LP ramp metering formulation

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Notation

- I... number of secions
- K... number of time steps
- $\mathcal{I} = [0...I-1]$
- $\mathcal{I}^- = [0...I-2]$
- $\mathcal{I}_m^- \subseteq \mathcal{I}$ ... segments with un-metered on-ramps
- $\mathcal{I}_m^+ \subseteq \mathcal{I}$ ... segments with metered on-ramps

Objective function

$$\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}} n_{(i,k)} + \sum_{i\in\mathcal{I}_m^+}\sum_{k\in K} l_{(i,k)} - \eta\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}} f_{(i,k)} - \eta\sum_{i\in\mathcal{I}_m^+}\sum_{k\in K} r_{(i,k)} \tag{1}$$

Cnst: ML conservation

for all $i \in \mathcal{I}$, $k \in \mathcal{K}$, define constaint (i,k),

$$n_{(i,k+1)}\underbrace{-n_{(i,k)}}_{k>0}\underbrace{-f_{(i-1,k)}}_{i>0}\underbrace{-r(i,k)}_{i\in\mathcal{I}_m^+}\underbrace{+\frac{1}{\overline{\beta}_{(i,k)}}f_{(i,k)}}_{\overline{\beta}_{(i,k)}>0} = 0\underbrace{+n_{(i,0)}}_{k=0}\underbrace{+d_{(i,k)}}_{i\in\mathcal{I}_m^-} \tag{2}$$

Cnst: OR conservation

for all $i \in \mathcal{I}_m^+$, $k \in \mathcal{K}$, define constaint (i,k),

$$l_{(i,k+1)}\underbrace{-l_{(i,k)}}_{k>0} + r_{(i,k)} = d_{(i,k)}\underbrace{+l_{(i,0)}}_{k=0} \tag{3}$$

Cnst: ML flow - free-flow

for all $i \in \mathcal{I}$, $k \in \mathcal{K}$, define constaint (i,k),

$$f_{(i,k)}\underbrace{-\overline{\beta}_{(i,k)}v_{(i)}n_{(i,k)}}_{k>0,\overline{\beta}_{(i,k)}>0}\underbrace{-\overline{\beta}_{(i,k)}v_{(i)}\gamma r_{(i,k)}}_{i\in\mathcal{I}_m^+,\overline{\beta}_{(i,k)}>0} \leq \underbrace{+\overline{\beta}_{(i,0)}v_{(i)}n_{(i,0)}}_{k=0}\underbrace{+\overline{\beta}_{(i,k)}v_{(i)}\gamma d_{(i,k)}}_{i\in\mathcal{I}_m^-} \tag{4}$$

Cnst: ML flow - congestion

for all $i \in \mathcal{I}^-$, $k \in \mathcal{K}$, define constaint (i,k),

$$f_{(i,k)} \underbrace{+ w_{(i+1)} n_{(i+1,k)}}_{k>0} \underbrace{+ w_{(i+1)} \gamma r_{(i+1,k)}}_{i+1 \in \mathcal{I}_m^+} \leq w_{(i+1)} \bar{n}_{(i+1)} \underbrace{- w_{(i+1)} n_{(i+1,0)}}_{k=0} \underbrace{- w_{(i+1)} \gamma d_{(i+1,k)}}_{i+1 \in \mathcal{I}_m^-} \tag{5}$$

Cnst: OR flow - demand

for all $i \in \mathcal{I}_m^+$, $k \in \mathcal{K}$, define constaint (i,k),

$$r_{(i,k)} \underbrace{- l_{(i,k)}}_{k>0} \leq d_{(i,k)} \underbrace{+ l_{(i,0)}}_{k=0} \tag{6}$$

Bnd: ML flow - capacity

for all $i \in \mathcal{I}$, $k \in \mathcal{K}$, define constaint (i,k),

$$f_{(i,k)} \leq \bar{f}_{(i)} \tag{7}$$

Bnd: OR max metering

for all $i \in \mathcal{I}_m^+$, $k \in \mathcal{K}$, define constaint (i,k),

$$r_{(i,k)} \leq \bar{r}_{(i)} \tag{8}$$

Bnd: OR flow positivity

for all $i \in \mathcal{I}_m^+$, $k \in \mathcal{K}$, define constaint (i,k),

$$r_{(i,k)} \geq 0 \tag{9}$$

Bnd: OR queue length bound

for all $i \in \mathcal{I}_m^+$, $k \in \mathcal{K}$, define constaint (i,k),

$$l_{(i,k+1)} \leq \bar{l}_{(i)} \tag{10}$$