



Il modello di Ising

Simulazione di Materia Condensata e Biosistemi

Filippo Negrini (Matricola: 47127A)





Table of Contents

1 Introduzione

- ▶ Introduzione
- ▶ Metodi numerici
- ▶ Simulazioni modello di Ising 1D
- ▶ Simulazioni modello di Ising 2D
- ▶ Modello XY
- ▶ Conclusioni
- ▶ Backup

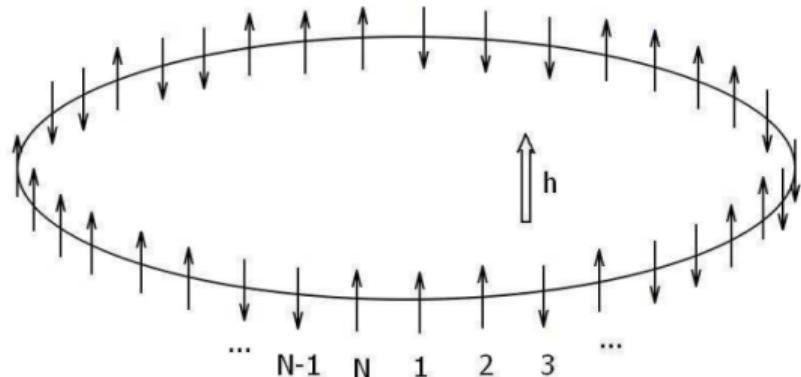


Hamiltoniana

1 Introduzione

$$H = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j - h \sum_i \sigma_i$$

- ◊ Interazione fra primi vicini
- ◊ Accoppiamento con un campo esterno



Modello di Ising 1D con condizioni periodiche.



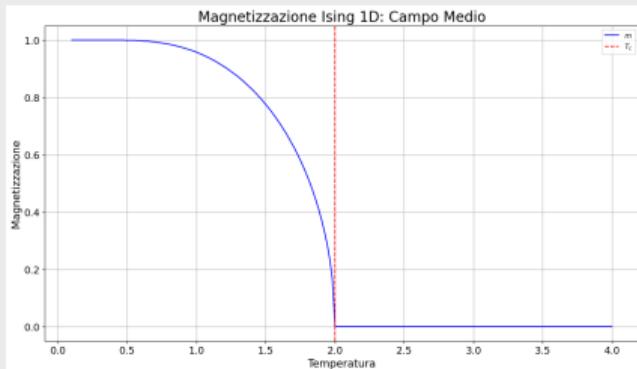
Modello di Ising 1D

1 Introduzione

Campo medio

- ◊ Transizione di fase a $T_c \neq 0$

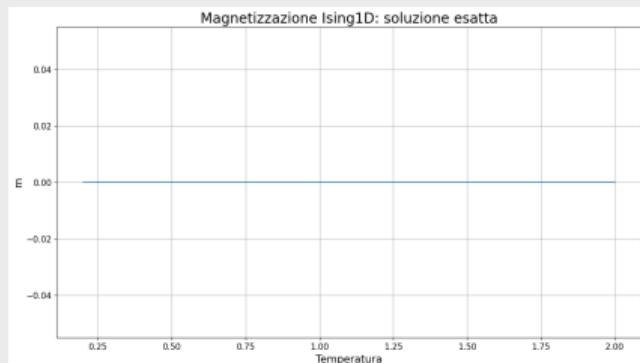
$$m = \tanh [\beta (h + Jn_{nn}m)]$$



Soluzione analitica

- ◊ No magnetizzazione se $T \neq 0, h = 0$

$$m = \frac{\sinh (\beta h)}{\sqrt{e^{-4\beta J} + \sinh^2 (\beta h)}}$$



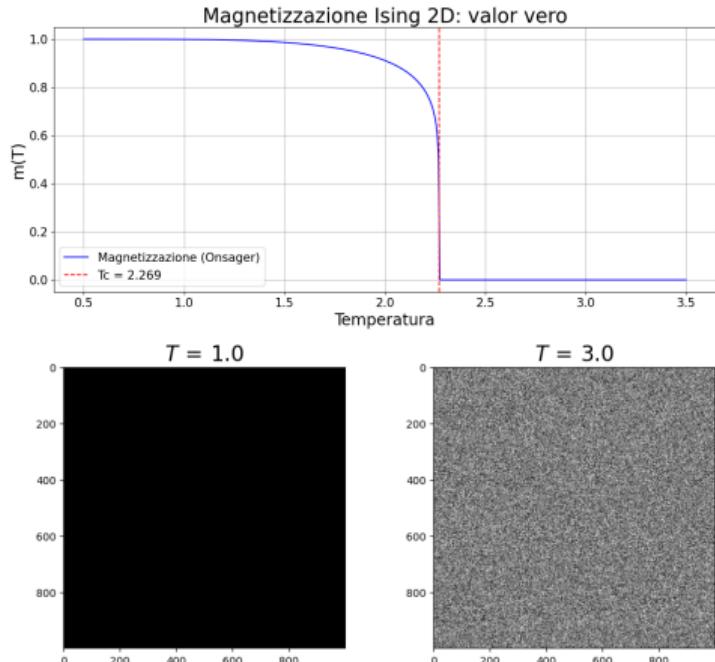


Modello di Ising 2D

1 Introduzione

- ◊ Soluzione analitica per $h = 0$
- ◊ Sistema presenta una transizione di fase a $T_c \neq 0$

$$m(\beta, h=0) = \begin{cases} \left[1 - \frac{1}{\sinh^4(2\beta J)}\right]^{\frac{1}{8}} & T < T_c \\ 0 & T > T_c \end{cases}$$





Modello XY

1 Introduzione

Hamiltoniana

$$H = -J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j - \sum_i \vec{h} \cdot \vec{s}_i$$

- ◊ Simmetria continua
- ◊ Difetti topologici (vortici)

Modello XY

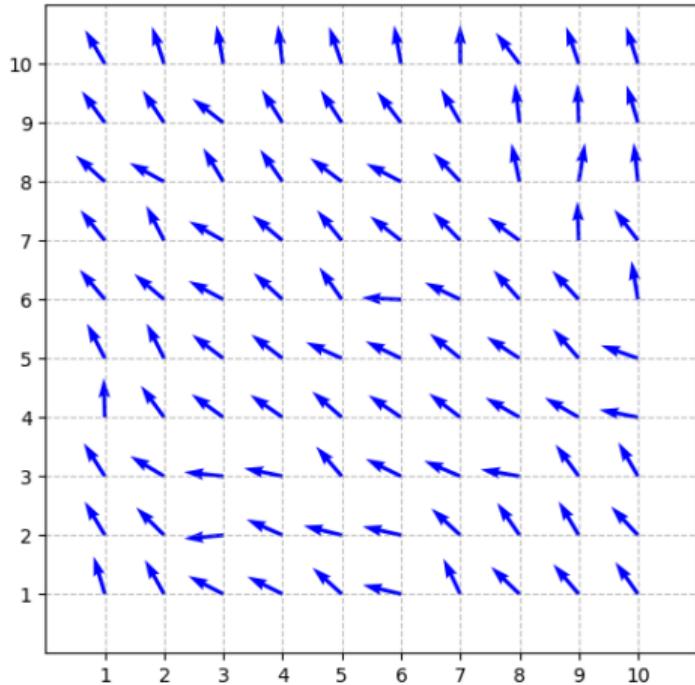




Table of Contents

2 Metodi numerici

- ▶ Introduzione
- ▶ Metodi numerici
- ▶ Simulazioni modello di Ising 1D
- ▶ Simulazioni modello di Ising 2D
- ▶ Modello XY
- ▶ Conclusioni
- ▶ Backup

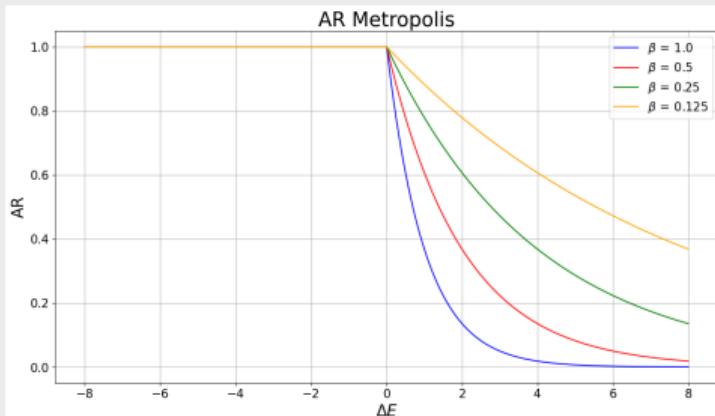


Metropolis vs Wolff

2 Metodi numerici

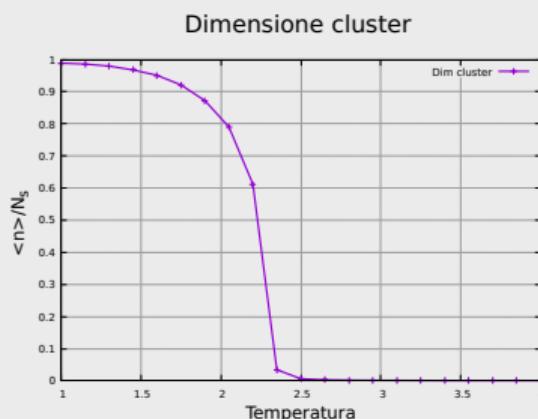
Metropolis

- ◊ Tentata inversione di un singolo spin
- ◊ $A(\nu | \mu) = \min [1, e^{-\beta(E_\nu - E_\mu)}]$
- ◊ Ottimo per $T \ll T_c$ oppure $T \gg T_c$



Wolff

- ◊ Algoritmo di clustering
- ◊ $P_{add} = 1 - \exp(-2\beta J)$
- ◊ Ottimo per $T \simeq T_c$

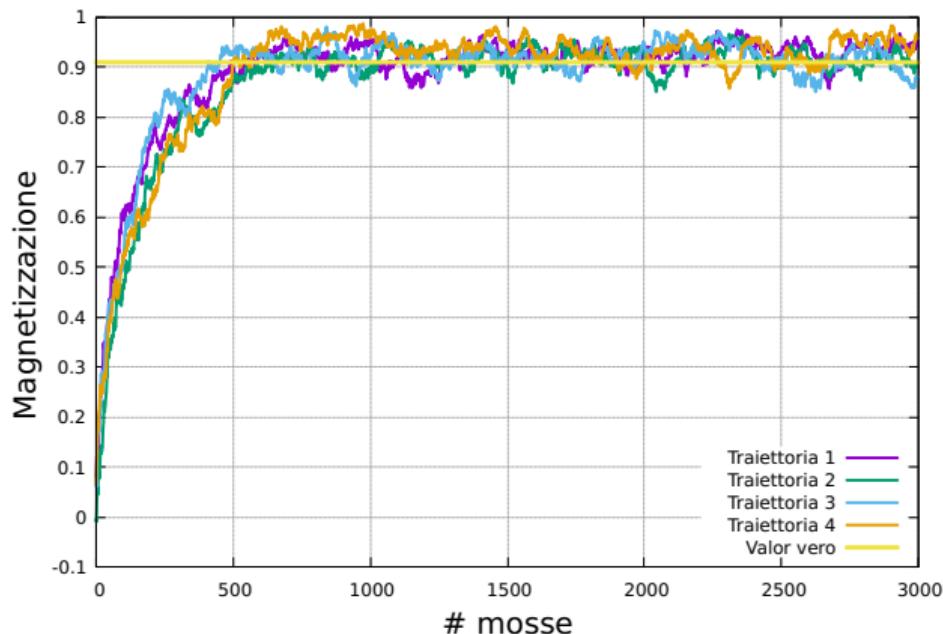




Termalizzazione

2 Metodi numerici

Termalizzazione: 3000 spin, $T = 0.5$



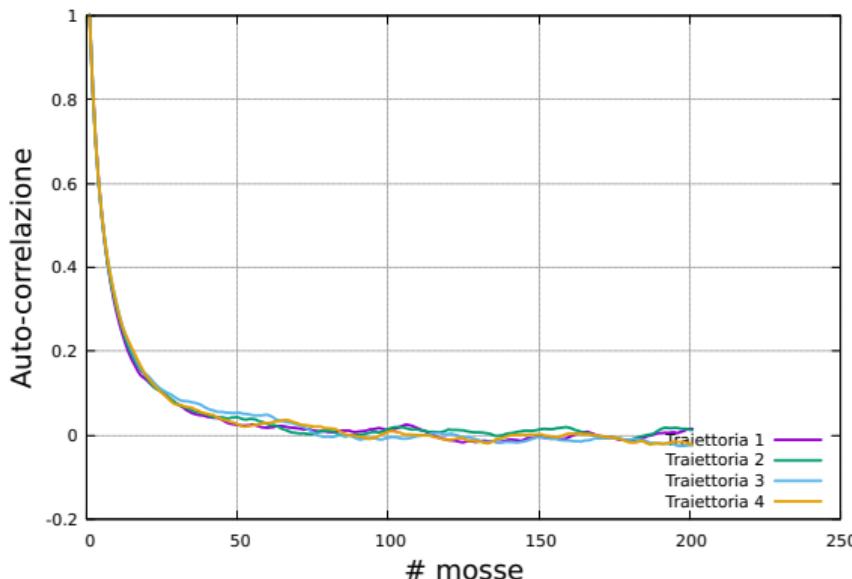
- ◊ Giungere all'equilibrio termodinamico
- ◊ Attenzione a stati metastabili
- ◊ Dipendenza dalla condizione iniziale



Auto-correlazione

2 Metodi numerici

Autocorrelazione m: N = 500, T = 2.0



Autocorrelazione per modello di Ising 2D.

