Problem 1 (40 points)

On the ALL data set, consider the ANOVA on the gene with the probe "109_at" expression values on B-cell patients in 5 groups: B, B1, B2, B3 and B4.

- (a) Conduct the one-way ANOVA. Do the disease stages affect the mean gene expression value?
- (b) From the linear model fits, find the mean gene expression value among B3 patients.
- (c) Which group's mean gene expression value is different from that of group B?
- (d) Use the pairwise comparisons at FDR=0.05 to find which group means are different. What is your conclusion?
- (e) Check the ANOVA model assumptions with diagnostic tests? Do we need to apply robust ANOVA tests here? If yes, apply the appropriate tests and state your conclusion.

Answer the question in each part directly. Relevant R outputs should be displayed to support your conclusion. Please submit your R commands separately, and label clearly which part the commands correspond to.

Soution:

1a)

```
Analysis of Variance Table

Response: y

Df Sum Sq Mean Sq F value Pr(>F)

ALLB1234$BT 4 2.1053 0.52632 3.4829 0.01082 *

Residuals 90 13.6006 0.15112

---

Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

As the value of p = 0.01082 is small, we reject the null-hypothesis and conclude that these disease stages affect the mean gene expression value.

```
1b)
lm(formula = y \sim ALLB1234\$BT - 1)
Residuals:
                1Q
                      Median
     Min
-1.09026 -0.27845
                     0.03999
                               0.26618
Coefficients:
               Estimate Std. Error t value Pr(>|t|)
                                                 <2e-16 ***
ALLB1234$BTB
                6.81021
                             0.17385
                                        39.17
                                                 <2e-16 ***
ALLB1234$BTB1
                6.57951
                             0.08918
                                        73.78
                                        99.94
                                                 <2e-16 ***
ALLB1234$BTB2 6.47503
                             0.06479
                                                 <2e-16 ***
ALLB1234$BTB3 6.68533
                             0.08106
                                        82.48
                             0.11222
                                        61.61
                                                 <2e-16 ***
ALLB1234$BTB4 6.91417
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.3887 on 90 degrees of freedom
Multiple R-squared: 0.9967, Adjusted R-squared: 0.9966 F-statistic: 5513 on 5 and 90 DF, p-value: < 2.2e-16
```

From the table, we can see that the mean gene expression value among B3 patient is 6.68533

1c)

```
Pairwise comparisons using t tests with pooled SD data: y and ALLB1234$BT

B B1 B2 B3
B1 1.00 - - - -
B2 0.52 1.00 - -
B3 1.00 1.00 0.37 -
B4 1.00 0.20 0.01 0.61

P value adjustment method: holm
```

Comparing the p-values from the table above, we can see that all are more than 0.05, which means the expressions are equal, thus we cannot reject the null hypothesis. So we can say that none of B1, B2, B3, B4 is different from mean expression of group B

1d)

```
Pairwise comparisons using t tests with pooled SD data: y and ALLB1234$BT

B B1 B2 B3
B1 0.40 - - -
B2 0.19 0.48 - -
B3 0.57 0.48 0.15 -
B4 0.62 0.11 0.01 0.20
```

P value adjustment method: fdr

Even after using FDR adjustment, we can see that mean expression of B is not different from that of B1, B2, B3, B4, as the p-values are more that 0.05.

Mean expression of B2, B4 are different as p< 0.01

1e)

Using shapiro test, we get p > 0.05, we accept the null hypothesis that the data follows Normal distribution.

Using Breusch pagan test, we get p>0.05, we accept the nnull hypothesis of equal variances.

As both the test for ANOVA assumptions of mean and variances were positive, we do not need to do the ROBUST TEST

Problem 2 (20 points)

Apply the nonparametric Kruskal-Wallis tests for every gene on the B-cell ALL patients in stage B, B1, B2, B3, B4 from the ALL data. (Hint: use the apply() function.)

- (a) Use FDR adjustments at 0.05 level. How many genes are expressed different in some of the groups?
- **(b)** Find the probe names for the top five genes with smallest p-values.

Please submit your R commands together with your answers to each part of the question.

Solutions:

At 0.05 level FDR adjustment, 423 genes have their p values less than 0.05. Hence we reject the null hypothesis of equal distribution for them and consider that the expression is different in them.

2b)

