BILKENT UNIVERSITY

ELECTRICAL AND ELECTRONICS ENGINEERING DEPARTMENT

EE321-02 LAB4 REPORT

14/11/2024

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INTRODUCTION:

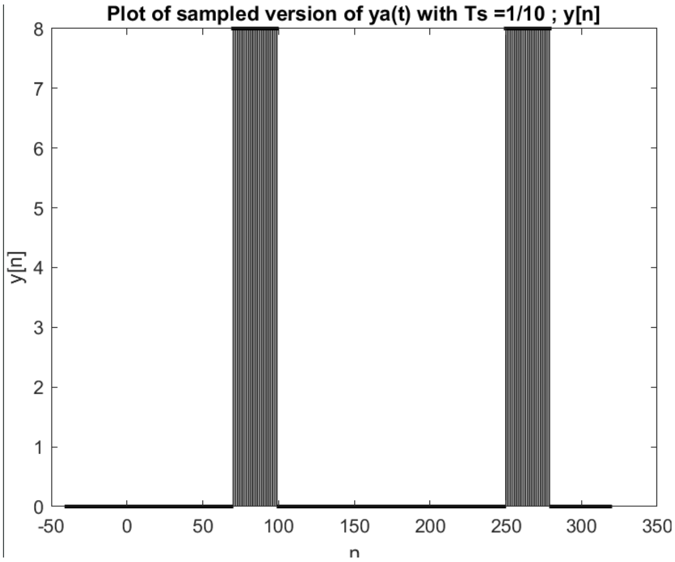
In this lab, we studied about Fourier series expansion and related approximations; with these we did some application both in MATLAB and numerical.

Q1-

a) The discretized signal ya(t) with a sample time of 1/10 seconds is shown below, along with its plot.

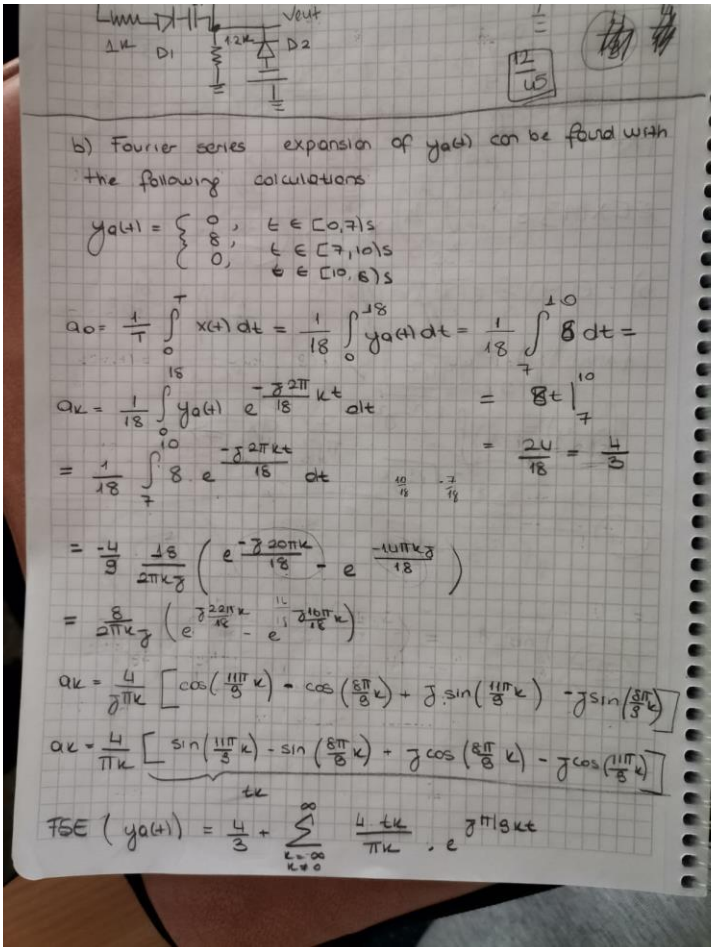
$$y_a(t) = \begin{cases} 0 & t \in [0,7)\text{s.} \\ 8 & t \in [7,10)\text{s.} \\ 0 & t \in [10,18)\text{s.} \end{cases}$$

[Figure 1: ya(t)]



[Figure 2:Plot of discrete ya(t with Ts= 1/10 s)]

b) You can observe the Fourier series expansion of ya(t).



b) Fourier series expansion of ya(t) can be found with the following calculations

$$y_a(t) = \begin{cases} 0, & t \in [0,7)s \\ 8, & t \in [7,10)s \\ 0, & t \in [10,18)s \end{cases}$$

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{18}\int_0^{18} y_a(t)\,dt = \frac{1}{18}\int_7^{10} 8\,dt = 8t\Big|_7^{10} = \frac{24}{18} = \frac{4}{3}$$

$$a_k = \frac{1}{18}\int_0^{18} y_a(t)\, e^{-j\frac{2\pi}{18}kt}\,dt$$

$$= \frac{1}{18}\int_7^{10} 8 \cdot e^{\frac{-j2\pi kt}{18}}\,dt$$

$$= \frac{-4}{9}\frac{18}{2\pi kj}\left(e^{\frac{-j20\pi k}{18}} - e^{\frac{-14\pi kj}{18}}\right)$$

$$= \frac{8}{2\pi kj}\left(e^{j\frac{22\pi k}{18}} - e^{j\frac{16\pi}{18}k}\right)$$

$$a_k = \frac{4}{j\pi k}\left[\cos\left(\frac{11\pi}{9}k\right) - \cos\left(\frac{8\pi}{9}k\right) + j\sin\left(\frac{11\pi}{9}k\right) - j\sin\left(\frac{8\pi}{9}k\right)\right]$$

$$a_k = \frac{4}{\pi k}\underbrace{\left[\sin\left(\frac{11\pi}{9}k\right) - \sin\left(\frac{8\pi}{9}k\right) + j\cos\left(\frac{8\pi}{9}k\right) - j\cos\left(\frac{11\pi}{9}k\right)\right]}_{t_k}$$

$$FSE(y_a(t)) = \frac{4}{3} + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \frac{4\,t_k}{\pi k} \cdot e^{j\pi/9\,kt}$$

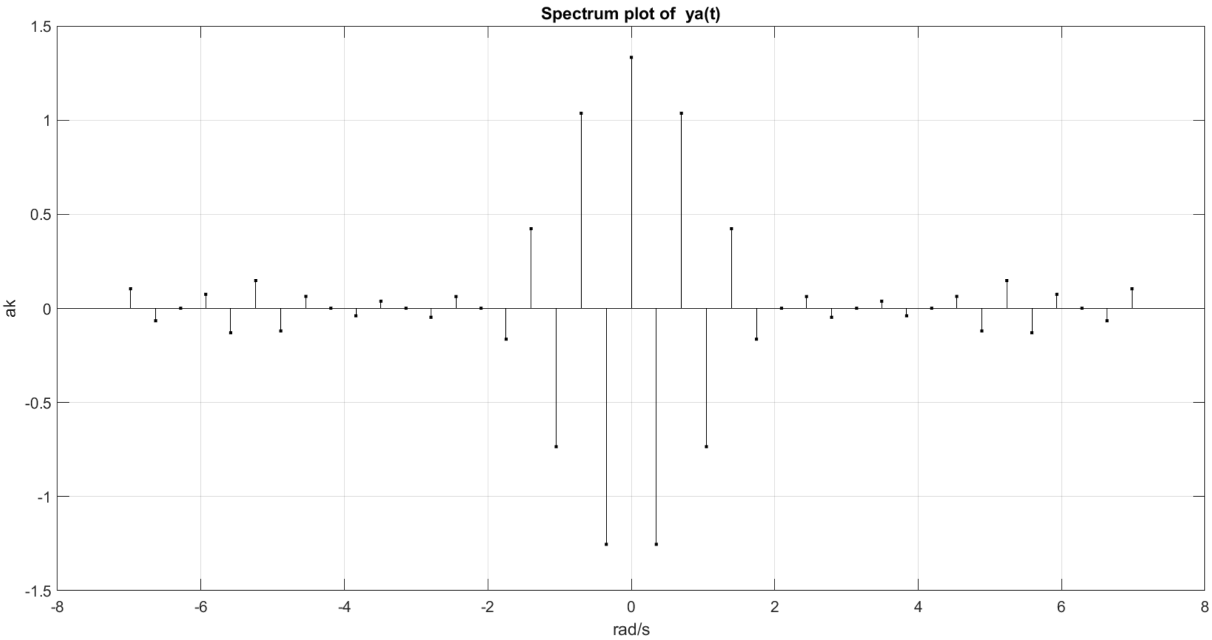
[Figure 3: F.S.E of ya(t)]

Overall, Figure 4 illustrates the link between the coefficients k and their corresponding ak.

$$a_k = \begin{cases} \frac{4}{3};\ k = 0 \\ \frac{4}{j\pi k}\cdot\left(e^{j\frac{22}{18}\pi k} - e^{j\frac{16}{18}\pi k}\right);\ k \neq 0 \end{cases}$$

[Figure 4: ak]

c) Here is the spectrum of coefficients of ak.



