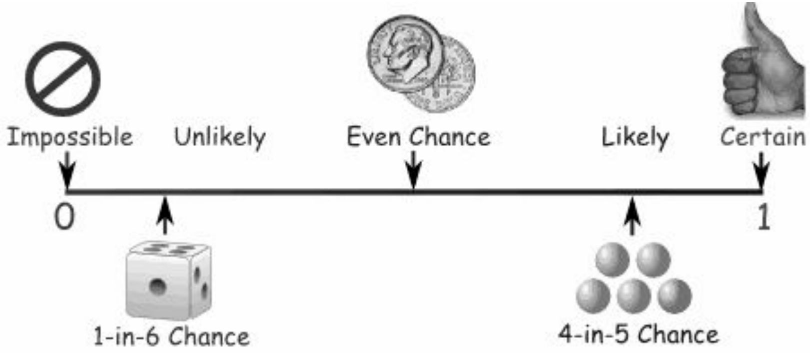
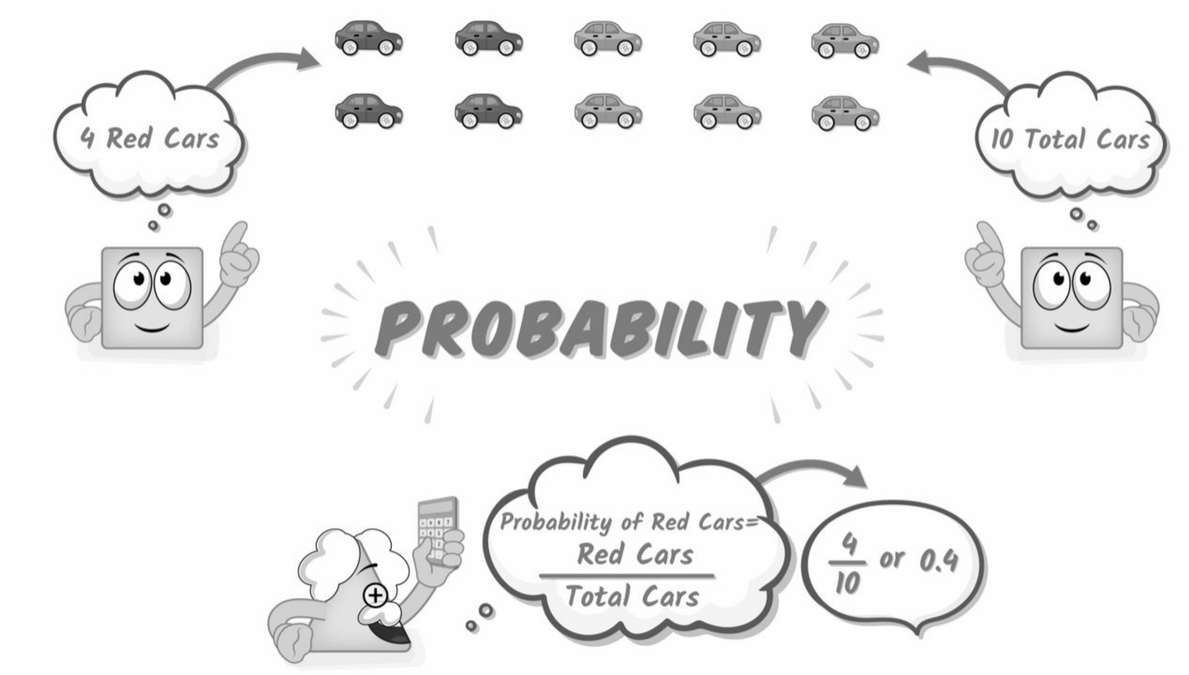
Probability

What is Probability?

- Probability is the chance that something will happen - how likely it is that some event will happen.
- Sometimes you can measure a probability with a number like "10% chance of rain", or you can use words such as impossible, unlikely, possible, even chance, likely and certain.
- Example: "It is unlikely to rain tomorrow".



Understanding with an Example



How Probability and ML are related?

