Project - Math 310

Neha Mumtaz Jafry (nj05165), Qazi Muhammad Talha(qt05099),
Saad Ahmed Khan (sk04293)

February 15, 2021

All plots are attached after the two page report.
Github Link: Project Repository

Task 1

For the expected distance of a 1-D discrete random walk of n steps in which a step can be either -1 or 1 or, -1, 0 or 1, the mathematical model is simply,

$$E[X] = \Sigma_n E[Z_i]$$

where Z is the discrete random variable for step and X is the discrete random variable for distance from starting point. Here, $E[Z_i] = \Sigma_z t P_Z(t)$.
For the simulation model we have used numpy.random's choice function to generate steps and plotted the position at each step against the number of steps. We saw a pattern that the expected average distance, when the probabilities of going left and going right are equal, the average distance from the initial position tends to zero. A sample run of 10,000 steps with difference scenarios is given in the table in **Figure 2**. As you can see in the table from figure 1 when the probabilities are unequal the expected distance changes in the direction of the biased or greater probability.

Task 2

In this task we took 2 random variables with a specific distance apart. Both of them have discrete steps, either they move right or left. The simulation of this task is shown in **Figure 3**. The histogram in **Figure 4** shows the simulation that we repeated 100 times, from which we noticed that the majority of the average steps were in between 0 and 100. The second histogram in **Figure 5** shows that simulations between 0 and 100 through trial and error.

Task 3

In this task we had to model a motion of a randomly moving particle in a disc. For the expected distance of a 2-D discrete random walk of n steps in we chose the distance r, and the angle θ, as discrete uniform variables that can take values 0, 0.5 and 1 and disrupted interval of -2pi. The coordinates at which the particle is moving inside the circle is calculated using the forumula $x = rcos\theta$ and $y = rsin\theta$. If it collides with the walls of the disc, it re-enters from the opposite co-ordinates of the wall. A sample run of 10,000 steps is given in **Figure 6**

Task 4

For the expected distance of a 1-D continuous random walk of n steps in which a step can be in the range [0,1], the mathematical model is given by,

$$E[X] = \Sigma_n E[Z_i]$$

where Z is the continuous random variable for step and X is the continuous random variable for distance from starting point. Here,
$E[Z_i = \int_0^1 t f_Z(t) = \frac{1}{2}$.
For the simulation model we have used numpy.random's uniform function to generate steps and plotted the position at each step against the number of steps. A sample run is given in **Figure 7**

Task 5

We used the same approach as Task 3 in this task. But this time the angle was uniformly distributed between 0 and 1, and 0 and 2π as a continuous random variable. A sample run steps is given in **Figure 8**

Task 7

In this task again, We used the same approach as Task 3 but now the angle was a uniformly distributed over 0 to 2π, as a continuous random variable. A sample run steps is given in **Figure 9**

Task 8

In task 8, for the random starting points we took random polar coordinates. For the radius r, we took a continuous random variable from 0 to 100. While for the orientation we took another continuous random variable from 0 to 2π. We then calculated the Cartesian coordinates x *and* y from these values.

Then we kept taking steps between 0 and 1 with a random orientation for each node. This was done until the absolute distance between the two nodes became zero. A sample simulation is given below in **Figure 10**
We repeated this simulation for a 100 times to find the average time for the nodes to be within 1 unit of each other. The histogram corresponding to the average is shown below in **Figure 11**. From the histogram we can see that the average number of steps for the nodes to be within 1 unit of each other will be around 100000 steps.

