

# Assignment 1

**AI1110:** Probability and Random Variables  
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22. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9.

**Answer:**  $\frac{4}{9}$ .

**Solution:**

X = Outcome of the first dice

Y = Outcome of the second dice

The outcome of each dice can be  $\{1, 2, 3, 4, 5, 6\}$

We first solve the question for a general case N, i.e the case where the product of numbers appearing on the die are less than N

$$F_Y(x) = P(X \leq x)$$

(1)

$$P(XY < N) = \sum_{k=1}^6 P(X = k) [F_Y\left(\frac{N}{k}\right) - P(Y = \frac{N}{k})]$$

(2)

$$F_Y\left(\frac{N}{k}\right) = F_Y\left(\frac{N}{k}\right) - P(Y = \frac{N}{k})$$

(3)

$$P(XY < N) = \sum_{k=1}^6 P(X = k) F_Y\left(\frac{N}{k}\right)$$

(4)

$$P(X = k) = \frac{1}{6}, k = 1, 2, 3, 4, 5, 6$$

(5)

$$F_Y(k) = \begin{cases} \frac{[k]}{6}, & \text{if } k \text{ in } [1, 6] \\ 1, & k > 6 \\ 0, & k < 1 \end{cases} \quad (6)$$

where  $P(X)$  is the probability mass function of X,  
 $F_Y(x)$  is the cumulative distribution function of Y.

From equation (4)

$P(XY < N) =$

$$\left\{ \frac{1}{6} F_Y 1\left(\frac{N}{1}\right) \right\} + \left\{ \frac{1}{6} F_Y 1\left(\frac{N}{2}\right) \right\} + \left\{ \frac{1}{6} F_Y 1\left(\frac{N}{3}\right) \right\} + \left\{ \frac{1}{6} F_Y 1\left(\frac{N}{4}\right) \right\} + \left\{ \frac{1}{6} F_Y 1\left(\frac{N}{5}\right) \right\} + \left\{ \frac{1}{6} F_Y 1\left(\frac{N}{6}\right) \right\}$$

Calculation for N=9:

$P(XY < 9) =$

$$\left\{ \frac{1}{6} F_Y 1(9) \right\} + \left\{ \frac{1}{6} F_Y 1\left(\frac{9}{2}\right) \right\} + \left\{ \frac{1}{6} F_Y 1\left(\frac{9}{3}\right) \right\} + \left\{ \frac{1}{6} F_Y 1\left(\frac{9}{4}\right) \right\} + \left\{ \frac{1}{6} F_Y 1\left(\frac{9}{5}\right) \right\} + \left\{ \frac{1}{6} F_Y 1\left(\frac{9}{6}\right) \right\}$$

Using formulae (5) and (6)

$P(XY < 9) =$

$$\left\{ \frac{1}{6} \times 1 \right\} + \left\{ \frac{1}{6} \times \frac{4}{6} \right\} + \left\{ \frac{1}{6} \times \frac{2}{6} \right\} + \left\{ \frac{1}{6} \times \frac{2}{6} \right\} + \left\{ \frac{1}{6} \times \frac{1}{6} \right\} + \left\{ \frac{1}{6} \times \frac{1}{6} \right\}$$

$P(XY < 9) = \frac{4}{9}$

**Conclusion :**

The probability that the product is less than 9 is  $\frac{4}{9}$