Definizione

$$\chi(t) = \frac{\langle m(t')m(t'+t) \rangle_{t'} - \langle m \rangle^2}{\sigma_m^2}$$

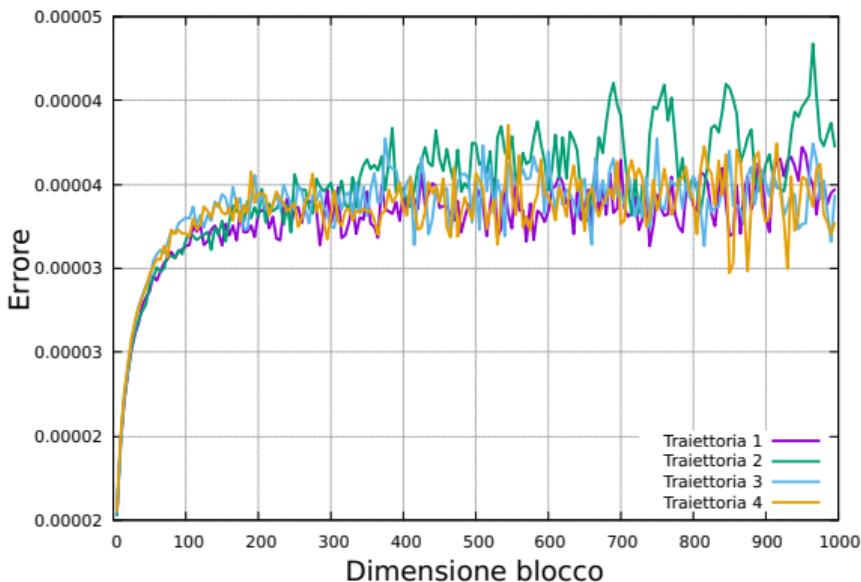
- ◊ $\chi(t) \propto e^{-t/t_c}$
- ◊ Indipendenza statistica fra configurazioni
- ◊ $n_{max} = \frac{t_{max}}{2t_c}$



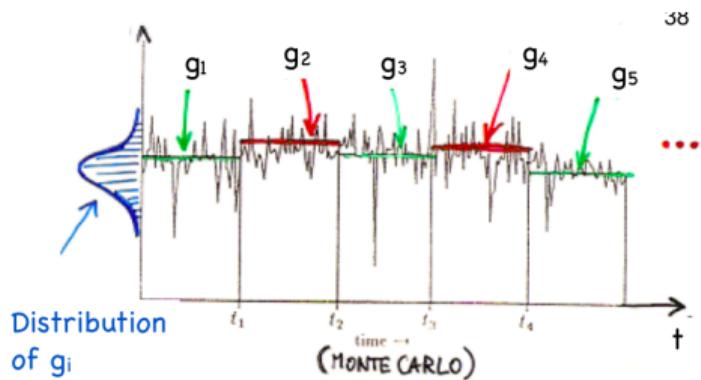
Data-blocking

2 Metodi numerici

Dimensione blocco: $N = 500$, $T = 2.0$



Analisi per dimensione blocchi nel caso di un modello di Ising 2D.



- ◊ Dati raggruppati in blocchi
- ◊ Errore satura quando raggiunta l_{lim}



Table of Contents

3 Simulazioni modello di Ising 1D

- ▶ Introduzione
- ▶ Metodi numerici
- ▶ Simulazioni modello di Ising 1D
- ▶ Simulazioni modello di Ising 2D
- ▶ Modello XY
- ▶ Conclusioni
- ▶ Backup

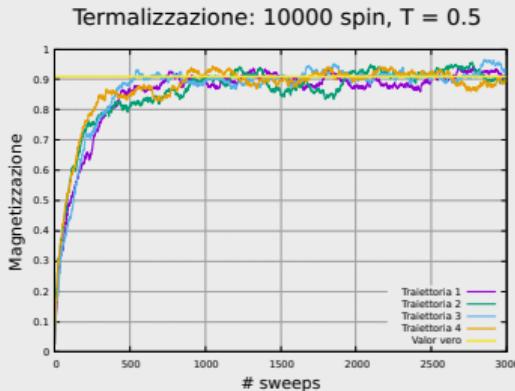


Caratterizzazione

3 Simulazioni modello di Ising 1D

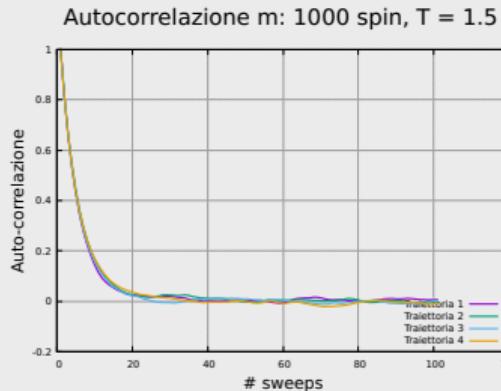
Termalizzazione

- ◊ Maggiore T, minore t_{ter}
- ◊ $t_{ter}^{max} \simeq 600$ sweeps



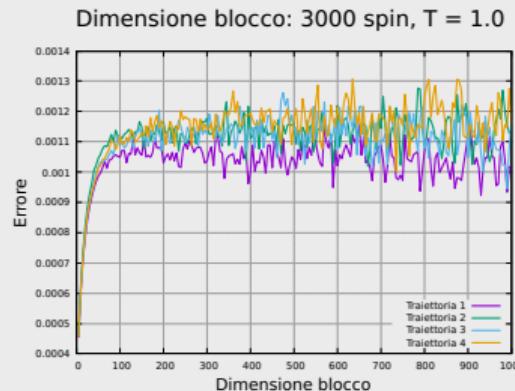
Auto-correlazione

- ◊ Maggiore T, minore t_c
- ◊ $t_c^{max} \simeq 500$ sweeps



Blocchi

- ◊ Maggiore T, minore l_{blk}
- ◊ $l_{blk}^{max} \simeq 1000$ sweeps

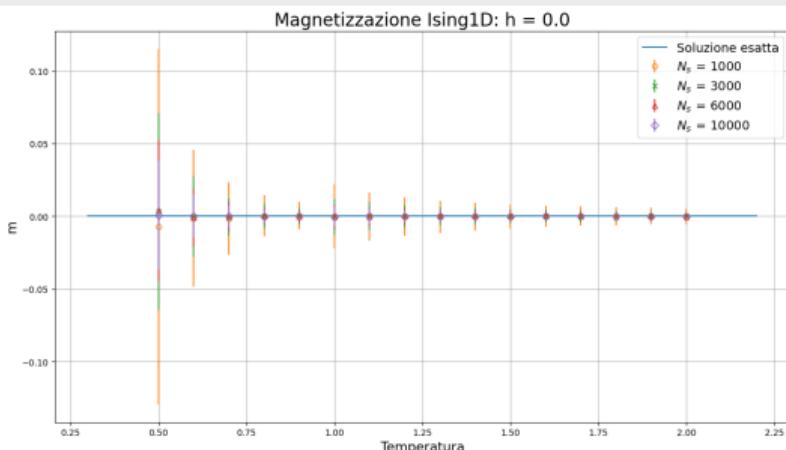




Magnetizzazione

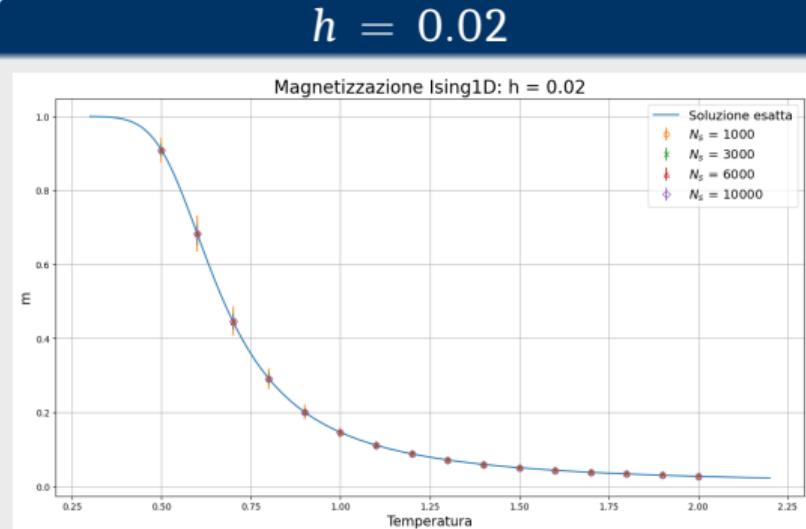
3 Simulazioni modello di Ising 1D

$$h = 0.0$$



◊ $m = 0$ per ogni $T \neq 0$

$$h = 0.02$$



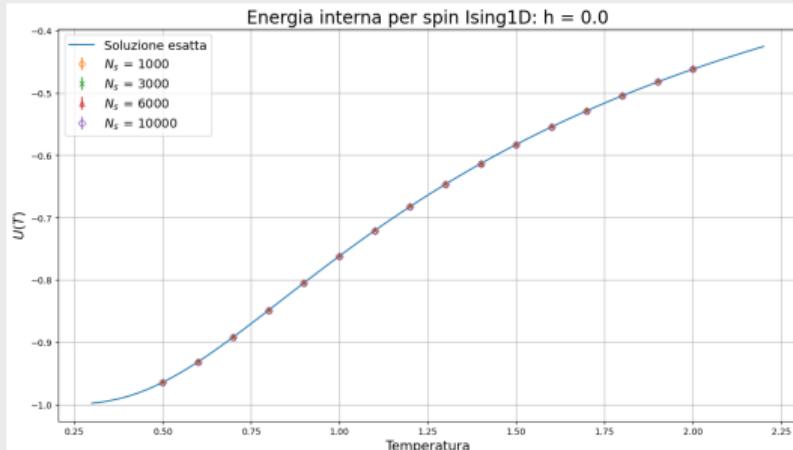
◊ campo magnetico impone ordine



Energia interna

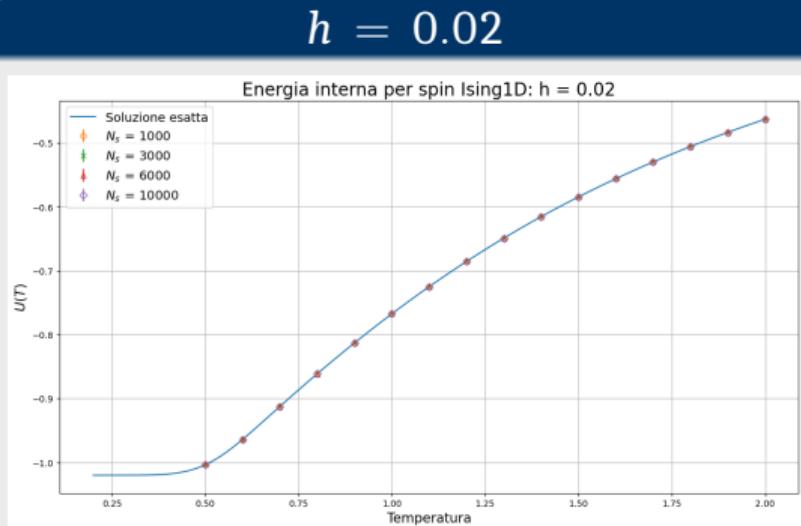
3 Simulazioni modello di Ising 1D

$$h = 0.0$$



◊ per $T \rightarrow 0$ l'energia $U(T) \rightarrow -1$

$$h = 0.02$$



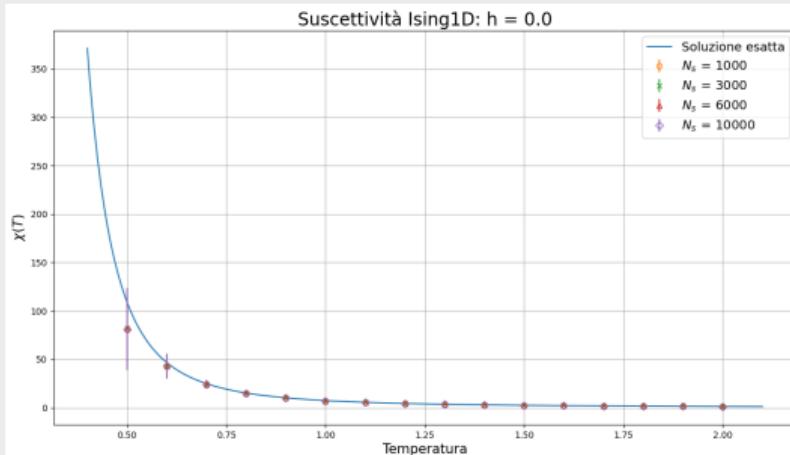
◊ per $T \rightarrow 0$ l'energia $U(T) \rightarrow -1.02$