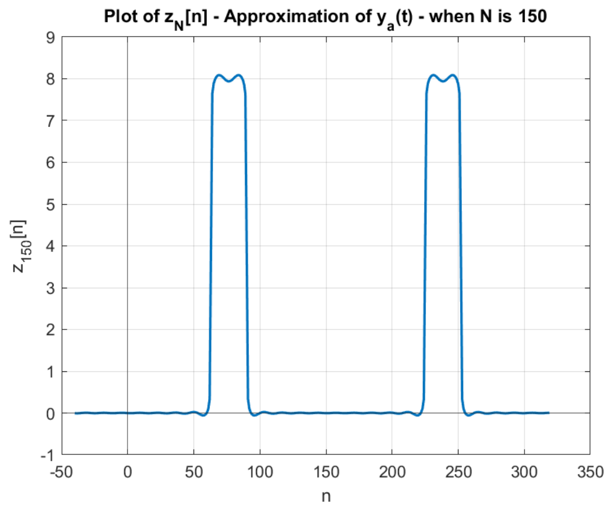
[Figure 5: Spectrum plot of ak]

d) Using the F.S.E from Part B, we may express zN[n] as in

$$z_N[n] = \frac{4}{3} + \sum_{k=-N\,;\,k\neq 0}^{N} a_k \cdot e^{j\frac{\pi}{9}\cdot k\cdot\frac{n}{9}}\ for\ n \in [-40\ ,\ 319]$$

[Figure 6: Z[n] equation]

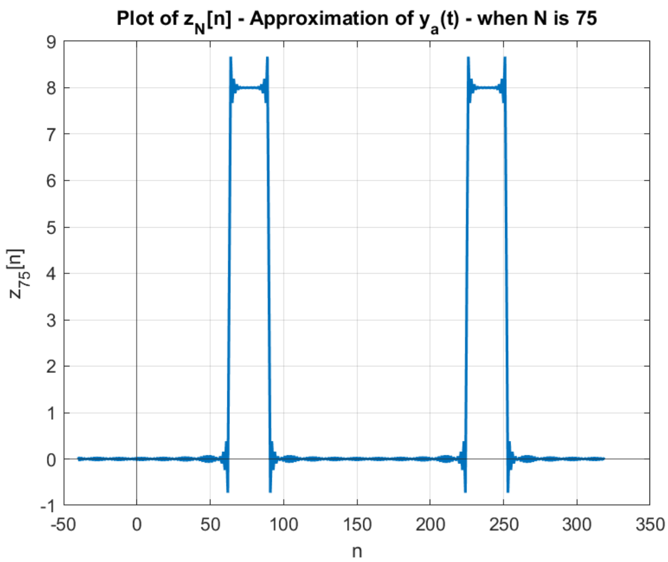
The zN[n] graphic for N = 150 is below:



[Figure 7: Plot of zN[n] when N = 150]

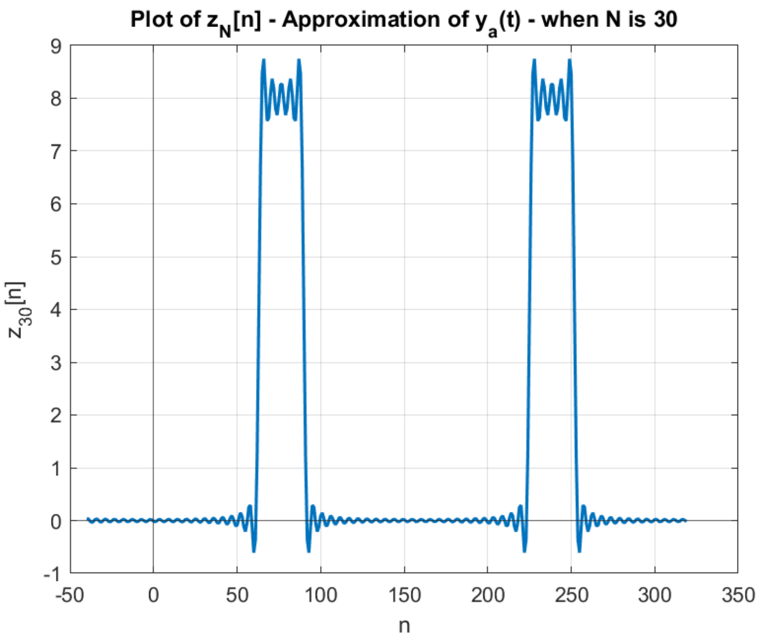
In several ways, the plot resembles the original function ya(t), although it is not exactly the same. This is because for our signal to increasingly resemble ya(t), N must go all the way to infinity. The approximation should begin to diverge further from the original function as N approaches zero.

e) The plot of zN[n] when N = 75



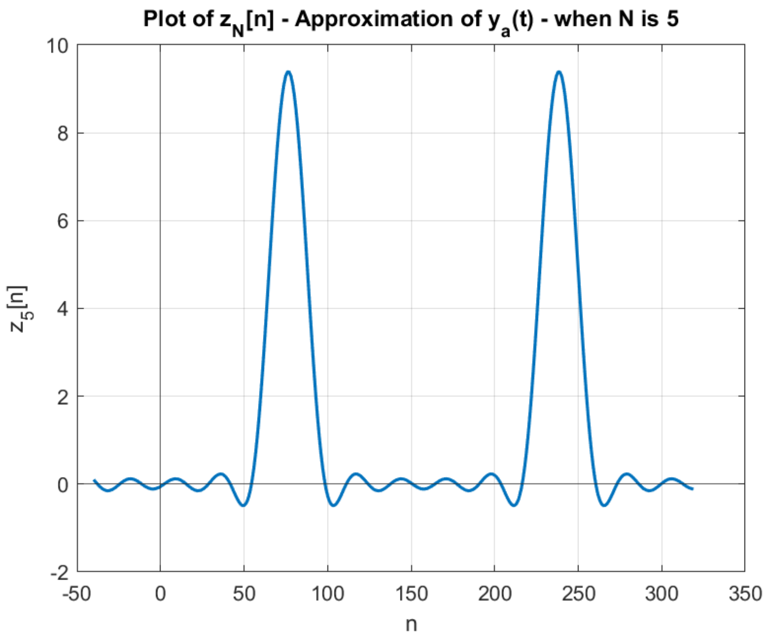
[Figure 8: Plot of zN[n] when N = 75]

f) The plot of zN[n] when N = 30



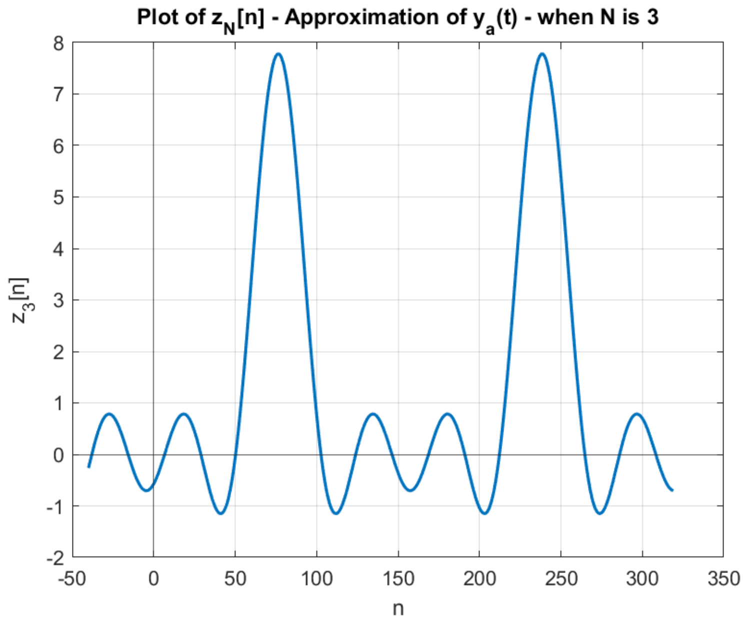
[Figure 9: Plot of zN[n] when N = 30]

g) The plot of zN[n] when N = 5



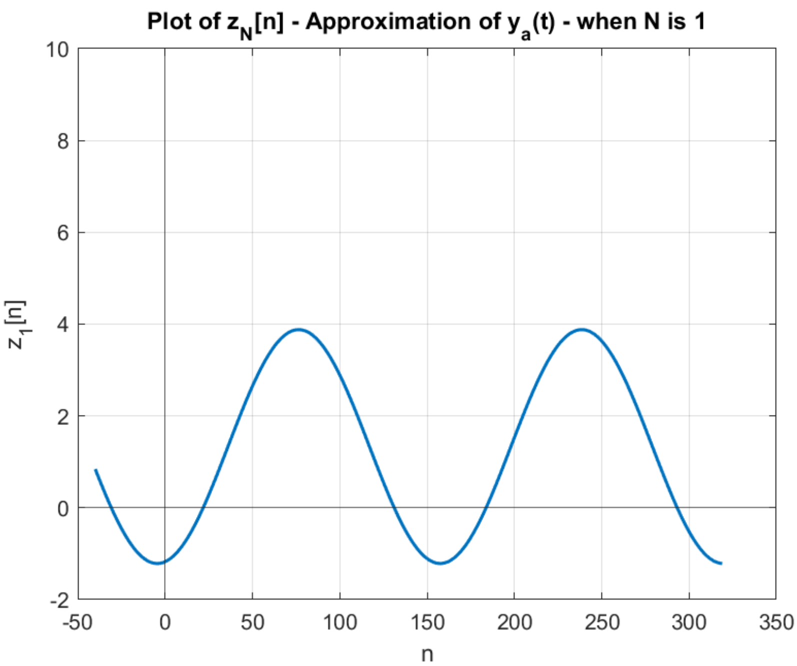
[Figure 10: Plot of zN[n] when N = 5]

h) The plot of zN[n] when N = 3



[Figure 11 :Plot of zN[n] when N = 3]

i) The plot of zN[n] when N = 1

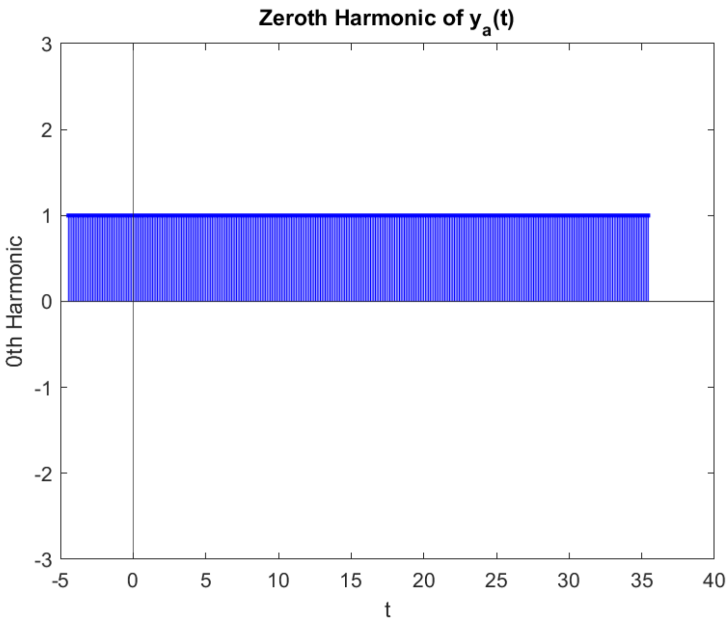


[Figure 12: Plot of zN[n] when N =1]

In fact, as we anticipated in part d, the quality of the approximation decreased as N approached zero. This is due to the fact that as N approaches 0, we are losing an increasing number of frequency components. The quality of the approximation is deteriorated by the missing components. Furthermore, we may state that the approximation is more impacted by components with comparatively greater aks than by those with lower coefficients.

