The .Q Simulator and Max Pressure Control of Network of Signalized Intersections

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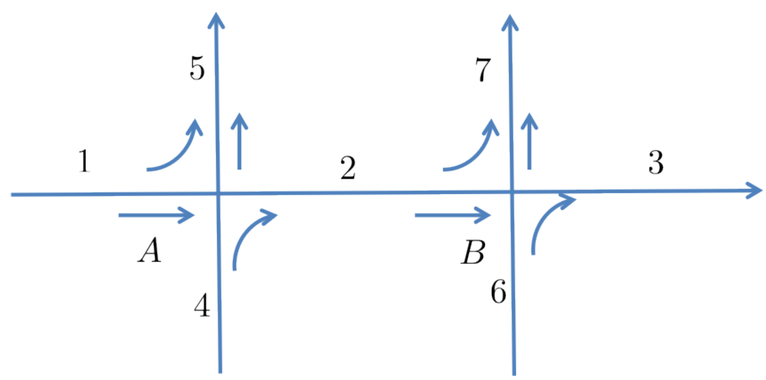

Figure 1: Modeling queue overflow.

We describe how .Q simulates queue overflow conditions in the context of the network of Fig. 1.

Network

The network has two intersections, A and B, and seven links, $1,\cdots,7$. There are four phases or turn movements at each intersection: $\{(1,2),(1,5),(4,2),(4,5)\}$ at A and $\{(2,3),(2,7),(6,3),(6,7)\}$ at B. Thus there are 8 queues corresponding to the 8 turn movements denoted by $q(1,2),q(1,5),\cdots$. The link queue is defined as the sum of all queues on the link: $q(1) = q(1,2) + q(1,5)$ and similarly for the other links. All movements are actuated.

The left and right turn queues have storage capacity limits, denoted by $\bar{q}(1,5),\bar{q}(4,2),\cdots$. Links have storage capacity limits $\bar{q}(1),\bar{q}(2),\cdots$. The link storage includes the turn queue storage, so that $\bar{q}(1) \geq \bar{q}(1,5)$, $\bar{q}(4) \geq \bar{q}(4,2)$, and similarly for $\bar{q}(2),\bar{q}(6)$. Links 1,4,6 are entry links with infinite storage, $\bar{q}(1) = \bar{q}(4) = \bar{q}(6) = \infty$.

Links 5, 7, and 3 are exit links with infinite storage, $\bar{q}(5) = \bar{q}(7) = \bar{q}(3) = \infty$. However, this is unnecessary since vehicles do not queue up on exit links.

Take

$$\bar{q}(1,5) = \bar{q}(4,2) = 30 \quad (1)$$
$$\bar{q}(2,7) = \bar{q}(6,3) = 30 \quad (2)$$
$$\bar{q}(2) = = 80 \quad (3)$$

The saturation rate for all movements (i, j) is the same, $\bar{c}(i, j) = 1$ per unit time. However these rates may decrease when the corresponding queue reaches its storage capacity or it is blocked. There are two forms of blocking:

Input blocking If a queue exceeds its capacity it reduces the saturation flow rate of the adjacent, movement as indicated in Fig.2.

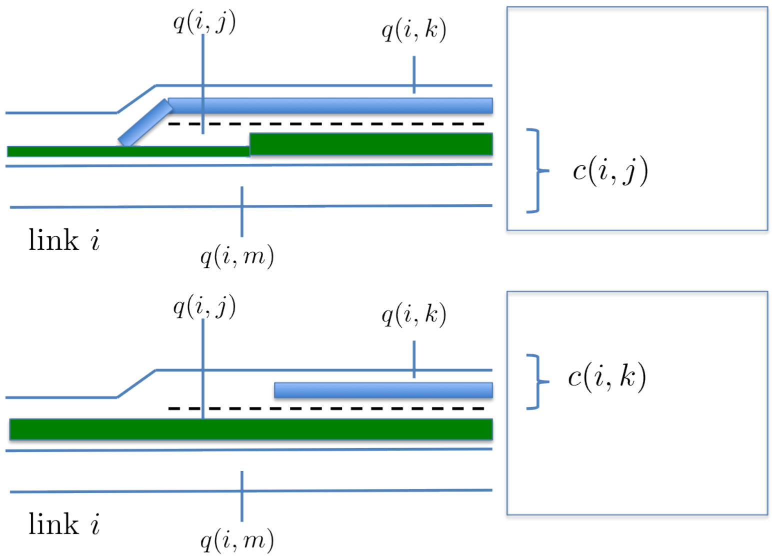

Figure 2: Input blocking.