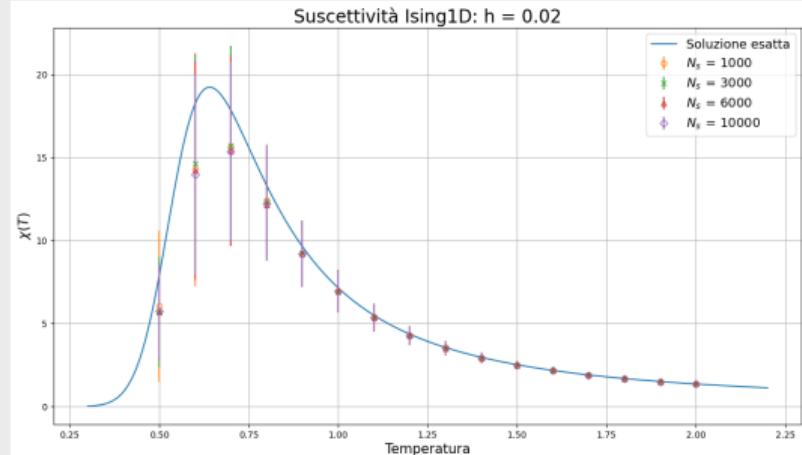
Suscettività magnetica

3 Simulazioni modello di Ising 1D

$h = 0.0$



$h = 0.02$



◊ aumento per $T \rightarrow 0$ perchè $T_c = 0$

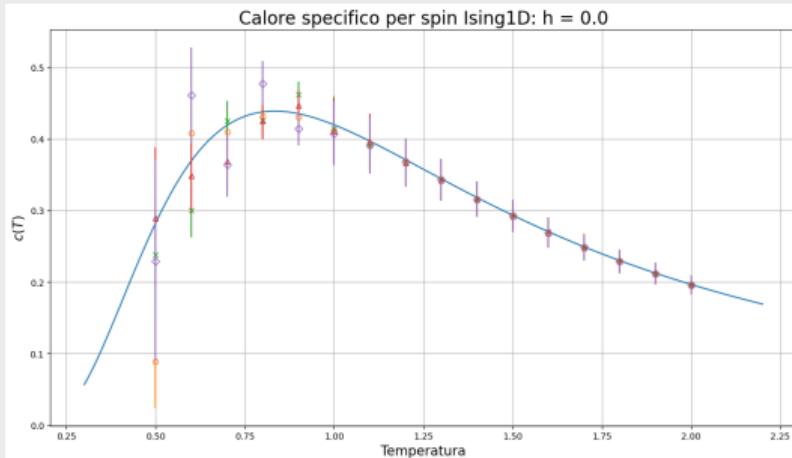
◊ picco a $T \neq 0$ dovuto ad h



Calore specifico

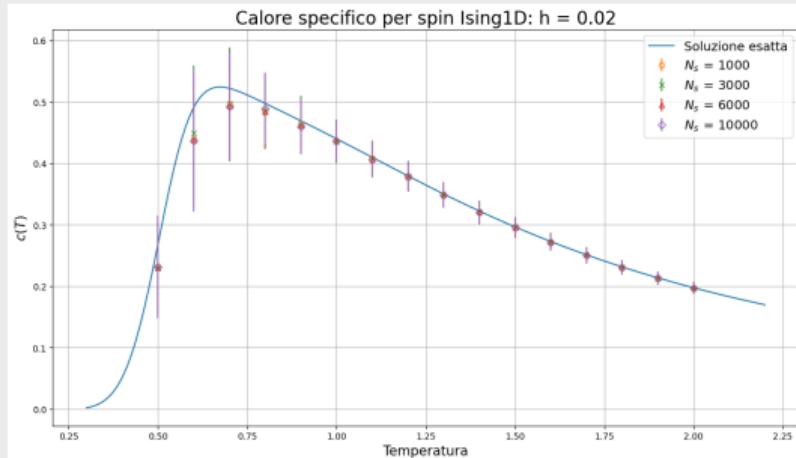
3 Simulazioni modello di Ising 1D

$h = 0.0$



◊ difficoltà a studiare il picco

$h = 0.02$



◊ campo magnetico semplifica lo studio



Table of Contents

4 Simulazioni modello di Ising 2D

- ▶ Introduzione
- ▶ Metodi numerici
- ▶ Simulazioni modello di Ising 1D
- ▶ Simulazioni modello di Ising 2D
- ▶ Modello XY
- ▶ Conclusioni
- ▶ Backup

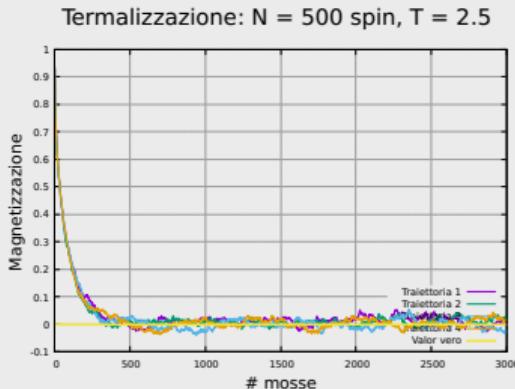


Caratterizzazione con metropolis

4 Simulazioni modello di Ising 2D

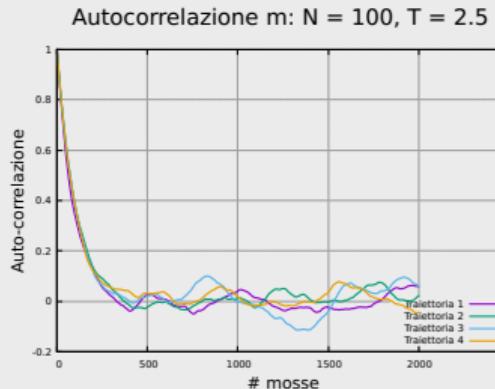
Termalizzazione

- ◊ t_{ter} maggiori per $T \simeq T_c$
- ◊ $t_{ter}^{max} \simeq 500$ sweeps



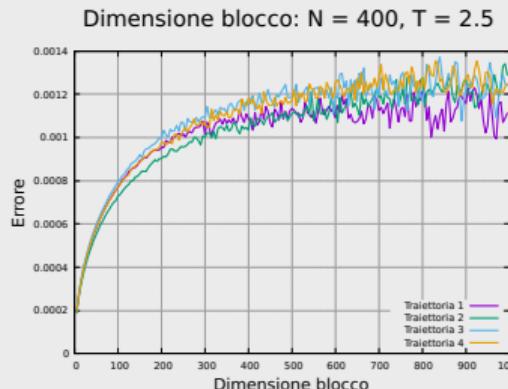
Auto-correlazione

- ◊ t_c maggiori per $T \simeq T_c$
- ◊ $t_c^{max} \simeq 400$ sweeps



Blocchi

- ◊ l_{blk} maggiori per $T \simeq T_c$
- ◊ $l_{blk}^{max} \simeq 1000$ sweeps



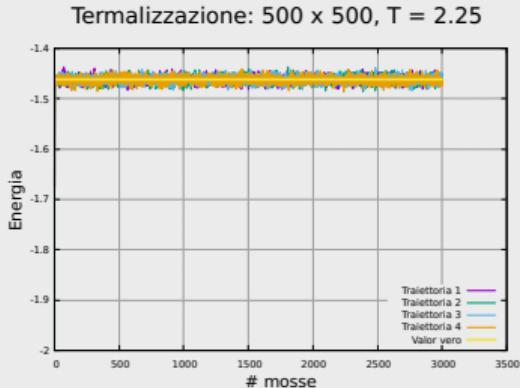


Caratterizzazione con Wolff

4 Simulazioni modello di Ising 2D

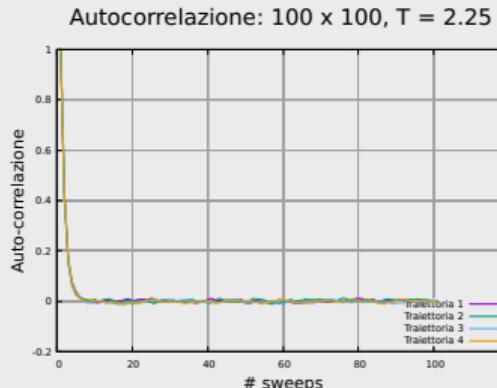
Termalizzazione

- ◊ Istantanea
- ◊ $t_{ter}^{max} \simeq 10$ sweeps



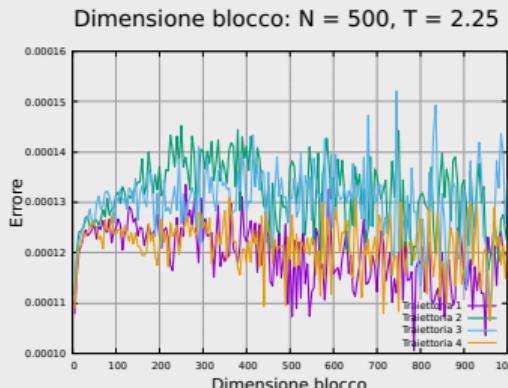
Auto-correlazione

- ◊ t_c maggiori per $T \simeq T_c$
- ◊ $t_c^{max} \simeq 40$ sweeps



Blocchi

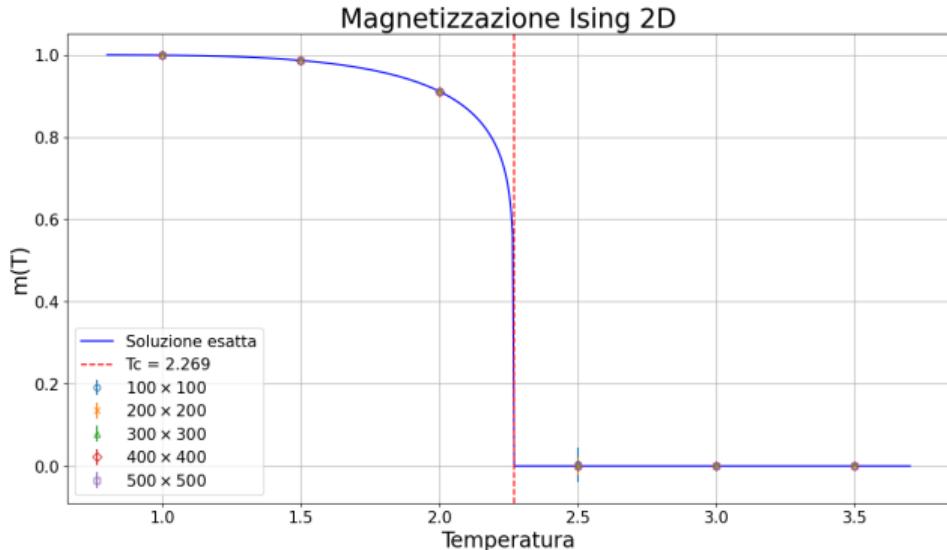
- ◊ l_{blk} maggiori per $T \simeq T_c$
- ◊ $l_{blk}^{max} \simeq 50$ sweeps



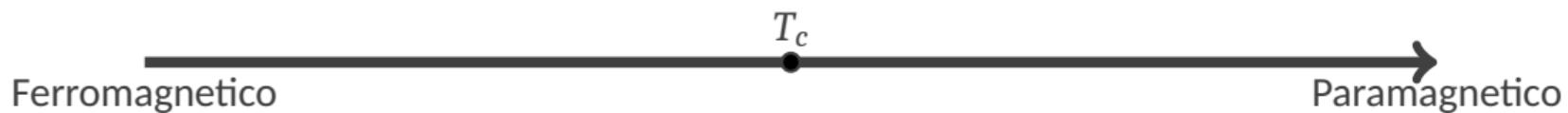


Magnetizzazione

4 Simulazioni modello di Ising 2D



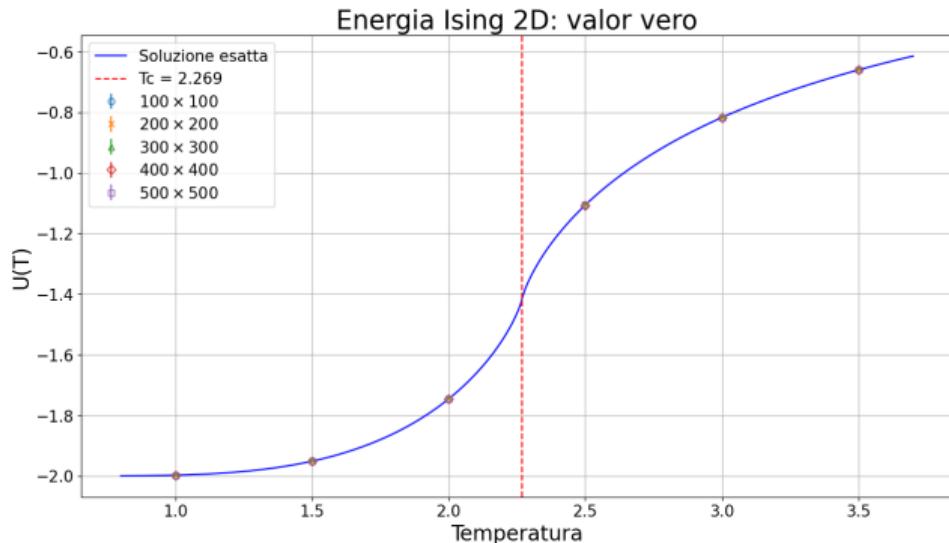
- ◊ Magnetizzazione spontanea per $T < T_c$
- ◊ Transizione di fase a T_c





Energia

4 Simulazioni modello di Ising 2D



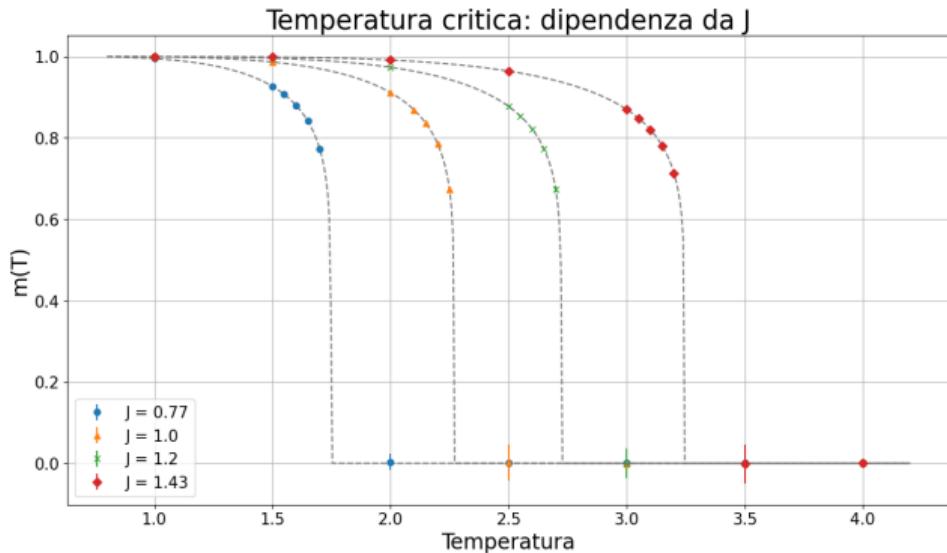
- ◊ copro tutto il reticolo con due legami per spin
- ◊ picco del calore specifico a T_c

$$U = -NJ \coth(2\beta J) \left\{ 1 + \frac{2}{\pi} [2 \tanh^2(2\beta J) - 1] \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - k^2 \sin^2(\phi)}} \right\}$$



Dipendenza da J

4 Simulazioni modello di Ising 2D

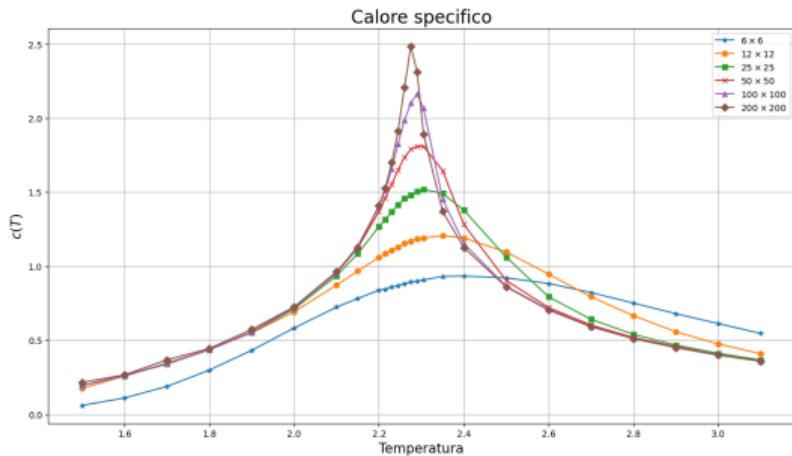


- ◊ Aumenta J , aumenta T_c
- ◊ Presenza o meno di ordine dipende dall'intensità dell'interazione



