

## Assignment 3 - Dimensionality Reduction

This assignment is based on content discussed in module 6 and will work with the famous MNIST dataset, which is a set of images of handwritten digits [https://en.wikipedia.org/wiki/MNIST\\_database](https://en.wikipedia.org/wiki/MNIST_database) ([https://en.wikipedia.org/wiki/MNIST\\_database](https://en.wikipedia.org/wiki/MNIST_database)). The dataset has been provided to you in a .csv file.

### Learning outcomes

- Apply a Random Forest classification algorithm to MNIST dataset
- Perform dimensionality reduction of features using PCA and compare classification on the reduced dataset to that of original one
- Apply dimensionality reduction techniques: t-SNE and LLE

### Questions (15 points total)

**Question 1 (1 point).** Load the MNIST dataset and split it into a training set and a test set (take the first 60,000 instances for training, and the remaining 10,000 for testing).

In [1]: `1 import pandas as pd`

In [2]: `1 df = pd.read_csv("mnist_dataset.csv", index_col=0)  
2 df.head()`

Out [2]:

	label	1x1	1x2	1x3	1x4	1x5	1x6	1x7	1x8	1x9	...	28x19	28x20	28x21	28x22	28x23
0	2	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0
1	5	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0
2	8	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0
4	4	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0

5 rows × 785 columns

```
In [3]: 1 df.isna().any().any()
```

```
Out[3]: False
```

```
In [4]: 1 target_column = "label"
2 df_features = df[[col for col in df.columns if col != target_column]]
3 df_target = df[target_column]
```

```
In [5]: 1 from sklearn.model_selection import train_test_split
```

```
In [6]: 1 df_train_X = df_features.iloc[0:60000, :]
2 df_train_y = df_target.iloc[0:60000]
3 df_test_X = df_features.iloc[60000:70000, :]
4 df_test_y = df_target.iloc[60000:70000]
```

**Question 2 (2 points).** Train a Random Forest classifier on the dataset and time how long it takes, then evaluate the resulting model on the test set.

```
In [7]: 1 from sklearn.model_selection import cross_val_score
2 from sklearn.ensemble import RandomForestClassifier
3 from sklearn.model_selection import GridSearchCV
4 import numpy as np
5 import time
6 import seaborn as sns
7 from sklearn.metrics import classification_report, confusion_matrix
```

```
In [8]: 1 def analyze_RF(rf):
2     list_DT = rf.estimators_
3     df_hyperparam = pd.DataFrame({"depth": [DT.get_depth() for DT in list_DT]})
4     df_hyperparam.plot.hist(subplots=True, legend=True, layout=(1, 2))
5     df_hyperparam.index.name = "Tree"
6     display(df_hyperparam)
```

The training of the RF CLF takes about 46 seconds.

```
In [9]: 1 n_estimators=100
2 time_start = time.time()
3 rf_clf = RandomForestClassifier(n_estimators=n_estimators)
4 rf_clf.fit(X=df_train_X, y=np.ravel(df_train_y))
5 print('Time elapsed: {} seconds'.format(time.time()-time_start))
```

Time elapsed: 44.44841694831848 seconds

The accuracy and classification report of the classifier are shown below. The classification report shows precision, recall, f1-score and support for each label. Since our data is almost balanced, in addition to accuracy, we're seeing a good result in precision, recall and f1-score too.

```
In [10]: 1 predictions = rf_clf.predict(df_test_X)
          2 print(classification_report(np.ravel(df_test_y), predictions))
```

	precision	recall	f1-score	support
0	0.98	0.99	0.98	959
1	0.99	0.99	0.99	1155
2	0.97	0.97	0.97	1007
3	0.96	0.95	0.95	1039
4	0.97	0.98	0.97	925
5	0.97	0.97	0.97	921
6	0.98	0.99	0.98	972
7	0.97	0.97	0.97	1048
8	0.95	0.96	0.96	953
9	0.96	0.94	0.95	1021
accuracy			0.97	10000
macro avg	0.97	0.97	0.97	10000
weighted avg	0.97	0.97	0.97	10000

