COMPUTING IN SCIENCES-II

END-SEMESTER EXAM

PART A (Theory)

Task-1: In fixed-point iteration, we solve equations of the form f(x)= 0 by rewriting them as x = g(x) and then iteratively computing $x_{n+1} = g(x_n)$. The method aims to converge to a value where this condition holds—that is, where the function value equals the input. Whether or not the iteration converges depends on the nature of g(x) near the fixed point.

You are given the function $f(x) = x^2 – 3x + 1$ which has the two real roots as $x = (3 + \sqrt{5})/2$ and $x = (3 - \sqrt{5})/2$. The equation f(x) = 0 can be rewritten in three different forms as $g_1(x) = x^2 – 2x + 1$, $g_2(x) = (x^2 + 1)/3$ and $g_3(x) = 3 – 1/x$. Each of these defines a fixed-point iteration scheme. Determine which of the given g(x) functions can be used to find which of the two roots.

Task-2: *"*The Newton-Raphson method is a widely used numerical technique to approximate the roots of a real-valued function. It begins with an initial guess for the root and improves it step by step using the slope (derivative) of the function.

The basic idea is to look at where the tangent to the function at the current guess intersects the x-axis. That intersection becomes the new guess. Repeating this process gradually leads us closer to the actual root, provided the initial guess is reasonable and the function behaves well in that region."

Q1. Graphically demonstrate the working of the Newton-Raphson method on a chosen function, showing the progression of approximations toward the root, and using this, derive the general expression for x_{n+1} in terms of x_n and f'(x) (slope) for a given f(x).

Q2. Show that if x_0 is chosen close to the actual root of the function f(x), then x_1, obtained using the Newton-Raphson method, is a better approximation to the root.

PART B (PROGRAMMING TASKS): TASK-3

Q1. For solving f(x) = 0, we first convert to g(x) = x. Now for the specific $f(x) = x^2 – 3x + 1$, we considered three cases for g(x) as $g_1(x) = x^2 – 2x + 1$, $g_2(x) = (x^2 + 1)/3$ and $g_3(x) = 3 – 1/x$. Plot phase plot showing demonstrations for each case. Since $n \to \infty$ iteration is not possible, design a 'termination condition' for stopping the iteration and demonstrate in the code.

Q2. Implement Newton-Raphson Method, for f(x) from above. Design appropriate 'termination condition', and demonstrate clearly in the code.