Regione critica

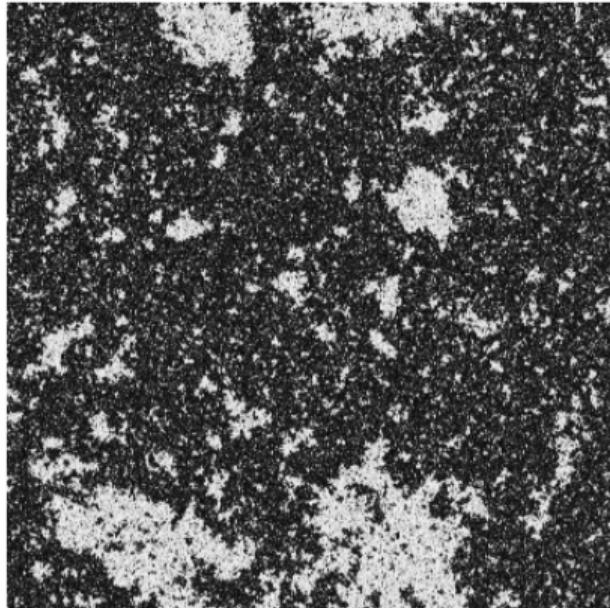
4 Simulazioni modello di Ising 2D



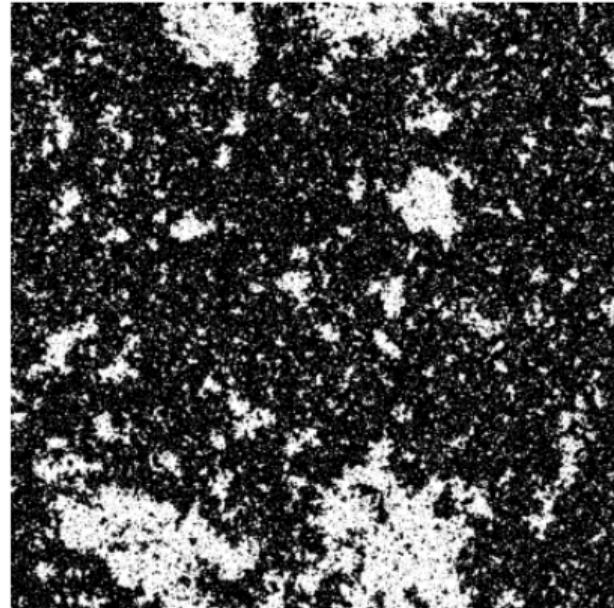


Coarse graining

4 Simulazioni modello di Ising 2D



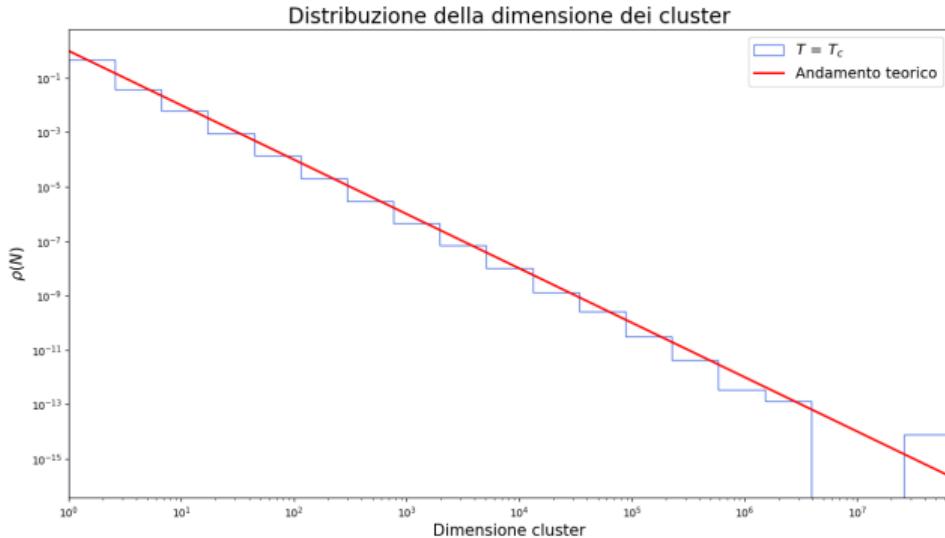
CG →





Dimensioni cluster

4 Simulazioni modello di Ising 2D



- ◊ $P(s) \propto s^{-\alpha}$
- ◊ $\alpha \simeq 2$
- ◊ perdita di un parametro di scala

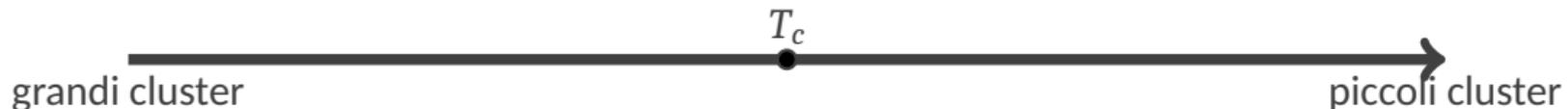




Table of Contents

5 Modello XY

- ▶ Introduzione
- ▶ Metodi numerici
- ▶ Simulazioni modello di Ising 1D
- ▶ Simulazioni modello di Ising 2D
- ▶ Modello XY
- ▶ Conclusioni
- ▶ Backup

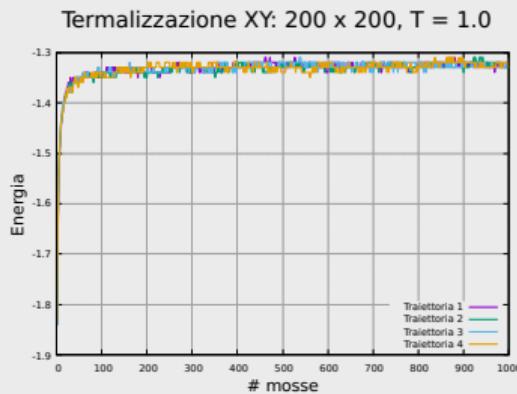


Caratterizzazione

5 Modello XY

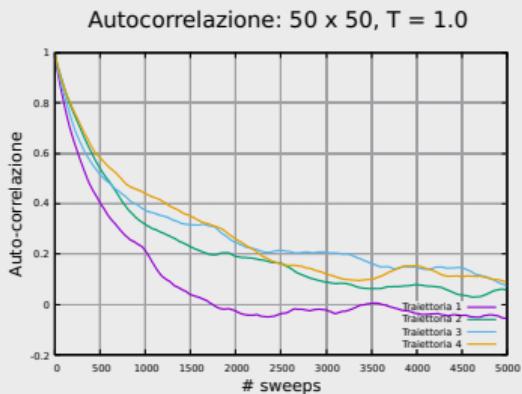
Termalizzazione

- ◊ t_{ter} maggiori per $T \rightarrow 0$
- ◊ $t_{ter}^{max} \simeq 300$ sweeps



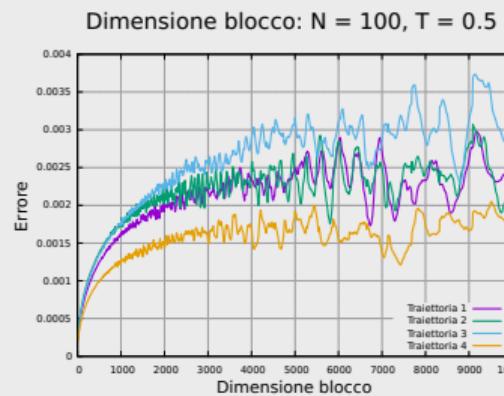
Auto-correlazione

- ◊ t_c maggiori per $T \rightarrow 0$
- ◊ $t_c^{max} \simeq 5000$ sweeps



Blocchi

- ◊ l_{blk} maggiori per $T \rightarrow 0$
- ◊ $l_{blk}^{max} \simeq 7000 - 1000$ sweeps



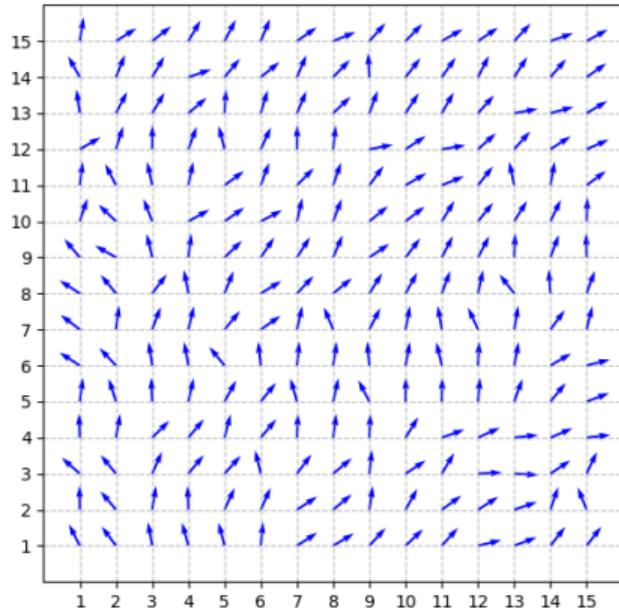


Configurazioni

5 Modello XY

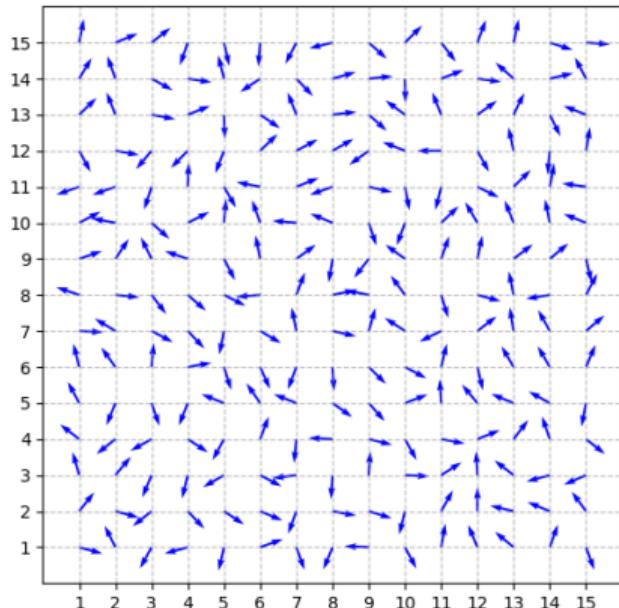
Bassa temperatura

Modello XY: $T = 0.5$



Alta temperatura

Modello XY: $T = 2.5$

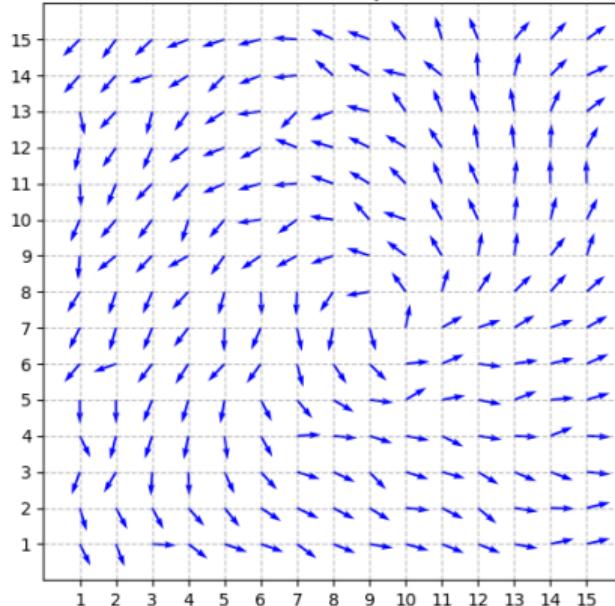




Vortice

5 Modello XY

Modello XY: esempio di vortice



- ◊ Winding number $w = 1$
- ◊ Transizione di Kosterlitz-Thouless

Temperatura critica

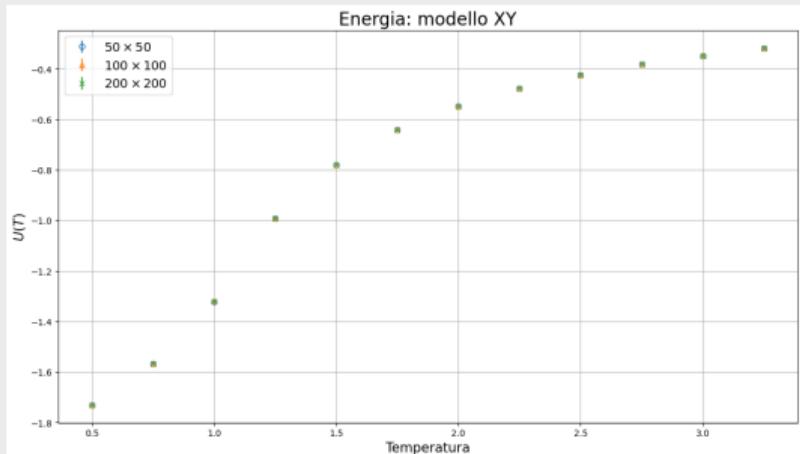
$$T_c = \frac{\pi J}{2}$$



Osservabili

5 Modello XY

Energia



Applicazione campo

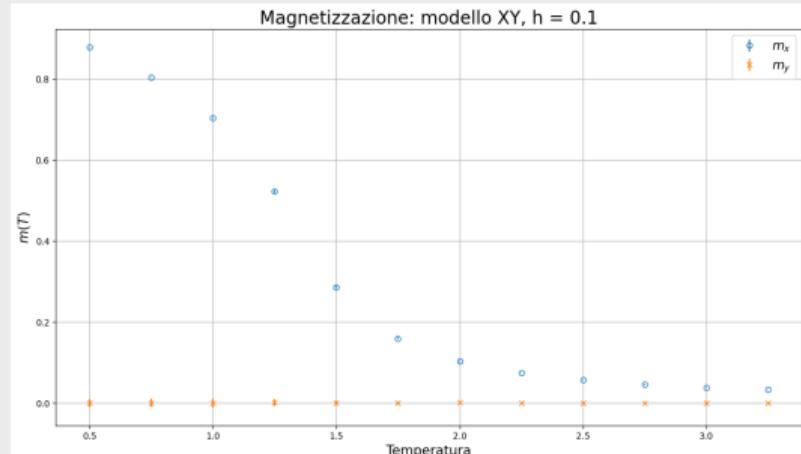




Table of Contents

6 Conclusioni

- ▶ Introduzione
- ▶ Metodi numerici
- ▶ Simulazioni modello di Ising 1D
- ▶ Simulazioni modello di Ising 2D
- ▶ Modello XY
- ▶ Conclusioni
- ▶ Backup



