

Assignment 1

AI1110: Probability and Random Variables
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22. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9.

Answer: $\frac{4}{9}$.

Solution:

X = Outcome of the first dice

Y = Outcome of the second dice

The outcome of each dice can be=[1,2,3,4,5,6]

We first solve the question for a general case N ,i.e the case where the product of numbers appearing on the die are less than N

$$P(XY < N) = \sum_{k=1}^6 P(X = k) [F_Y(\frac{N}{k}) - P(Y = \frac{N}{k})] \quad (1)$$

$$F_Y(\frac{N}{k}) = F_Y(\frac{N}{k}) - P(Y = \frac{N}{k}) \quad (2)$$

$$P(XY < N) = \sum_{k=1}^6 P(X = k) F_Y(\frac{N}{k}) \quad (3)$$

$$P(X = k) = \frac{1}{6}, k = 1, 2, 3, 4, 5, 6 \quad (4)$$

$$F_Y(k) = \begin{cases} \frac{[k]}{6}, & \text{if } k \text{ in } [1,6] \\ 1, & k > 6 \\ 0, & k < 1 \end{cases} \quad (5)$$

where P(X) is the probability mass function of X,
 $F_Y(x)$ is the cumulative distribution function of Y.

From equation (3)

$$P(XY < N) =$$

$$(\frac{1}{6}) F_Y(1(N)) + (\frac{1}{6}) F_Y(1(\frac{N}{2})) + (\frac{1}{6}) F_Y(1(\frac{N}{3})) + (\frac{1}{6}) F_Y(1(\frac{N}{4})) + (\frac{1}{6}) F_Y(1(\frac{N}{5})) + (\frac{1}{6}) F_Y(1(\frac{N}{6}))$$

Calculation for N=9:

Using formulae (4) and (5)

$$P(XY < 9) = (\frac{1}{6}) F_Y(1(9)) + (\frac{1}{6}) F_Y(1(\frac{9}{2})) + (\frac{1}{6}) F_Y(1(\frac{9}{3})) + (\frac{1}{6}) F_Y(1(\frac{9}{4})) + (\frac{1}{6}) F_Y(1(\frac{9}{5})) + (\frac{1}{6}) F_Y(1(\frac{9}{6}))$$

$$P(XY < 9) = (\frac{1}{6}) (1) + (\frac{1}{6}) (\frac{4}{6}) + (\frac{1}{6}) (\frac{2}{6}) + (\frac{1}{6}) (\frac{2}{6}) + (\frac{1}{6}) (\frac{1}{6}) + (\frac{1}{6}) (\frac{1}{6})$$

$$P(XY < 9) = \frac{4}{9}$$

Conclusion :

The probability that the product is less than 9 is $\frac{4}{9}$