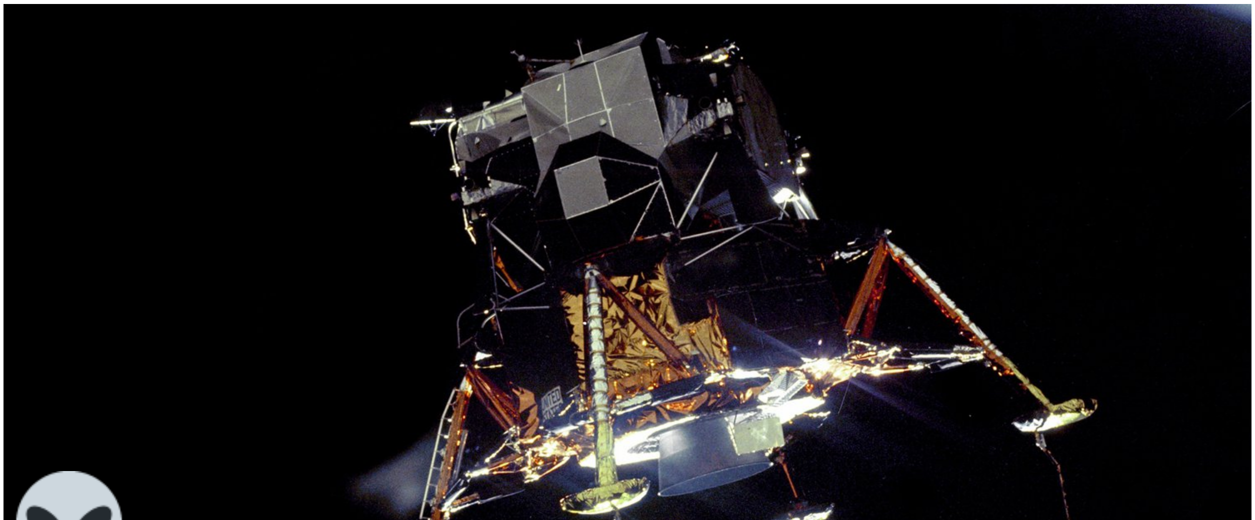


Single Qubit!

Tags	Part I	SubUnit II

Date: @Feb 11, 2020

Topic: Single Qubit!