```
> order.ALLfdr <- order(ALLData.fdr, decreasing=FALSE)
> k=1
> names = NULL
> for (i in order.ALLfdr[1:5]){names[k] <- names(ALLData.fdr[i])
+ k=k+1}
> print('Top five genes with smallest p-values =')
[1] "Top five genes with smallest p-values ="
> print(names)
[1] "1389_at" "38555_at" "40268_at" "1866_g_at" "40155_at"
```

The top five genes with smallest p-values is: 1389_at, 38555_at, 40268_at, 1866_g_at, 40155_at

Problem 3 (20 points)

On the ALL data set, we consider the ANOVA on the gene with the probe "38555_at" expression values on two factors. The first factor is the disease stages: B1, B2, B3 and B4 (we only take patients from those four stages). The second factor is the gender of the patient (stored in the variable ALL\$sex).

- (a) Conduct the appropriate ANOVA analysis. Does any of the two factors affects the gene expression values? Are there interaction between the two factors?
- **(b)**Check the ANOVA model assumption with diagnostic tests? Are any of the assumptions violated?

Please submit your R commands together with your answers to each part of the question. Relevant R outputs should be displayed to support your conclusion.

Solutions:

3a)

Analysis of Variance Table

```
Response: ALL.dataSex

Df Sum Sq Mean Sq F value Pr(>F)

B.cell 3 24.436 8.1453 19.1179 1.818e-09 ***

ALL.sex 1 0.032 0.0319 0.0748 0.7851

B.cell:ALL.sex 3 0.230 0.0768 0.1803 0.9095

Residuals 81 34.511 0.4261

---

Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

As the p-value is less than 0.05 for Bcell, we can say that B group affects gene expression

As the p value for ALL.sex is more that 0.05, we say that gender does not effect gene expression.

For B.cell:ALL.sex has a p-value that is more than 0.05, we conclude that there is no statistically significant interaction

```
3b)
Shapiro-Wilk normality test
data: residuals(lm(ALL.dataSex ~ B.cell + ALL.sex))
W = 0.971, p-value = 0.04335
```

For shapiro-wilk normality test, the p-value is less than 0.05, we can reject the null hypothesis. Thus the normal distribution assumption is violated.

```
Breusch-Pagan test

data: lm(ALL.dataSex ~ B.cell + ALL.sex)
BP = 4.5761, df = 4, p-value = 0.3336
```

For Breusch-pagan we accept the null hypothesis of equal variances as the p-value is greter than 0.05. Thus the normal distribution assumption is not violated

Problem 4 (20 points)

We wish to conduct a permutation test for ANOVA on $(y_1,...,y_N)$, with the group identifiers stored in the vector 'group'. We wish to use $\frac{1}{g-1}\sum_{j=1}^g (\hat{\mu}_j - \hat{\mu})^2$ as the test statistic. Here $\hat{\mu}_j$ is the j-th group sample mean, and $\hat{\mu} = \frac{1}{g}\sum_{j=1}^g \hat{\mu}_j$.

- (a) Program this permutation test in R.
- (b) Run this permutation test on the Ets2 repressor gene 1242_at on the patients in stage B1, B2, and B3 from the ALL data set.

Submit your R script for (a) and the answer and R outputs for (b).

Hint: the sample group means can be found by R command by(y,group,mean).

Solutions:

4a) the R code for the problem is given below

```
> stages <- #Example: c("B1","B2","B3")
+ gene <- #Example: "109_at"
+ Statistics <- function(stages,gene) {
    ALL.B <- ALL[,which(ALL$BT %in% stages)]
+ data <- exprs(ALL.B)[gene,]
+ group <- ALL.B$BT[,drop=T]
+ g <- length(stages)
+ Means <- summary(lm(data ~ group-1))[["coefficients"]][1:g]
+ Total.Mean <- (1/g)*sum(Means)
+ MUj_MU <- NULL
+ for (i in 1:g){
+ MUj_MU[i] <- (Means[i]-Total.Mean)^2
+ }
+ T.obs <- (1/(g-1))*sum(MUj_MU) #Observed statistic
+ n <- length(data)
+ n.perm = 2000
+ T.perm = NULL
+ for(i in 1:n.perm) {
+ data.perm = sample(data, n, replace=F)
+ Means.Perm <- summary(lm(data.perm ~ ALL.B$BT-1))[["coefficients"]][1:g]
+ Total.MeanPerm <- (1/g)*sum(Means.Perm)
+ MUj_MU1 <- NULL
+ for (k in 1:g){
+ MUj_MU1[k] <- (Means.Perm[k]-Total.MeanPerm)^2
+ }
+ T.perm[i] = (1/(g-1))*sum(MUj_MU1) #Permuted statistic
+ }</pre>
```

```
+ mean(T.perm>=T.obs) #p-value
+ }

4b)
> stages <- c("B1","B2","B3")
> gene <- "1242_at"
> Statistics(stages,gene)
[1] 0.5425
```

As the p-value is greater than 0.05, we accept the null hypothesis, of equal distribution of expression values.