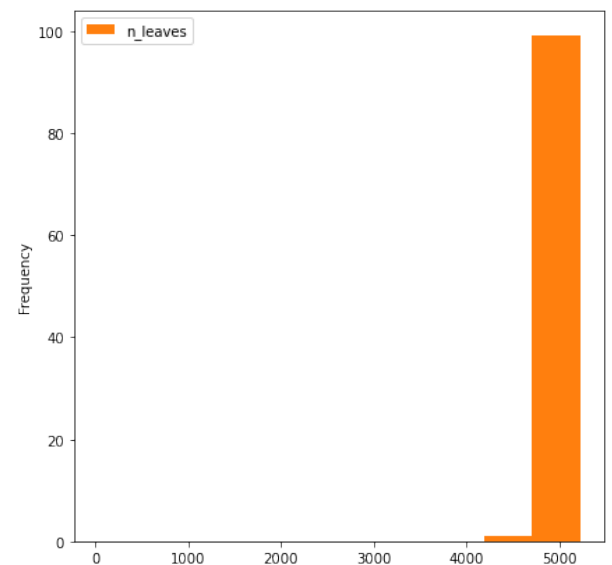
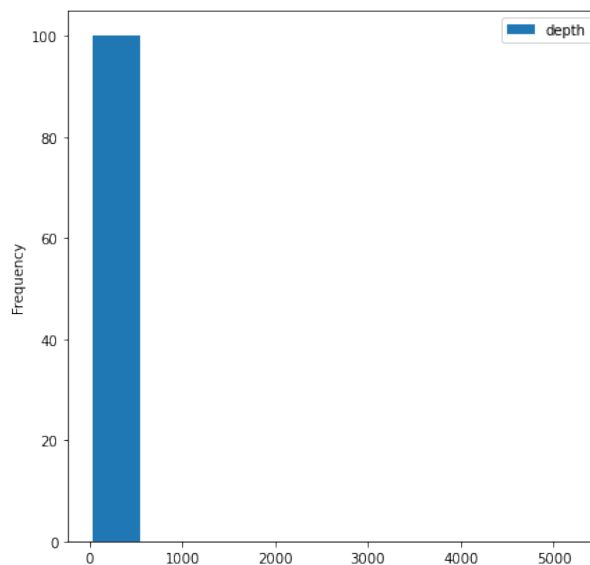
```
In [11]: 1 from sklearn import metrics
          2 print("Accuracy of RF CLF without any reduction:", metrics.accuracy_score(df_test_y, predictions))
```

Accuracy of RF CLF without any reduction: 0.9698

In [12]: `1 analyze_RF(rf_clf)`

	depth	n_leaves
<b>Tree</b>		
<b>0</b>	39	5081
<b>1</b>	37	4838
<b>2</b>	36	4969
<b>3</b>	33	5050
<b>4</b>	33	5069
...	...	...
<b>95</b>	34	4940
<b>96</b>	34	5034
<b>97</b>	36	4836
<b>98</b>	33	4858
<b>99</b>	30	4909

100 rows × 2 columns



**Question 3 (4 points).** Next, use PCA to reduce the dataset's dimensionality, with an explained variance ratio of 95%. Train a new Random Forest classifier on the reduced dataset and see how long it takes. Was training much faster? Next evaluate the classifier on the test set: how does it compare to the previous classifier?

As you can see below, the time that training takes is 107 seconds which is almost twice the time that training without the reduction takes, but there are other factors that can effect the RF such as the rotation of the dataset.

For the accuracy, the accuracy is decrease by 2.18 percent which I think can be ignored compare to how much memory we're saving and how simpler the data have become.

```
In [13]: 1 from sklearn.decomposition import PCA
```

```
In [14]: 1 time_start = time.time()
2 pca = PCA(0.95)
3 df_features_pca_reduced = pca.fit_transform(df_features)
4 print('PCA done! Time elapsed: {} seconds'.format(time.time()-time_start))
```

PCA done! Time elapsed: 19.303411960601807 seconds

```
In [15]: 1 df_train_X_pca_reduced = df_features_pca_reduced[0:60000]
2 df_test_X_pca_reduced = df_features_pca_reduced[60000:70000]
```

```
In [16]: 1 time_start = time.time()
2 rf_clf_PCA_reduced = RandomForestClassifier(n_estimators=n_estimators)
3 rf_clf_PCA_reduced.fit(X=df_train_X_pca_reduced, y=np.ravel(df_train_y))
4 print('PCA reduced RF CLF trained! Time elapsed: {} seconds'.format(time.time()-time_start))
```

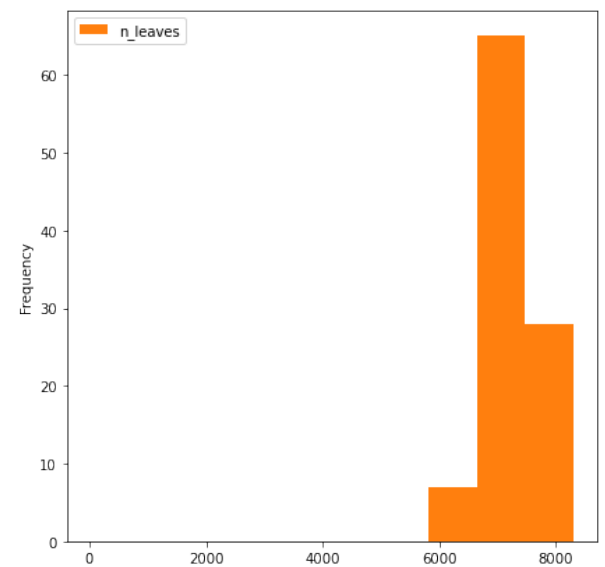
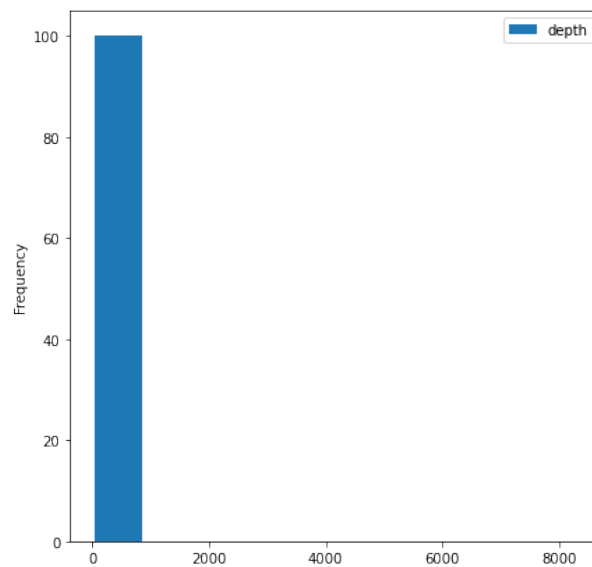
PCA reduced RF CLF trained! Time elapsed: 101.72174000740051 seconds