Fine

6 Conclusioni

Grazie per l'attenzione



Table of Contents

7 Backup

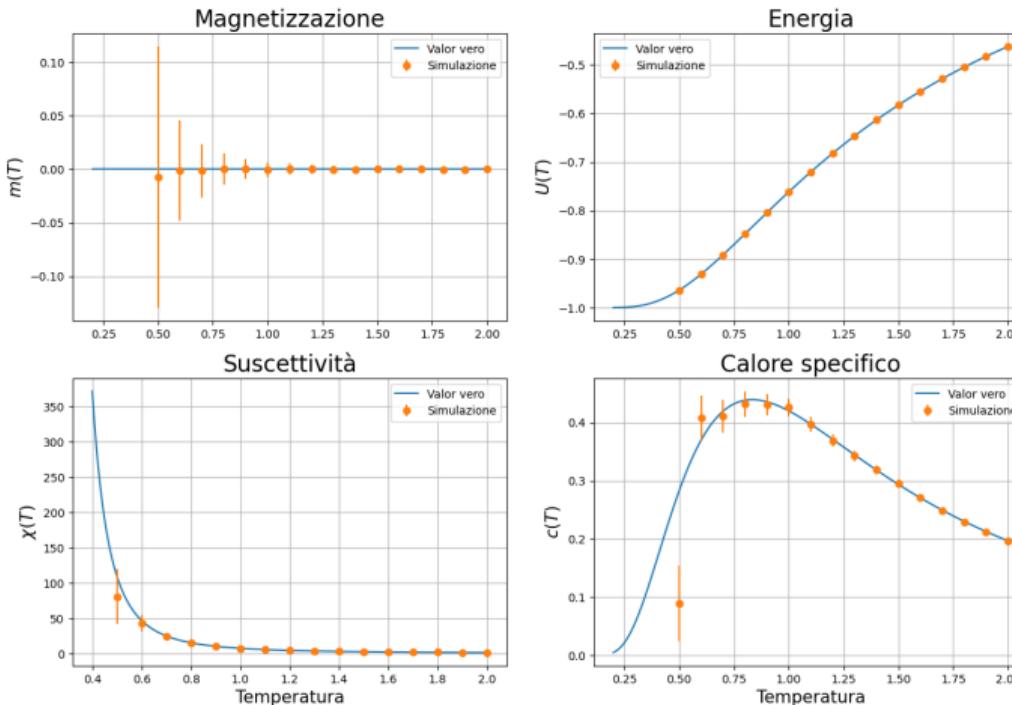
- ▶ Introduzione
- ▶ Metodi numerici
- ▶ Simulazioni modello di Ising 1D
- ▶ Simulazioni modello di Ising 2D
- ▶ Modello XY
- ▶ Conclusioni
- ▶ Backup



