$H(Y)$ ~~$H(Y|X)$~~

IIIT-H
EC5.102: Information and Communication
Summer-2025

Exam: End-Sem examination — Date: 1-May-2025

Marks: 100 — Time: 9am to 12pm

Instructions:

- Answering ALL the questions is compulsory. $H(Y) = p(Y) \log \frac{1}{p(Y)}$
- State the question number correctly and clearly. $P(Y=0) = \frac{(1-p)}{2}$ $P(Y=1) = \frac{(1-p)}{2}$
- Calculators are allowed.
- Note that the questions have varying number of points. Manage your time appropriately. $P(Y=?) = p$
- All steps should be justified in detail.
- Clearly state the assumptions (*if any*) made that are not specified in the questions.

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1. Answer whether the following sentences are true or false, without justification. [$10\times1=10$]

   (i) The set of vectors $\{00, 01, 10, 11\}$ cannot be a linear block code but it can be a block code.

   (ii) Source code $\{110, 11, 10\}$ is not uniquely decodable.

   (iii) Consider a linear block code $\mathcal{C}$ with generator matrix $G \in \mathbb{F}_2^{k\times n}$ and parity check matrix $H \in \mathbb{F}_2^{(n-k)\times n}$. Then the rows of G provide a basis of code $\mathcal{C}$ and the columns of H provide a basis for the dual code $\mathcal{C}^{\perp}$.

   (iv) $I(X;Y) = H(X,Y) - H(X) - H(Y)$.

   (v) Hamming code of length 15 can correct all error patterns of weight 1.

   (vi) Fourier transform of the signal $m(t) = 30\sin(300\pi t)$ is conjugate symmetric.

   (vii) Consider a Huffman code designed for $\mathcal{X} = \{a, b, c\}$ with pmf $[0.3 \;\; 0.2 \;\; 0.5]$. This code will be unique.

   (viii) The random variable with the pmf $[0.3 \;\; 0.25 \;\; 0.25 \;\; 0.2]$ contains 2 bits of information.

   (ix) Capacity of a binary symmetric channel is equal to $1 + p\log_2 p + (1-p)\log_2(1-p)$.

   (x) Consider an antenna emitting an electromagnetic wave. The distance covered by an electromagnetic wave is inversely proportional to its frequency.

2. The following questions carry 8 points each. [$8\times5=40$]

   (i) A message signal $m(t) = 10\cos(5000\pi t) + 8\cos(2000\pi t)$ is used to modulate a carrier signal of the form $c(t) = 20\cos(100000\pi t)$ using DSB-SC modulation.

      (a) Write the mathematical expression for the modulated signal.

      (b) Determine the bandwidth of the modulated signal.

      (c) Sketch or describe the frequency spectrum of the modulated signal, labeling key frequencies.

$\cos(\omega t) + \sin(\omega t) \Rightarrow \sqrt{2}\left(\sin\frac{\pi}{4}\cos(\omega t) + \sin(\omega t)\cos\frac{\pi}{4}\right)$

$\Rightarrow \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)\cos(\omega t) + \sin\left(\frac{\pi}{4}\right)\sin \omega t\right) \Rightarrow \sqrt{2}\cos\left(\omega t - \frac{\pi}{4}\right.$

(ii) How many distinct linear block codes of length $n = 6$ and $k = 4$ are possible? Justify your answer in detail.

(iii) Describe BPSK and QPSK modulation schemes. Elaborate on the phase changes for the carrier wave for these modulation schemes.

(iv) Write down the precise statement of Shannon's source coding theorem.

(v) Consider a linear block code C with generator matrix G given below.

$\cos\left(\omega t - \frac{\pi}{4}\right) = \cos\cos + \sin\sin$

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

$\cos\left(\frac{3\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

$\cos\left(\omega t + \frac{\pi}{4}\right) = \cos\cos - \sin\sin$

(a) What is the rate of this code?

(b) What is the error correction capability of this code?

(c) Write down the set of codewords in the dual code $C^{\perp}$ of this code.

3. The following questions carry 10 points each. [10×5=50]

(i) Show that for prefix codes, $L(C) \geq H(X)$.

Kraft:

$\sum \frac{1}{2^{l_i}} \leq 1$

(ii) Consider a code with parity check matrix H given below. For this code, decode the received vector $\mathbf{y} = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$ using standard array decoding if the cross-over probability of the binary symmetric channel is $p = 0.75$.

$$H = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

(iii) Determine the Fourier transform of $x(t) = e^{-3t}u(t)$ and sketch the following:

(a) $|X(\omega)|$

(b) $\angle X(\omega)$

(c) $Re\{X(\omega)\}$

(d) $Im\{X(\omega)\}$

(iv) Derive the capacity of a binary erasure channel with erasure probability p.

(v) The following table shows current undergraduate and MEng enrollments for the School of Engineering.

$y = \frac{1}{\sqrt{9+x^2}}$

$y^2 = \frac{1}{9+x^2}$

Course (Department)	number of of students	probability
I (Civil & Env.)	121	0.07
II (Mech. Eng.)	389	0.23
III (Mat. Sci.)	127	0.07
VI (EECS)	645	0.38
X (Chem. Eng.)	237	0.13
XVI (Aero & Astro)	198	0.12
Total	1717	1

$y = \frac{-x}{9+x^2}$

$\frac{dy}{dx} = \frac{(-1)(9+x^2) - (-x)(2x)}{(9+x^2)^2}$

(a) Determine which student department provides the least amount of information.

(b) Design a variable-length Huffman code to minimize the average number of bits used to encode department data. Provide the Huffman tree and encodings.

(c) Compute the average message length when encoding departments for groups of 100 randomly chosen students.

$-9 - x^2 + 2x^2 = 0$

$= -9 + 3x^2 = 0$

$3x^2 = 9$

$\frac{3}{9+9} = \frac{3}{18} = \frac{1}{6}$