Because of the random forest algorithm, when the 'max\_features' parameter is not set, then 'max\_features' will be equal to  $\sqrt{n\_features}$  which for the main dataset is  $\sqrt{784} = 28$ . But when the PCA is applied, the number of features is reduced to 154 so the 'max\_features' parameter will be  $\sqrt{154} = 12$ . So the number of features is reduced from 28 to 12 which as we can see, doesn't effect the time that much and it's because other things such as rotation of the data is effecting the random forest, but the PCA is helping a lot with memory here.

```
In [17]: 1 analyze_RF(rf_clf_PCA_reduced)
```

	depth	n_leaves
<b>Tree</b>		
<b>0</b>	38	7854
<b>1</b>	37	6578
<b>2</b>	33	7273
<b>3</b>	32	8129
<b>4</b>	50	6482
...	...	...
<b>95</b>	41	7102
<b>96</b>	38	7277
<b>97</b>	34	6912
<b>98</b>	37	6597
<b>99</b>	34	7288

100 rows × 2 columns



```
In [18]: 1 pca_predictions = rf_clf_PCA_reduced.predict(df_test_X_pca_reduced)
          2 print(classification_report(np.ravel(df_test_y), pca_predictions))
```

	precision	recall	f1-score	support
0	0.97	0.98	0.98	959
1	0.99	0.99	0.99	1155
2	0.94	0.95	0.94	1007
3	0.93	0.92	0.93	1039
4	0.94	0.96	0.95	925
5	0.92	0.94	0.93	921
6	0.97	0.98	0.97	972
7	0.96	0.95	0.95	1048
8	0.93	0.91	0.92	953
9	0.94	0.91	0.92	1021
accuracy			0.95	10000
macro avg	0.95	0.95	0.95	10000
weighted avg	0.95	0.95	0.95	10000

```
In [19]: 1 print("Accuracy of RF CLF using PCA:", metrics.accuracy_score(np.r
Accuracy of RF CLF using PCA: 0.9487
```

Since the time wasn't reduced the time, I was wondering can the time reduction be seen in another classification method. As you can see in below, with a KNN classifier, the total time of training and testing is almost half in the PCA reduced data and we have higher accuracy too.

```
In [20]: 1 from sklearn.neighbors import KNeighborsClassifier
```

```
In [21]: 1 time_start = time.time()
          2 knn = KNeighborsClassifier(n_neighbors=20)
          3 knn.fit(X=df_train_X, y=np.ravel(df_train_y))
          4 print('KNN CLF trained! Time elapsed: {} seconds'.format(time.time
```

KNN CLF trained! Time elapsed: 0.034976959228515625 seconds

```
In [22]: 1 time_start = time.time()
2 predictions_knn = knn.predict(df_test_X)
3 print(classification_report(np.ravel(df_test_y), predictions_knn))
4 print('KNN CLF tested! Time elapsed: {} seconds'.format(time.time() - time_start))
```

	precision	recall	f1-score	support
0	0.97	0.99	0.98	959
1	0.94	1.00	0.97	1155
2	0.99	0.94	0.96	1007
3	0.96	0.96	0.96	1039
4	0.98	0.96	0.97	925
5	0.97	0.96	0.97	921
6	0.97	0.99	0.98	972
7	0.95	0.96	0.96	1048
8	0.99	0.92	0.96	953
9	0.93	0.95	0.94	1021
accuracy			0.96	10000
macro avg	0.97	0.96	0.96	10000
weighted avg	0.96	0.96	0.96	10000

KNN CLF tested! Time elapsed: 48.07457494735718 seconds

```
In [23]: 1 print("Accuracy of KNN CLF:", metrics.accuracy_score(np.ravel(df_test_y), predictions_knn))
Accuracy of KNN CLF: 0.964
```

```
In [24]: 1 time_start = time.time()
2 knn_pca = KNeighborsClassifier(n_neighbors=20)
3 knn_pca.fit(X=df_train_X_pca_reduced, y=np.ravel(df_train_y))
4 print('PCA reduced KNN CLF trained! Time elapsed: {} seconds'.format(time.time() - time_start))
```

