## Model Selection

Import baseball database We are using statistics to predict salary of players.

```
library(ISLR)
summary(Hitters)
```

```
HmRun
##
        AtBat
                          Hits
                                                           Runs
##
    Min.
           : 16.0
                     Min.
                                    Min.
                                            : 0.00
                                                     Min.
                                                             : 0.00
                             : 1
##
    1st Qu.:255.2
                     1st Qu.: 64
                                    1st Qu.: 4.00
                                                     1st Qu.: 30.25
##
    Median :379.5
                     Median: 96
                                    Median: 8.00
                                                     Median: 48.00
            :380.9
                                                             : 50.91
##
    Mean
                     Mean
                             :101
                                    Mean
                                            :10.77
                                                     Mean
                                    3rd Qu.:16.00
##
    3rd Qu.:512.0
                     3rd Qu.:137
                                                     3rd Qu.: 69.00
            :687.0
                             :238
                                            :40.00
                                                             :130.00
##
    Max.
                     Max.
                                    Max.
                                                     Max.
##
##
         RBI
                          Walks
                                             Years
                                                               CAtBat
##
    Min.
            : 0.00
                              : 0.00
                                        Min.
                                                : 1.000
                                                                  :
                                                                       19.0
                      Min.
                                                           Min.
##
    1st Qu.: 28.00
                      1st Qu.: 22.00
                                         1st Qu.: 4.000
                                                           1st Qu.:
                                                                     816.8
##
    Median : 44.00
                      Median : 35.00
                                        Median : 6.000
                                                           Median: 1928.0
##
    Mean
           : 48.03
                              : 38.74
                                                : 7.444
                                                                  : 2648.7
                      Mean
                                         Mean
                                                           Mean
    3rd Qu.: 64.75
                      3rd Qu.: 53.00
                                                           3rd Qu.: 3924.2
##
                                         3rd Qu.:11.000
##
    Max.
           :121.00
                      Max.
                              :105.00
                                         Max.
                                                :24.000
                                                           Max.
                                                                  :14053.0
##
##
        CHits
                           CHmRun
                                             CRuns
                                                                CRBI
##
           :
                4.0
                              : 0.00
                                                :
                                                                  :
                                                                       0.00
    Min.
                      Min.
                                         Min.
                                                    1.0
                                                           Min.
                      1st Qu.: 14.00
                                                                     88.75
##
    1st Qu.: 209.0
                                         1st Qu.: 100.2
                                                           1st Qu.:
                                        Median : 247.0
                                                           Median : 220.50
##
    Median : 508.0
                      Median: 37.50
##
    Mean
           : 717.6
                      Mean
                              : 69.49
                                         Mean
                                                : 358.8
                                                           Mean
                                                                  : 330.12
    3rd Qu.:1059.2
                      3rd Qu.: 90.00
##
                                         3rd Qu.: 526.2
                                                           3rd Qu.: 426.25
            :4256.0
                              :548.00
##
    Max.
                      Max.
                                        Max.
                                                :2165.0
                                                           Max.
                                                                  :1659.00
##
##
        CWalks
                                             PutOuts
                                                               Assists
                       League
                               Division
##
    Min.
           :
                0.00
                       A:175
                                E:157
                                          Min.
                                                 :
                                                     0.0
                                                            Min.
                                                                    : 0.0
##
    1st Qu.: 67.25
                       N:147
                                W:165
                                          1st Qu.: 109.2
                                                            1st Qu.: 7.0
##
    Median: 170.50
                                          Median : 212.0
                                                            Median: 39.5
##
    Mean
           : 260.24
                                          Mean
                                                 : 288.9
                                                                   :106.9
                                                            Mean
##
    3rd Qu.: 339.25
                                          3rd Qu.: 325.0
                                                            3rd Qu.:166.0
            :1566.00
##
    Max.
                                          Max.
                                                 :1378.0
                                                            Max.
                                                                    :492.0
##
##
                                       NewLeague
        Errors
                          Salary
##
           : 0.00
                            : 67.5
                                       A:176
    Min.
                     Min.
##
    1st Qu.: 3.00
                     1st Qu.: 190.0
                                       N:146
    Median: 6.00
                     Median: 425.0
##
           : 8.04
                             : 535.9
    Mean
                     Mean
##
    3rd Qu.:11.00
                     3rd Qu.: 750.0
##
            :32.00
                             :2460.0
    Max.
                     Max.
##
                     NA's
                             :59
```