Osservabili per $N_s = 1000$, $h = 0.0$

Ising 1D

Ising 1D: $N_s = 1000$, $h = 0.0$

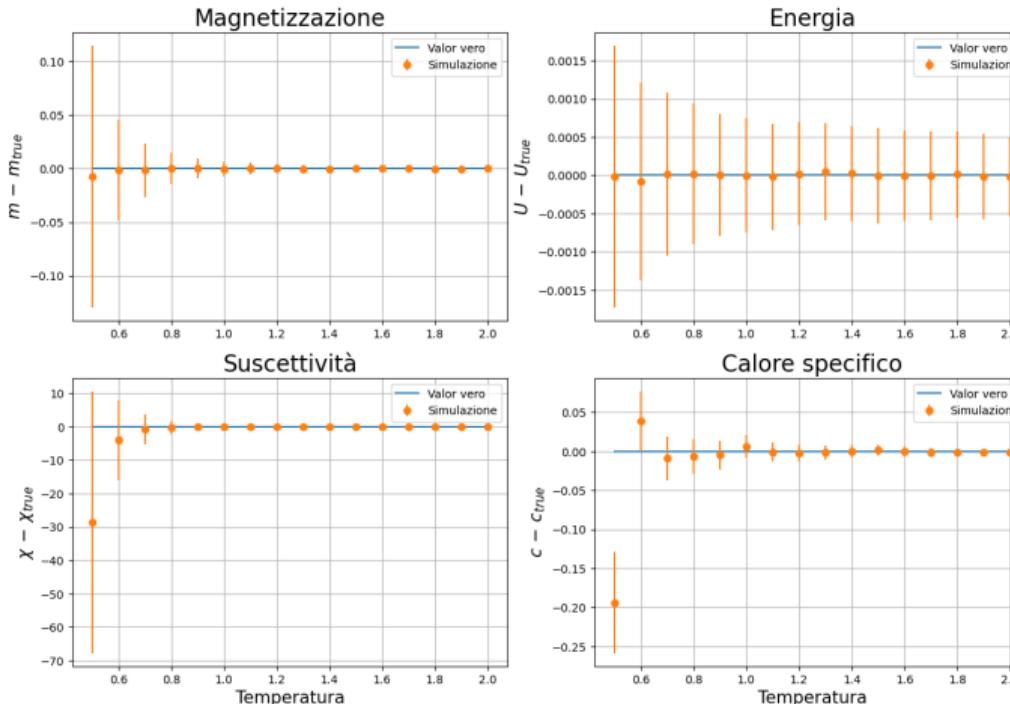




Differenza dal valor vero per $N_s = 1000$, $h = 0.0$

Ising 1D

Ising 1D: $N_s = 1000$, $h = 0.0$

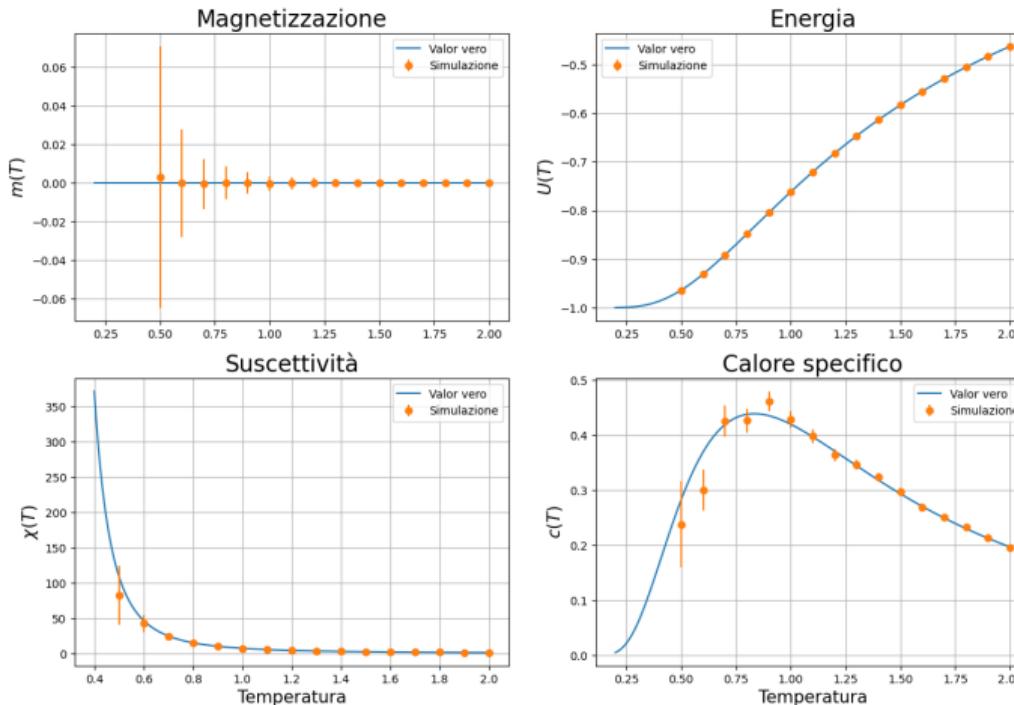




Osservabili per $N_s = 3000$, $h = 0.0$

Ising 1D

Ising 1D: $N_s = 3000$, $h = 0.0$

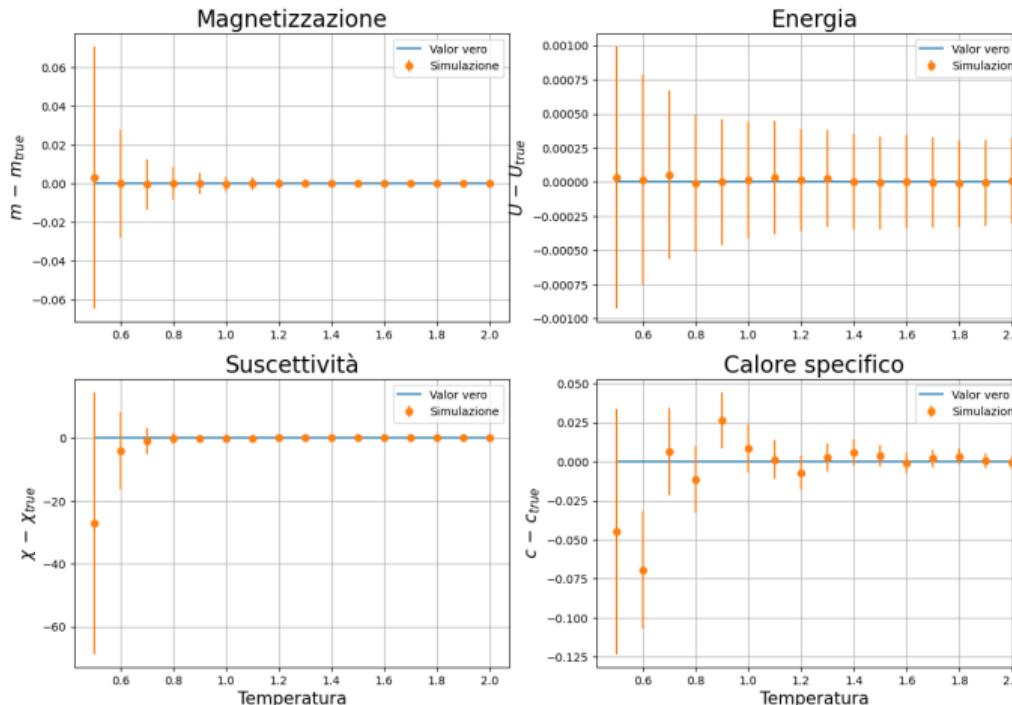




Differenza dal valor vero per $N_s = 3000$, $h = 0.0$

Ising 1D

Ising 1D: $N_s = 3000$, $h = 0.0$

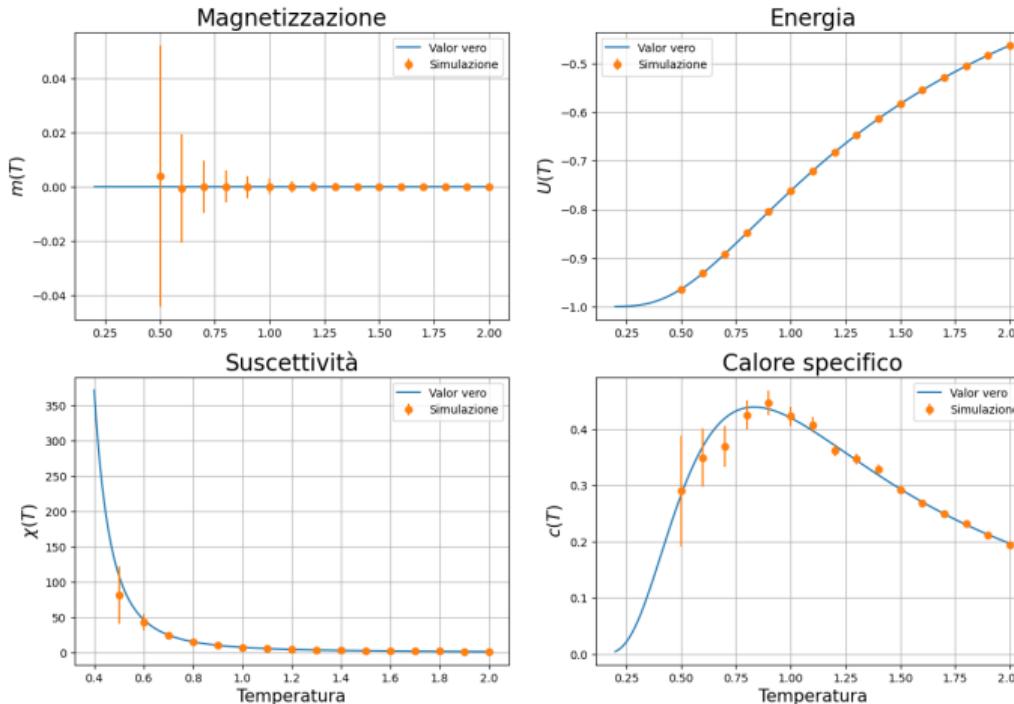




Osservabili per $N_s = 6000$, $h = 0.0$

Ising 1D

Ising 1D: $N_s = 6000$, $h = 0.0$

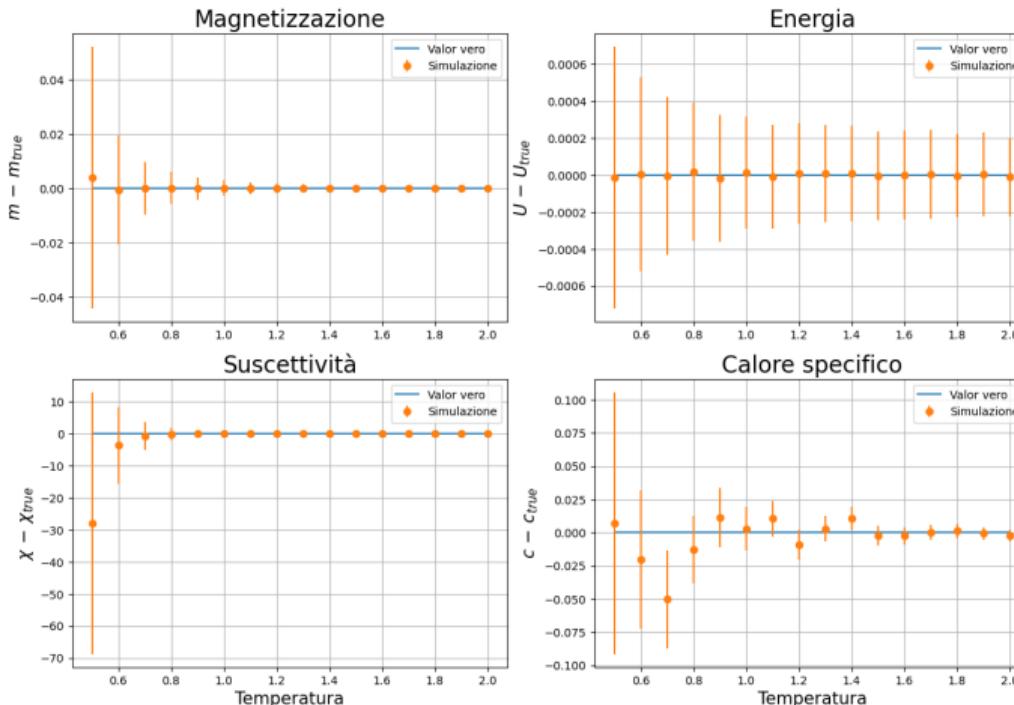




Differenza dal valor vero per $N_s = 6000$, $h = 0.0$

Ising 1D

Ising 1D: $N_s = 6000$, $h = 0.0$

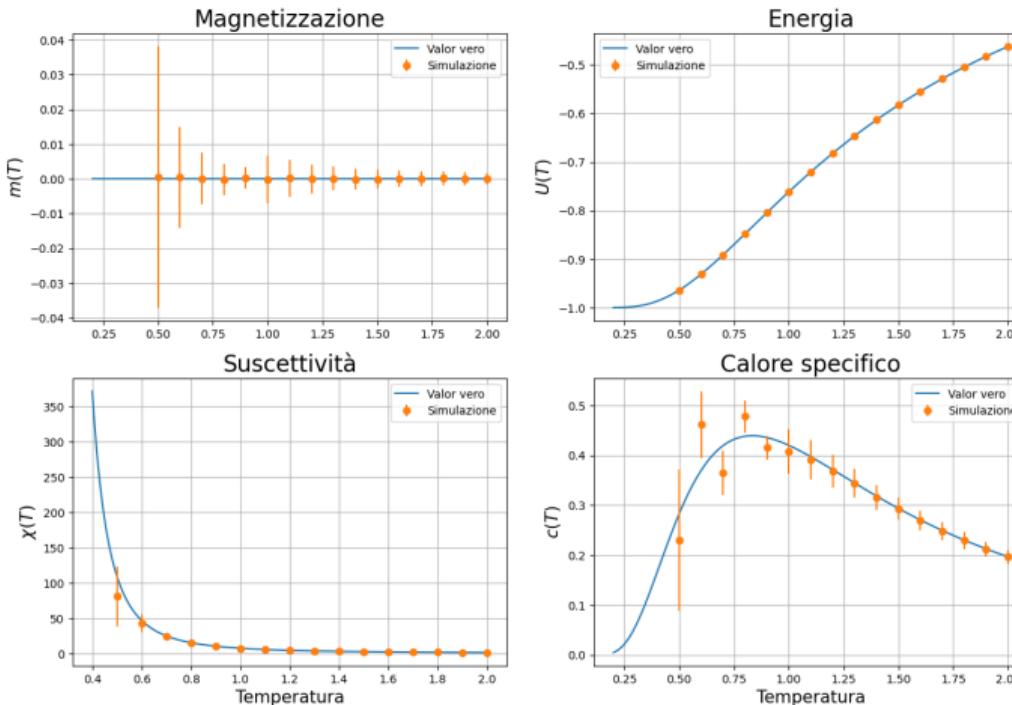




Osservabili per $N_s = 10000$, $h = 0.0$

Ising 1D

Ising 1D: $N_s = 10000$, $h = 0.0$

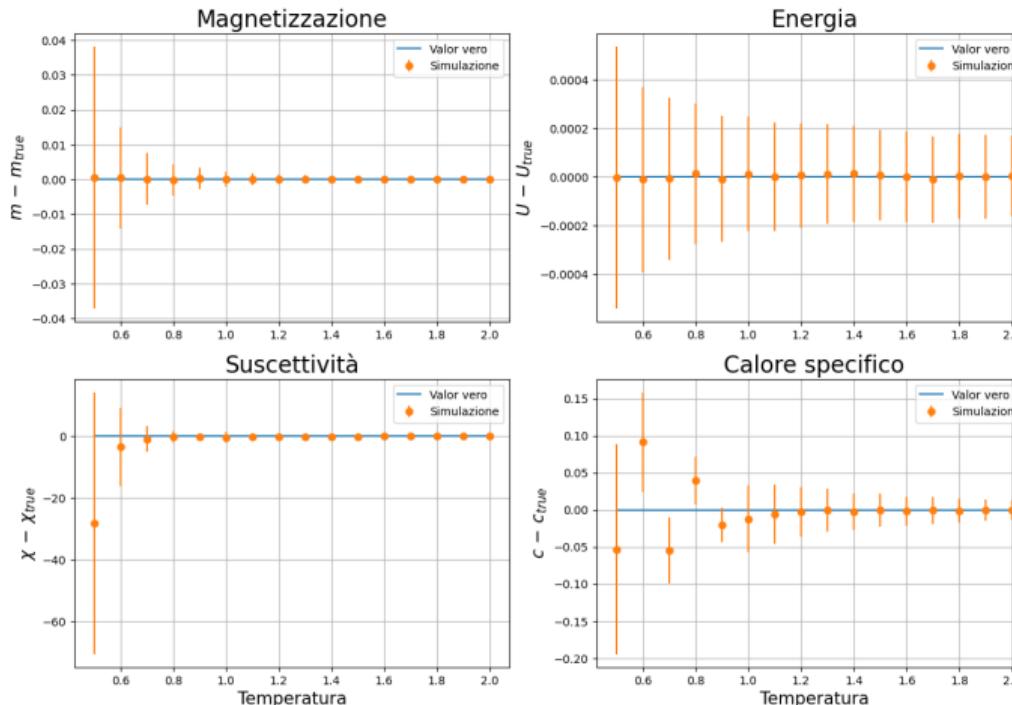




Differenza dal valor vero per $N_s = 10000$, $h = 0.0$

Ising 1D