PCA reduced KNN CLF trained! Time elapsed: 0.1246650218963623 seconds



```
In [25]: 1 time_start = time.time()
2         pca_predictions_knn = knn_pca.predict(df_test_X_pca_reduced)
3         print(classification_report(np.ravel(df_test_y), pca_predictions_k
4         print('PCA reduced KNN CLF tested! Time elapsed: {} seconds'.format
```

	precision	recall	f1-score	support
0	0.98	0.99	0.98	959
1	0.95	0.99	0.97	1155
2	0.99	0.95	0.97	1007
3	0.97	0.96	0.96	1039
4	0.98	0.96	0.97	925
5	0.97	0.96	0.97	921
6	0.97	0.99	0.98	972
7	0.95	0.96	0.95	1048
8	0.99	0.93	0.96	953
9	0.94	0.95	0.94	1021
accuracy			0.97	10000
macro avg	0.97	0.97	0.97	10000
weighted avg	0.97	0.97	0.97	10000

PCA reduced KNN CLF tested! Time elapsed: 29.460464239120483 seconds

```
In [27]: 1 print("Accuracy of KNN CLF using PCA:", metrics.accuracy_score(np.
Accuracy of KNN CLF using PCA: 0.9656
```

**Question 4 (4 points).** Use t-SNE to reduce the MNIST dataset, show result graphically.

Using the t-SNE, the time of the reduction was way more than the time of reduction using PCA but the accuracy was a little less than the PCA. The time of training the model on the reduced dataset is almost the same with the pca reduced data training time.

```
In [28]: 1 from sklearn.manifold import TSNE
2         import matplotlib.pyplot as plt
```

```
In [29]: 1 time_start = time.time()
2         tsne = TSNE(n_components=2, verbose=1, perplexity=40, n_iter=300)
3         tsne_result = tsne.fit_transform(df_features.values)
4         print('t-SNE done! Time elapsed: {} seconds'.format(time.time()-ti
```

```
[t-SNE] Computing 121 nearest neighbors...
[t-SNE] Indexed 70000 samples in 0.044s...
[t-SNE] Computed neighbors for 70000 samples in 350.887s...
[t-SNE] Computed conditional probabilities for sample 1000 / 70000
[t-SNE] Computed conditional probabilities for sample 2000 / 70000
```

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```
[t-SNE] Computed conditional probabilities for sample 53000 / 70000
[t-SNE] Computed conditional probabilities for sample 54000 / 70000
[t-SNE] Computed conditional probabilities for sample 55000 / 70000
[t-SNE] Computed conditional probabilities for sample 56000 / 70000
[t-SNE] Computed conditional probabilities for sample 57000 / 70000
[t-SNE] Computed conditional probabilities for sample 58000 / 70000
[t-SNE] Computed conditional probabilities for sample 59000 / 70000
[t-SNE] Computed conditional probabilities for sample 60000 / 70000
[t-SNE] Computed conditional probabilities for sample 61000 / 70000
[t-SNE] Computed conditional probabilities for sample 62000 / 70000
[t-SNE] Computed conditional probabilities for sample 63000 / 70000
[t-SNE] Computed conditional probabilities for sample 64000 / 70000
[t-SNE] Computed conditional probabilities for sample 65000 / 70000
[t-SNE] Computed conditional probabilities for sample 66000 / 70000
[t-SNE] Computed conditional probabilities for sample 67000 / 70000
[t-SNE] Computed conditional probabilities for sample 68000 / 70000
[t-SNE] Computed conditional probabilities for sample 69000 / 70000
[t-SNE] Computed conditional probabilities for sample 70000 / 70000
[t-SNE] Mean sigma: 357.459602
[t-SNE] KL divergence after 250 iterations with early exaggeration: 9
9.212646
[t-SNE] KL divergence after 300 iterations: 5.192997
t-SNE done! Time elapsed: 810.6054530143738 seconds
```

