SECTION A

1. Answer the following questions: **(True /False)**

(a) For a limear transform **T:** $V \to V$, dim (**Ker(T)**)-dim(**Im** (T)) = dim (V) --**True /False**
(b) All Hermitian matrices are unitary --**True /False**
(c) If a square matrix **A** has a zero row, then **detA=1--True /False**
(d) If **A** is a Hermitian matrix, then the eigen values of A are complex--**True /False**
(e) The identity matrix of any order is positive definete--**True /False**
(f) Singular values of orthogonal matrix are always equal to 1--**True /False**
(g) The right singular vector represents perpendicular distance from the data point to the best fit line --**True /False**
(h) If A is orthogonally diagonalizable, then A is skew symmetric --**True /False**
(i) Any set of **m** vectors in $\mathbf{R}^n$ is linearly dependent if m>n --**True /False**
(j) If the matrix A is invertible, then the system of equation **Ax=0** has only trivial solution--**True /False**

[1x10=10]

SECTION B

2. Answer the following questions:

(a) If **A** is similar to **B**, then show that $\mathbf{A}^T$ is similar to $\mathbf{B}^T$.

(b) Find the conjugate transpose of the matrix **A**= $\begin{pmatrix} 1+i & -i & 1+5i \\ 1 & 4-i & 11 \\ 3+7i & -9i & 4-3i \end{pmatrix}$

(c) Find the inverse of the elementary matrix $\begin{pmatrix} 1 & 0 & 0 \\ -3/5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(d) Prove that the distance **d(u,v)**= $\sqrt{\|u\|^2+\|v\|^2}$ if and only if **u** and **v** are orthogonal.

(e) Is the matrix **A**= $\begin{pmatrix} 1 & 1-i & 0 \\ 1+i & 1 & i \\ 0 & -i & 1 \end{pmatrix}$ Hermatian? Justify

[5x2=10]

SECTION C

3. a) Find the orthogonal diagonalization of the following matrix $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ b) Determine wether the following is linear transformationor not $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x-y \\ x+2y \end{pmatrix}$ **[7+3=10]**

4. a) Find the pseudo inverse of the matrix $A=\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ b) Find the symmetric matrix **A** associated with given quadratic form $5a^2-b^2+2c^2+2ab-4ac+4bc$. c) Let **B** be an invertible matrix, show that B^TB is posotive definite. **[4+2+4=10]**

5. a) Compute $A=\begin{pmatrix} 0 & 3 \\ 1 & 2 \end{pmatrix}^k$ b) Diagonalize the matrix $M=\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ **[4+6=10]**

6. Find the eigen value, eigen vector, the charecterastic plynomial, geometric multiplity and algebric multiplicity of the matrix $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 3 & 0 \\ -2 & 1 & 2 & -1 \end{pmatrix}$. b) Find out whether the following set of vectors spans $\mathbb{R}^3$ or not: $S=\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \text{ and } \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \right\}$ **[(2 .5+ 2 .5+1+1+1)+2=10]**

7. a) Find the QR factorization of the matrix $\begin{pmatrix} 2 & 8 & 2 \\ 1 & 7 & -1 \\ -2 & -2 & 1 \end{pmatrix}$ b) Show that $W=\begin{pmatrix} a \\ -a \\ 2a \end{pmatrix}$ is a subspace of the vector space $\mathbb{R}^3$ with respect to standard vector addition and scalar multiplication in $\mathbb{R}^3$. c) Prove that every vector space has an unique zero vector. **[5+3+2=10]**

8. a) Find the outer product form of the SVD of the matrix $A=\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix}$ b) If **A** is a **nxn** matrix, show that **adj A** is also invertible and $(\text{adj } A)^{-1} = A/\det A = \text{adj}(A^{-1})$ **[7+3=10]**

9. a) Apply Gram Schmidt process to obtain the othonormal basis from the set of vectors{ $\begin{pmatrix}1\\-1\\1\\1\end{pmatrix}$,

$\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\\0\\1\end{pmatrix}$ }in $\mathbf{R}^4$ 20 **b)** Consider the set $\mathbf{M}_{n\times n}$ **(R)** is the set of all real square matrices. Find out whether $\langle \mathbf{A},\mathbf{B}\rangle = \mathbf{Tr}(\mathbf{AB}^T)$ for all $A, B \in M_{n\times n}(R)$ is an inner product or not. **[5+5=10]**

10. a) Find a unitary matrix **U** and a diagonal matrix **D** such that $\mathbf{U}^*\mathbf{AU}=\mathbf{D}$ for $\mathbf{A}=\begin{pmatrix}-1 & 1+i\\ 1-i & 0\end{pmatrix}$ **b)** Solve the following system of equations by Cramer's rule **2x+y+3z=1, y+z=1,z=1.** [7+3=10]

Note: R is the set of Real numbers and C is the set of complex numbers