

COMPUTING IN SCIENCES – II

QUIZ

Newton-Raphson Method:

The Newton-Raphson method is a widely used numerical technique to approximate the roots of a real-valued function. It begins with an initial guess for the root and improves it step by step using the slope (derivative) of the function.

The basic idea is to look at where the tangent to the function at the current guess intersects the x-axis. That intersection becomes the new guess. Repeating this process gradually leads us closer to the actual root, provided the initial guess is reasonable and the function behaves well in that region

Based on this, answer the following questions:

Q1. Graphically demonstrate the working of the Newton-Raphson method on a chosen function, showing the progression of approximations toward the root.

Q2. Given a function $f(x)$ and its slope $f'(x)$, derive the expression for the next approximation x_{n+1} in terms of the current approximation x_n , using this concept.

Q3. Let $f(x) = x^2 - 2$ and take the starting value $x_0 = 1.5$

Using the Newton-Raphson method, derive the next two approximations x_1 and x_2

Q4. Show that if x_0 is chosen close to the actual root of the function $f(x)$, then x_1 , obtained using the Newton-Raphson method, is a better approximation to the root.

Hint: Use the Taylor expansion of $f(x)$ around the root to support your argument.