```

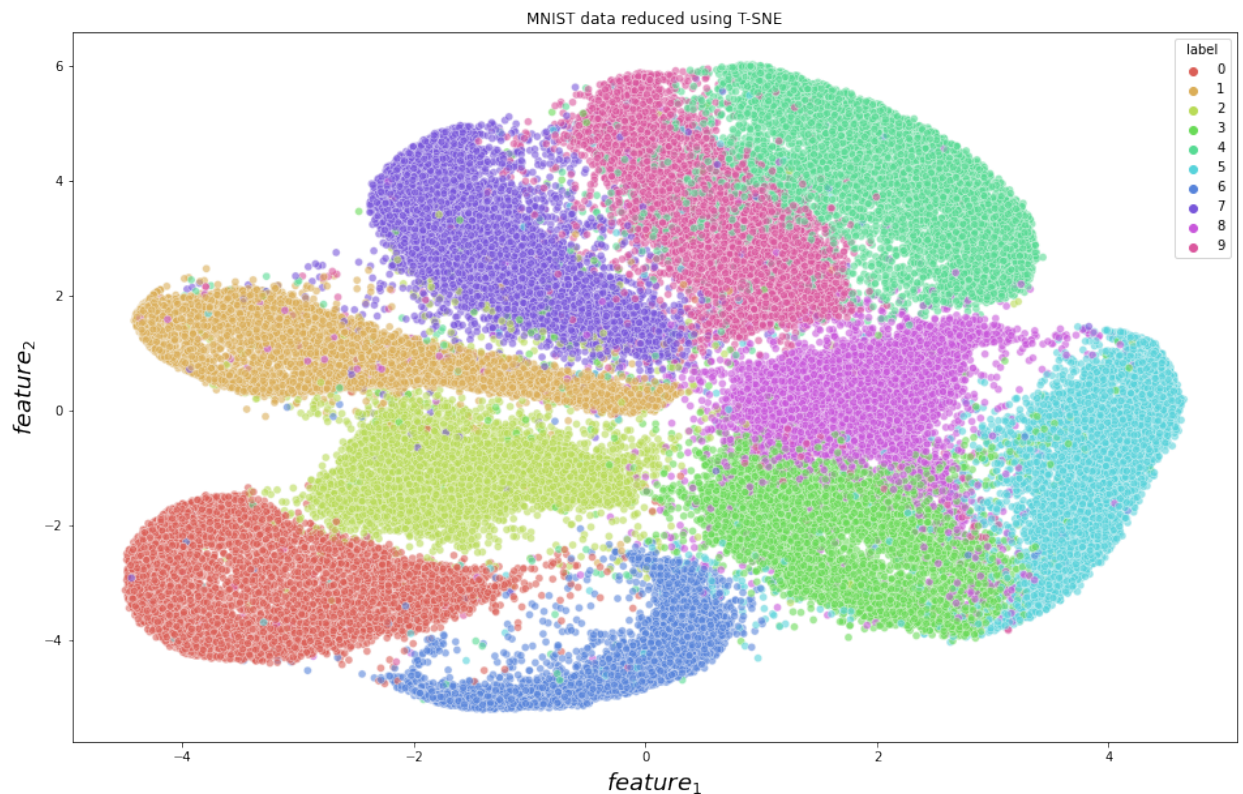
In [30]: 1 df['tsne-2d-one'] = tsne_result[:, 0]
          2 df['tsne-2d-two'] = tsne_result[:, 1]
          3
          4 plt.figure(figsize=(16,10))
          5 plt.title("MNIST data reduced using T-SNE")
          6 plt.xlabel("$feature_1$", fontsize=18)
          7 plt.ylabel("$feature_2$", fontsize=18)
          8 sns.scatterplot(x=tsne_result[:, 0], y=tsne_result[:, 1], hue=df['
          9                             palette=sns.color_palette('hls', 10),
          10                             alpha=0.6)

```

```

Out[30]: <AxesSubplot:title={'center': 'MNIST data reduced using T-SNE'}, xlabel=
l='$feature_1$', ylabel='$feature_2$'>

```



```

In [31]: 1 df_train_X_tsne_reduced = df_features_pca_reduced[0:60000]
          2 df_test_X_tsne_reduced = df_features_pca_reduced[60000:70000]

```

```

In [32]: 1 time_start = time.time()
          2 rf_clf_tsne_reduced = RandomForestClassifier(n_estimators=n_estima
          3 rf_clf_tsne_reduced.fit(X=df_train_X_tsne_reduced, y=np.ravel(df_t
          4 print('TSNE reduced RF CLF trained! Time elapsed: {} seconds'.form

```

TSNE reduced RF CLF trained! Time elapsed: 113.5741548538208 seconds

```
In [33]: 1 tsne_predictions = rf_clf_tsne_reduced.predict(df_test_X_tsne_reduced)
2 print(classification_report(np.ravel(df_test_y), tsne_predictions))
```

	precision	recall	f1-score	support
0	0.98	0.98	0.98	959
1	0.98	0.99	0.98	1155
2	0.94	0.95	0.95	1007
3	0.93	0.93	0.93	1039
4	0.94	0.96	0.95	925
5	0.93	0.94	0.94	921
6	0.96	0.97	0.97	972
7	0.96	0.95	0.96	1048
8	0.93	0.92	0.93	953
9	0.94	0.91	0.93	1021
accuracy			0.95	10000
macro avg	0.95	0.95	0.95	10000
weighted avg	0.95	0.95	0.95	10000

```
In [34]: 1 print("Accuracy of RF CLF with T-SNE:", metrics.accuracy_score(np.
```

Accuracy of RF CLF with T-SNE: 0.9503

Again, I compare the reduced data with a KNN model too, the total time of testing and training the model is less than the training time of the main dataset in part 2. The accuracy was almost the same.