### Remove missing values

```
#delete all rows with missing values
Hitters = na.omit(Hitters)
#check if na left
with(Hitters, sum(is.na(Salary)))
```

## [1] 0

### BEST SUBSET SELECTION

go through all the predictors and select the best subset of models. For each subset size a star is put next to the important features.

```
library(leaps)
regfit.full = regsubsets(Salary~.,data=Hitters)
summary(regfit.full)
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = Hitters)
## 19 Variables (and intercept)
              Forced in Forced out
## AtBat
                   FALSE
                              FALSE
## Hits
                   FALSE
                              FALSE
## HmRun
                   FALSE
                              FALSE
## Runs
                   FALSE
                              FALSE
## RBI
                   FALSE
                              FALSE
## Walks
                   FALSE
                              FALSE
## Years
                   FALSE
                              FALSE
## CAtBat
                   FALSE
                              FALSE
## CHits
                   FALSE
                              FALSE
## CHmRun
                   FALSE
                              FALSE
## CRuns
                   FALSE
                              FALSE
## CRBI
                   FALSE
                              FALSE
## CWalks
                   FALSE
                              FALSE
## LeagueN
                   FALSE
                              FALSE
## DivisionW
                   FALSE
                              FALSE
## PutOuts
                   FALSE
                              FALSE
## Assists
                   FALSE
                              FALSE
## Errors
                              FALSE
                   FALSE
## NewLeagueN
                   FALSE
                              FALSE
## 1 subsets of each size up to 8
## Selection Algorithm: exhaustive
##
            AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns
## 1 (1)""
                   11 11
                        11 11
                               11 11
                                    11 11 11 11
                                               11 11
                                                     11 11
                                                             11 11
                                                                   11 11
## 2 (1)""
                   11 * 11
                        11 11
                                    11 11 11 11
                                               11 11
## 3 (1)""
                                    11 11 11 11
                                               11 11
                                    11 11 11 11
## 4
     (1)
            11 11
                   11 * 11
                               11 11
## 5 (1) "*"
                                    " " "*"
                        11 11
                               11 11
                                               11 11
                                                     11 11
## 6 (1) "*"
                                                                           11 11
## 7 (1)""
                                                                   "*"
```

```
" " " " *"
     (1)"*"
##
            CRBI CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1
## 2
      (1
                                11 11
                                          "*"
## 3
       1
## 4
      (1
                                "*"
                                          "*"
## 6
      (1
                                "*"
## 7
      (1
     (1)""
## 8
```

Default subset size is 8 but we can push it up to 19 (as many variables as we have) Cp = prediction error Pick the model with the minimum <math>Cp

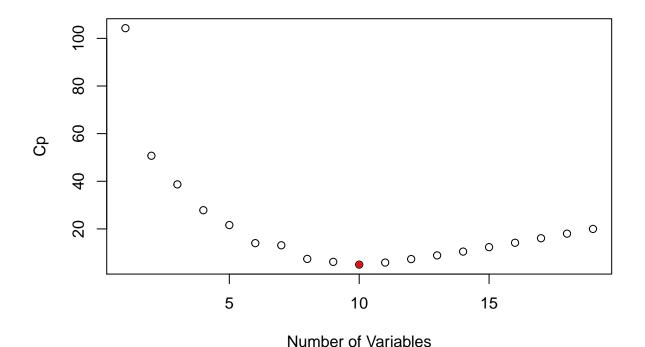
```
regfit.full=regsubsets(Salary~.,data=Hitters, nvmax=19)
reg.summary=summary(regfit.full)
names(reg.summary)

## [1] "which" "rsq" "rss" "adjr2" "cp" "bic" "outmat" "obj"

plot(reg.summary$cp,xlab="Number of Variables",ylab="Cp")
which.min(reg.summary$cp)

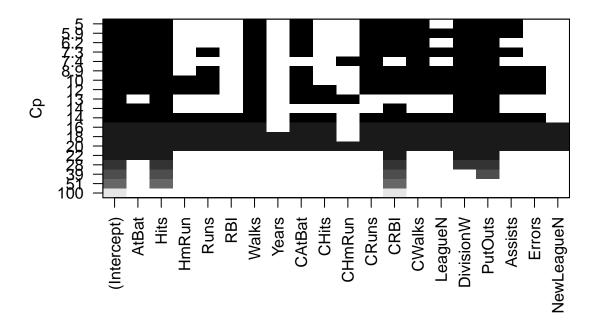
## [1] 10

points(10,reg.summary$cp[10],pch=20,col="red")
```