Ising 1D: $N_s = 10000$, $h = 0.0$

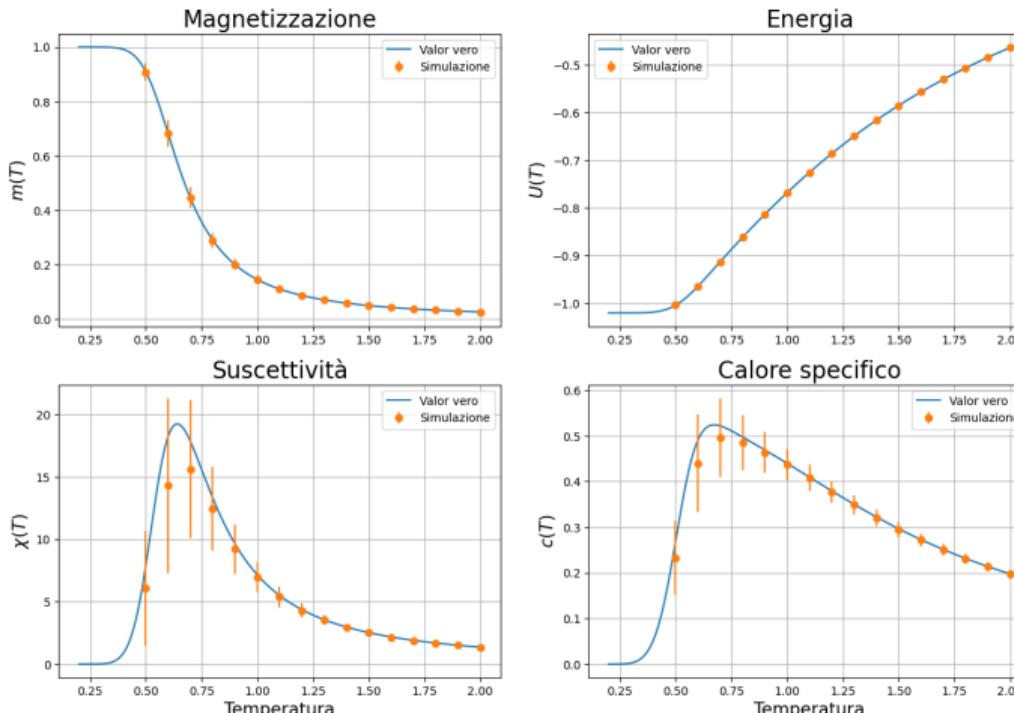




Osservabili per $N_s = 1000$, $h = 0.02$

Ising 1D

Ising 1D: $N_s = 1000$, $h = 0.02$

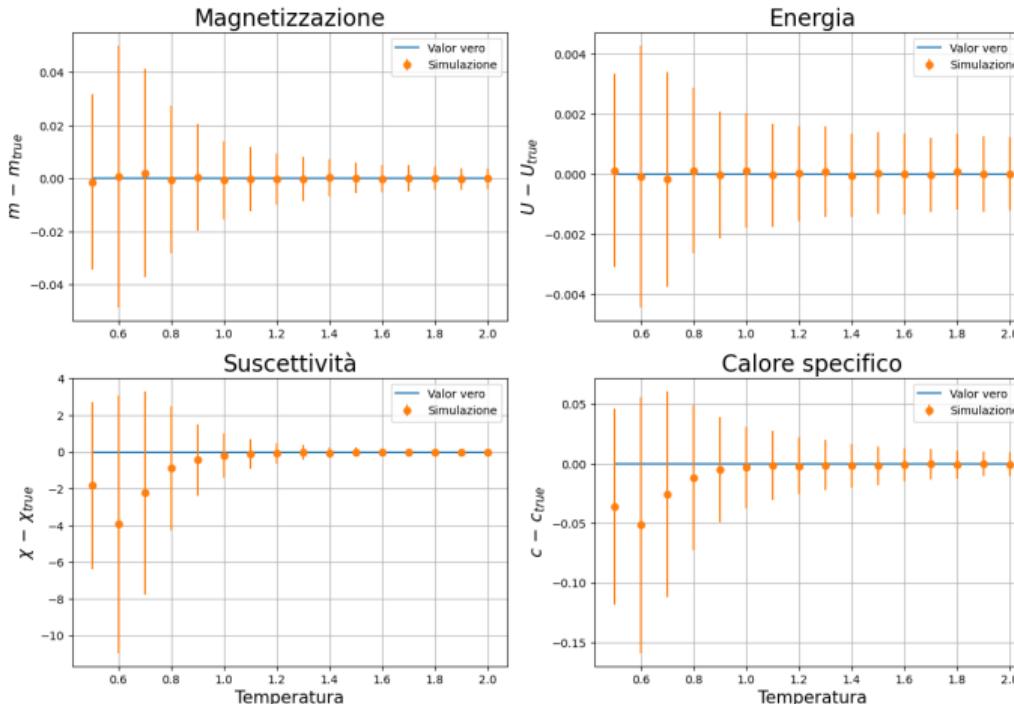




Differenza dal valor vero per $N_s = 1000$, $h = 0.02$

Ising 1D

Ising 1D: $N_s = 1000$, $h = 0.02$

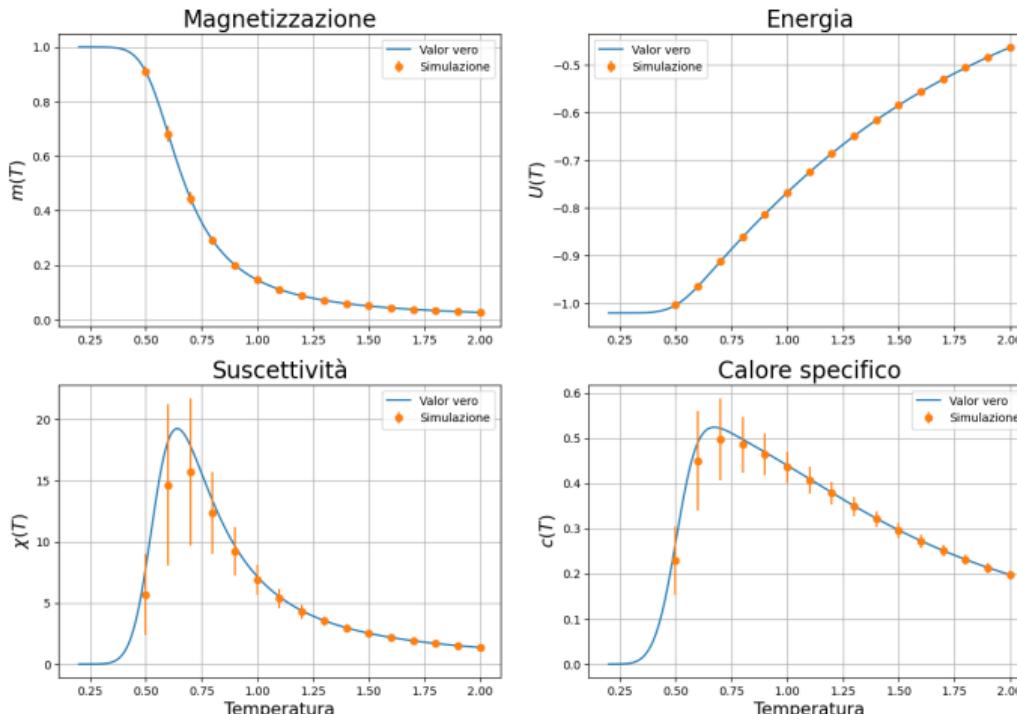




Osservabili per $N_s = 3000$, $h = 0.02$

Ising 1D

Ising 1D: $N_s = 3000$, $h = 0.02$

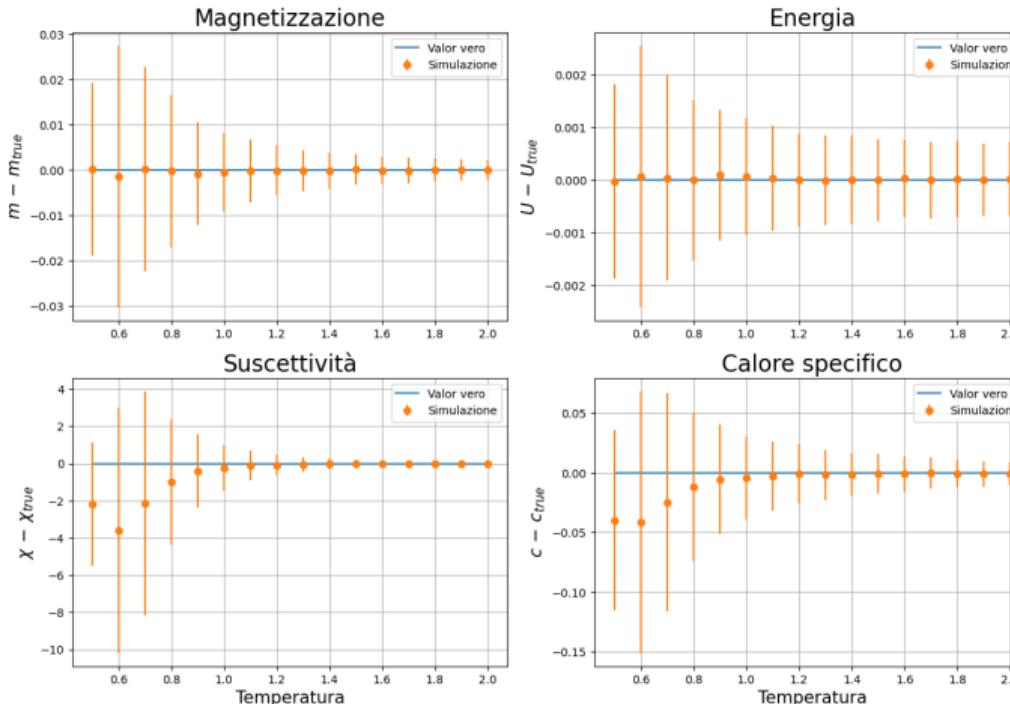




Differenza dal valor vero per $N_s = 3000$, $h = 0.02$

Ising 1D

Ising 1D: $N_s = 3000$, $h = 0.02$

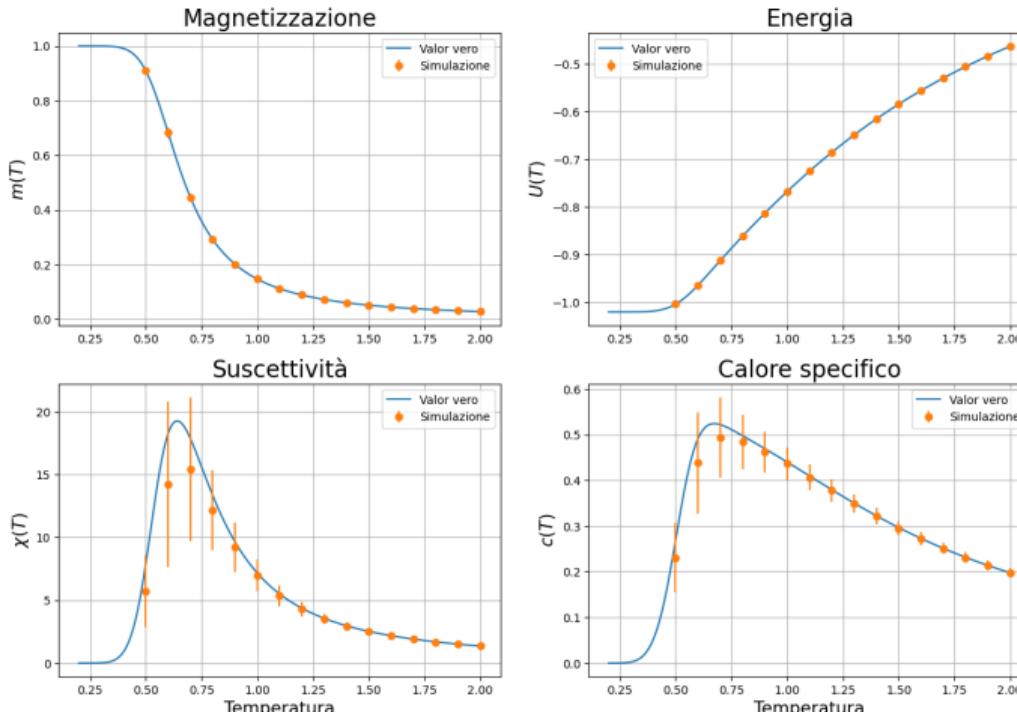




Osservabili per $N_s = 6000$, $h = 0.02$

Ising 1D

Ising 1D: $N_s = 6000$, $h = 0.02$

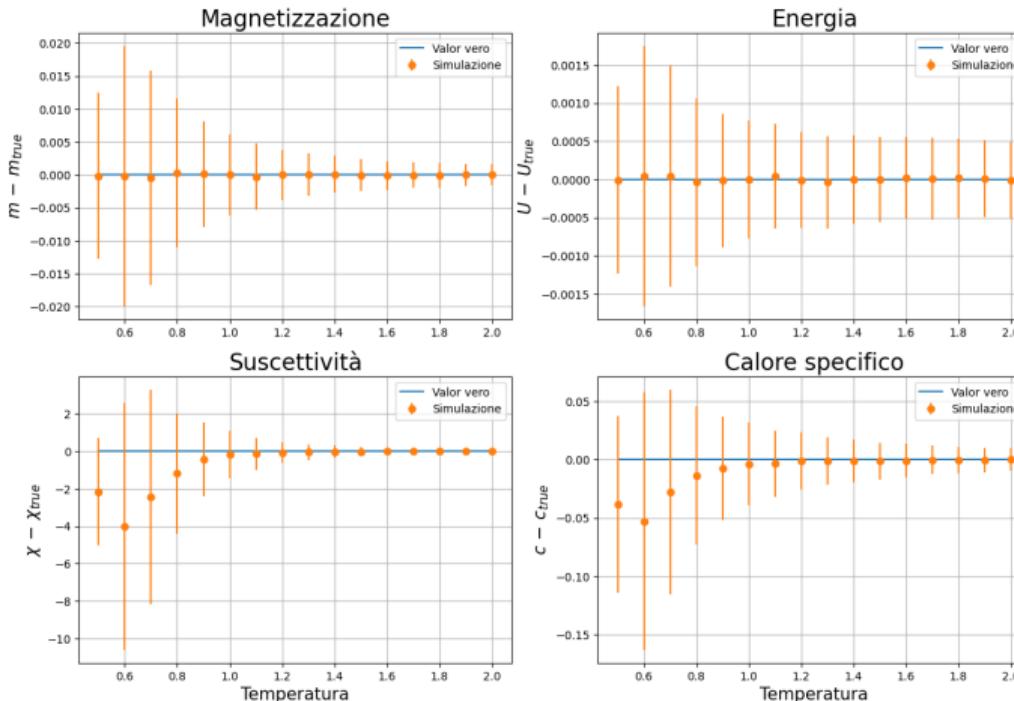




Differenza dal valor vero per $N_s = 6000$, $h = 0.02$

Ising 1D

Ising 1D: $N_s = 6000$, $h = 0.02$

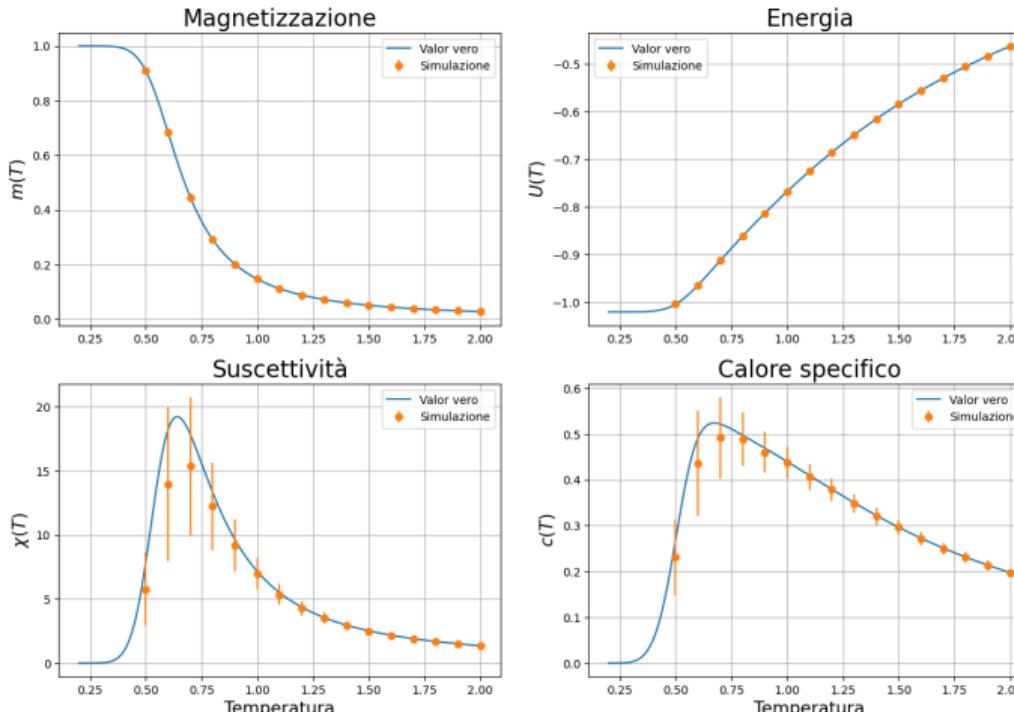




Osservabili per $N_s = 10000$, $h = 0.02$

Ising 1D

Ising 1D: $N_s = 10000$, $h = 0.02$

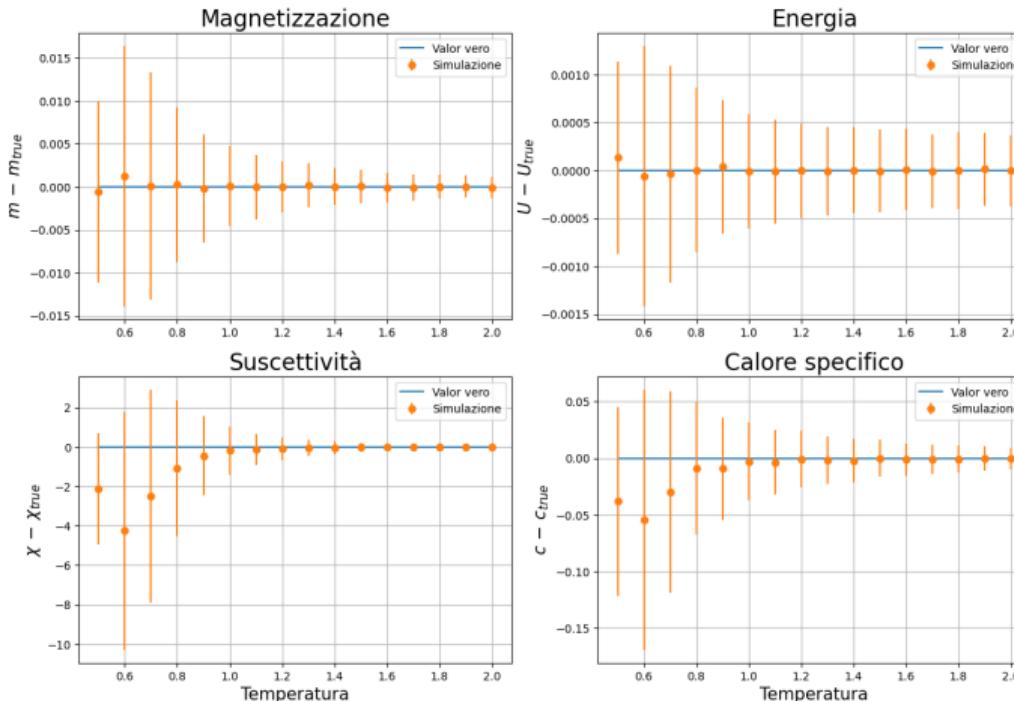