- Most of machine learning is about prediction; how likely is the occurrence of something - an event, outbreak of a disease, diagnosing cancer, population growth etc.
- Combining statistics with probability can help in achieving better predictions.

What is the likelihood of the person in this picture being identified as a boy or a cat by a bot?

Total : 100
Men : 80
Women: 20

P(Men) = 80/100 = 0.8

P(Women) = 20/100 = 0.2

P (Getting > 20k Salary) = P (Men), P (Women)

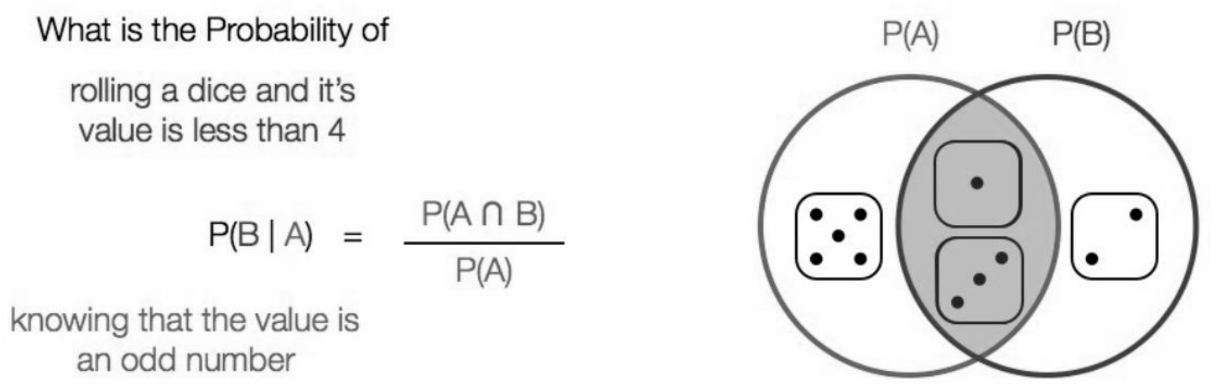
General Defenition:

Total = A + B

P(A) = No.of occurences of A/ Total Obeservations

Types of Probability

- Simple Probability
- Conditional Probability



Simple Probability

Conditional Probability

What is a Random Experiment?

- An experiment in which the outcome varies in an unpredictable fashion when the experiment is repeated under the same conditions.
- Examples:
    - Tossing a die.
    - Tossing a coin.
    - Cricket betting on Dream 11 and etc.

What is a Sample Space?

- Sample space is a set of ALL possible outcomes of an (random) experiment.
- It's denoted by S.
- If a single die rolling experiment,

  The sample space will be $S = \{1,2,3,4,5,6\}$.
- What are the different types of Sample Spaces?
  - A sample space can be finite or infinite.
  - A sample space can be discrete or continuous.
  - A sample space can be countable or uncountable.

What is an Event?

- An event is the outcome of an experiment.
- Formally, any subset of the sample space is an event.
- What are various types of events?
    - Simple a.k.a Elementary Event
    - Compound Event
    - Certain Event
    - Impossible Event
    - Equivalent Events a.k.a Identical Events
    - Equally Likely Events
    - Mutually Exclusive Events
    - Independent Events

Simple Event a.k.a Elementary Event

- Any event which consists of a single outcome in the sample space is called an elementary or simple event.
- Example: if we throw a die, then the sample space, S = {1, 2, 3, 4, 5, 6}. Now the event of 2 appearing on the die is simple and is given by E = {2}.

Compound Event

- Any event which consists of two or more outcomes in the sample space is called an compound event.
- A compound event consists of two or more simple events.
- Example: if we throw a die, having S = {1, 2, 3, 4, 5, 6}, the event of a odd number being shown is given by E = {1, 3, 5}.

Certain Event

- A certain event is an event that is sure to happen.
- E is a certain event if and only if $P(E) = 1$.
- Example. In flipping a coin once, a certain event would be getting a head or a tail.

Impossible Event

- An impossible event is an event that is impossible to happen.
- E is an impossible event if and only if $P(E) = 0$.
- In other words, An event which cannot occur at any performance of the experiment is called an impossible event.
- Example. Probability of getting 7 while tossing a dice once.

