Quantum Communication Complexity

Negar Sakhaei

August 30, 2020

Outline

1 Classic Communication Complexity

2 Quantum Communication Complexity

3 Simulations

Communication Complexity

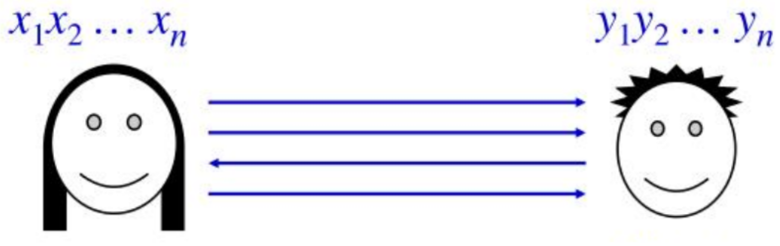


Figure: Alice and Bob each have an n bit number.

- They want to compute a function $f(x, y)$
- But with the least amount of communication possible.

Communication Complexity

Figure: Alice and Bob each have an n bit number.

- They want to compute a function $f(x, y)$
- But with the least amount of communication possible.

Equality

Example

We want to find the least amount of communication necessary to compute the equality function, i.e.,

$$EQ_n(x,y) = \begin{cases} 1 & \mathsf{x} = \mathsf{y} \\ 0 & \text{o.w.} \end{cases}$$

Deterministic CC: $O(n)$.

We can prove - using the fooling set technique - that communicating the entire input is required to compute the function.

Disjointness

Example

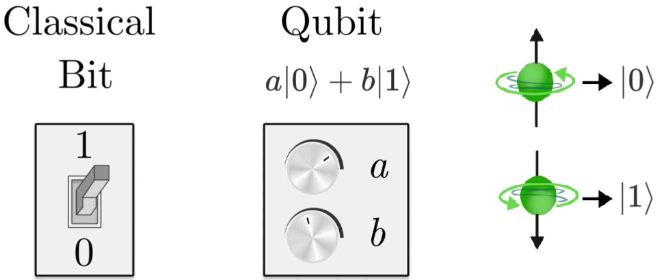
Another problem that we discuss is the set disjointness problem. We intrepret the inputs as subsets of $\{1, \ldots, n\}$ and we have:

$$DISJ_n(x, y) = \begin{cases} 1 & \text{x and y are disjoint} \\ 0 & \text{o.w.} \end{cases}$$

Deterministic CC: $O(n)$.

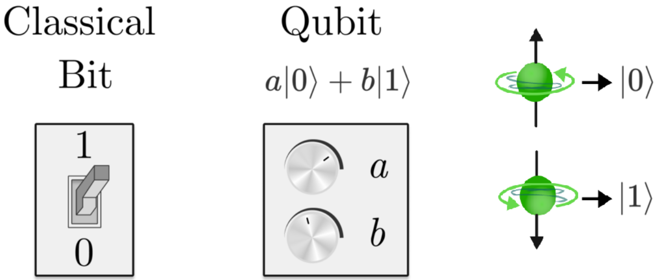
We can prove - using the rank technique - that this is the lower bound for the communication needed.

Qubits



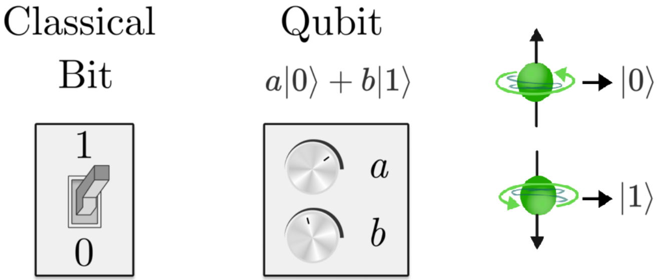
- Prob $(state = |0\rangle) = a^2$
- Prob $(state = |1\rangle) = b^2$
- $a, b \in \mathbb{C}$

