

Quiz 1

(MA6.102) Probability and Random Processes, Monsoon 2025

29 August, 2025

Max. Duration: 45 Minutes

- Question 1** (5 marks). Consider a family that has two children. Assume that every birth results in a boy with probability $\frac{1}{2}$, independent of other births and also that the parents in the family had decided to have exactly two children. Assume that if a child is a girl, her name will be Lilly with probability α independently from other child's name. If the child is a boy, his name will not be Lilly.
- (a) Given that both children are girls, what is the probability that at least one of them is named Lilly?
 - (b) What is the probability that at least one child is a girl named Lilly.
 - (c) Given that the family has at least one child named Lilly, what is the probability that both children are girls?

Question 2 (5 Marks). Let the sample space be $\Omega = \{-3, -2, -1, 0, 1, 2, 3\}$, and let $X : \Omega \rightarrow \mathbb{R}$ be a function defined as $X(\omega) = |\omega|$.

- (a) For each $x \in \mathbb{R}$, list the events $\{X \leq x\} \triangleq \{\omega \in \Omega : X(\omega) \leq x\}$.
- (b) Determine the cardinality of the smallest σ -field with respect to which X is a random variable.
- (c) Find that smallest σ -field.

Question 3 (5 Marks). Let $F : \mathbb{R} \rightarrow [0, 1]$ be a function satisfying:

- If $x < y$, then $F(x) \leq F(y)$.
- $\lim_{x \rightarrow -\infty} F(x) = 0$ and $\lim_{x \rightarrow \infty} F(x) = 1$.
- $\lim_{\epsilon \downarrow 0^+} F(x + \epsilon) = F(x)$, for every $x \in \mathbb{R}$.

Show that there exists a probability space (Ω, \mathcal{F}, P) and a random variable $X : \Omega \rightarrow \mathbb{R}$ such that the cumulative distribution function (CDF) of X is equal to F , i.e., $P(X \leq x) = F(x)$, for all $x \in \mathbb{R}$.

Hint: $\Omega = \mathbb{R}$.