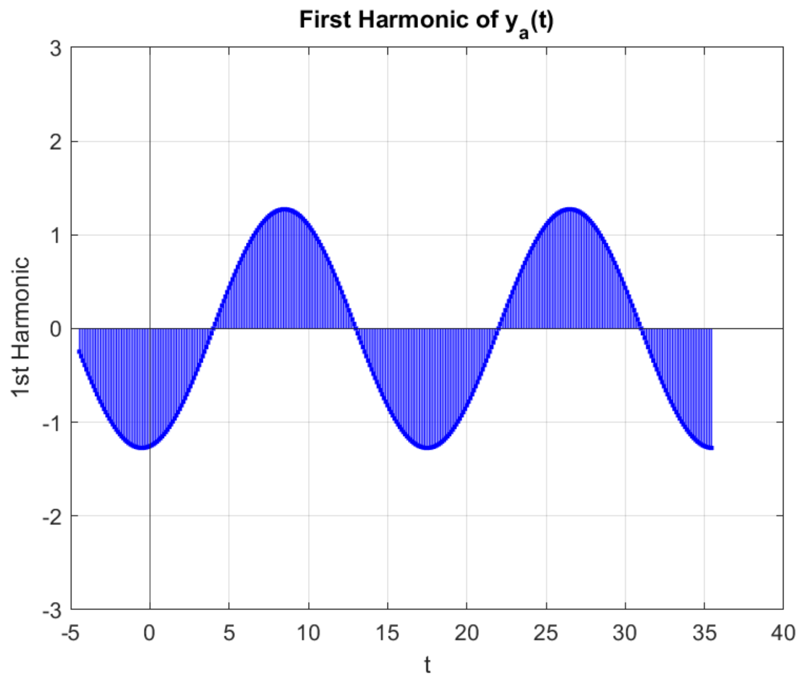
Furthermore, the Gibbs phenomenon states that the jump discontinuity cannot vanish as the number of components added to our estimated function increases. Because MATLAB itself fits the points to the lines smoothly when they get very near to one another, the graph for N=150 shows a smoother curve than the one for N=75. Consequently, the initial approximation graph ought to seem differently than it does in MATLAB. **The irregularities in the plot where N is 150, it must be sharper.**

j) 0 th Harmonic:



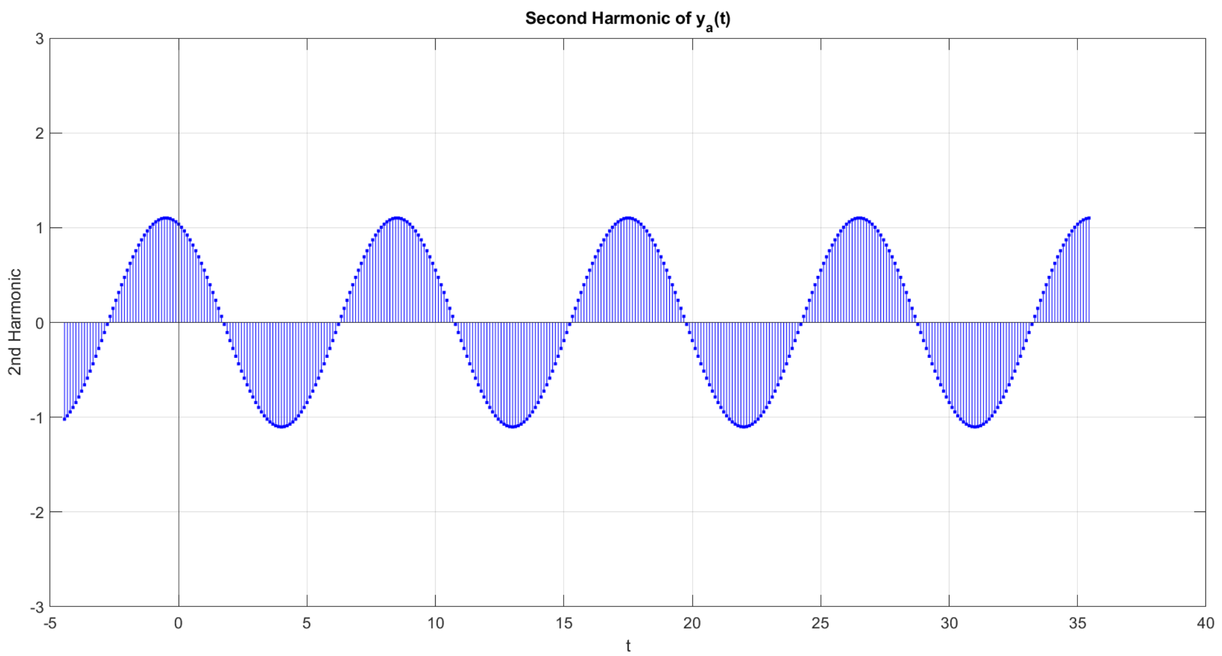
[Figure 13: 0 th harmonic]

1st Harmonic:



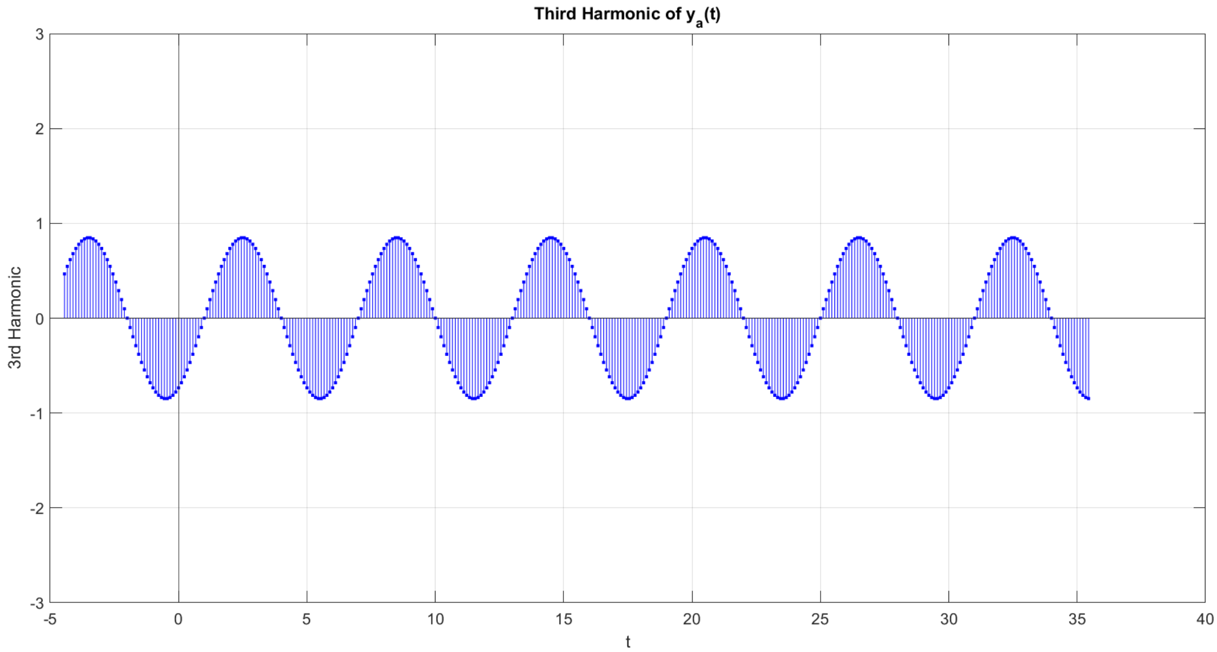
[Figure 14: 1st harmonic]

2nd Harmonic:



[Figure 15: 2nd Harmonic]

3rd Harmonic:



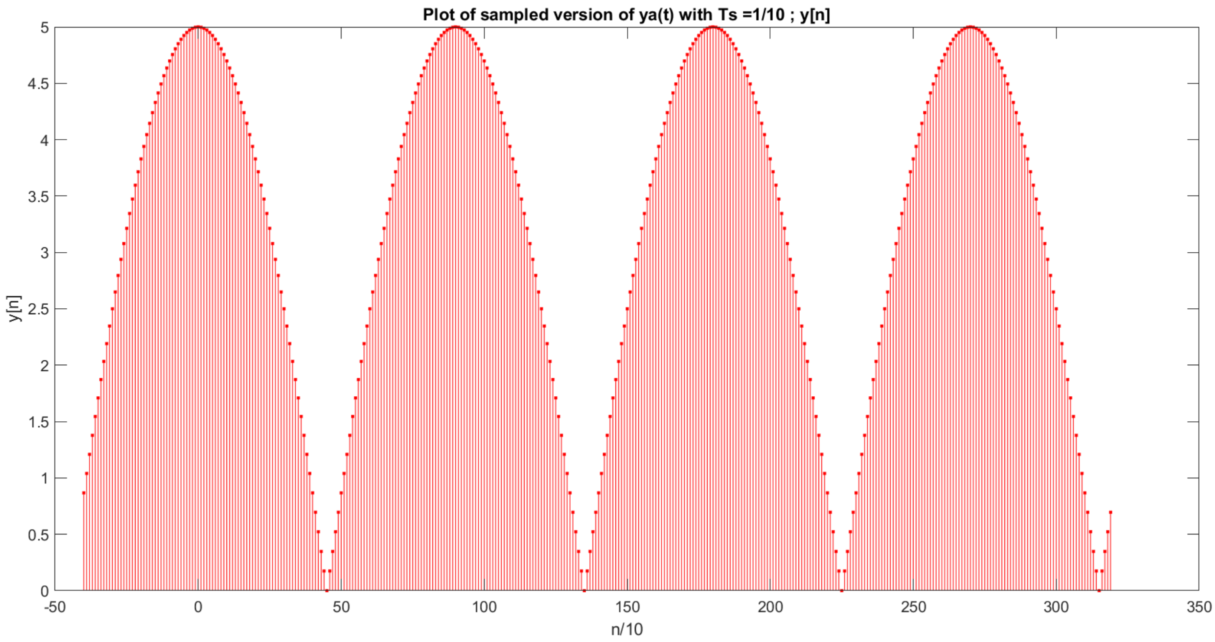
[Figure 16: 3rd Harmonic]

Q2-

a) Figures 17 and 18 show the discretized signal ya(t) and its plot with a sample period of 1/10 seconds. This signal's fundamental period is 18 units of time.