Qubits



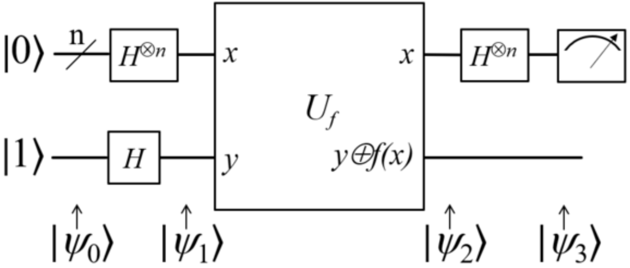
- Prob $(state = |0\rangle) = a^2$
- Prob $(state = |1\rangle) = b^2$
- $a, b \in \mathbb{C}$

Qubits



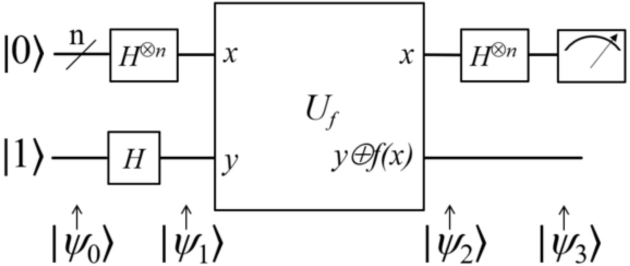
- Prob $(state = |0\rangle) = a^2$
- Prob $(state = |1\rangle) = b^2$
- $a, b \in \mathbb{C}$

Deutsch-Jozsa Algorithm



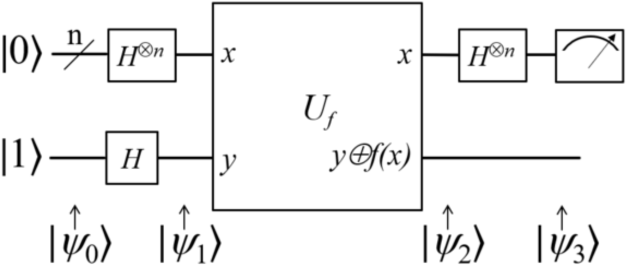
- Input: A binary function $f : \{0,1\}^n \rightarrow \{0,1\}$
- Promised that function is either constant or balanced.
- Output: Which?
- Can be done in polytime with qc, but needs exponential time in a cc.

Deutsch-Jozsa Algorithm



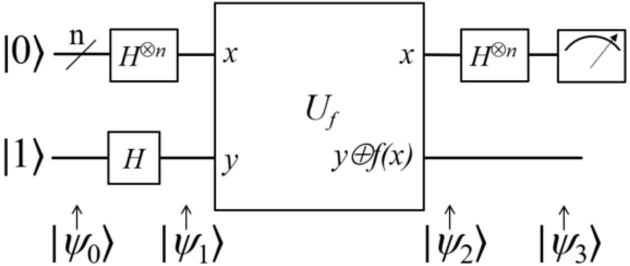
- Input: A binary function $f : \{0,1\}^n \to \{0,1\}$
- Promised that function is either constant or balanced.
- Output: Which?
- Can be done in polytime with qc, but needs exponential time in a cc.

Deutsch-Jozsa Algorithm



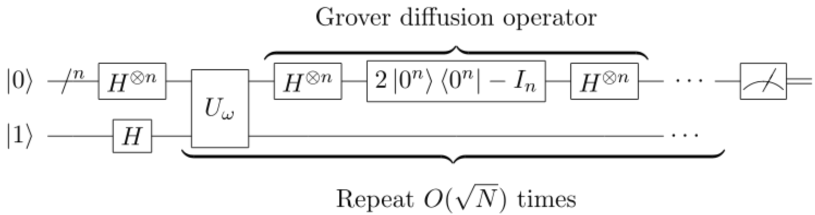
- Input: A binary function $f : \{0,1\}^n \to \{0,1\}$
- Promised that function is either constant or balanced.
- Output: Which?
- Can be done in polytime with qc, but needs exponential time in a cc.

