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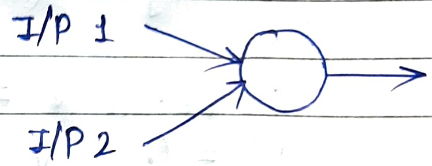
OR function [McCulloch-Pitts Model]

$W_1 = 1$ threshold $\theta = 0$
$W_2 = 1$

Input 1	Input 2
0	0
0	1
1	0
1	1



1) compute total weighted inputs
2) calculate output using logistic sigmoid activation function.

$1/1+e^{-x}$

I/P1	I/P2	$x_1w_1 + x_2w_2$	Y
0	0	$(0)(1) + (0)(1) = 0 \ngeq \theta$	0
0	1	$(0)(1) + (1)(1) = 1 \geq \theta$	1
1	0	$(1)(1) + (0)(1) = 1 \geq \theta$	1
1	1	$(1)(1) + (1)(1) = 2 \geq \theta$	1

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① Example of Hopfield Network

pattern [0 1 1 0 1]

weight matrix

$$\begin{bmatrix} 0 & W_{12} & W_{13} & W_{14} & W_{15} \\ W_{21} & 0 & W_{23} & W_{24} & W_{25} \\ W_{31} & W_{32} & 0 & W_{34} & W_{35} \\ W_{41} & W_{42} & W_{43} & 0 & W_{45} \\ W_{51} & W_{52} & W_{53} & W_{54} & 0 \end{bmatrix}$$

$$W_{ij} = (2V_i - 1)(2V_j - 1)$$

$\therefore$ for $V = 0\ 1\ 1\ 0\ 1$

$V_1' = 0 \quad V_2' = 1 \quad V_3' = 1 \quad V_4' = 0 \quad V_5' = 1$

$$W_{12} = (2V_1 - 1)(2V_2 - 1) = (0-1)(2-1) = (-1)(1) = -1$$
$$W_{13} = (2V_1 - 1)(2V_3 - 1) = (0-1)(2-1) = (-1)(1) = -1$$
$$W_{14} = (2V_1 - 1)(2V_4 - 1) = (0-1)(0-1) = (-1)(-1) = 1$$
$$W_{15} = (2V_1 - 1)(2V_5 - 1) = (0-1)(2-1) = (-1)(1) = -1$$

$$W_{23} = (2V_2 - 1)(2V_3 - 1) = (2-1)(2-1) = (1)(1) = 1$$
$$W_{24} = (2V_2 - 1)(2V_4 - 1) = (2-1)(0-1) = (1)(-1) = -1$$
$$W_{25} = (2V_2 - 1)(2V_5 - 1) = (2-1)(2-1) = (1)(1) = 1$$

$$W_{34} = (2V_3 - 1)(2V_4 - 1) = (2-1)(0-1) = (1)(-1) = -1$$
$$W_{35} = (2V_3 - 1)(2V_5 - 1) = (2-1)(2-1) = (1)(1) = 1$$

$$W_{45} = (2V_4 - 1)(2V_5 - 1) = (0-1)(2-1) = (-1)(1) = -1$$

$\therefore$ weighted matrix.

$$\begin{bmatrix} 0 & -1 & -1 & 1 & -1 \\ -1 & 0 & 1 & -1 & 1 \\ -1 & 1 & 0 & -1 & 1 \\ 1 & -1 & -1 & 0 & -1 \\ -1 & 1 & 1 & -1 & 0 \end{bmatrix}$$

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(2) Example 2 of Hopfield Network

$V^1 = 0\ 1\ 1\ 0\ 1$
$V^2 = 1\ 0\ 1\ 0\ 1$

$W_{12} = (2V_1^1 - 1)(2V_2^1 - 1) + (2V_1^2 - 1)(2V_2^2 - 1) = (-1)(1) + 1(-1) = -2$
$W_{13} = (2V_1^1 - 1)(2V_3^1 - 1) + (2V_1^2 - 1)(2V_3^2 - 1) = (-1)(1) + 1(1) = 0$
$W_{14} = (2V_1^1 - 1)(2V_4^1 - 1) + (2V_1^2 - 1)(2V_4^2 - 1) = (-1)(-1) + 1(-1) = 0$
$W_{15} = (2V_1 - 1)(2V_5^1 - 1) + (2V_1^2 - 1)(2V_5^2 - 1) = (-1)(1) + 1(1) = 0$

$W_{23} = (2V_2^1 - 1)(2V_3^1 - 1) + (2V_2^2 - 1)(2V_3^2 - 1) = (1)(1) + (-1)(1) = 0$
$W_{24} = (2V_2^1 - 1)(2V_4^1 - 1) + (2V_2^2 - 1)(2V_4^2 - 1) = (1)(-1) + (-1)(-1) = 0$
$W_{25} = (2V_2 - 1)(2V_5 - 1) + (2V_2^2 - 1)(2V_5^2 - 1) = (1)(1) + (-1)(1) = 0$

