Assignment 1

AI1110: Probability and Random Variables
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22. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9.

Answer: $\frac{4}{9}$**.**

Solution:

X = Outcome of the first dice
Y = Outcome of the second dice

The out come of each dice can be=[1,2,3,4,5,6]

$$P(XY < N) = \sum_{k=1}^{m} P(X = k)F_Y(N/k)$$

where P(X=k) is the probability mass function of X, $F_Y(x)$ is the cumulative distribution function of Y, and m is the largest integer such that m < N.

$$P(X = k) = 1/6, k = 1, 2, 3, 4, 5, 6 \quad (2)$$

$$\mathrm{F}_Y(k) = P(Y < k) = (k-1)/6 \text{ , k = 1, 2, 3, 4, 5, 6}$$

$$\mathrm{F}_Y(k) = P(Y < k) = [k]/6 \text{ ,k in [1,6] except 1,2,3,4,5,6}$$

$$\mathrm{F}_Y(k) = 1 \text{ for k > 6}$$

$$\mathrm{F}_Y(k) = 0 \text{ for k<1} \quad (3)$$

$P(XY < N) = (1/6)F_Y(N) + (1/6)F_Y(N/2) + (1/6)F_Y(N/3) + (1/6)F_Y(N/4) + (1/6)F_Y(N/5) + (1/6)F_Y(N/6)$

Calculation for N=9:

$P(XY < 9) = (1/6)F_Y(9) + (1/6)F_Y(9/2) + (1/6)F_Y(9/3) + (1/6)F_Y(9/4) + (1/6)F_Y(9/5) + (1/6)F_Y(9/6)$

$P(XY < 9) = (1/6)(1) + (1/6)(4/6) + (1/6)(2/6) + (1/6)(2/6) + (1/6)(1/6) + (1/6)(1/6)$

$P(XY < 9) = 4/9$

Conclusion : The probability that the product is less than 9 = $\frac{4}{9}$