Deutsch-Jozsa Algorithm



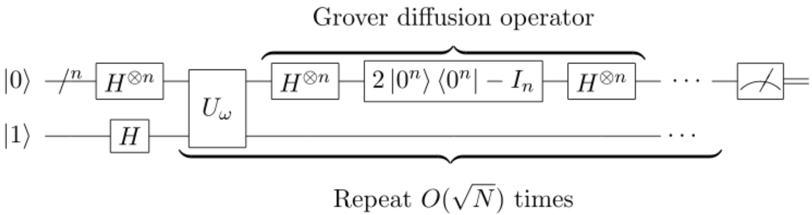
- Input: A binary function $f : \{0,1\}^n \to \{0,1\}$
- Promised that function is either constant or balanced.
- Output: Which?
- Can be done in polytime with qc, but needs exponential time in a cc.

Grover's Algorithm



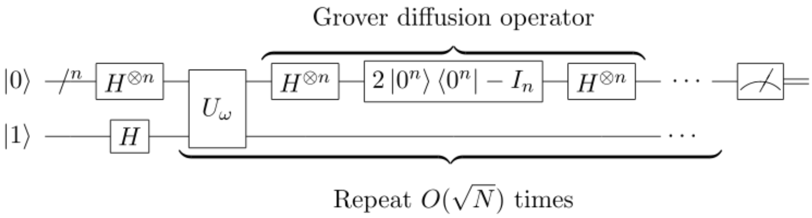
- Input: A function $f : \{0,1\}^n \to \{0,1\}^n$
- Output: Find x, where $f(x) = y$.
- Can be done in $O(\sqrt{n})$ with a quantum computer using Grover's Algorithm, but needs $O(n)$ time in the classical setting.

Grover's Algorithm



- Input: A function $f : \{0,1\}^n \to \{0,1\}^n$
- Output: Find x, where $f(x) = y$.
- Can be done in $O(\sqrt{n})$ with a quantum computer using Grover's Algorithm, but needs $O(n)$ time in the classical setting.

Grover's Algorithm



- Input: A function $f : \{0,1\}^n \to \{0,1\}^n$
- Output: Find x, where $f(x) = y$.
- Can be done in $O(\sqrt{n})$ with a quantum computer using Grover's Algorithm, but needs $O(n)$ time in the classical setting.

Can we do better?

- We need $2^n - 1$ states to describe an n qubit state
- **Holevo's Theorem** says that with direct transmission, only n bits can be communicated with n qubits.
- So, to no avail?

Can we do better?

- We need $2^n - 1$ states to describe an n qubit state
- **Holevo's Theorem** says that with direct transmission, only n bits can be communicated with n qubits.
- So, to no avail?

Can we do better?

- We need $2^n - 1$ states to describe an n qubit state
- **Holevo's Theorem** says that with direct transmission, only n bits can be communicated with n qubits.
- So, to no avail?

Equality

- Distributed Deutsch-Jozsa
- Promise version of Equality. Two inputs are either equal, or different at exactly half of the bits.
- Has $O(\log n)$ CC, Classic is $O(n)$!

Equality

- Distributed Deutsch-Jozsa
- Promise version of Equality. Two inputs are either equal, or different at exactly half of the bits.
- Has $O(\log n)$ CC, Classic is $O(n)$!

Equality

- Distributed Deutsch-Jozsa
- Promise version of Equality. Two inputs are either equal, or different at exactly half of the bits.
- Has $O(\log n)$ CC, Classic is $O(n)$!

Disjointness

- $z = x \wedge y, \quad z_i = x_i \wedge y_i$
- z_i is 1 iff $x_i = y_i$
- Solving disjointness is finding i where $z_i = 1$.
- Distributed Grover's
- Has $O(\log n\sqrt{n})$ CC, Classic is $O(n)$!