$W_{34} = (2V_3^1 - 1)(2V_4^1 - 1) + (2V_3^2 - 1)(2V_4^2 - 1) = (1)(-1) + 1(-1) = -2$
$W_{35} = (2V_3^1 - 1)(2V_5^1 - 1) + (2V_3^2 - 1)(2V_5^2 - 1) = (1)(1) + 1(1) = 2$

$W_{45} = (2V_4^1 - 1)(2V_5^1 - 1) + (2V_4^2 - 1)(2V_5^2 - 1) = (-1)(1) + (-1)(1) = -2$

weight matrix

$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & -2 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 2 \\ 0 & 0 & -2 & 0 & -2 \\ 0 & 0 & 2 & -2 & 0 \end{bmatrix} \end{array}$$

now,

input pattern $[1\ 1\ 1\ 1\ 1]$
node: 1 2 3 4 5

ordering → 3 1 5 2 4 node

node 3 $V_3in = (0\ 0\ 0\ -2\ 2)(1\ 1\ 1\ 1\ 1)$
$= (0 \times 1) + (0 \times 1) + (0 \times 1) + (-2 \times 1) + (2 \times 1)$
$= 0$

as $0 \geq 0$

$\therefore V_3 = 1$ did not change

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node 1,

$V_1 \text{ in} = (0\ -2\ 0\ 0\ 0)(1\ 1\ 1\ 1\ 1)$
$= -2 \quad < 0$
$\therefore V_1 = 0 \qquad$ changed.

node 5,

$V_5 \text{ in} = (0\ 0\ 2\ -2\ 0)(0\ 1\ 1\ 1\ 1)$
$= 0 \quad \geq 0$
$\therefore V_5 = 1 \qquad$ did not change.

node 2,
$V_2 \text{ in} = (-2\ 0\ 0\ 0\ 0)(0\ 1\ 1\ 1\ 1)$
$= 0 \quad \geq 0$
$V_2 = 1 \qquad$ did not change

node 4
$V_4 \text{ in} = (0\ 0\ -2\ 0\ -2)(0\ 1\ 1\ 1\ 1)$
$= -4 \quad < 0$
$\therefore V_4 = 0 \qquad$ changed

~~next iteration~~ traversing again.

			change
	$\begin{array}{ccccc}1 & 2 & 3 & 4 & 5\end{array}$		
node 3	$V_3 \text{ in} = (0\ 0\ 0\ -2\ 2)(0\ 1\ 1\ 0\ 1) = 2$	$\therefore V_3 = 1$	×
node 1	$V_1 \text{ in} = (0\ -2\ 0\ 0\ 0)(0\ 1\ 1\ 0\ 1) = -2$	$\therefore V_1 = 0$	×
node 5	$V_5 \text{ in} = (0\ 0\ 2\ -2\ 0)(0\ 1\ 1\ 0\ 1) = 2$	$\therefore V_5 = 1$	×
node 2	$V_2 \text{ in} = (-2\ 0\ 0\ 0\ 0)(0\ 1\ 1\ 0\ 1) = 0$	$\therefore V_2 = 1$	×
node 4	$V_4 \text{ in} = (0\ 0\ -2\ 0\ -2)(0\ 1\ 1\ 0\ 1) = -4$	$\therefore V_4 = 0$	×

all nodes did not changed.
$\therefore$ output $\Rightarrow [0\ 1\ 1\ 0\ 1]$

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now the sequence 2 4 3 5 1

node 2 $V_2 in = (-2\ 0\ 0\ 0\ 0\ 0)(1\ 1\ 1\ 1\ 1) = -2 < 0$

$\therefore V_2 = 0$ changed

node 4 $V_4 in = (0\ 0\ -2\ 0\ -2)(1\ 0\ 1\ 1\ 1) = -4 < 0$

$\therefore V_4 = 0$ changed

node 3 $V_3 in = (0\ 0\ 0\ -2\ 2)(1\ 0\ 1\ 0\ 1) = 2 \geq 0$

$\therefore V_3 = 1$ did not changed.

node 5 $V_5 in = (0\ 0\ 2\ -2\ 0)(1\ 0\ 1\ 0\ 1) = 2 \geq 0$

$\therefore V_5 = 1$ did not changed.

node 1 $V_1 in = (0\ -2\ 0\ 0\ 0)(1\ 0\ 1\ 0\ 1) = 0 \geq 0$

$\therefore V_1 = 1$ did not changed.

again traversing

node 2 $V_2 in = (-2\ 0\ 0\ 0\ 0)(1\ 0\ 1\ 0\ 1) = -2 < 0$

$\therefore V_2 = 0$ did not changed

node 4 $V_4 in = (0\ 0\ -2\ 0\ -2)(1\ 0\ 1\ 0\ 1) = -4 < 0$

$\therefore V_4 = 0$ did not changed.

updated each node in network without changing them, so stopped.

$\therefore$ output pattern = $[1\ 0\ 1\ 0\ 1]$