References

- https://www.coursera.org/lecture/probability-theory-statistics/generating-discrete-random-variables-with-python-4UYIY
- http://www.pas.rochester.edu/ stte/phy104-F00/n9/notes-9a.html
- https://www.coursera.org/lecture/probability-theory-statistics/generating-and-visualizing-continuous-random-variables-with-python-A0WoC
- https://medium.com/@dreamume/leetcode-478-generate-random-point-in-a-circle-efc5590c5065

P (Going left)	P (Going right)	Expected Avg distance from initial
0.5	0.5	24.2
0.4	0.6	2006
0.3	0.7	4005.5
0.2	0.8	5968.8
0.1	0.9	8020.333

Figure 1: Task 1

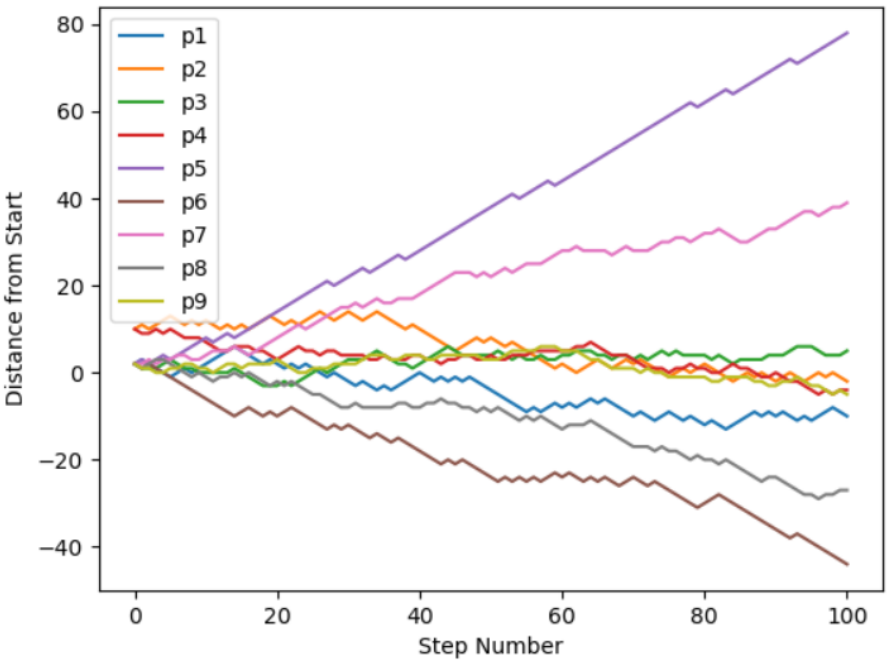

Figure 2: Task 1

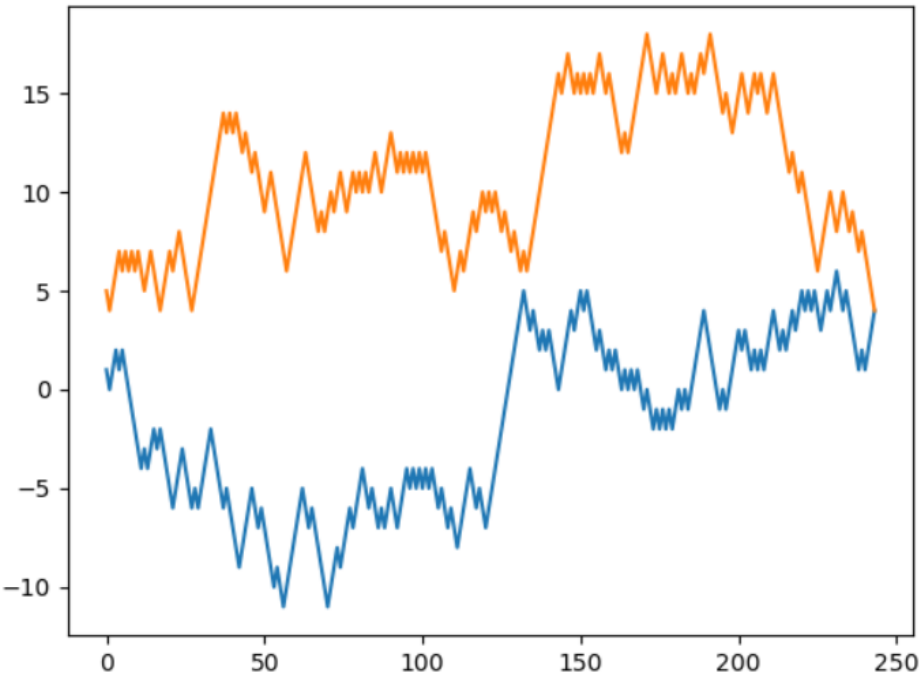

Figure 3: Task 2