Disjointness

- $z = x \wedge y, \quad z_i = x_i \wedge y_i$
- z_i is 1 iff $x_i = y_i$
- Solving disjointness is finding i where $z_i = 1$.
- Distributed Grover's
- Has $O(\log n\sqrt{n})$ CC, Classic is $O(n)$!

Disjointness

- $z = x \wedge y, \quad z_i = x_i \wedge y_i$
- z_i is 1 iff $x_i = y_i$
- Solving disjointness is finding i where $z_i = 1$.
- Distributed Grover's
- Has $O(\log n \sqrt{n})$ CC, Classic is $O(n)$!

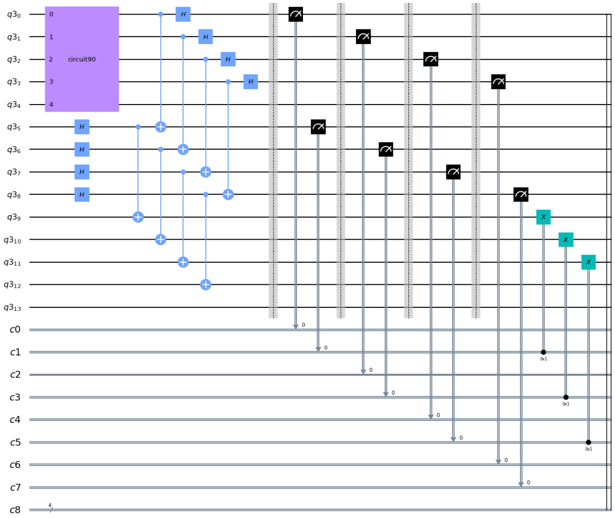
Disjointness

- $z = x \wedge y, \quad z_i = x_i \wedge y_i$
- z_i is 1 iff $x_i = y_i$
- Solving disjointness is finding i where $z_i = 1$.
- Distributed Grover's
- Has $O(\log n\sqrt{n})$ CC, Classic is $O(n)$!

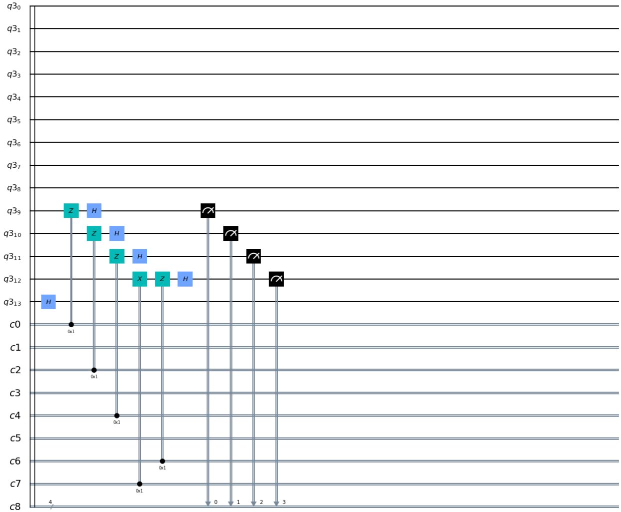
Disjointness

- $z = x \wedge y, \quad z_i = x_i \wedge y_i$
- z_i is 1 iff $x_i = y_i$
- Solving disjointness is finding i where $z_i = 1$.
- Distributed Grover's
- Has $O(\log n\sqrt{n})$ CC, Classic is $O(n)$!

Circuit - Alice



Circuit - Bob


$q3_0$
$q3_1$
$q3_2$
$q3_3$
$q3_4$
$q3_5$
$q3_6$
$q3_7$
$q3_8$
$q3_9$
$q3_{10}$
$q3_{11}$
$q3_{12}$
$q3_{13}$
c0
c1
c2
c3
c4
c5
c6
c7
c8
H
Z
X
0x1
4
0
1
2
3

Thank You!