chosen plant, in red the unknown parameters

1.2 equation of motion

$$\begin{cases} m_1\ddot{x}_1 = +k_1\left(x_2 - x_1\right) + c_{12}\left(\dot{x}_2 - \dot{x}_1\right) - c_1\dot{x}_1 + k_v v \\ m_2\ddot{x}_2 = +k_1\left(x_1 - x_2\right) + k_2\left(x_3 - x_2\right) + c_{12}\left(\dot{x}_1 - \dot{x}_2\right) + c_{23}\left(\dot{x}_3 - \dot{x}_2\right) - c_2\dot{x}_2 \\ m_3\ddot{x}_3 = +k_2\left(x_2 - x_3\right) + c_{23}\left(\dot{x}_2 - \dot{x}_3\right) - c_3\dot{x}_3 - k_3x_3 \end{cases} \tag{1}$$

In the classical matrix form:

$$\boldsymbol{M}\ddot{x} + \boldsymbol{C}\dot{x} + K\dot{x} = \left[\ f\ 0\ 0\ \right]^\top \tag{2}$$

where:

$$K = cacca \tag{3a}$$

$$C = dampers \tag{3b}$$

$$\boldsymbol{M} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \tag{3c}$$

1.3 experimental setup

1.4 parameters and data available

1.5 initial hypothesis and approximations

- rectilinear motion (all perfect aligned)

- inerta and damping of the motor are merged respectvely into m_1 and c_1.

$$\begin{cases} m_1 = m_{block} + \frac{J_{motor}|_{zz}}{r^2} \\ c_2 = c_{block} + \frac{c_{motor}}{r^2} \end{cases} \tag{4}$$

  where r is the radius of the gear-rack coupling (gear wheel), $J_{motor}|_{zz}$ is the inertia of the motor , c_{motor} the rotational damping and "block" quantities are the ones stricly related to the physical first mass.

2 System identification

2.1 step response analysis

First of all, the step response analysis can be performed. In this analysis the "static" coefficients can be estimated, they are:

- voltage to force k_v
- springs' stiffness k_i with $i \in 1, 2, 3$

The coefficient to be estimated is the *voltage-to-force* coefficient

$$f = (\boldsymbol{k_a} \cdot \boldsymbol{k_t} \cdot \boldsymbol{k_{mp}})v = k_v v \tag{5}$$

2.2 voltage-to-force coefficient estimation

2.3 Parameters estimation

$$ciao \tag{6a}$$

$$ciaone \tag{6b}$$