```
In [35]: 1 time_start = time.time()
2 knn_tsne = KNeighborsClassifier(n_neighbors=20)
3 knn_tsne.fit(X=df_train_X_tsne_reduced, y=np.ravel(df_train_y))
4 print('t-SNE reduced KNN CLF trained! Time elapsed: {} seconds'.format(time.time() - time_start))
```

t-SNE reduced KNN CLF trained! Time elapsed: 0.030025959014892578 seconds

```
In [38]: 1 time_start = time.time()
2 tsne_predictions_knn = knn_tsne.predict(df_test_X_tsne_reduced)
3 print(classification_report(np.ravel(df_test_y), tsne_predictions_
4 print('t-SNE reduced KNN CLF tested! Time elapsed: {} seconds'.for
```

	precision	recall	f1-score	support
0	0.98	0.99	0.98	959
1	0.95	0.99	0.97	1155
2	0.99	0.95	0.97	1007
3	0.97	0.96	0.96	1039
4	0.98	0.96	0.97	925
5	0.97	0.96	0.97	921
6	0.97	0.99	0.98	972
7	0.95	0.96	0.95	1048
8	0.99	0.93	0.96	953
9	0.94	0.95	0.94	1021
accuracy			0.97	10000
macro avg	0.97	0.97	0.97	10000
weighted avg	0.97	0.97	0.97	10000

t-SNE reduced KNN CLF tested! Time elapsed: 22.261780977249146 seconds

```
In [39]: 1 print("Accuracy of KNN CLF with t-SNE:", metrics.accuracy_score(np
Accuracy of KNN CLF with t-SNE: 0.9656
```

From what I realized, PCA and t-SNE reduced the data in a way that made the training time of the model more. But they both had a good accuracy that was near the main dataset accuracy with a much simpler dataset which shows how powerful they are. The time problem didn't exist with the KNN model.

**Question 5 (4 points).** Compare with other dimensionality methods: *Locally Linear Embedding* (LLE) or *Multidimensional scaling* (MDS).

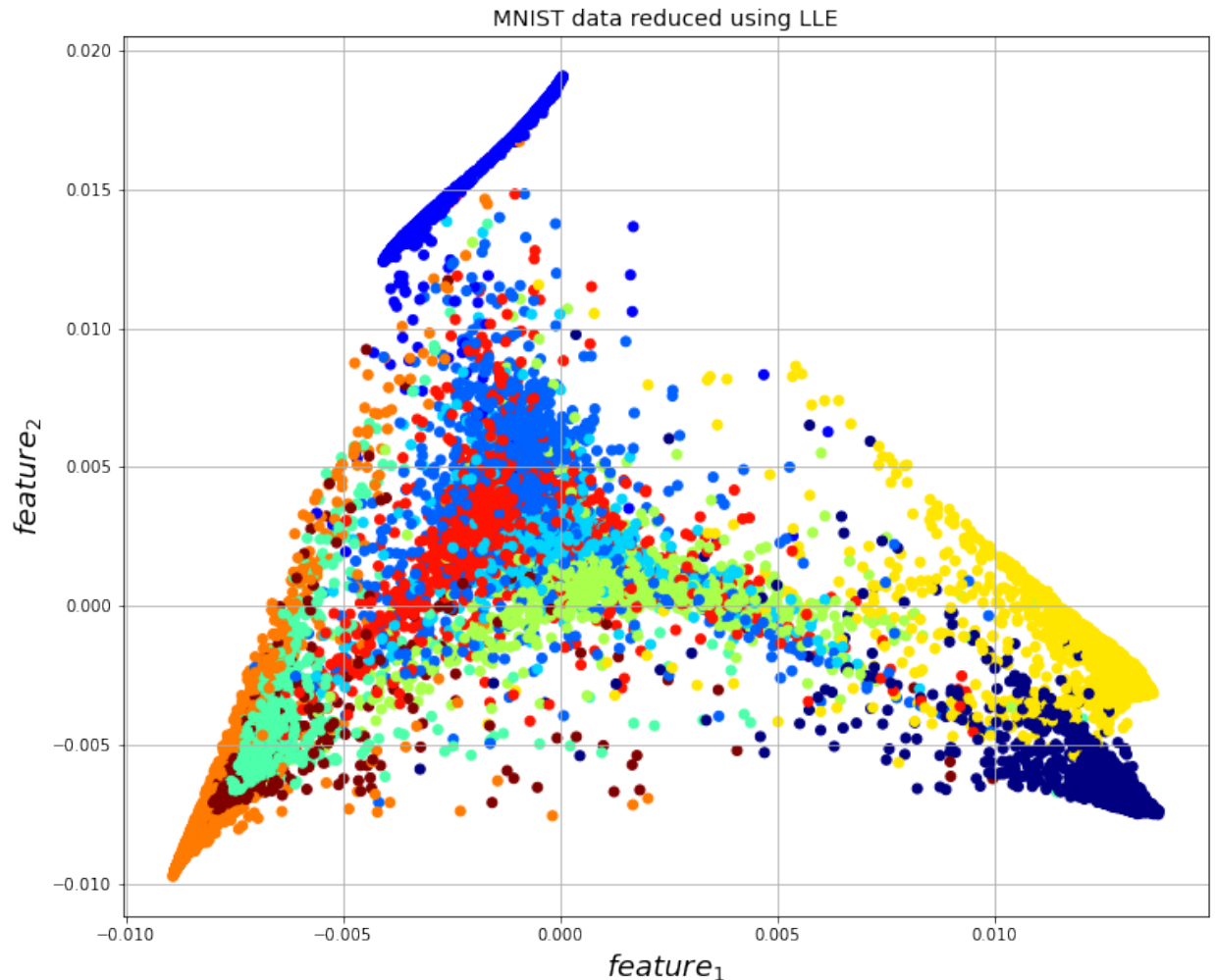
```
In [40]: 1 N=20000
2 df_subset = df.loc[:N, :].copy()
3 df_subset_features = df_subset[[col for col in df.columns if col !=
4 df_subset_target = df_subset[target_column]
```