Input blocking is modeled as

$$c(i, j) = \theta(i, j)\bar{c}(i, j),$$

with

$$\theta(i, j) = \begin{cases} 1, & q(i,k) < \bar{q}(i,k) \\ 0.5 & q(i,k) \geq \bar{q}(i,k) \end{cases} . \tag{4}$$

For example if $q(1,5) \geq \bar{q}(1,5)$, $\theta(1,2) = 0.5$ so the saturation rate of the through movement $(1,2)$ drops to $c(1,2) = \theta(1,2) \times \bar{c}(1,2) = 0.5$.

Similarly, if $q(1,2) \geq \bar{q}(1,2)$, then $\theta(1,2) = 0.5$ and the left turn saturation flow rate drops to $c(1,5) = 0.5$.

Output blocking If the queue on link 2 exceeds its limit, i.e. $q(2) \geq \bar{q}(2)$, it will block the movement $(1,2)$ entirely. That is:

$$\theta(1,2) = \begin{cases} 1, & q(2) < \bar{q}(2) \\ 0 & q(2) \geq \bar{q}(2) \end{cases} . \tag{5}$$

Thus $c(1,2) = 0$ when $q(2) = q(2,3) + q(2,7) \geq \bar{q}(2)$. There is no output blocking in any other links because their downstream links are all exit links.

Control

There are two stages for each intersection. Stage 1 at A actuates $\{(1,2),(1,5)\}$ and at B actuates $\{(2,3),(2,7)\}$. Stage 2 at A actuates $\{(4,5),(4,2)\}$ and at B actuates $\{(6,7),(6,3)\}$.

The control is fixed time, the cycle time is 100 time units, and each stage is actuated for 50 time units. There is no lost time.

Since the saturation rate for each movement is 1, and each movement is actuated for 50 time units in a cycle, in the absence of blocking, the saturation flow rate for each movement is 50 v/cycle.

Demand

We want to see how throughput is affected by blocking. Take the demand entering links 1,4, 6 to be

$$d(1) = d(4) = d(6) = 90\text{veh/cycle}$$

or 0.9 veh/time unit. Take all the routing probabilities to be 0.5, i.e. $r(1,2) = r(1,5) = \cdots = 0.5$. Then, in steady state, all flows will equal 45 v/cycle: e.g.

$$f(1,2) = r(1,2) \times d(1) = 0.5 \times 90 = 45.$$

If there is no blocking, all saturation rates equal 50 veh/cycle and so, if the queues had infinite capacity the system would be stable.

We now investigate blocking with this demand. Consider link 1. Within one cycle of 100s, 45 vehicles want to take the left turn $(1,5)$. During 50s while movement $(1,5)$ is actuated, one-half or 22.5 of the arriving vehicles will leave, the other half or 22.5 vehicles will leave from $q(1,5)$, so at the end of green this queue will be empty. (In fact, since the saturation flow rate is 1, the queue will empty before the end of green.) During the next 50s, while movement $(1,2)$ is not permitted (signal is red), an additional 22.5 vehicles will arrive and join $q(1,5)$, which, therefore will equal 22.5 at the end of the cycle. Fig. 3 shows the evolution over tune of $q(1,5)$, the flow $f(1,5)(t)$ and the constant demand $d(1,5)(t)$. **As can be seen, the system is stable.**

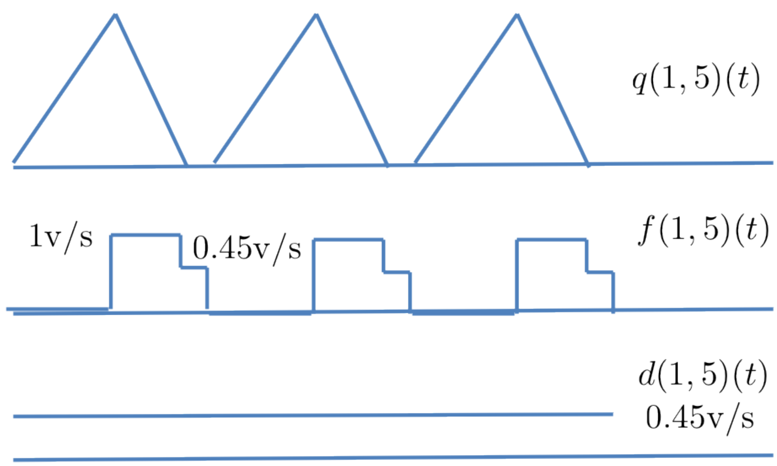

Figure 3: Evolution of queue (1,5).

Suppose now all queue storage capacities $\bar{q}(i,j)$ are reduced from 30 to 20. **Then the system will be unstable.**