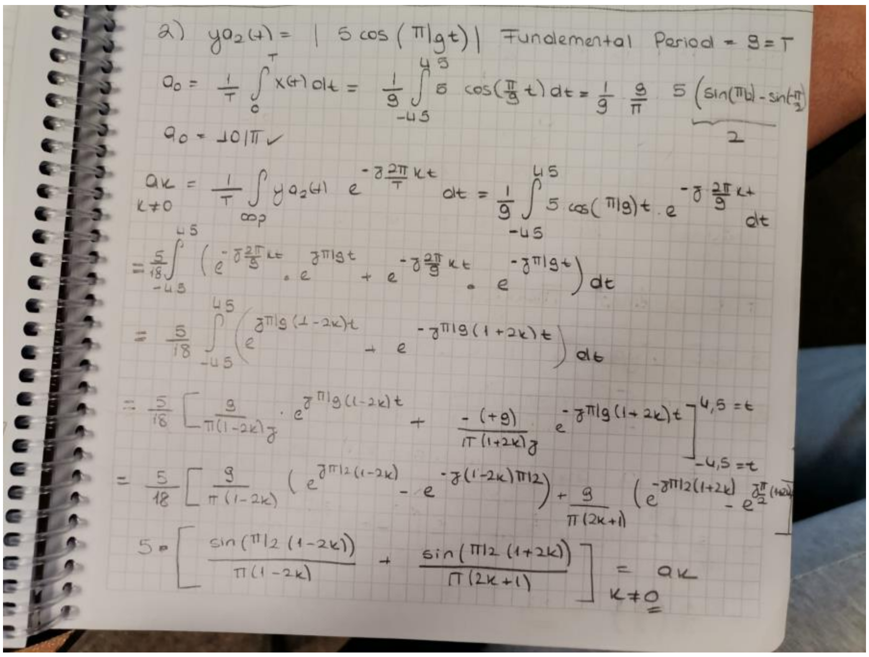
$$y_a(t) = \left|5\cos\left(\frac{\pi}{9}t\right)\right|$$

[Figure 17: ya(t)]



[Figure 18: Plot of the Discrete ya(t) with Ts = 1/10 seconds]

b) You can see the Fourier series expansion of ya(t);



a) $ya_2(t) = |5\cos(\pi/9 t)|$ Fundemental Period $= 9 = T$

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{9}\int_{-4.5}^{4.5} 5\cos(\tfrac{\pi}{9}t)\,dt = \frac{1}{9}\,\frac{9}{\pi}\,5\,\underbrace{\left(\sin(\pi/2) - \sin(-\tfrac{\pi}{2})\right)}_{2}$$

$a_0 = 10/\pi$ ✓

$$\underset{k\neq 0}{a_k} = \frac{1}{T}\int_{\infty p} ya_2(t)\, e^{-j\frac{2\pi}{T}kt}\,dt = \frac{1}{9}\int_{-4.5}^{4.5} 5\cos(\pi/9)t \cdot e^{-j\frac{2\pi}{9}kt}\,dt$$

$$= \frac{5}{18}\int_{-4.5}^{4.5}\left(e^{-j\frac{2\pi}{9}kt}\cdot e^{j\pi/9 t} + e^{-j\frac{2\pi}{9}kt}\cdot e^{-j\pi/9 t}\right)dt$$

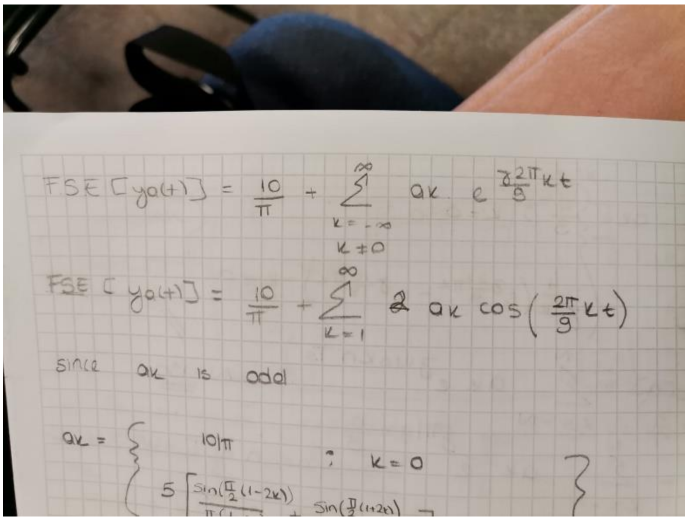
$$= \frac{5}{18}\int_{-4.5}^{4.5}\left(e^{j\pi/9(1-2k)t} + e^{-j\pi/9(1+2k)t}\right)dt$$

$$= \frac{5}{18}\left[\frac{9}{\pi(1-2k)j}\cdot e^{j\pi/9(1-2k)t} + \frac{-(+9)}{\pi(1+2k)j}\, e^{-j\pi/9(1+2k)t}\right]_{-4,5=t}^{4,5=t}$$

$$= \frac{5}{18}\left[\frac{9}{\pi(1-2k)}\left(e^{j\pi/2(1-2k)} - e^{-j(1-2k)\pi/2}\right) + \frac{9}{\pi(2k+1)}\left(e^{-j\pi/2(1+2k)} - e^{j\frac{\pi}{2}(1+2k)}\right)\right]$$

$$5\left[\frac{\sin(\pi/2\,(1-2k))}{\pi(1-2k)} + \frac{\sin(\pi/2\,(1+2k))}{\pi(2k+1)}\right] = \underset{k\neq 0}{a_k}$$

[Figure 19: F.S.E. of ya(t)1]



$$FSE[ya(t)] = \frac{10}{\pi} + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} a_k \, e^{j\frac{2\pi}{9}kt}$$

$$FSE[ya(t)] = \frac{10}{\pi} + \sum_{k=1}^{\infty} 2 a_k \cos\left(\frac{2\pi}{9}kt\right)$$

since a_k is odd

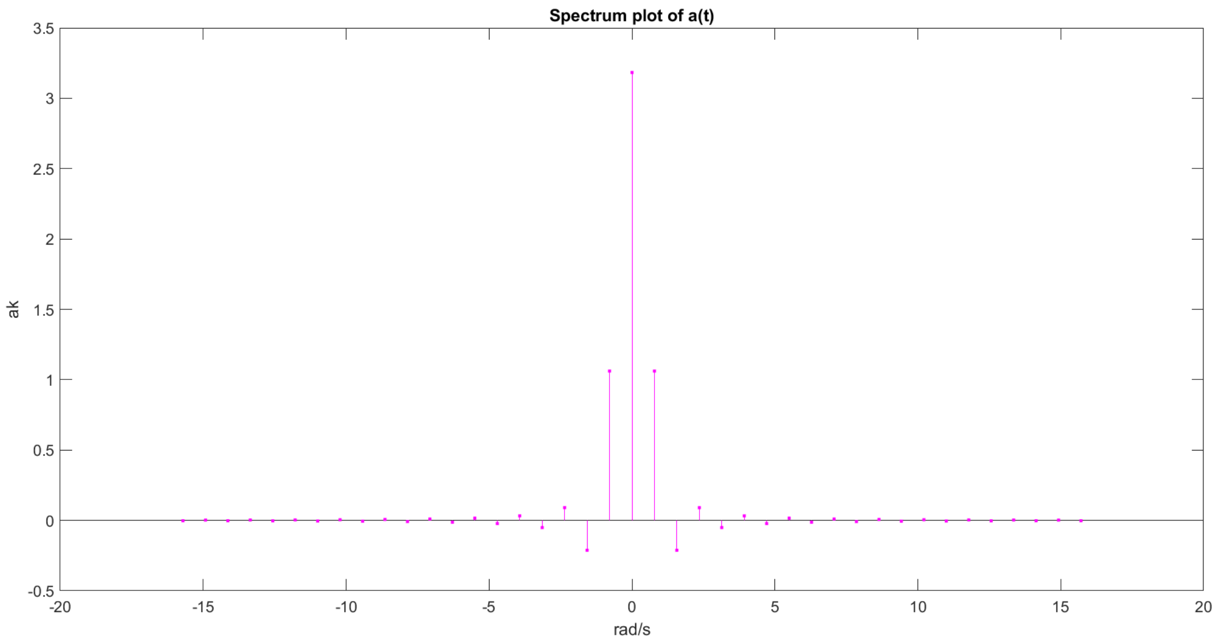
$$a_k = \begin{cases} 10/\pi & ; k = 0 \\ 5\left[\frac{\sin(\frac{\pi}{2}(1-2k))}{\pi(1 \ldots} + \frac{\sin(\frac{\pi}{2}(1+2k))}{\ldots}\right] \end{cases}$$

[Figure 20: F.S.E. of ya(t)2]

c) Figure 21 and 22 demonstrates how the coefficients k and their corresponding ak are related. Here is the spectrum of coefficients as well.

$$a_k = \begin{cases} \dfrac{10}{\pi} & ; k = 0 \\ 5 \cdot \left[\dfrac{\sin\left(\frac{\pi}{2} \cdot (1-2k)\right)}{\pi \cdot (1-2k)} + \dfrac{\sin\left(\frac{\pi}{2} \cdot (1+2k)\right)}{\pi \cdot (1+2k)} \right] & ; k \neq 0 \end{cases}$$

[Figure 21: The equation of ak]



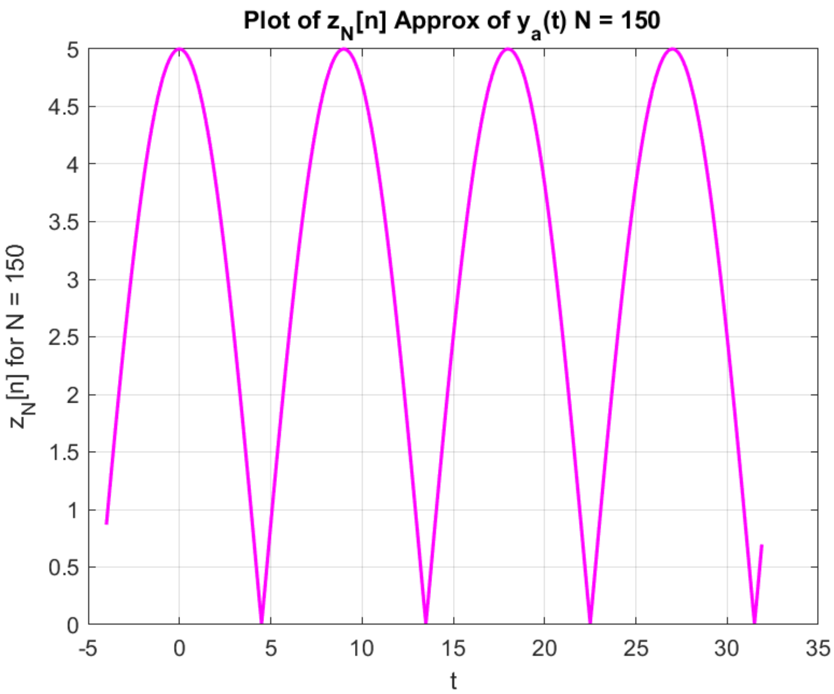
[Figure 22: Spectrum Plot of ya(t)]

d) By using the F.S.E from part b,

$$z_N[n] = \frac{10}{\pi} + \sum_{k=1\,;\,k\neq 0}^{N} 2 \cdot a_k \cdot \cos\left(\frac{2\pi}{9} \cdot k \cdot \frac{n}{9}\right) \; for\, n \in [-40\,,\,319]$$