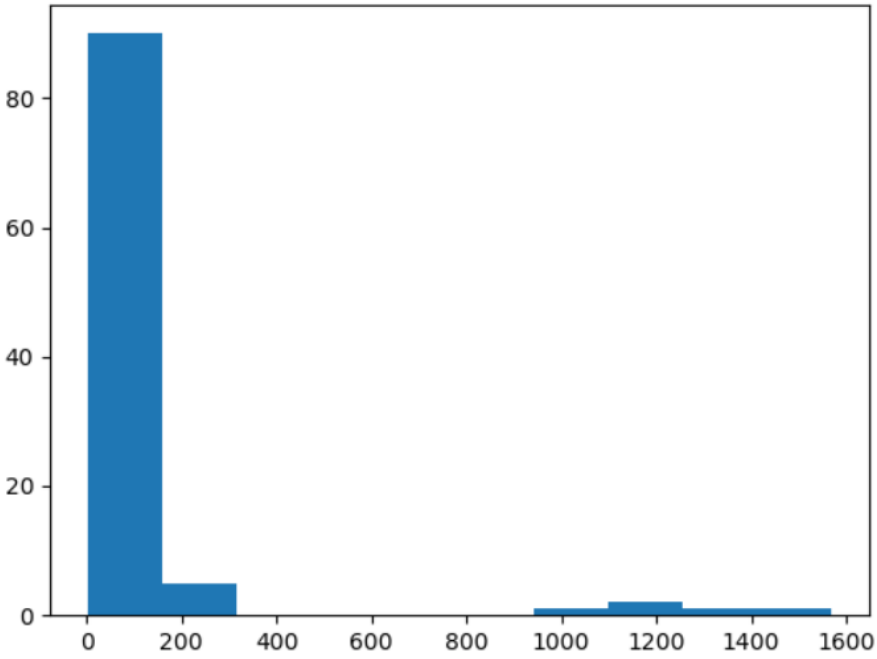

Figure 4: Task 2

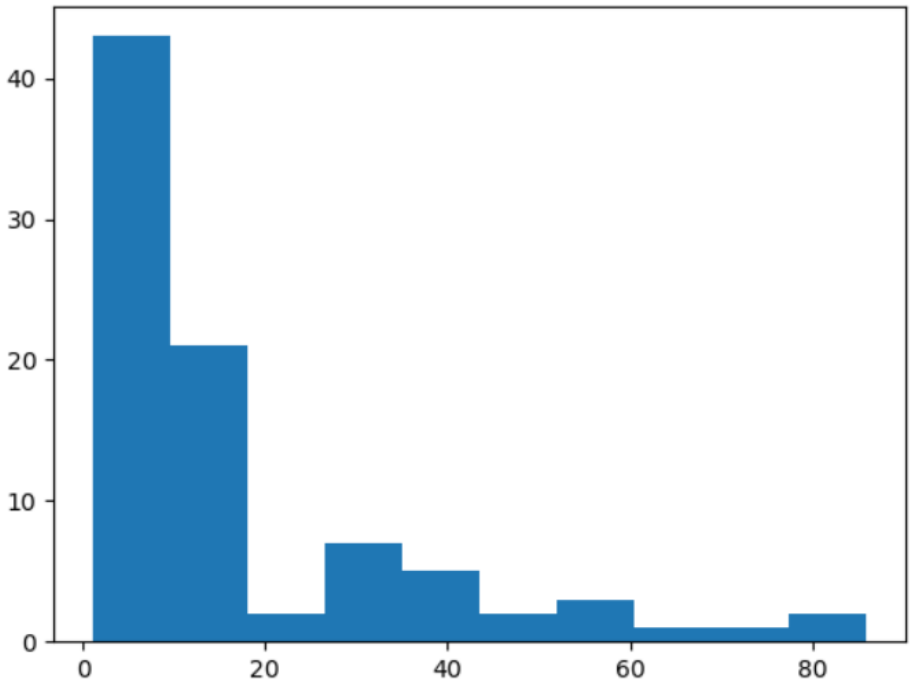

Figure 5: Task 2

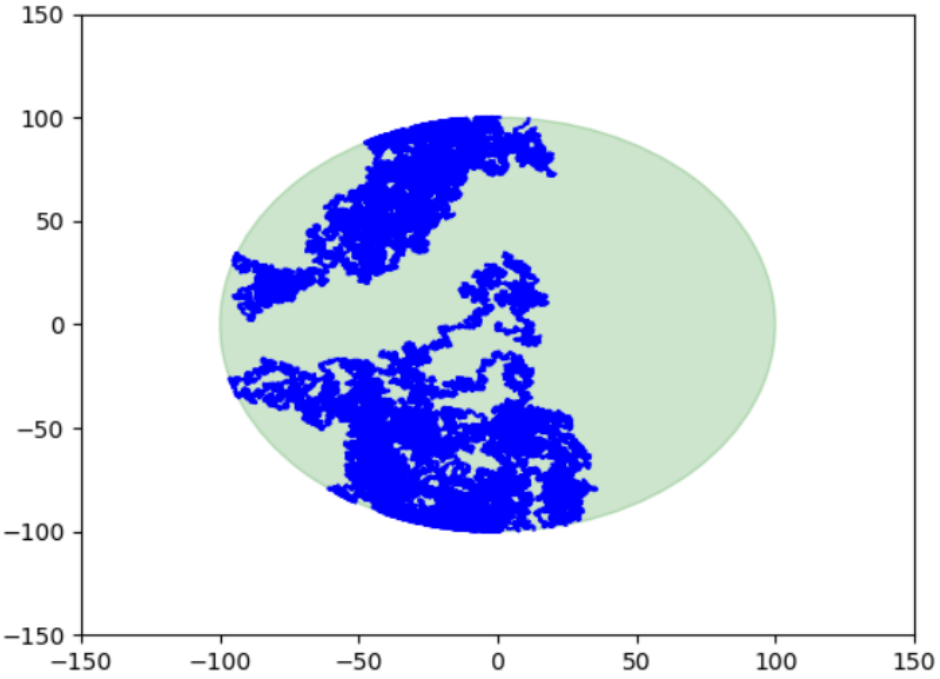

Figure 6: Task 3

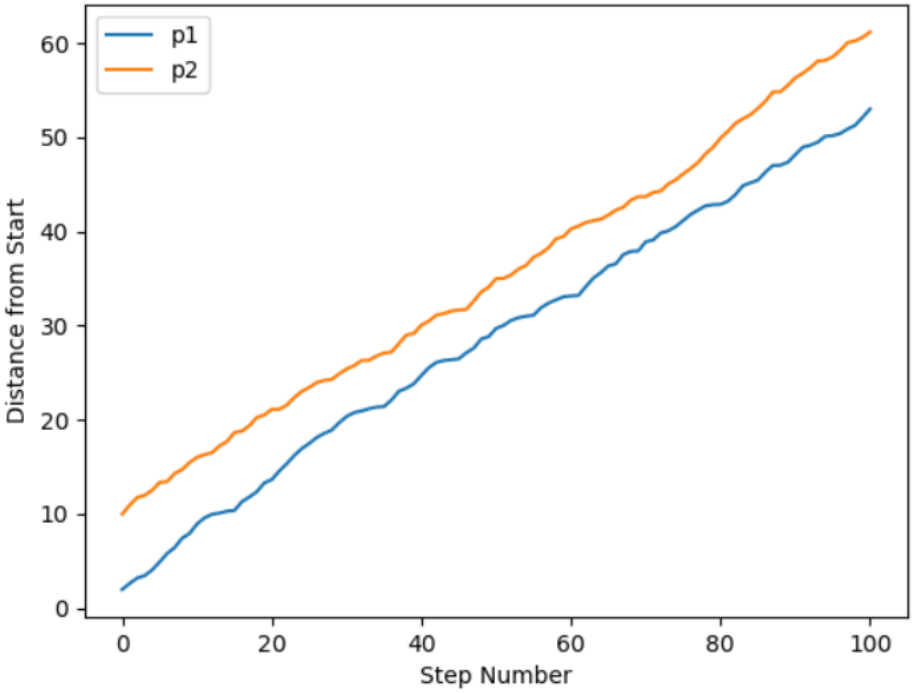

Figure 7: Task 4

