Classical Mechanics(H1) (SC1.102)

IIIT-H, Semester Winter 25, Quiz 1

Full Marks: 30, Duration: 45min, January 31st, 2025

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1. Two identical pendulums, each with length ℓ and mass m, are suspended from the same height at a horizontal distance d from each other. Both pendulums oscillate in a vertical X-Y plane. The two masses are connected by a massless spring with spring constant k. Answer the following questions:

   (a) Write down the constraint equations for the system. **[3]**

   (b) What is the number of degrees of freedom of the system? **[1]**

   (c) Choose appropriate generalized coordinates for the system. **[3]**

   (d) Write the Lagrangian of the system. **[6]**

   (e) Write the Euler-Lagrange equations of motion. (You do not need to solve the equations.) **[3]**

2. Consider two points in the X-Y plane that are separated by a distance. Prove that the shortest distance between these two points is a straight line. *Hint: Begin by expressing the distance between the two points. Then, consider the total path length between the points and transform the problem into a variational form to minimize the distance.* **[10]**

3. Suppose there is a Lagrangian $L(q, \dot{q}, t)$, and a function $F(q, t)$, which is a differentiable function of the generalized coordinates q. We define a new Lagrangian as

$$L'(q, \dot{q}, t) = L(q, \dot{q}, t) + \frac{dF(q, t)}{dt}.$$

Prove that the Lagrangians $L'(q, \dot{q}, t)$ and $L(q, \dot{q}, t)$ lead to the same equations of motion. *You may construct the actions for the two Lagrangian and then think. Another way is to directly apply the Euler-Lagrange EoM.* **[4]**