Reversible Gates

Tags	Part I	SubUnit I

Circuits and Entropy

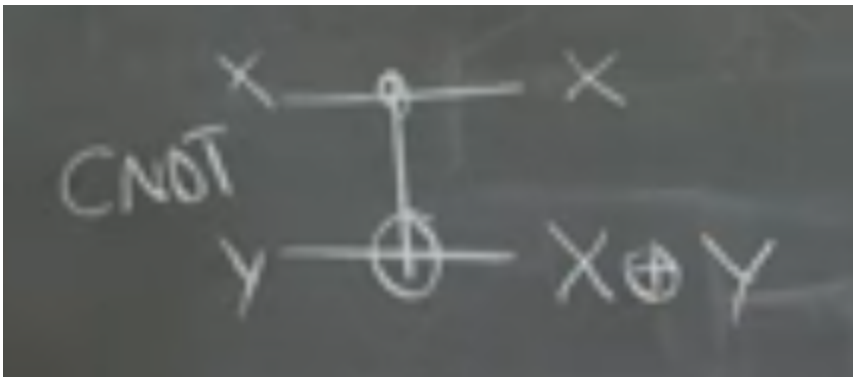
- Classic gates destroy randomness → Consume energy!
- By Landauer's principle:

$$kT \ln 2 \text{ energy needed per bit!}$$

- Need reversible gates to preform low energy computing.

Reversible Gates

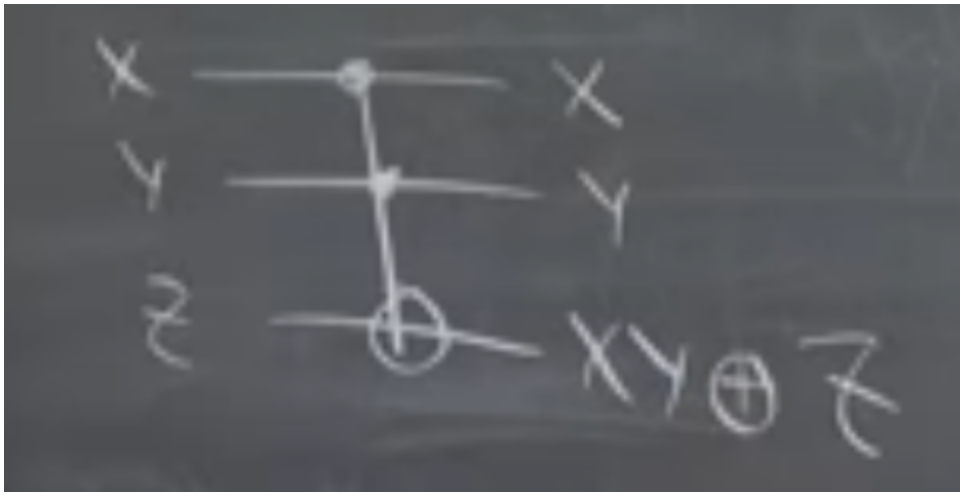
1. **CNOT**: Controlled NOT



Controlled Not, if X is true, output is NOT Y . Otherwise, all stay the same.

2. **Toffoli gate** or CCNOT: Controlled Controlled NOT.

   This gate is universal by itself, because it can simulate both AND and NOT gates.



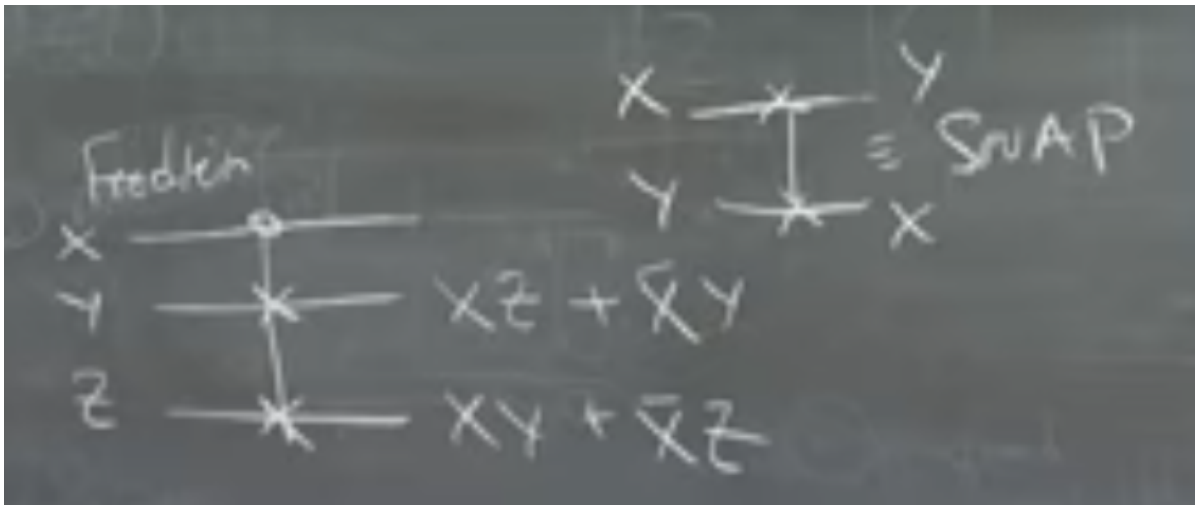
If both X and Y are true, output is NOT Z. Otherwise, all stay the same.

3. **Fredkin gate** or CSWAP: Controlled Swap.

   This gate is universal by itself, because it can simulate both AND and NOT gates.

   If we set Y =1 and Z = 0, it simulates the NOT gate!

   Fredkin gate cannot change the Hamming weight (number of one's) of its inputs.



If X is true, Y and Z get swapped,.Otherwise, all stay the same.

💡 **Theorem**: The number of garbage outputs in these gates to simulate a circuit is O(width of circuits).

💡 **Theorem**: Any Boolean circuit of size n can be simulated by a reversible circuit of order of polynomial
in n reversible gates. (approx. O(n^2))

Def. : “in-place reversible" : No garbage bits are allowed, and no extra ancilla bits are provided (constant zero or one).