There is a particular library to plot these graphs Black indicates in variables and white squares are out

```
plot(regfit.full,scale="Cp")
```



# #coefficients of the 10th model coef(regfit.full,10)

```
##
    (Intercept)
                        AtBat
                                       Hits
                                                   Walks
                                                                CAtBat
##
    162.5354420
                   -2.1686501
                                 6.9180175
                                               5.7732246
                                                            -0.1300798
##
                         CRBI
                                     CWalks
                                               DivisionW
                                                               PutOuts
          CRuns
                    0.7743122
                                -0.8308264 -112.3800575
##
      1.4082490
                                                             0.2973726
##
        Assists
      0.2831680
```

#### FORWARD STEPWISE SELECTION

Use regsubset with method=forward

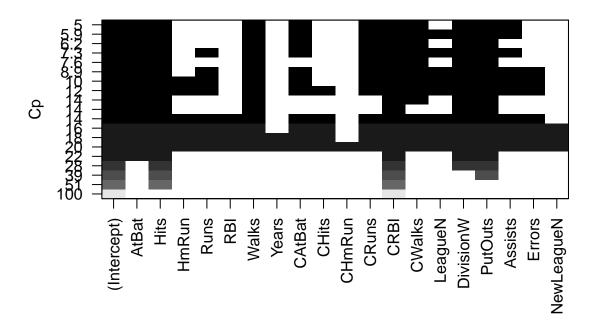
```
regfit.fwd=regsubsets(Salary~.,data=Hitters,nvmax=19,method="forward")
summary(regfit.fwd)
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = Hitters, nvmax = 19, method = "forward")
```

```
## 19 Variables (and intercept)
##
                Forced in Forced out
                                 FALSE
## AtBat
                    FALSE
## Hits
                    FALSE
                                 FALSE
## HmRun
                    FALSE
                                 FALSE
## Runs
                    FALSE
                                 FALSE
## RBI
                    FALSE
                                 FALSE
## Walks
                    FALSE
                                 FALSE
## Years
                    FALSE
                                 FALSE
## CAtBat
                    FALSE
                                 FALSE
## CHits
                    FALSE
                                 FALSE
## CHmRun
                    FALSE
                                 FALSE
## CRuns
                                 FALSE
                    FALSE
## CRBI
                                 FALSE
                    FALSE
## CWalks
                    FALSE
                                 FALSE
## LeagueN
                    FALSE
                                 FALSE
## DivisionW
                    FALSE
                                 FALSE
## PutOuts
                    FALSE
                                 FALSE
## Assists
                    FALSE
                                 FALSE
## Errors
                    FALSE
                                 FALSE
## NewLeagueN
                    FALSE
                                 FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: forward
               AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns
## 1
      (1)
                                   11 11
                                        11 11 11
   2
      (1)
                      "*"
##
   3
      (1)
                      "*"
##
   4
       (1
                      "*"
               "*"
## 5
      (1)
## 6
       (1
                      "*"
## 7
               "*"
       (1
           )
                                   11 11
## 8
       (1
           )
               "*"
                      "*"
                            11 11
                                           11
                                                                                   "*"
                      "*"
## 9
       (1)
               "*"
                                                                                  "*"
                                   11 11
                                                                          .. ..
                                                                                  "*"
## 10
        (1)
                                                                                   "*"
                                                           اليواا
            )
               "*"
## 11
        ( 1
                      "*"
                            11 11
                                   "*"
                                           11
                                                    11 11
                                                                   11 11
                                                                                  "*"
##
   12
        (1
                                                                                  "*"
## 13
        ( 1
                                                           11 * 11
## 14
        (1)
               "*"
                                   "*"
                                                           "*"
                                                                                   "*"
               "*"
                            11 * 11
                                   11 * 11
                                                           11 * 11
                                                                   11 + 11
                                                                                   "*"
## 15
        (1
            )
                      11 * 11
                            "*"
                                   "*"
                                                                                   "*"
##
   16
        (1
            )
               "*"
                                                           "*"
                                                                                  "*"
                            "*"
                                   "*"
                                                    .. ..
                                                                          11 11
   17
        (1
                                                           "*"
                                                                   11 * 11
                                                                                   "*"
               "*"
                      "*"
                            "*"
                                   "*"
                                         "*" "*"
                                                    "*"
                                                           "*"
                                                                   "*"
## 18
        ( 1
            )
##
   19
                      "*"
                            "*"
                                   "*"
                                        "*" "*"
                                                    "*"
                                                           "*"
                                                                   "*"
                                                                          "*"
                                                                                  "*"
##
               CRBI
                    CWalks LeagueN DivisionW
                                                 PutOuts Assists Errors NewLeagueN
                                                  ......
       (1)
                                      11 11
                                                  11 11
                                                                             .. ..
## 2
      (1)
               "*"
                                      11 11
                                                                             .. ..
                                                  "*"
                                                                     11 11
##
   3
       (1
               "*"
           )
## 4
               "*"
                                      "*"
                                                  "*"
      (1)
## 5
               "*"
                                      "*"
      (1)
                                      "*"
                                                  "*"
## 6
      ( 1
               "*"
           )
## 7
               "*"
                                      "*"
                                                  "*"
       (1
           )
                                      "*"
                                                  "*"
               "*"
## 8
      ( 1
           )
                                      "*"
                                                  "*"
                                                                             11 11
## 9
       (1)
                                      "*"
                                                  "*"
## 10 (1) "*"
                     11 * 11
                                                           11 * 11
```

```
"*"
                                                   "*"
                                                            "*"
## 11
        (1
## 12
                                       "*"
                                                   "*"
##
   14
                                                   "*"
##
##
   16
                                       "*"
                                                   "*"
                                                                      "*"
## 18
## 19
        ( 1
                                       "*"
```

