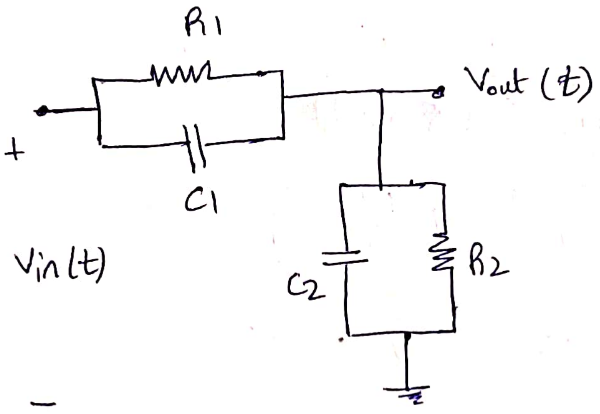
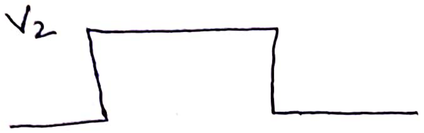


$$Z_1 = R_1 \| \frac{1}{sC_1} = \frac{R_1}{1+sCR_1}$$

$$Z_2 = R_2 \| \frac{1}{sC_2} = \frac{R_2}{1+sCR_2}$$

$V_{in}(t)$:

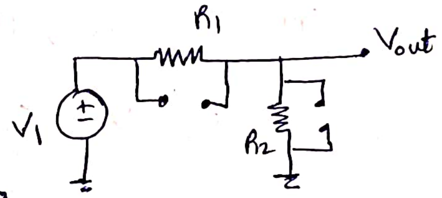


$(T_0 \gg R_1C_1, R_2C_2)$

Ⓐ

for $t<0$: Circuit is at steady state with $V_{in} = V_1$, C_1, C_2 are open

∴ Equivalent ckt is:



$$\boxed{V_{out}(0^-) = \frac{V_1 R_2}{R_1+R_2}} = V_{C_2}, \quad V_{C_1}(0^-) = V_1 - V_{C_2}(0^-)$$

$$= V_1 - \frac{V_1 R_2}{R_1+R_2}$$

$$V_{C_1}(0^-) = \frac{V_1 R_1}{R_1+R_2}$$

$$\Rightarrow I_{R_1} = \frac{V_{C1}}{R_1} = \frac{V_1}{R_1+R_2}$$

$$I_{R_2} = \frac{V_{C2}}{R_2} = \frac{V_1}{R_1+R_2}$$

for $t=0^+$: $V_{in} = V_2$

On assuming $V_{C_1}(0^-) = V_{C_1}(0^+)$, $V_{C_2}(0^-) = V_{C_2}(0^+)$, solving KVL gives

$$V_{in} = V_{C_1} + V_{C_2}$$

$$V_2 = \left(\frac{V_1 R_1}{R_1+R_2}\right) + \left(\frac{V_1 R_2}{R_1+R_2}\right)$$

$V_2 = V_1$ [contradiction]

∴ $V_{C_1}(0^+)$, $V_{C_2}(0^+)$ will not be same as $V_{C_1}(0^-)$, $V_{C_2}(0^-)$ respectively.

On applying charge conservation at o/p node:

$$Q_{initial} = Q_{final} \Rightarrow \quad Q_{initial} = -C_1(V_{C_1}(0^-)) + C_2(V_{C_2}(0^-))$$

$$= -C_1\left[\frac{V_1 R_1}{R_1+R_2}\right] + C_2\left[\frac{V_1 R_2}{R_1+R_2}\right]$$

$$= \frac{V_1}{R_1+R_2}(R_2 C_2 - R_1 C_1) \quad -① $$

$$Q_{final} = -C_1(V_{C_1}(0^+)) + C_2(V_{C_2}(0^+))$$

$$= -C_1\left[V_2 - V_{C_2}(0^+)\right] + C_2(V_{C_2}(0^+))$$

$$= -C_1 V_2 + (C_1+C_2)V_{C_2}(0^+) \quad -② $$

eq ① = ② :

$$-C_1 V_2 + (C_1+C_2)V_{C_2}(0^+) = \frac{V_1}{R_1+R_2}(R_2 C_2 - R_1 C_1)$$

$$V_{C_2}(0^+) = \frac{C_1}{C_1+C_2}\cdot V_2 + \frac{V_1}{(C_1+C_2)(R_1+R_2)}(R_2 C_2 - R_1 C_1) = V_{out}(0^+)$$

$$\therefore V_{C_1}(0^+) = V_2 - V_{C_2}(0^+) = \frac{C_2}{C_1+C_2}\cdot V_2 + \frac{V_1}{(C_1+C_2)(R_1+R_2)}(R_1 C_1 - R_2 C_2)$$

$$I_{R_1}(0^+) = \frac{V_{C_1}(0^+)}{R_1}, \quad I_{R_2}(0^+) = \frac{V_{C_2}(0^+)}{R_2}$$