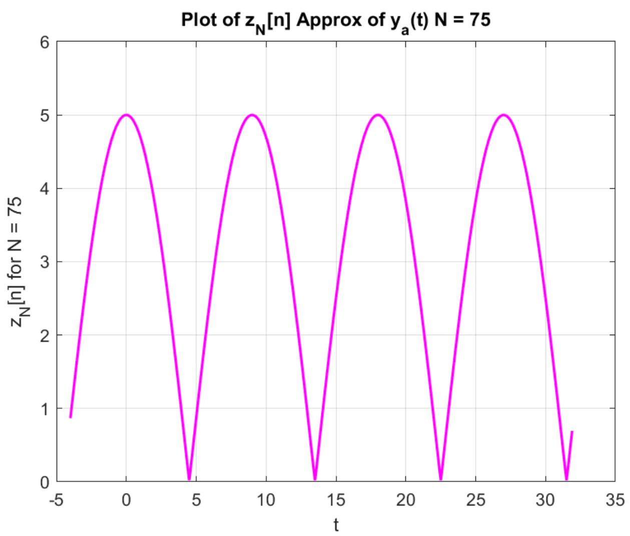
[Figure 23: zN[n]]

Here is the plot of zN[n] when N = 150



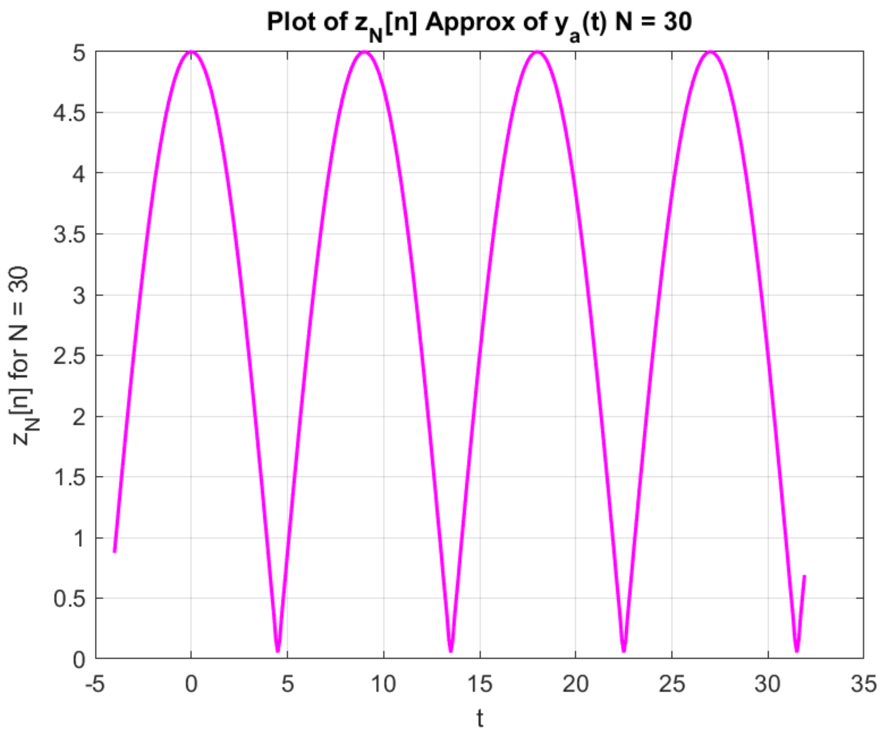
[Figure 24: Plot of zN[n] when N = 150]

e) The plot of zN[n] when N = 75



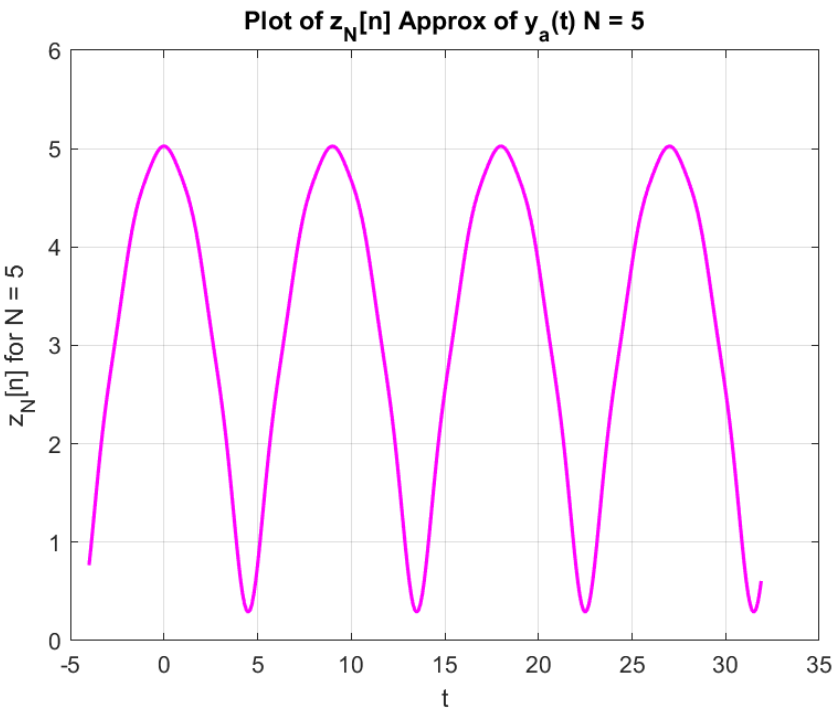
[Figure 25: Plot of zN[n] when N = 75]

f) The plot of zN[n] when N = 30



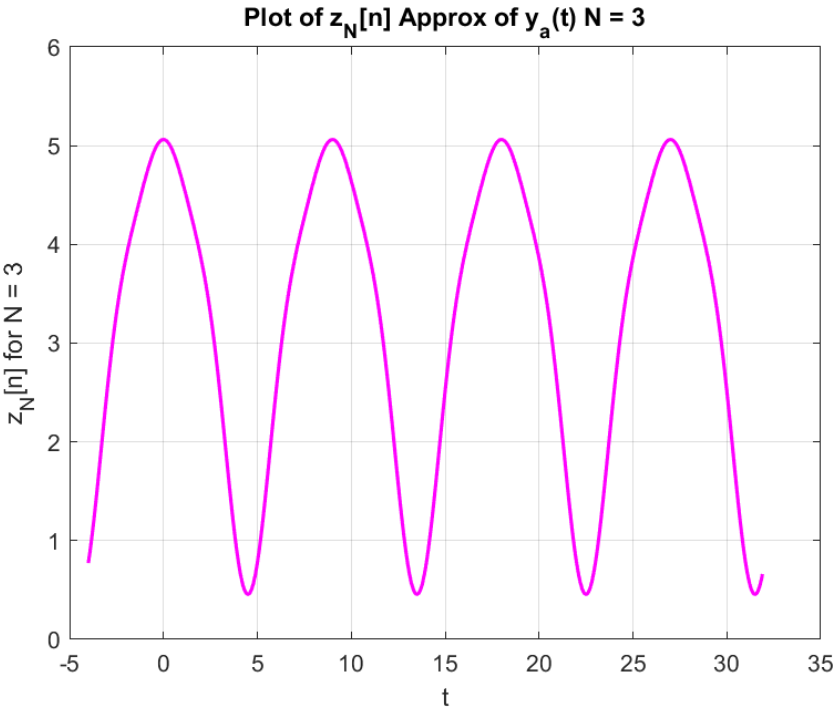
[Figure 26:Plot of zN[n] when N = 30]

g) The plot of zN[n] when N = 5



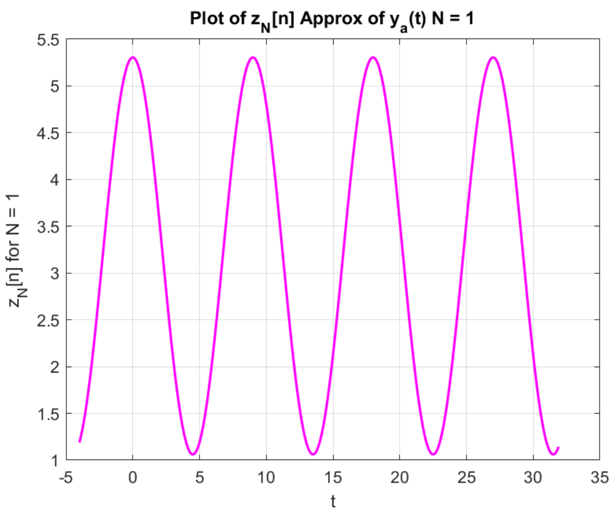
[Figure 27:Plot of zN[n] when N = 5]

h) The plot of zN[n] when N = 3



[Figure 28: Plot of zN[n] when N = 3]