Recall

Quantum States

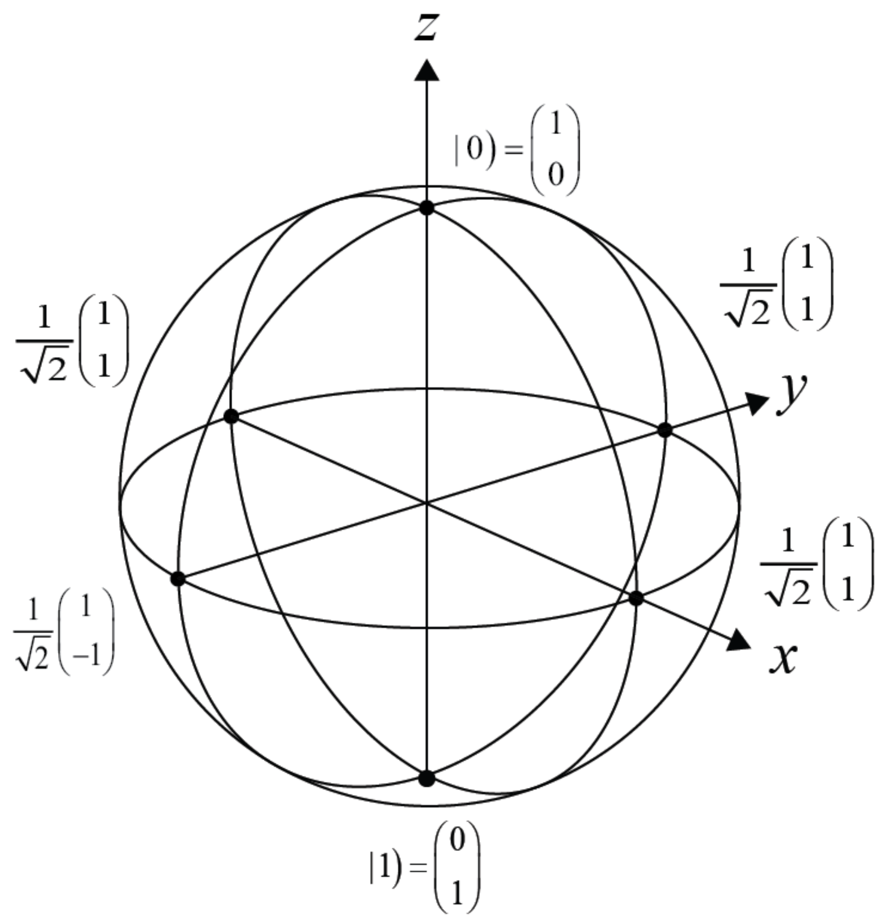
Notes

- Are vectors
- ***Global*** phases don't change states!
- Any pure state of a two-level quantum system can be written as a superposition of the basis vectors $|0\rangle$ and $|1\rangle$, where the coefficient or amount of each basis vector is a complex number.
- Any state:

$$|\psi\rangle = \cos(\theta/2)\,|0\rangle \,+\, e^{i\phi}\sin(\theta/2)\,|1\rangle$$

- Main points:

Bloch Sphere



- Also note:

[Understanding the Bloch sphere](https://physics.stackexchange.com/a/205209)
https://physics.stackexchange.com/a/205209

[Introduction to quantum computing: Bloch sphere.](http://akyrillidis.github.io/notes/quant_post_7)
http://akyrillidis.github.io/notes/quant_post_7

Rotations of the sphere

- Each rotate around the axis!

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Single qubit gates

- Gates!

Operator	Gate(s)		Matrix
Pauli-X (X)	$\boxed{\mathbf{X}}$	$\oplus$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y (Y)	$\boxed{\mathbf{Y}}$		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)	$\boxed{\mathbf{Z}}$		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)	$\boxed{\mathbf{H}}$		$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)	$\boxed{\mathbf{S}}$		$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)	$\boxed{\mathbf{T}}$		$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$

The Hadamard gate acts on a single qubit. It maps the basis state $|0\rangle$ to $|0\rangle + |1\rangle / 2$ and $|1\rangle$ to $|0\rangle - |1\rangle / 2$, which means that a measurement will have equal probabilities to become 1 or 0 (i.e. creates a superposition)

Relations?

$$S^2 = \sigma_z,\ T^2 = S$$

$$\sigma_x \sigma_y = i\sigma_z = -\sigma_y \sigma_x$$

$$\sigma_y \sigma_z = i\sigma_x = -\sigma_z \sigma_y$$

$$\sigma_z \sigma_x = i\sigma_y = -\sigma_x \sigma_z$$

- **Result**: The three Pauli vectors are anti-commute pairs.

$$U\ |i\rangle = |C_i\rangle$$

Unitary Transitions

Ci is of unit length. Also, Ci is the ith column of U.

$$U = \sum_{i=1}^{d-1} |C_i\rangle\langle i|$$

Where d is the dimension of the state space. Now:

$$U|j\rangle = \sum_{i=1}^{d-1} |C_i\rangle\langle i|j\rangle = \sum_{i=1}^{d-1} |C_i\rangle\delta_{ij} = C_j$$

Def 1: Takes a complex unit vector to another.

Def 2: All columns are unitary vectors and orthogonal. (takes a basis to another)

Def 3: All rows are unitary vectors and orthogonal.

Def 4:

$$U^{-1} = U^\dagger$$

Two Kinds of Processes:

- Unitary
- Measurement

Copenhagen interpretation

$$e^{i\theta\sigma_z/2}, ...$$

Rotation of Theta around Axes?

$$U = e^{i\alpha}\begin{pmatrix} \cos\theta & e^{i\phi_2}\sin\theta \\ e^{i\phi_1}\sin\theta & -e^{i(\phi_2-\phi_1)}\cos\theta \end{pmatrix}$$

3 Degrees of freedom!

Note: Above formula is diff in lecture, why am I wrong?

Every Transformation can be done by a transformation around x, z, and x. (So 3 deg. of freedom again!)

Degrees of Freedom for Rotation?

(2×2 unitary)

> *On the Bloch sphere, the axis of rotation of a single-qubit gate U is the line going from the center of the Bloch sphere, and a point $\vec{p}$ = (px,py,pz) on the Bloch sphere, corresponding to an eigenstate of U.*

📌 **SUMMARY: Bloch Sphere, Unitary Matrices and Rotations! ⇒ Quantum Gates.**