```
In [41]: 1 df_subset_train_X = df_subset_features.iloc[0:16000, :]  
2 df_subset_train_y = df_subset_target.iloc[0:16000]  
3 df_subset_test_X = df_subset_features.iloc[16000:20000, :]  
4 df_subset_test_y = df_subset_target.iloc[16000:20000]
```

```
In [45]: 1 from sklearn.manifold import LocallyLinearEmbedding  
2  
3 time_start = time.time()  
4 lle = LocallyLinearEmbedding(n_components=2, n_neighbors=10, random_state=42)  
5 df_features_reduced_lle = lle.fit_transform(df_subset_features)  
6 print('LLE done! Time elapsed: {} seconds'.format(time.time()-time_start))
```

LLE done! Time elapsed: 371.15496492385864 seconds

```
In [46]: 1 plt.figure(figsize=(12,10))
2 plt.title("MNIST data reduced using LLE", fontsize=14)
3 plt.scatter(df_features_reduced_lle[:, 0], df_features_reduced_lle[:, 1])
4 plt.xlabel("$feature_1$", fontsize=18)
5 plt.ylabel("$feature_2$", fontsize=18)
6 plt.grid(True)
7
8 plt.show()
```



```
In [47]: 1 time_start = time.time()
2 rf_lle = RandomForestClassifier(n_estimators=n_estimators)
3 rf_lle.fit(X=df_features_reduced_lle[0:16000], y=np.ravel(df_subse
4 print('LLE reduced RF CLF trained! Time elapsed: {} seconds'.format
```

LLE reduced RF CLF trained! Time elapsed: 2.1599090099334717 seconds



```
In [48]: 1 time_start = time.time()
2 lle_predictions_rf = rf_lle.predict(df_features_reduced_lle[16000:
3 print(classification_report(np.ravel(df_subset_test_y), lle_predic
4 print('LLE reduced RF CLF tested! Time elapsed: {} seconds'.format
```

	precision	recall	f1-score	support
0	0.96	0.95	0.96	396
1	0.97	0.98	0.97	459
2	0.51	0.46	0.48	384
3	0.62	0.61	0.62	405
4	0.83	0.91	0.87	371
5	0.73	0.79	0.76	384
6	0.94	0.96	0.95	396
7	0.89	0.87	0.88	423
8	0.51	0.51	0.51	374
9	0.86	0.82	0.84	408
accuracy			0.79	4000
macro avg	0.78	0.79	0.78	4000
weighted avg	0.79	0.79	0.79	4000

LLE reduced RF CLF tested! Time elapsed: 0.13289403915405273 seconds

```
In [49]: 1 print("Accuracy of RF CLF with LLE:", metrics.accuracy_score(np.ra
Accuracy of RF CLF with LLE: 0.7905
```

