Classical Mechanics(H1) (SC1.102)

IIIT-H, Semester Winter 25, ~~Midterm~~ Exam (H1 End)

7 questions, Full Marks: 60, Duration: 120 minutes

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1. A particle moves in a plane along a curve $y = y(x)$ with velocity $v = \lambda x$ (λ is a constant with dimension of inverse time). It goes from point $(0, 0)$ to a point $(1, 1)$ such that the travel time is minimum. Find the equation of motion of the path. *Hint: You may apply calculus of variation. The $\lambda = 1$ can be assumed.* [6]

2. A block of mass M can move on a frictionless horizontal surface along the x-direction. A simple pendulum of mass m and massless wire of length ℓ is suspended from the block. *i)* How many degrees of freedom does the system have? *ii)* Write down the Lagrangian of the system with suitable generalized coordinates assuming gravitational acceleration g. *iii)* Show that for small amplitude oscillation (in vertical plane) the time period of the pendulum is $T = 2\pi\sqrt{\frac{\ell}{g}}\sqrt{\frac{M}{M+m}}$ [1+3+5]

3. A particle moves under a central force about the point $r = 0$. The equation of the orbit is given by $r = e^{-\theta}$. Show that the force is inversely proportional to r^3. Obtain the total energy of the system. [5+1]

4. An object is in motion under a central force $U(r)$ such that the force is $F(r) = -3a/r^4$. Draw the effective potential $V'(r) = \frac{\ell^2}{2\mu r^2} + U(r)$ as a function of r and indicate the r_{min} and r_{max} of the motion. *i)* Draw the trajectories of the particle (in $r - \theta$ plane) in the following scenarios *ii)* the motion starts at a distance $r > r_{\text{max}}$, and *iii)* it starts at a distance $r < r_{\text{min}}$. *iv)* Is motion allowed for $r_{\text{min}} < r < r_{\text{max}}$? [3+2+2+2]

5. Consider two identical pendulums (of mass m and length ℓ) suspended from a ceiling at a distance r from each other. The masses are connected by a spring of force constant k. The system oscillate in a vertical plane such that the angular displacement of each pendulum is very small. *i)* Write down the Lagrangian of the system and obtain the equations of motion, *ii)* obtain the normal frequencies, *iii)* explain the motion for each of the normal frequencies, and *iv)* obtain the normal modes. [2+4+4+4]

6. There is a mass point m that is constrained to move on the surface of cylinder of radius R whose axis is along the z direction. The Lagrangian of the system in cylindrical coordinate is

$$L = \frac{1}{2}m(R^2\dot{\theta}^2 + \dot{z}^2) - \frac{1}{2}k(R^2 + z^2),$$

where k is a constant. *i)* Obtain the generalized momentum corresponding to θ and z. *ii)* Write down the Hamiltonian of the system and obtain the Hamilton's equations of motion. *iii)* Show that the angular momentum about the z-axis is a constant of motion. *iv)* Show that it executes simple harmonic motion along the z direction. [1+3+2+3]

7. A tennis ball is dropped from height h. *i)* Assuming no energy loss during the collision with the surface, draw the phase space diagram with height on the horizontal axis and momentum on the vertical axis. *ii)* Assume a fraction of energy is lost in each collision. Draw the phase space diagram again. [3+4]