plot(regfit.fwd,scale="Cp")



Model Selection Using a Validation Set

Make a training and validation set, so that we can choose a good subset model.

```
## [1] 263 20

set.seed(1)
#seq creates a sequence from 1 to n
```

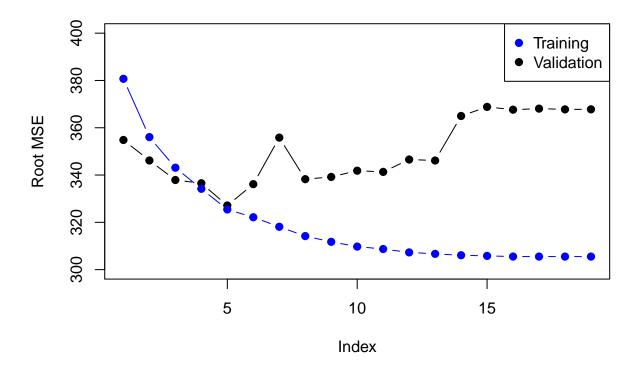
```
#180 is indexes of observations
train=sample(seq(263),180,replace=FALSE)
train
```

```
[1] 70 98 150 237 53 232 243 170 161
##
                                         16 259 45 173 97 192 124 178
##
   [18] 245 94 190 228 52 158 31 64 92
                                           4 91 205 80 113 140 115 43
## [35] 244 153 181 25 163 93 184 144 174 122 117 251
                                                      6 104 241 149 102
## [52] 183 224 242 15 21 66 107 136 83 186 60 211 67 130 210 95 151
   [69] 17 256 207 162 200 239 236 168 249 73 222 177 234 199 203 59 235
##
## [86] 37 126 22 230 226 42 11 110 214 132 134 77
                                                     69 188 100 206
## [103] 44 159 101 34 208 75 185 201 261 112 54
                                                 65
                                                     23
## [120] 154 142 71 166 221 105 63 143 29 240 212 167 172
                                                          5 84 120 133
## [137] 72 191 248 138 182 74 179 135 87 196 157 119
                                                     13 99 263 125 247
## [154] 50 55 20 57
                        8 30 194 139 238 46 78 88 41
                                                          7 33 141 32
## [171] 180 164 213 36 215 79 225 229 198 76
```

```
regfit.fwd=regsubsets(Salary~.,data=Hitters[train,],nvmax=19,method="forward")
```