```
In [50]: 1 from sklearn.manifold import MDS
```

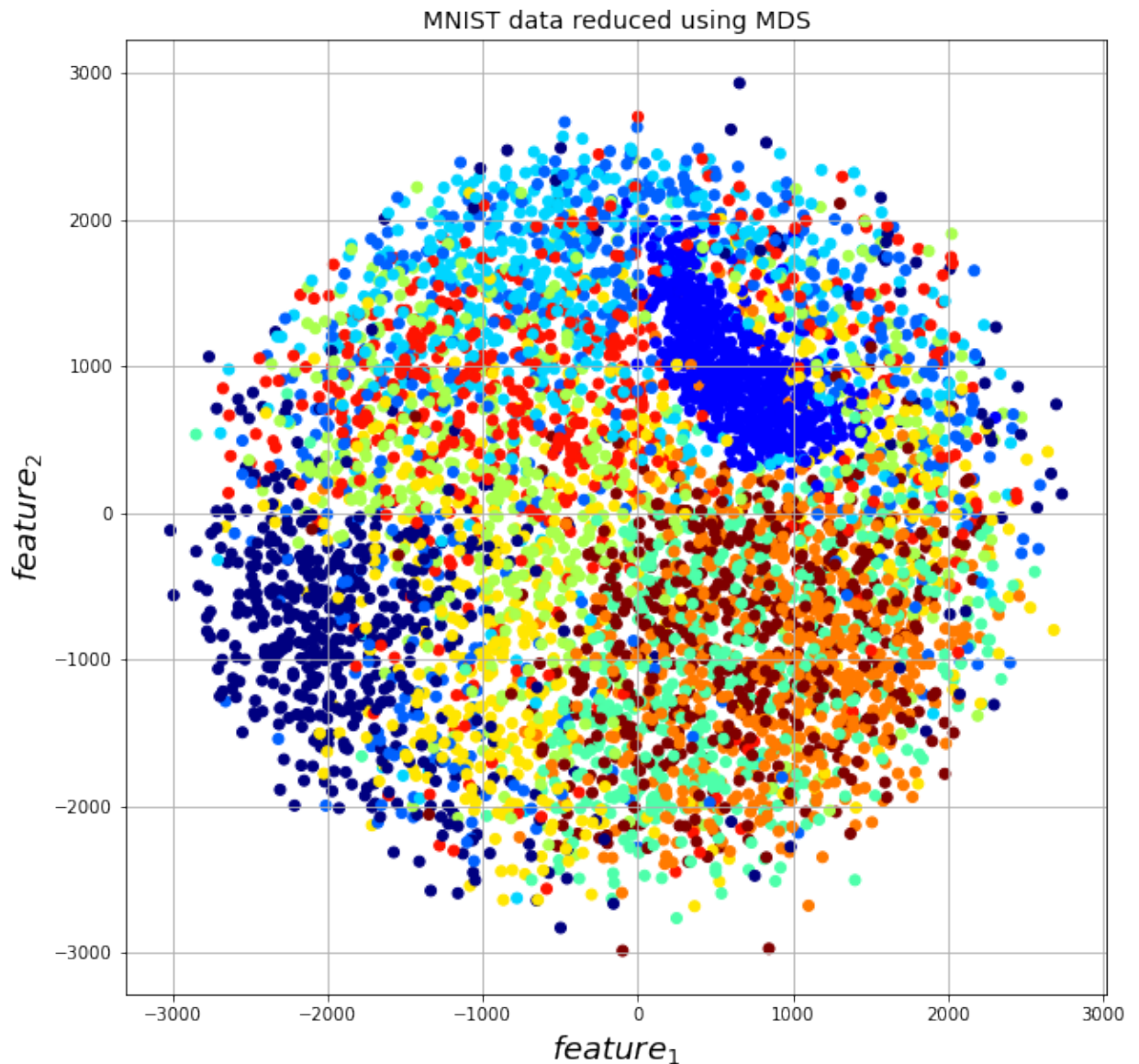
```
In [51]: 1 N=5000
2 df_subset_mds = df.loc[:N, :].copy()
3 df_subset_features_mds = df_subset_mds[[col for col in df.columns
4 df_subset_target_mds = df_subset_mds[target_column]
```

```
In [52]: 1 df_subset_mds_train_X = df_subset_features_mds.iloc[0:4000, :]
2 df_subset_mds_train_y = df_subset_target_mds.iloc[0:4000]
3 df_subset_mds_test_X = df_subset_features_mds.iloc[4000:5000, :]
4 df_subset_mds_test_y = df_subset_target_mds.iloc[4000:5000]
```

```
In [53]: 1 time_start = time.time()
2 mds = MDS(n_components=2, random_state=41)
3 df_features_reduced_mds = mds.fit_transform(df_subset_features_mds
4 print('MDS done! Time elapsed: {} seconds'.format(time.time()-time
```

MDS done! Time elapsed: 1270.186173915863 seconds

```
In [54]: 1 plt.figure(figsize=(10,10))
2 plt.title("MNIST data reduced using MDS", fontsize=14)
3 plt.scatter(df_features_reduced_mds[:, 0], df_features_reduced_mds[:, 1])
4 plt.xlabel("$feature_1$", fontsize=18)
5 plt.ylabel("$feature_2$", fontsize=18)
6 plt.grid(True)
7
8 plt.show()
```



```
In [55]: 1 time_start = time.time()
2 rf_lle = RandomForestClassifier(n_estimators=n_estimators)
3 rf_lle.fit(X=df_features_reduced_mds[0:4000], y=np.ravel(df_subset))
4 print('MDS reduced RF CLF trained! Time elapsed: {} seconds'.format(time.time() - time_start))
```

MDS reduced RF CLF trained! Time elapsed: 0.7660119533538818 seconds

```
In [56]: 1 time_start = time.time()
2 mds_predictions_rf = rf_lle.predict(df_features_reduced_mds[4000:5000])
3 print(classification_report(np.ravel(df_subset_mds_test_y), mds_predictions_rf))
4 print('MDS reduced RF CLF tested! Time elapsed: {} seconds'.format(time.time() - time_start))
```

	precision	recall	f1-score	support
0	0.61	0.69	0.65	93
1	0.82	0.88	0.85	105
2	0.26	0.23	0.25	99
3	0.34	0.38	0.36	99
4	0.27	0.24	0.25	93
5	0.35	0.30	0.32	100
6	0.25	0.26	0.26	93
7	0.40	0.48	0.44	121
8	0.29	0.27	0.28	96
9	0.33	0.29	0.31	101
accuracy			0.41	1000
macro avg	0.39	0.40	0.40	1000
weighted avg	0.40	0.41	0.40	1000

MDS reduced RF CLF tested! Time elapsed: 0.0734710693359375 seconds

```
In [57]: 1 print("Accuracy of RF CLF with MDS:", metrics.accuracy_score(np.ravel(mds_predictions_rf), df_subset_mds_test_y))
```

Accuracy of RF CLF with MDS: 0.406

For both LLE and MDS, I had to take a subset of the dataset. LLE worked better than MDS but none of them performed as good as PCA and t-SNE overall on this dataset.

In [ ]:

1