Consider the following discrete-time system:

$$x(k+1) = \begin{bmatrix} 0.9863 & 0.0528 \\ -0.1189 & 1.1680 \end{bmatrix} x(k) + \begin{bmatrix} 0.0024 \\ 0.0628 \end{bmatrix} u(k) \quad (1)$$

Where $x(k) = [x_1(k), x_2(k)]^T$.

a) Using dynamic programming, find the optimal input for the above system in such a way that the following cost function is minimized. Also, plot the control input.

$$J_1 = (x_1(3))^2 + \sum_{k=0}^{2} \left((x_1(k))^2 + 0.5(x_2(k))^2 + (u(k))^2 \right) \quad (2)$$

b) Similarly, using dynamic programming, find the optimal input for the above system in such a way that the following cost function is minimized. Also, plot the control input.

$$J_2 = (x_1(200))^2 + \sum_{k=0}^{199} \left((x_1(k))^2 + 0.5(x_2(k))^2 + (u(k))^2 \right) \quad (3)$$