Assignment 1

AI1110: Probability and Random Variables
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22. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9.

Answer: $\frac{4}{9}$.

Solution:

X = Outcome of the first dice

Y = Outcome of the second dice

The outcome of each dice can be={1, 2, 3, 4, 5, 6}

We first solve the question for a general case N ,i.e the case where the product of numbers appearing on the die are less than N

$$F_Y(x) = P(X \leqslant x) \tag{1}$$

$$P(XY < N) = \sum_{k=1}^{6} P(X = k)[F_Y(\frac{N}{k}) - P(Y = \frac{N}{k})] \tag{2}$$

$$F_Y1(\frac{N}{k}) = F_Y(\frac{N}{k}) - P(Y = \frac{N}{k}) \tag{3}$$

$$P(XY < N) = \sum_{k=1}^{6} P(X = k)F_Y1(\frac{N}{k}) \tag{4}$$

$$P(X = k) = \frac{1}{6}, k = 1, 2, 3, 4, 5, 6 \tag{5}$$

$$F_Y(k) = \begin{cases} \frac{[k]}{6}, & \text{if k in [1,6]} \\ 1, & \text{k> 6} \\ 0, & \text{k <1} \end{cases} \tag{6}$$

where P(X) is the probability mass function of X, $F_Y(x)$ is the cumulative distribution function of Y.

From equation (4)

P(XY < N) =

$$\left\{\frac{1}{6}F_Y1(N)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{N}{2}\right)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{N}{3}\right)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{N}{4}\right)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{N}{5}\right)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{N}{6}\right)\right\}$$

Calculation for N=9:

P(XY < 9) =

$$\left\{\frac{1}{6}F_Y1(9)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{9}{2}\right)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{9}{3}\right)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{9}{4}\right)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{9}{5}\right)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{9}{6}\right)\right\}$$

Using formulae (5) and (6)

P(XY<9) =

$$\left\{\frac{1}{6} \times 1\right\} + \left\{\frac{1}{6} \times \frac{4}{6}\right\} + \left\{\frac{1}{6} \times \frac{2}{6}\right\} + \left\{\frac{1}{6} \times \frac{2}{6}\right\} + \left\{\frac{1}{6} \times \frac{1}{6}\right\} + \left\{\frac{1}{6} \times \frac{1}{6}\right\}$$

P(XY<9)=$\frac{4}{9}$

Conclusion :

The probability that the product is less than 9 is $\frac{4}{9}$