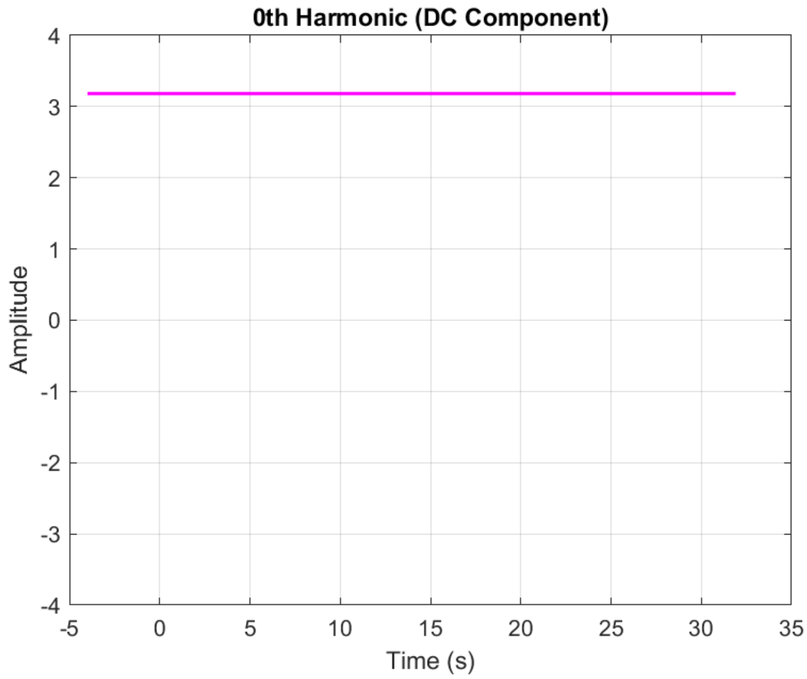
i) The plot of zN[n] when N = 1



[Figure 29: Plot of zN[n] when N =1]

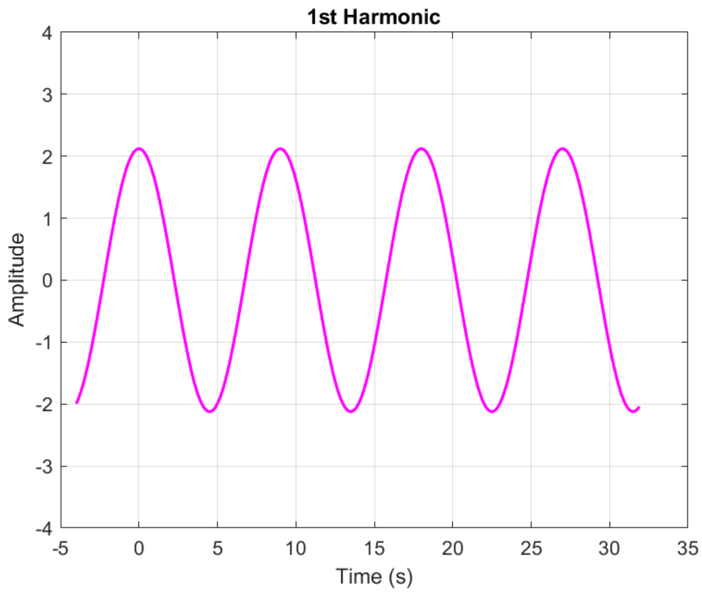
As with the first inquiry, the quality of the approximation did decrease as N approached zero. This is due to the fact that as N approaches 0, we are losing an increasing number of frequency components. The quality of the approximation is affected by the missing components. Additionally, we may state that the elements with comparatively higher aks have an impact on the greater approximation than those with smaller coefficients. However, since there were no abrupt jumps or discontinuities, this question differs from the first one. We were unable to see the Gibbs effect in this area of the lab during the original function.

j) Oth Harmonic



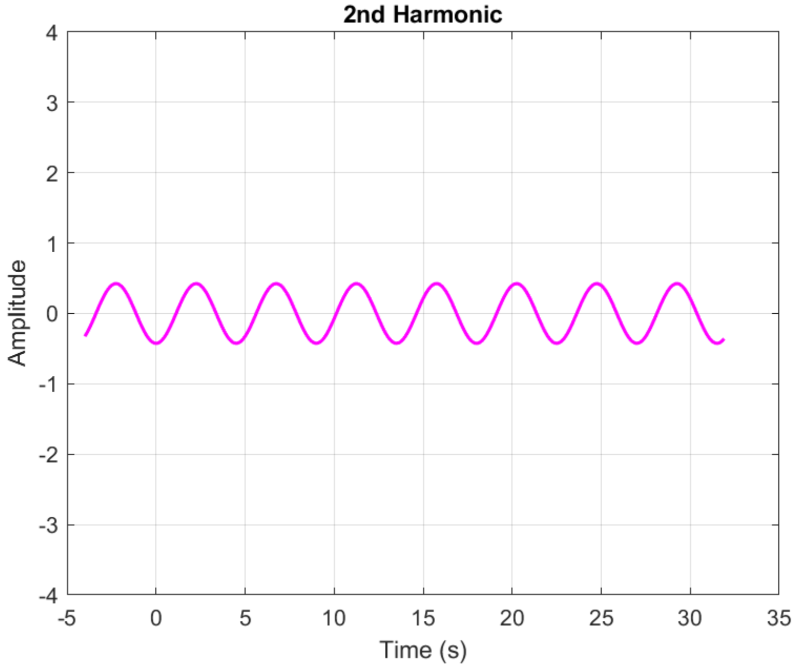
[Figure 30: Zeroth Harmonic]

First Harmonic:



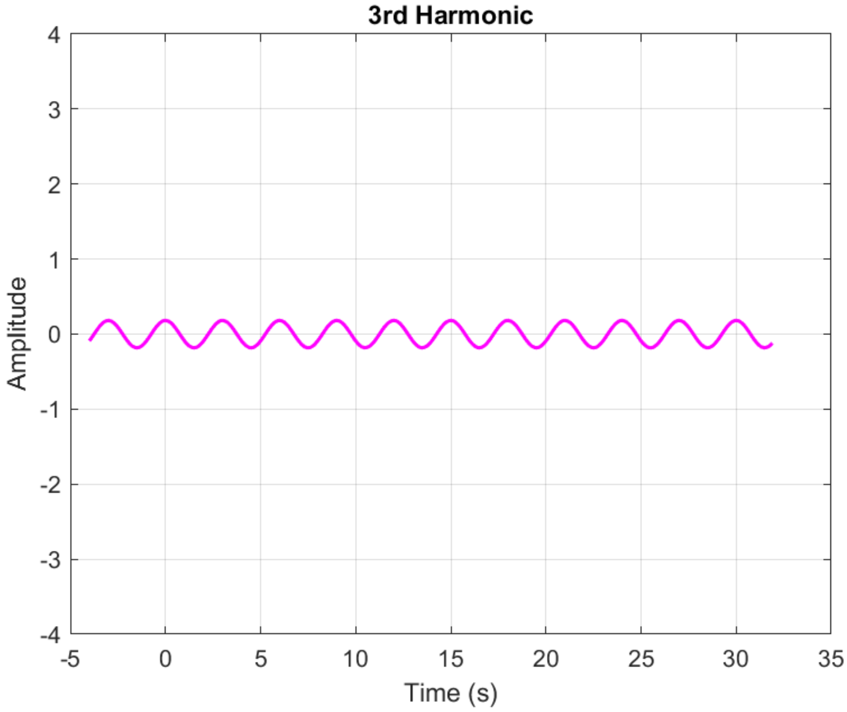
[Figure 31: 1st Harmonic]

2nd Harmonic:



[Figure 32: 2nd Harmonic]

3rd Harmonic:



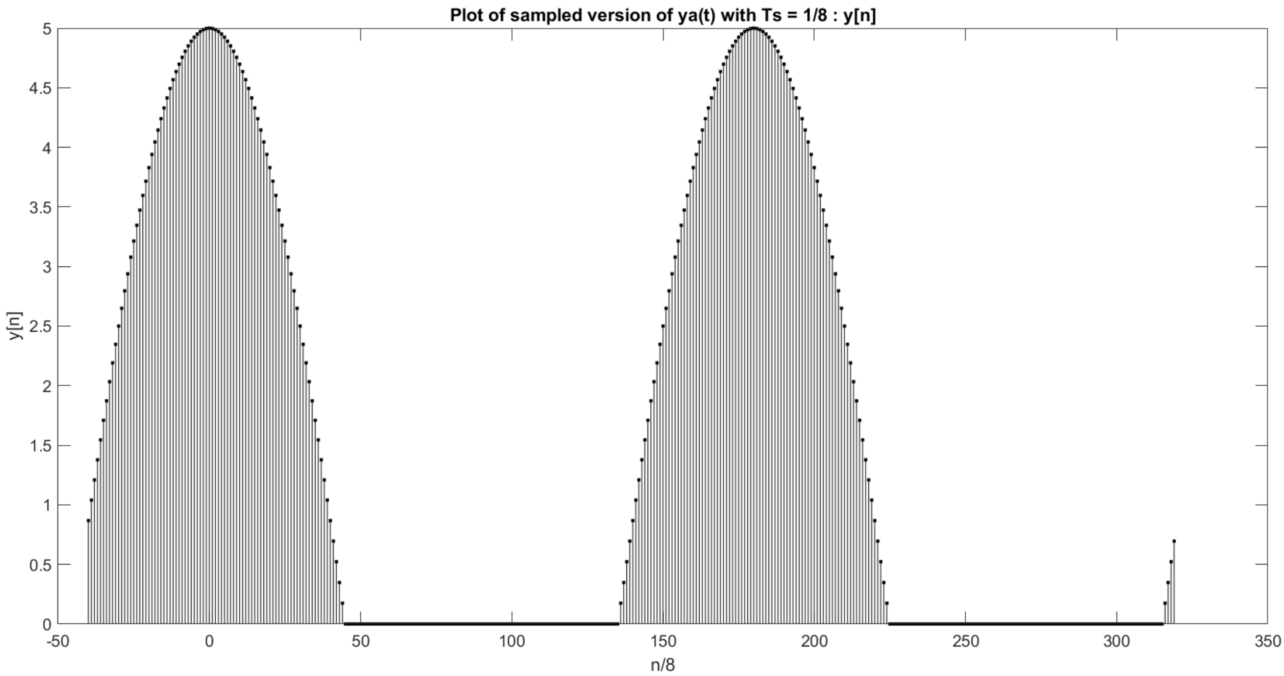
[Figure 33: 3rd Harmonic]

Q3-

a) Figures 34 and 35 show the discretized signal ya(t) and its plot with a sample time of 1/10 seconds. This signal's fundamental period is 18 units of time.

