Assignment 1

AI1110: Probability and Random Variables
Indian Institute of Techonology Hyderabad

Bolla Nehasree
CS22BTECH11012

22. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9.

Answer: $\frac{4}{9}$.

Solution:

X = Outcome of the first dice

Y = Outcome of the second dice

The outcome of each dice can be={1, 2, 3, 4, 5, 6}

We first solve the question for a general case N ,i.e the case where the product of numbers appearing on the die are less than N

$$F_Y(x) = P(X \leqslant x) \tag{1}$$

$$P(XY < N) = \sum_{k=1}^{6} P(X = k)[F_Y(\frac{N}{k}) - P(Y = \frac{N}{k})] \tag{2}$$

$$F_Y1(\frac{N}{k}) = F_Y(\frac{N}{k}) - P(Y = \frac{N}{k}) \tag{3}$$

$$P(XY < N) = \sum_{k=1}^{6} P(X = k)F_Y1(\frac{N}{k}) \tag{4}$$

$$P(X = k) = \frac{1}{6}, k = 1, 2, 3, 4, 5, 6 \tag{5}$$

$$F_Y(k) = \begin{cases} \frac{[k]}{6}, & \text{if k in [1,6]} \\ 1, & \text{k> 6} \\ 0, & \text{k <1} \end{cases} \tag{6}$$

where P(X) is the probability mass function of X, $F_Y(x)$ is the cumulative distribution function of Y.

From equation (3)

P(XY < N) =

$$\left\{\frac{1}{6}F_Y1(N)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{N}{2}\right)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{N}{3}\right)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{N}{4}\right)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{N}{5}\right)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{N}{6}\right)\right\}$$

Calculation for N=9:

Using formulae (4) and (5)

P(XY < 9) =

$$\left\{\frac{1}{6}F_Y1(9)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{9}{2}\right)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{9}{3}\right)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{9}{4}\right)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{9}{5}\right)\right\} + \left\{\frac{1}{6}F_Y1\left(\frac{9}{6}\right)\right\}$$

P(XY¡9) =

$$\left\{\frac{1}{6} \times 1\right\} + \left\{\frac{1}{6} \times \frac{4}{6}\right\} + \left\{\frac{1}{6} \times \frac{2}{6}\right\} + \left\{\frac{1}{6} \times \frac{2}{6}\right\} + \left\{\frac{1}{6} \times \frac{1}{6}\right\} + \left\{\frac{1}{6} \times \frac{1}{6}\right\}$$

P(XY<9)=$\frac{4}{9}$

Conclusion :

The probability that the product is less than 9 is $\frac{4}{9}$