

# Real analysis

## Quiz 2 (Feb 24)

Duration 1 hour

Question 1: (8 marks) Define what it means for a function to be uniformly continuous on a set.

Question 2: (9 marks) Give examples with justification for each of the following.

1. A bounded sequence  $(n)$  for which  $\limsup_{n \rightarrow \infty} z_n \neq \liminf_{n \rightarrow \infty} z_n$ .
2. A function  $f: [0,1] \rightarrow \mathbb{R}$  which is discontinuous at each  $x \in [0,1]$ .
3. A continuous function which is not uniformly continuous.

Question 3: (8 marks)

Show the following statements:

1. A bounded monotone sequence is convergent.
2. Every sequence has a monotone subsequence.

Question 4 (6 marks)

Let us say that a sequence  $(c_n)$  of real numbers "converges to  $c$ " (where  $c \in \mathbb{R}$ ) if and only if there is an  $N \in \mathbb{N}$  such that for all  $n > N$  and all  $\epsilon > 0$ , we have  $|c_n - c| < \epsilon$ .

1. If a sequence  $(c_n)$  converges to  $c$ , does  $(G)$  converge to  $c$ ? Explain and if not, give an example.
2. If a sequence  $(C)$  converges to  $c$ , does  $(c_n)$  converge to  $c$ ? Explain and if not, give an example.

Question 5: (9 marks)

Let  $X$  be a metric space such that  $X \subset Y$ , where  $Y$  is a complete metric space. Let  $(n)$  be a Cauchy sequence in  $X$  such that  $(z_n)$  contains a convergent subsequence in  $X$ . Then  $(n)$  converges in  $X$ .

Question 6: (10 marks)

Let  $Z$  be a metric space and let  $Y$  be a dense subset of  $Z$ . Suppose that every Cauchy sequence in  $Y$  converges in  $Z$ . Prove that  $Z$  is complete.