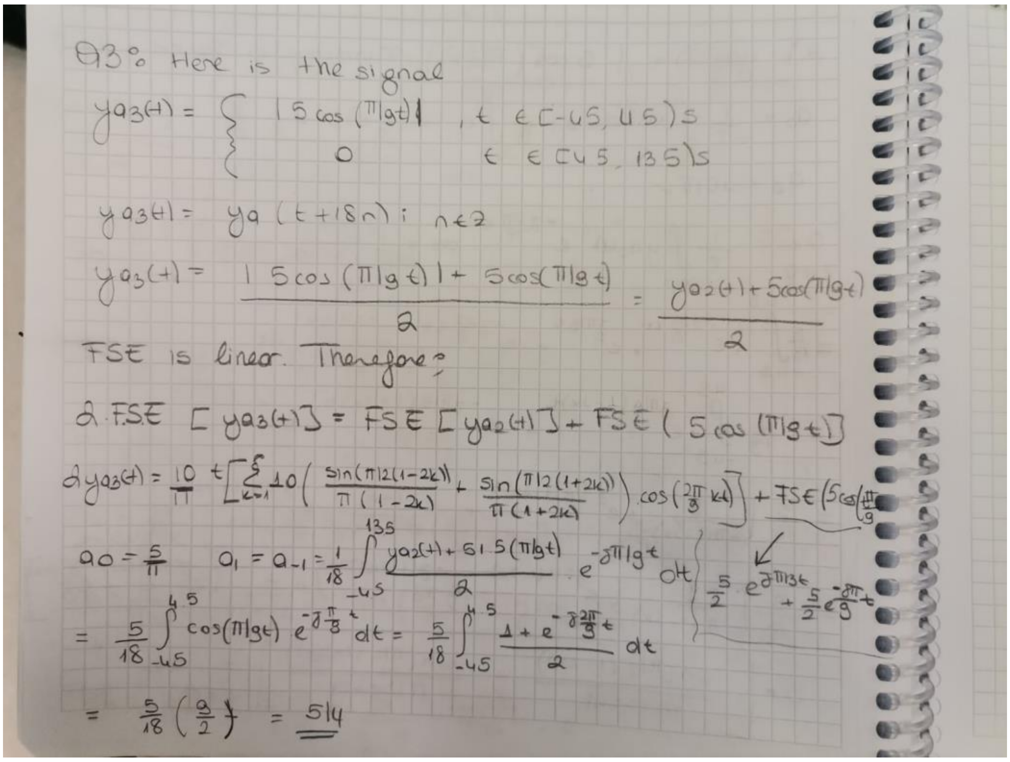
$$y_a(t) = \begin{cases} \left|5\cos\left(\frac{\pi}{9}t\right)\right| & t \in [-4.5, 4.5)\,\mathrm{s}\,. \\ 0 \quad t \in [4.5, 13.5)\mathrm{s}. \end{cases}$$

[Figure 34: ya(t)]



[Figure 35: Plot of the Discrete ya(t) with Ts = 1/10 seconds]

b) The Fourier series expansion of ya(t) is shown here.



Q3: Here is the signal

$$ya_3(t) = \begin{cases} |5\cos(\pi/9t)| & , t \in [-4.5, 4.5)s \\ 0 & t \in [4.5, 13.5)s \end{cases}$$

$ya_3(t) = ya(t+18n) ; n \in Z$

$$ya_3(t) = \frac{|5\cos(\pi/9t)| + 5\cos(\pi/9t)}{2} = \frac{ya_2(t) + 5\cos(\pi/9t)}{2}$$

FSE is linear. Therefore:

$2 \cdot FSE[ya_3(t)] = FSE[ya_2(t)] + FSE(5\cos(\pi/9t))$

$$2ya_3(t) = \frac{10}{\pi} + \left[\sum_{k=1}^{\infty} 10\left(\frac{\sin(\pi/2(1-2k))}{\pi(1-2k)} + \frac{\sin(\pi/2(1+2k))}{\pi(1+2k)}\right)\cos\left(\frac{2\pi}{9}kt\right)\right] + FSE\left(5\cos\left(\frac{\pi}{9}t\right)\right)$$

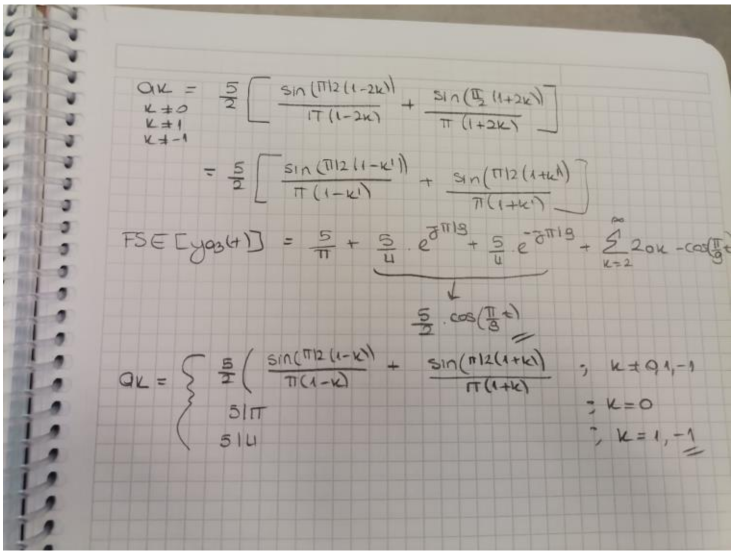
$\frac{5}{2}e^{j\pi/9t} + \frac{5}{2}e^{-j\pi/9t}$

$a_0 = \frac{5}{\pi}$ $\quad a_1 = a_{-1} = \frac{1}{18}\int_{-4.5}^{13.5} \frac{ya_2(t) + 5\cos(\pi/9t)}{2} e^{-j\pi/9t} dt$

$$= \frac{5}{18}\int_{-4.5}^{4.5} \cos(\pi/9t)\, e^{-j\frac{\pi}{9}t} dt = \frac{5}{18}\int_{-4.5}^{4.5} \frac{1 + e^{-j\frac{2\pi}{9}t}}{2} dt$$

$$= \frac{5}{18}\left(\frac{9}{2}\right) = \underline{\underline{5/4}}$$

[Figure 36: F.S.E. of ya(t)1]



$$a_k = \frac{5}{2}\left[\frac{\sin(\pi/2\,(1-2k))}{\pi(1-2k)} + \frac{\sin(\frac{\pi}{2}(1+2k))}{\pi(1+2k)}\right] \quad k \neq 0,\ k \neq 1,\ k \neq -1$$

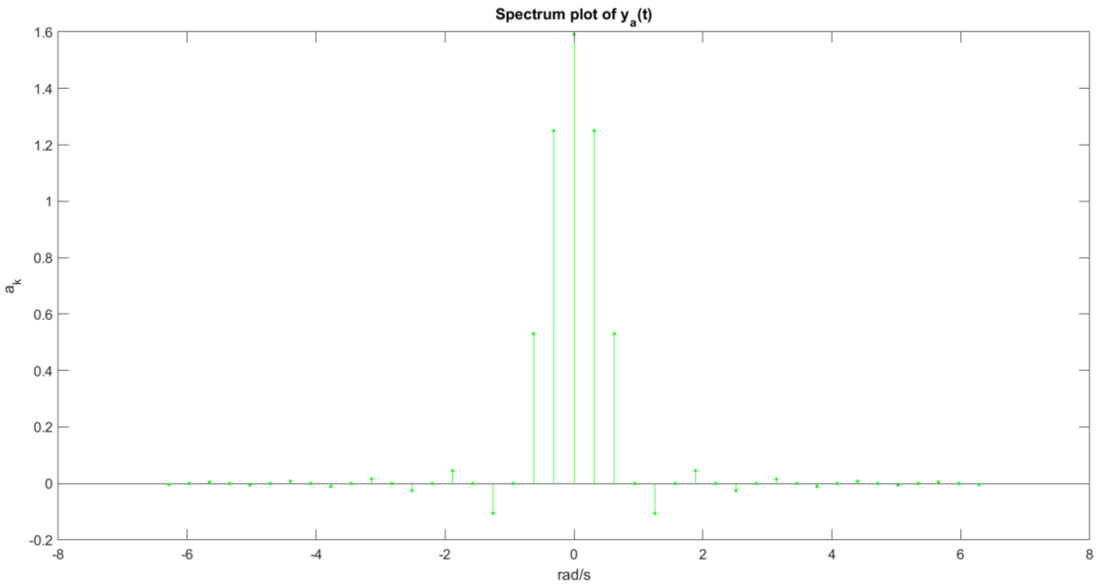
$$= \frac{5}{2}\left[\frac{\sin(\pi/2\,(1-k'))}{\pi(1-k')} + \frac{\sin(\pi/2\,(1+k'))}{\pi(1+k')}\right]$$

$$FSE[y_{a3}(t)] = \frac{5}{\pi} + \underbrace{\frac{5}{4}\cdot e^{j\pi t/9} + \frac{5}{4}\cdot e^{-j\pi t/9}}_{\frac{5}{2}\cdot\cos(\frac{\pi}{9}t)} + \sum_{k=2}^{\infty} 2a_k \cdot \cos(\tfrac{\pi}{9}\ldots$$

$$a_k = \begin{cases} \frac{5}{2}\left(\frac{\sin(\pi/2\,(1-k))}{\pi(1-k)} + \frac{\sin(\pi/2\,(1+k))}{\pi(1+k)}\right) & ;\ k \neq 0, 1, -1 \\ 5/\pi & ;\ k = 0 \\ 5/4 & ;\ k = 1, -1 \end{cases}$$

[Figure 37: F.S.E. of ya(t)2]

c) At the end of Figure 37, the relationship between the corresponding ak and the coefficients k. Here is the spectrum of coefficients as well.