Differenza dal valor vero per $N_s = 10000$, $h = 0.02$

Ising 1D

Ising 1D: $N_s = 10000$, $h = 0.02$

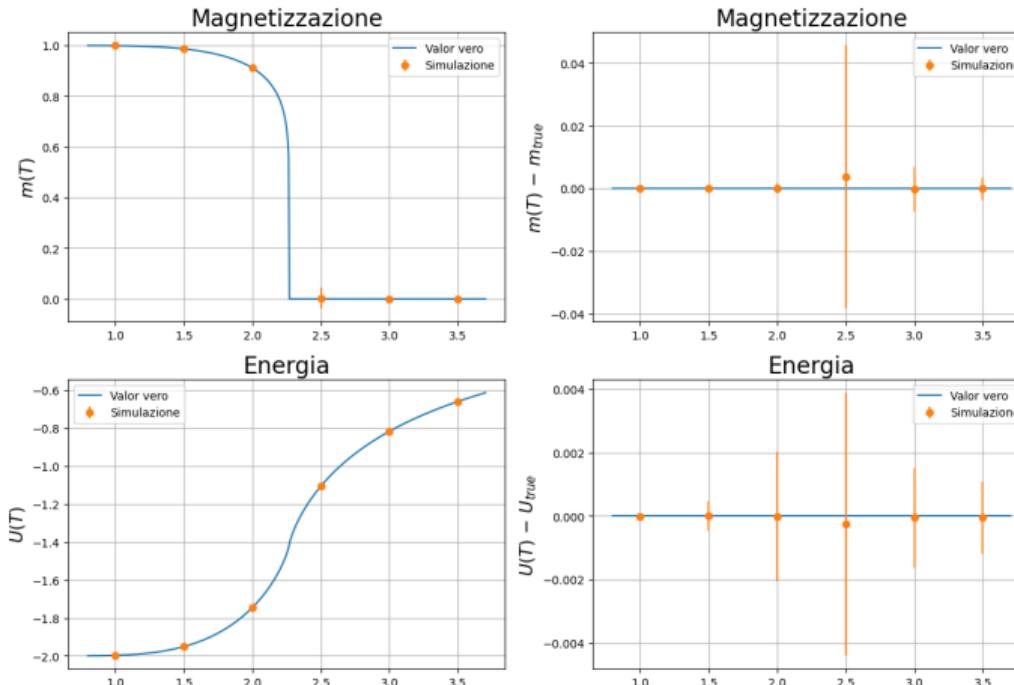




Osservabili per reticolo 100×100

Ising 2D

Ising 2D: reticolo 100×100

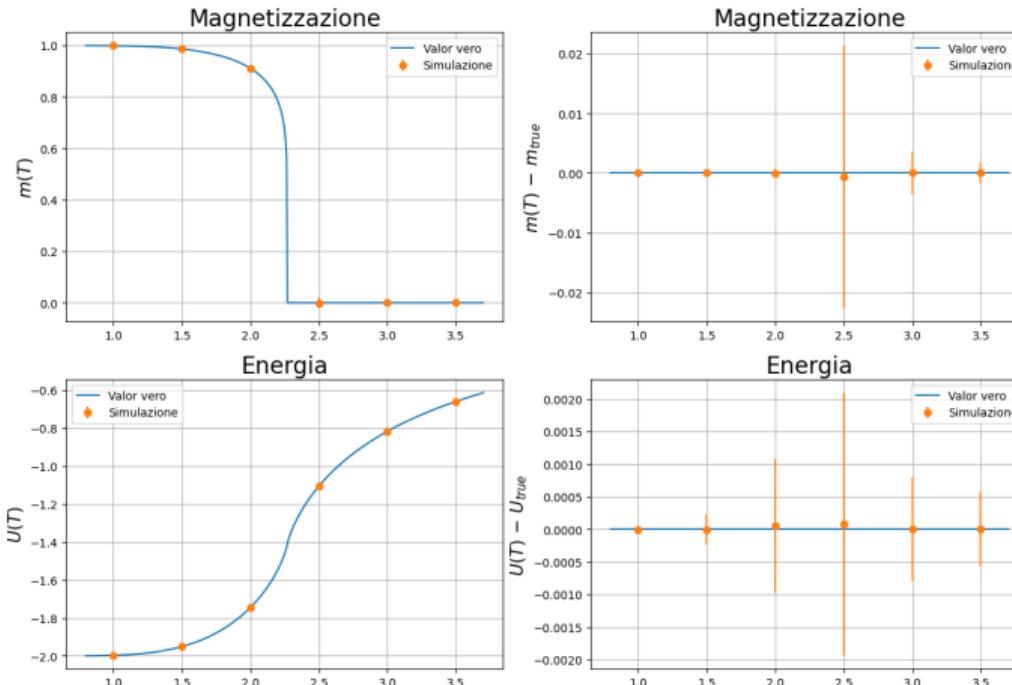




Osservabili per reticolo 200×200

Ising 2D

Ising 2D: reticolo 200×200

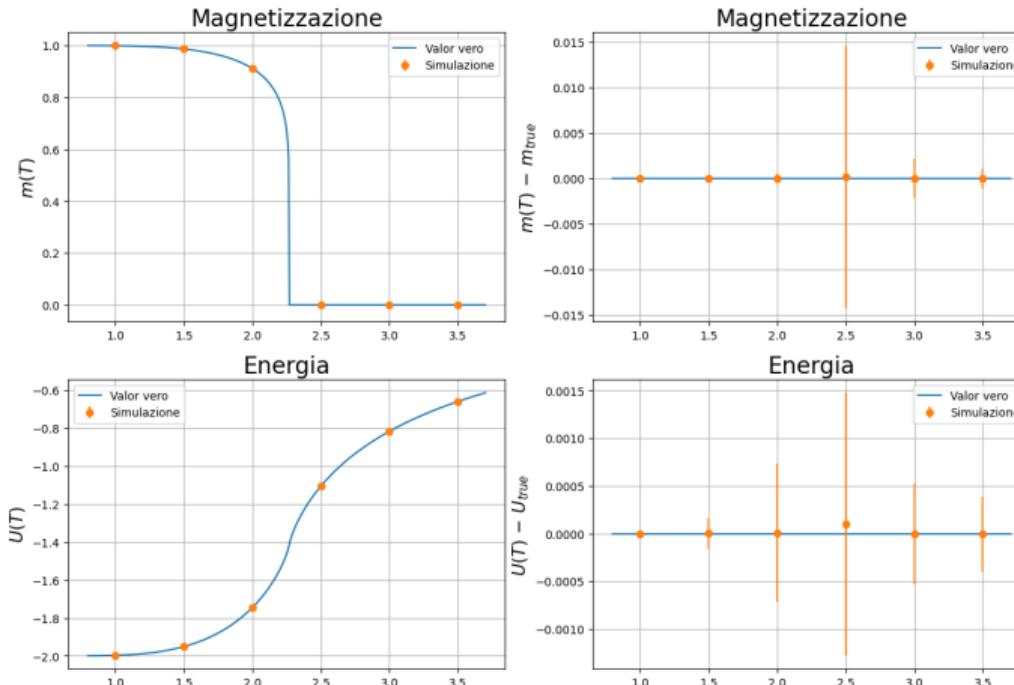




Osservabili per reticolo 300×300

Ising 2D

Ising 2D: reticolo 300×300

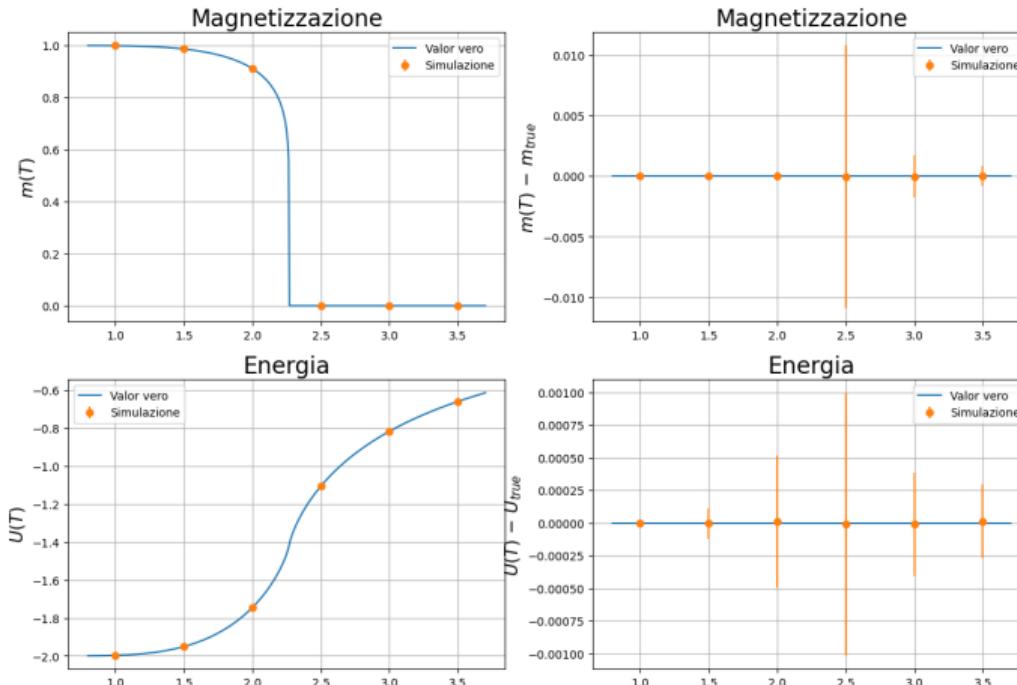




Osservabili per reticolo 400×400

Ising 2D

Ising 2D: reticolo 400×400

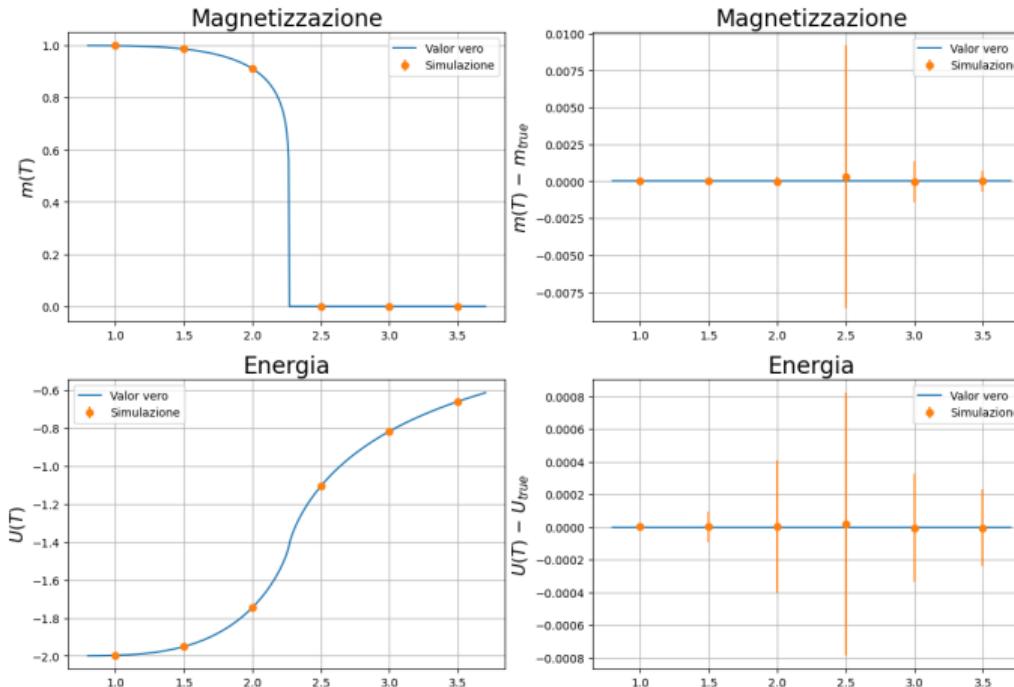




Osservabili per reticolo 500×500

Ising 2D

Ising 2D: reticolo 500×500



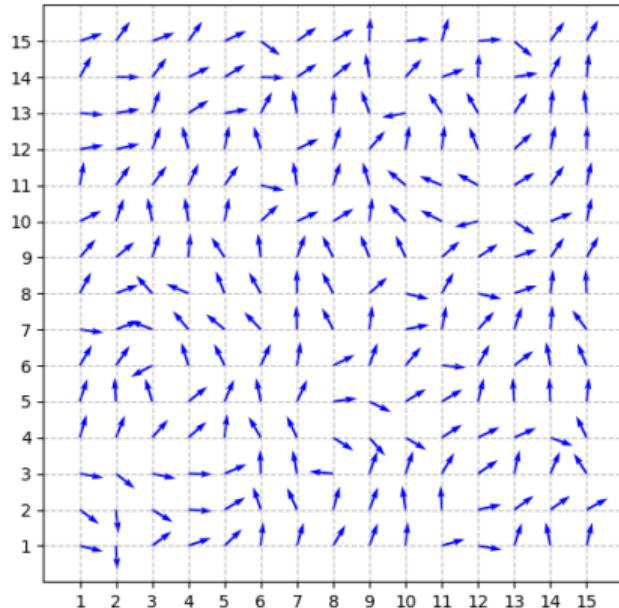


Configurazioni

Modello XY

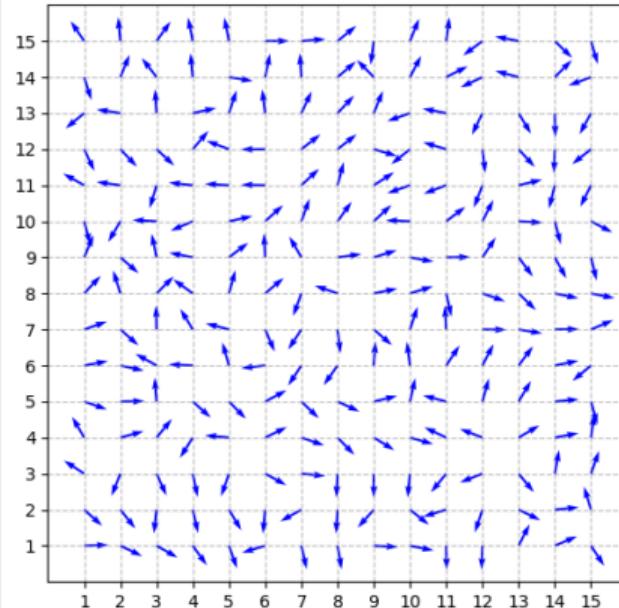
$T = 1.0$

Modello XY: $T = 1.0$



$T = 1.5$

Modello XY: $T = 1.5$



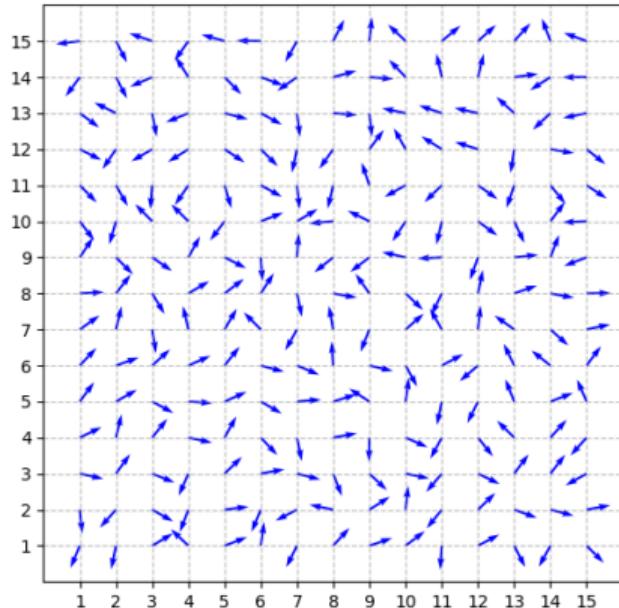


Configurazioni

Modello XY

$T = 2.0$

Modello XY: $T = 2.0$



$T = 3.0$

Modello XY: $T = 3.0$