there are 19 models, so we set up some vectors to record the errors

```
val.errors=rep(NA,19)
#create a matrix where training data is removed from set
x.test=model.matrix(Salary~.,data=Hitters[-train,])
#for all the parameters
for(i in 1:19){
    #size of sample is i
    coefi=coef(regfit.fwd,id=i)
    pred=x.test[,names(coefi)]%*%coefi
    val.errors[i]=mean((Hitters$Salary[-train]-pred)^2)
}
plot(sqrt(val.errors),ylab="Root MSE",ylim=c(300,400),pch=19,type="b")
points(sqrt(regfit.fwd$rss[-1]/180),col="blue",pch=19,type="b")
legend("topright",legend=c("Training","Validation"),col=c("blue","black"),pch=19)
```



model prediction method for regsubset

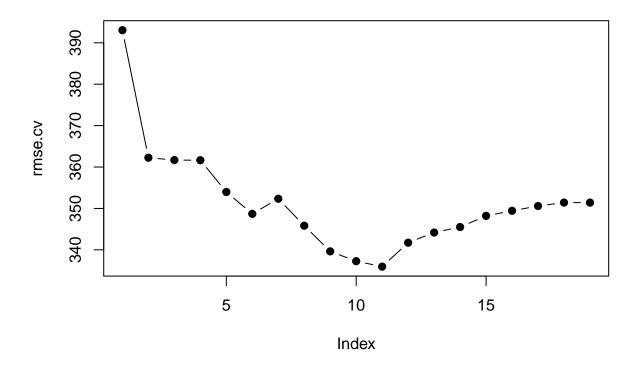
```
predict.regsubsets=function(object,newdata,id,...){
  form=as.formula(object$call[[2]])
  mat=model.matrix(form,newdata)
  coefi=coef(object,id=id)
  mat[,names(coefi)]%*%coefi
}
```

### MODEL SELECTION WITH CROSS VALIDATION

10 fold cross validation

```
set.seed(10)
#take 10 samples
folds = sample(rep(1:10,length=nrow(Hitters)))
folds
##
     [1]
                        3
                            9
                                      8
                                             5
                                                              7
                                                                 3
                                                                    5
                                                                        8
                                        10
                                                         10
                        7
                                                                 3
                                                                    2
##
##
    [47]
                  9
                     2
                        6
                            7
                               2 10
                                      1
                                                9
                                                   2
                                                              4
                                                                 5
                                                                   10
                                                                        6
                                                                                     7
              5
                                                              7
##
    [70]
           3
              5
                  3
                     3
                       10
                            8
                               2
                                   6
                                      9
                                         8
                                             9
                                                5
                                                   5
                                                       9
                                                          7
                                                                 7
                                                                    2
                                                                        9 10
##
    [93]
           2
              6
                  6
                     1
                        2
                            9
                               4
                                  5
                                      6 10 10
                                                6
                                                   6
                                                       8
                                                          5
                                                              5
                                                                 8 10 10
## [116]
                 6
                     2
                        1
                            6
                               5
                                   6 10
                                             1
                                                7
                                                   7
                                                          8
                                                              9
                                                                 3
                                                                    2
                     9
                        2
                            1
                               8
                                      6
                                             9
                                                                 3 10
                                                                        3
## [139]
              9 10
                                  5
                                                4 10
```

```
## [162] 3 7 1 3 6 8 1 4 6 8 7 8 10 10 1 4 1 9 10 8 7 5 8
## [185] 10 7 7 3 9 3 6 3 8 6 3 6 3 9 8 3 3 4 2 2 9
## [208] 4 3 8 7 2 8 10 3 7 3 5 7 5 4 3 8 7 7 8 4 9 1 4
## [231] 4 6 7 2 1 3 5 3 10 3 8 10 2 2 1 5 7 2 2 4 8 8 4
## [254] 2 5 8 4 2 1 2 4 4 3
table(folds)
## folds
## 1 2 3 4 5 6 7 8 9 10
## 27 27 27 26 26 26 26 26 26 26
#make a matrix to store errors for each model on a fold
cv.errors=matrix(NA,10,19)
#two loops
for(afold in 1:10){
 #fit a regsubset model
 best.fit=regsubsets(Salary~.,data=Hitters[folds!=afold,],nvmax=19,method="forward")
 for(param in 1:19){
   #predict the best fit from selected
   pred=predict(best.fit,Hitters[folds==afold,],id=param)
   cv.errors[afold,param]=mean( (Hitters$Salary[folds==afold]-pred)^2)
 }
}
rmse.cv=sqrt(apply(cv.errors,2,mean))
plot(rmse.cv,pch=19,type="b")
```



its not as jittery as the validation test curve