Equivalent Events a.k.a Identical Events

- Two events are said to be equivalent or identical if one of them implies and implied by other. That is, the occurrence of one event implies the occurrence of the other and vice versa.
- For example, “even face” and “face-2” or “face-4” or “face-6” are two identical events.

Equally likely Events

- When there is no reason to expect the happening of one event in preference to the other, then the events are known equally likely events.
- For example; when an unbiased coin is tossed the chances of getting a head or a tail are the same.

E1 , E2

RE - > Outcome/Event -> S={All Events /Outcome}

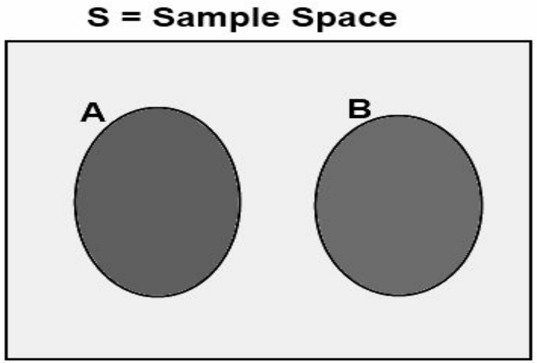
E1 intersection E2 is not an empty set then that is that event?

Mutually Exclusive Events

- If there be no element common between two or more events, i.e., between two or more subsets of the sample space, then these events are called mutually exclusive events.
- If E1 and E2 are two mutually exclusive events, then E1 ∩ E2 = ∅
- For example, in connection with throw a die “even face” and “odd face” are mutually exclusive.
    - But” odd-face” and “multiple of 3” are not mutually exclusive, because when “face-3” occurs both the events “odd face” and “multiply of 3” are said to be occurred simultaneously.
    - We see that two simple-events are always mutually exclusive while two compound events may or may not mutually exclusive.

Independent Event

- Independent Events. When two events are said to be independent of each other, what this means is that the probability that one event occurs in no way affects the probability of the other event occurring.
- An example of two independent events is as follows; say you rolled a die and flipped a coin.



If we have two events E1 and E2 from the same sample space S, does the information about the occurrence of one of the events affect the probability of the other event?

Conditional Probability

- For Dependent Events:
  - If events A and B are not independent, then the probability of the intersection of A and B (the probability that both events occur) is defined by
  - $P(A \text{ and } B) = P(A)\,P(B|A)$
- For Independent Events:
  - If events A and B are independent, then the probability of the intersection of A and B (the probability that both events occur) is defined by
  - $P(A \text{ and } B) = P(A)\,P(B)$

Conditional Probability - Example

If a family has two children. If one of them is know to be a boy, then what is the probability that both are boys? i.e P(B/B)??

P(B/B) = P(B)P(B/B)

Sample Space (S)= {BB, BG, GB, GG}

E = {BB, BG, GB}

F = {BB}

$$P(F/E) = \frac{P\ (E \text{ and } F)}{P(E)}$$

Total Probability

Law of Total Probability:

If $B_1, B_2, B_3, \cdots$ is a partition of the sample space S, then for any event A we have

$$P(A) = \sum_i P(A \cap B_i) = \sum_i P(A|B_i)P(B_i).$$

Total Probability - Example

There are 2 bags Bag A and Bag B.
Bag A has 2 Green Balls and 3 Red Balls.
Bag B has 3 Green Balls and 2 Red Balls.
We draw a ball from A and add it B and then draw the ball from B. What is the probability that the ball which we have drawn is Red. i.e P(R)?

Sample Space (S) =

P(R) = P(G1 and R2) + P (R1 and R2)
= P (G1) P(R2/G1) + P(R1) P(R2/R1)

Bayes Theorem

- P(D) is the probability of a person having Diabetes. It's value is 0.01 or in other words, 1% of the general population has diabetes(Disclaimer: these values are assumptions and are not reflective of any medical study).
- P(+) is the probability of getting a positive test result.
- P(-) is the probability of getting a negative test result.
- P(+|D) is the probability of getting a positive result on a test done for detecting diabetes, given that you have diabetes. This has a value 0.9. In other words the test is correct 90% of the time.
- P(-|~D) is the probability of getting a negative result on a test done for detecting diabetes, given that you do not have diabetes. This also has a value of 0.9 and is therefore correct, 90% of the time.

