

# Assignment 1

**AI1110:** Probability and Random Variables  
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22. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9.

**Answer:**  $\frac{4}{9}$ .

**Solution:**

X = Outcome of the first dice

Y = Outcome of the second dice

The outcome of each dice can be=[1,2,3,4,5,6]

We first solve the question for a general case N ,i.e the case where the product of numbers appearing on the die are less than N

$$P(XY < N) = \sum_{k=1}^6 P(X = k)(F_Y(N/k) - P(Y = N/k)) \quad (1)$$

$$F_Y1(N/k) = F_Y(N/k) - P(Y = N/k) \quad (2)$$

$$P(XY < N) = \sum_{k=1}^6 P(X = k)F_Y1(N/k) \quad (3)$$

$$P(X = k) = 1/6, k = 1, 2, 3, 4, 5, 6 \quad (4)$$

$$F_Y(k) = \begin{cases} [k]/6, & \text{if } k \text{ in } [1,6] \\ 1, & k > 6 \\ 0, & k < 1 \end{cases} \quad (5)$$

where P(X) is the probability mass function of X,  
 $F_Y(x)$  is the cumulative distribution function of Y.

From equation (3)

$$P(XY < N) = (1/6) F_Y1(N) + (1/6)F_Y1(N/2) + (1/6)F_Y1(N/3) +$$

$$(1/6)F_Y1(N/4) + (1/6)F_Y1(N/5) + (1/6)F_Y1(N/6)$$

Calculation for N=9:

Using formulae (4) and (5)

$$P(XY < 9) = (1/6) F_Y1(9) + (1/6)F_Y1(9/2) + (1/6)F_Y1(9/3) + (1/6)F_Y1(9/4) + (1/6)F_Y1(9/5) + (1/6)F_Y1(9/6)$$

$$P(XY < 9) = (1/6)(1) + (1/6)(4/6) + (1/6)(2/6) + (1/6)(2/6) + (1/6)(1/6) + (1/6)(1/6)$$

$$P(XY < 9) = 4/9$$

**Conclusion :**

The probability that the product is less than 9 is  $\frac{4}{9}$