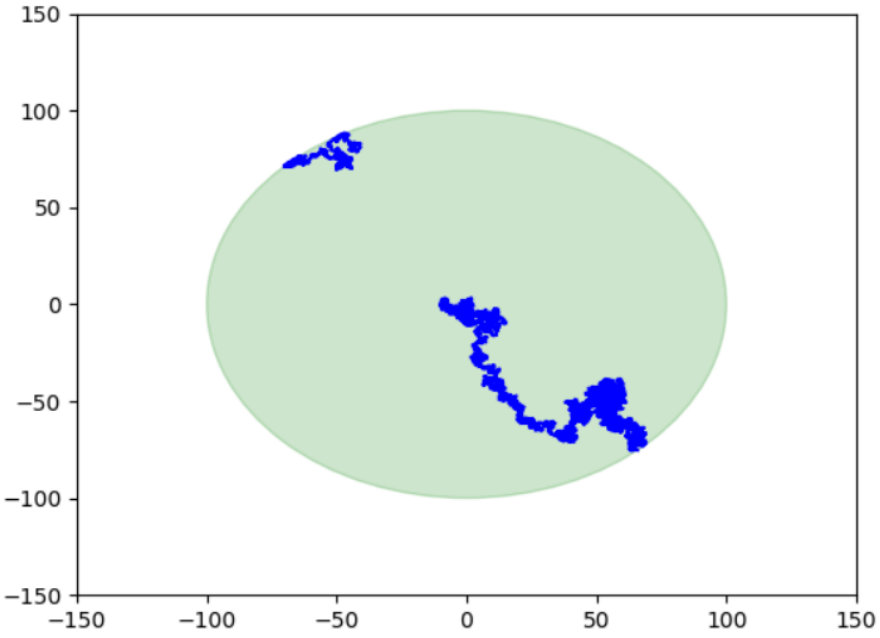

Figure 8: Task 5

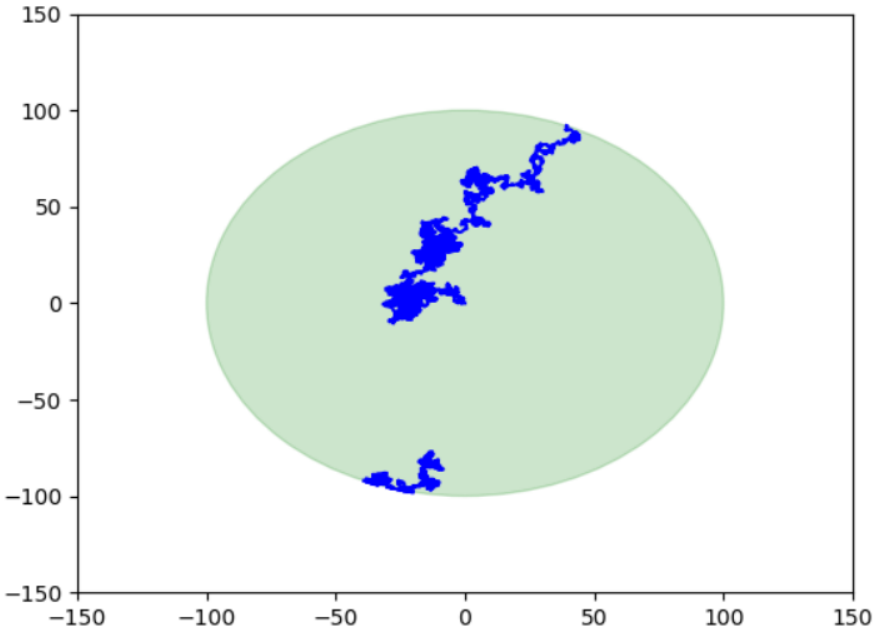

Figure 9: Task 7

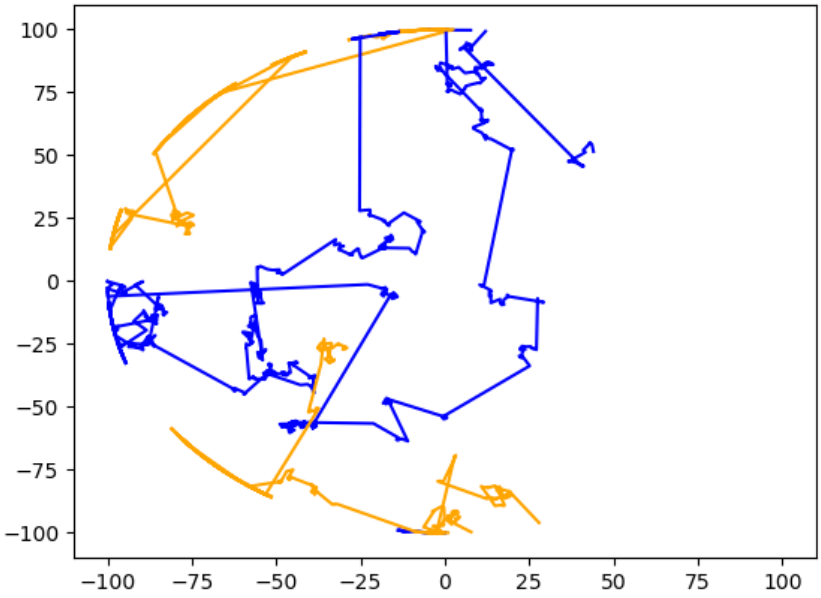

Figure 10: Task 8

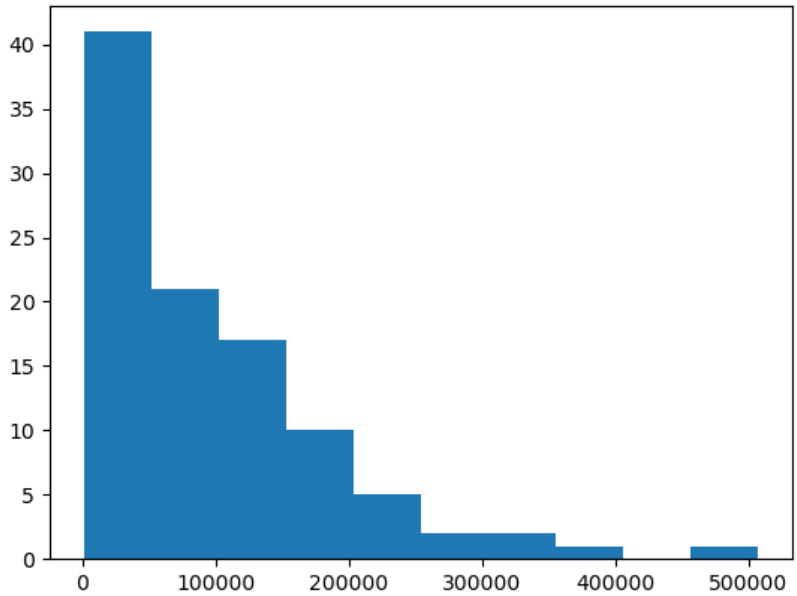

Figure 11: Task 8