Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987
   Ans: B

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ =6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
   Ans: False: There are more employees older than 44 than there are between 38 and 44.

   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.
   Ans: False : If you want to attract employees under 30, you might get around 37 employees, not exactly 36. So, it's not exactly 36, but close to it.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.
   Ans: So, both 2X1 and X1 + X2 follow normal distributions with the same mean (2μ), but 2X1 has a larger variance (4σ^2) compared to X1 + X2 (2σ^2). This means that 2X1 is more spread out compared to X1 + X2, which is a bit narrower.

4. Let X ~ N(100, 20²). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

   Ans: Range (9.900000000000002 – 9.8.1) in Millions

   B. Specify the 5th percentile of profit (in Rupees) for the company

   Ans: The 5th percentile of profit for the company is 17.0 Crore Rupees

   C. Which of the two divisions has a larger probability of making a loss in a given year?

   Ans: The Probability of division #1 making a loss is 4.78 %
   The probability of division #2 making a loss is 4.01 %
   The Division 1 has a larger Probability of making a loss