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Bayes Theorem

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

- P(A) is the prior probability of A occurring independently. In our example this is P(D). This value is given to us.
- P(B) is the prior probability of B occurring independently. In our example this is P(Pos).
- P(A|B) is the posterior probability that A occurs given B. In our example this is P(D|Pos). That is, the probability of an individual having diabetes, given that, that individual got a positive test result. This is the value that we are looking to calculate.
- P(B|A) is the likelihood probability of B occuring, given A. In our example this is P(Pos|D). This value is given to us.
- Putting our values into the formula for Bayes theorem we get:
- P(D|Pos) = (P(D) * P(Pos|D) / P(Pos)

Random Variables

A random variable is a real valued function whose domain is the sample space of a random experiment.

S = {HH, HT, TH, TT}, need exactly one head, X=1, S = {HT, TH}, P(X=1) = 2/4 = ½

P(X=0)?, S = {TT}, P(X=0) = ¼, X>=1, P(X>=1)=3/4

Let X be a random variable whose possible values $x_1, x_2, x_3, \ldots, x_n$ occur with probabilities $p_1, p_2, p_3, \ldots p_n$ respectively. The mean of X, denoted by μ, is

the number $\sum_{i=1}^{n} x_i p_i$.

The mean of a random variable X is also called the expectation of X, denoted by E (X).

Bernoulli Trials and Binomial Distribution

Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions :

(i) There should be a finite number of trials.

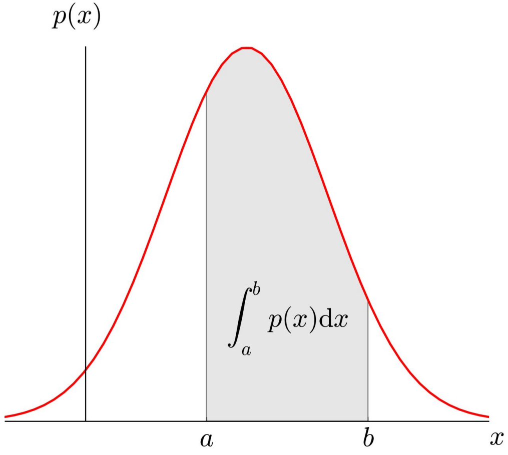
(ii) The trials should be independent.

(iii) Each trial has exactly two outcomes : success or failure.

(iv) The probability of success remains the same in each trial.

For Binomial distribution B (n, p), P $(X = x) = {}^{n}C_{x}\, q^{n-x} p^{x}$, $x = 0, 1, \ldots, n$ $(q = 1 - p)$

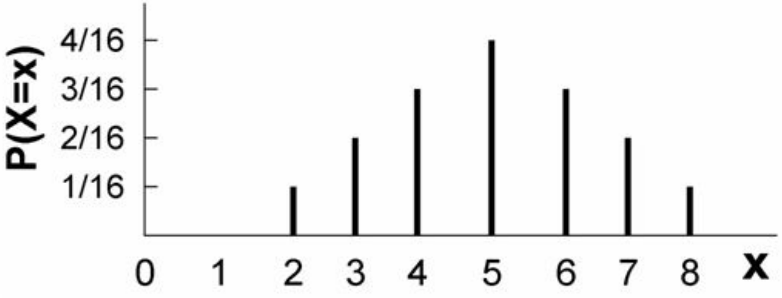
Probability Density Function



Probability Mass Function

When a 4 sided dice is rolled twice. X sum of two throws.

x	P(x)
2	1/16
3	2/16
4	3/16
5	4/16
6	3/16
7	2/16
8	1/16



What is the difference between PDF and PMF?

Probability Mass Function is associated to discrete random variables, while the Probability Distribution Function is associated to continuous random variables.