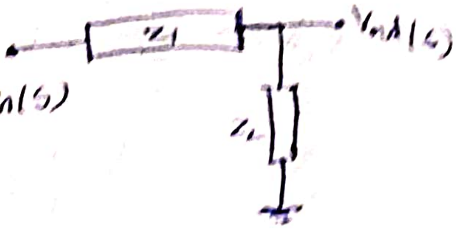
$\therefore$ Summary:

	V_{C_1}	V_{C_2}	I_{R_1}	I_{R_2}
$t=0^-$	$V_1 R_1/R_1+R_2$	$V_1 R_2/R_1+R_2$	V_1/R_1+R_2	V_1/R_1+R_2
$t=0^+$	$\frac{V_2 C_2}{C_1+C_2} + \frac{V_1(R_1C_1-R_2C_2)}{(C_1+C_2)(R_1+R_2)}$	$\frac{V_2 C_1}{C_1+C_2} + \frac{V_1(R_2C_2-R_1C_1)}{(C_1+C_2)(R_1+R_2)}$	V_{C_1}/R_1	V_{C_2}/R_2

if $V_1 = 0$, then

	V_{C_1}	V_{C_2}	I_{R_1}	I_{R_2}
$t=0^-$	$V_1 R_1/R_1+R_2 = 0$	0	0	0
$t=0^+$	$\frac{V_2 C_2}{C_1+C_2}$	$\frac{V_2 C_1}{C_1+C_2}$	$\frac{V_2 C_2}{R_1(C_1+C_2)}$	$\frac{V_2 C_1}{R_2(C_1+C_2)}$

(B) $Z_1 = \dfrac{R_1}{1+sR_1C_1}$ $Z_2 = \dfrac{R_2}{1+sR_2C_2}$



$$V_{out}(s) = \frac{Z_2}{Z_1+Z_2} \cdot V_{in}(s)$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{Z_2}{Z_1+Z_2} = \frac{\dfrac{R_2}{1+sR_2C_2}}{\dfrac{R_1}{1+sR_1C_1} + \dfrac{R_2}{1+sR_2C_2}} = \frac{R_2(1+sR_1C_1)}{R_1 + sR_1R_2C_2 + R_2 + sR_1R_2C_1}$$

$$\therefore H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R_2(1+sR_1C_1)}{(R_1+R_2) + sR_1R_2(C_1+C_2)} \Rightarrow H(0) = \frac{R_2}{R_1+R_2} < 1$$

Zeros : $z = -\dfrac{1}{R_1C_1}$

Poles : $p = -\dfrac{(R_1+R_2)}{R_1R_2(C_1+C_2)} = -\dfrac{1}{R_1C_1} \dfrac{\left(1+\frac{R_1}{R_2}\right)}{\left(1+\frac{C_2}{C_1}\right)}$

$$p = z \frac{(1+R_1/R_2)}{(1+C_2/C_1)} \quad \text{— (1)}$$

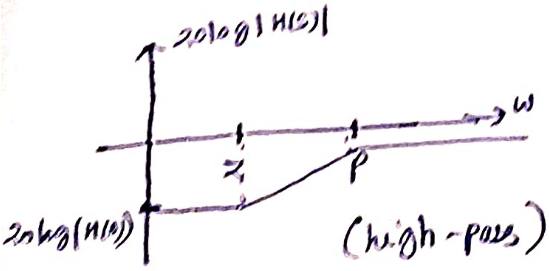
if $R_1C_1 > R_2C_2$

$\dfrac{R_1}{R_2} > \dfrac{C_2}{C_1}$

$\left(\dfrac{1+R_1/R_2}{1+C_2/C_1}\right) > 1$

$p/z > 1$

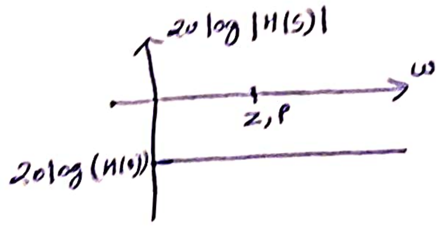
i.e., $\boxed{p > z}$



(high-pass)

if $R_1C_1 = R_2C_2$

$\Rightarrow p = z$

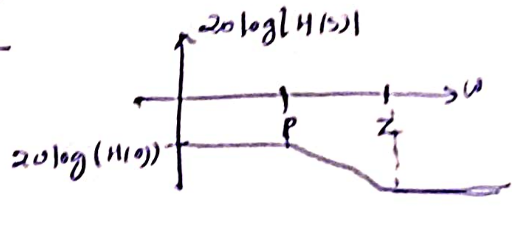


(all-pass)

if $R_1C_1 < R_2C_2$

$\Rightarrow p < z$

as $\dfrac{R_1}{R_2} < \dfrac{C_2}{C_1}$



(low-pass)