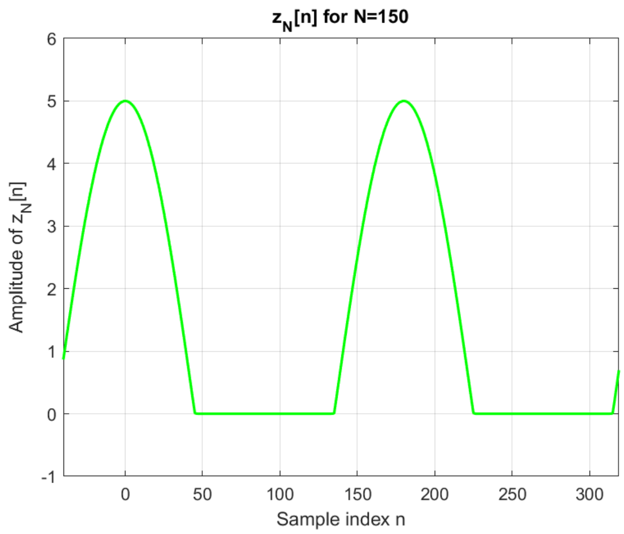
[Figure 38: Spectrum Plot of ya(t)]

d) By using the F.S.E from part b,

$$z_N[n] \;=\; \frac{5}{\pi} + \frac{5}{2}\cdot \cos\left(\frac{\pi}{9}\cdot\frac{n}{9}\right) \;+\; \sum_{k=1\,;\,k\neq 0}^{N} 2\cdot a_k \cdot \cos\left(\frac{\pi}{9}\cdot k\cdot\frac{n}{9}\right) \;\; for\; n \in [-40\,,\,319]$$

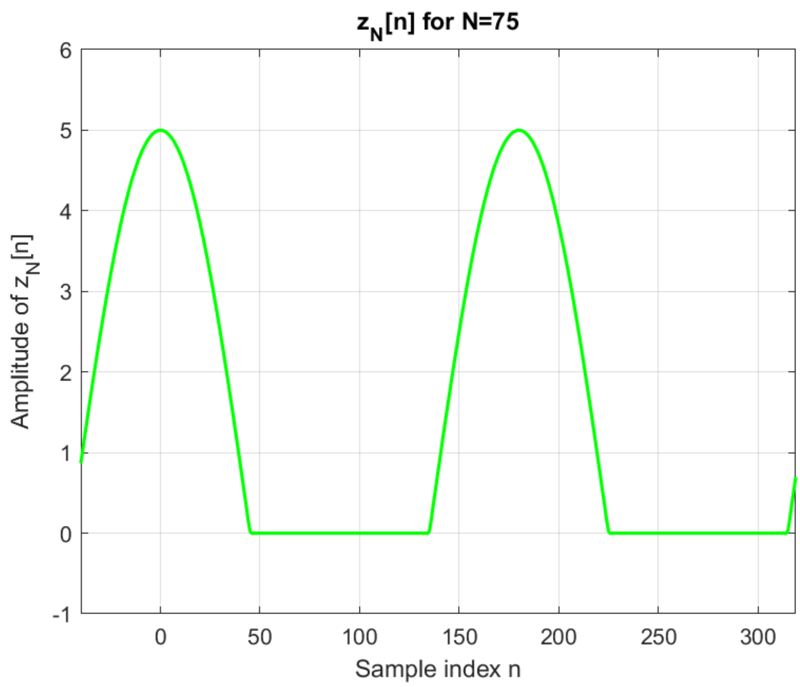
[Figure 39: zN[n]]

The plot of zN[n] when N = 150



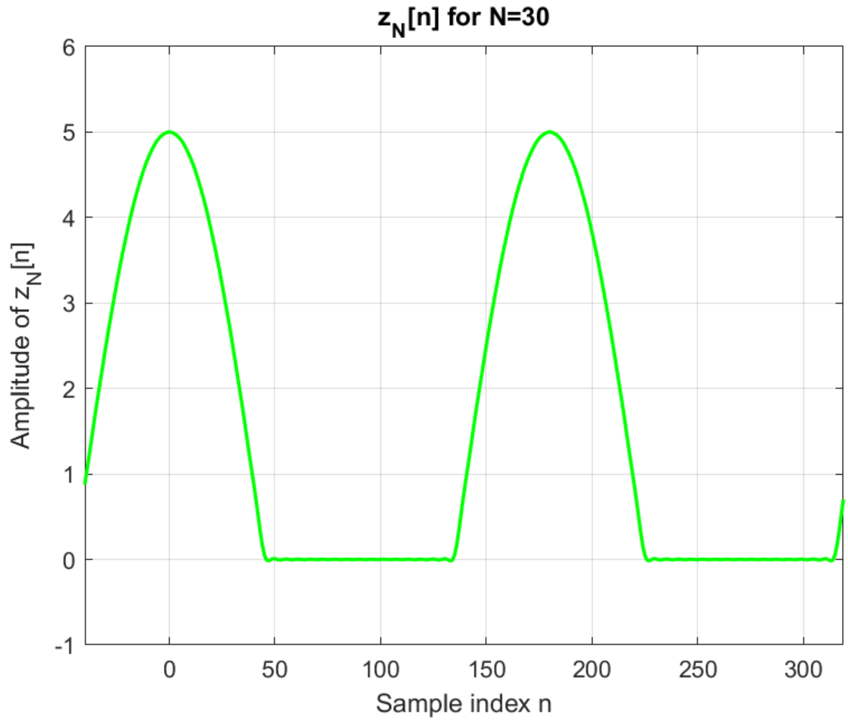
[Figure 40: Plot of zN[n] when N = 150] 1

e) Plot of zN[n] when N = 75



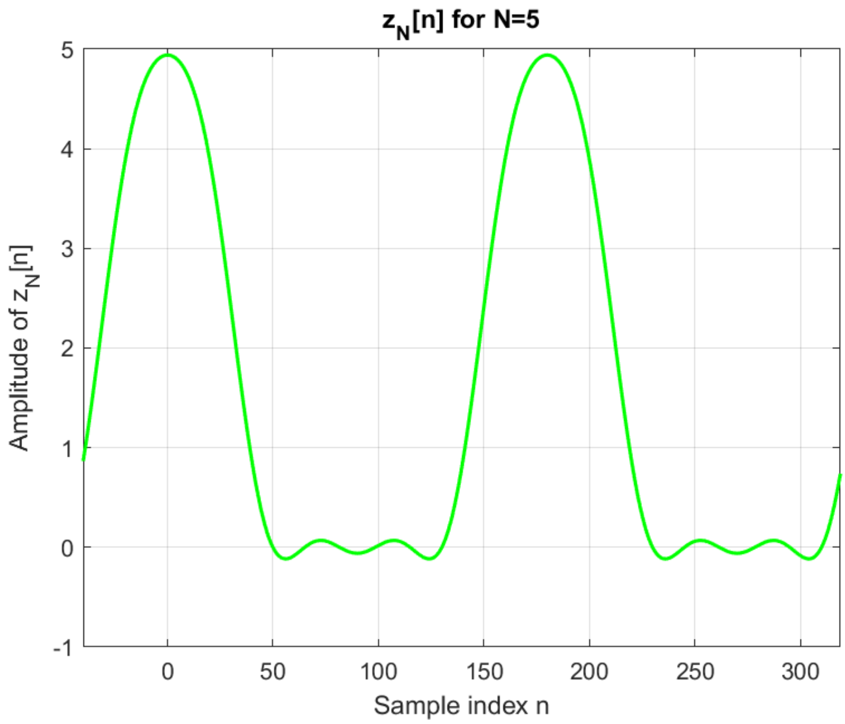
[Figure 41: Plot of zN[n] when N = 75]

f) Plot of zN[n] N= 30



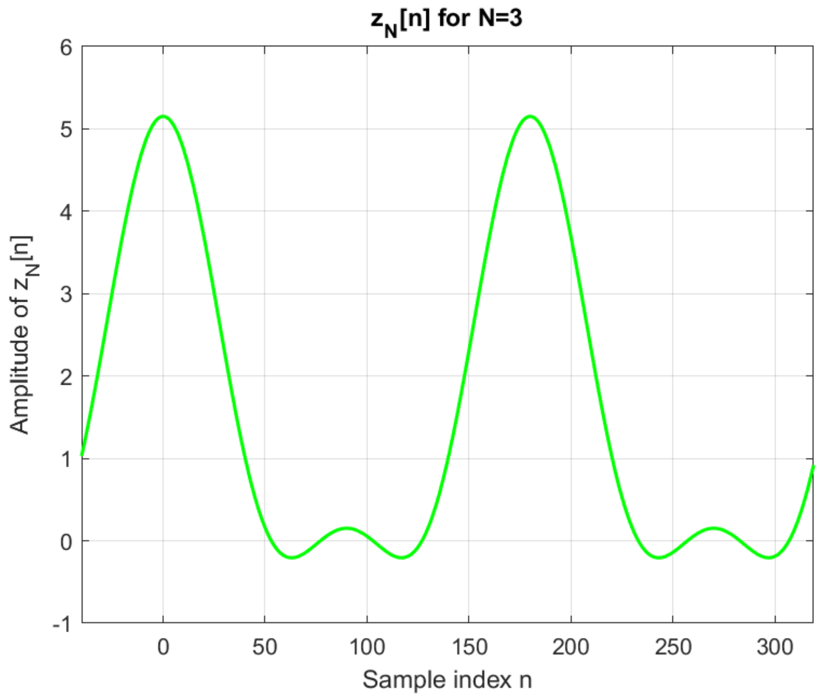
[Figure 42: Plot of zN[n] when N = 30]

g) Plot of zN[n] N= 5



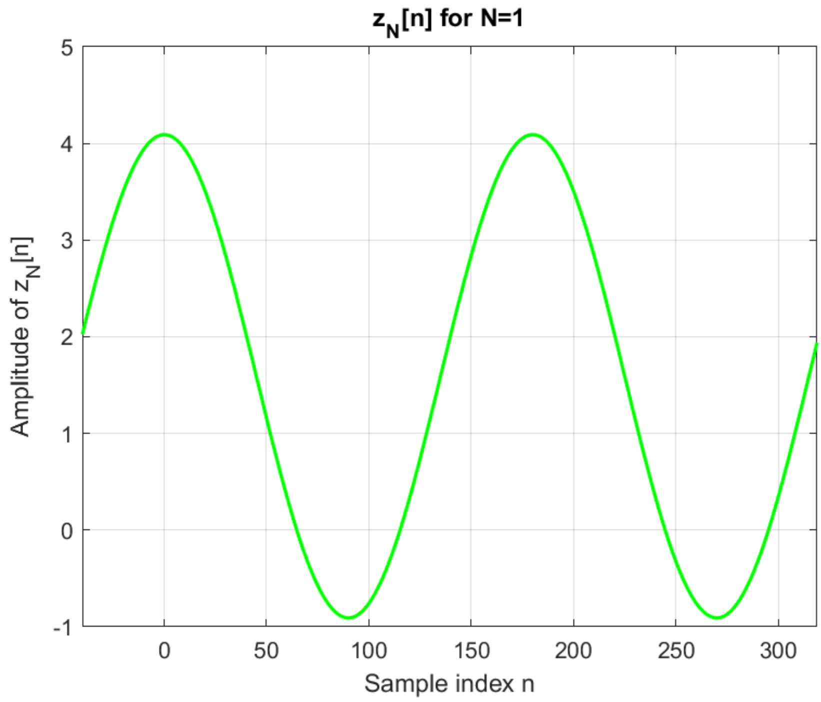
[Figure 43 : Plot of zN[n] when N = 5]

h) Plot of zN[n] when N = 3



[Figure 44: Plot of zN[n] when N = 3]

i) Here is the plot of zN[n] when N = 1

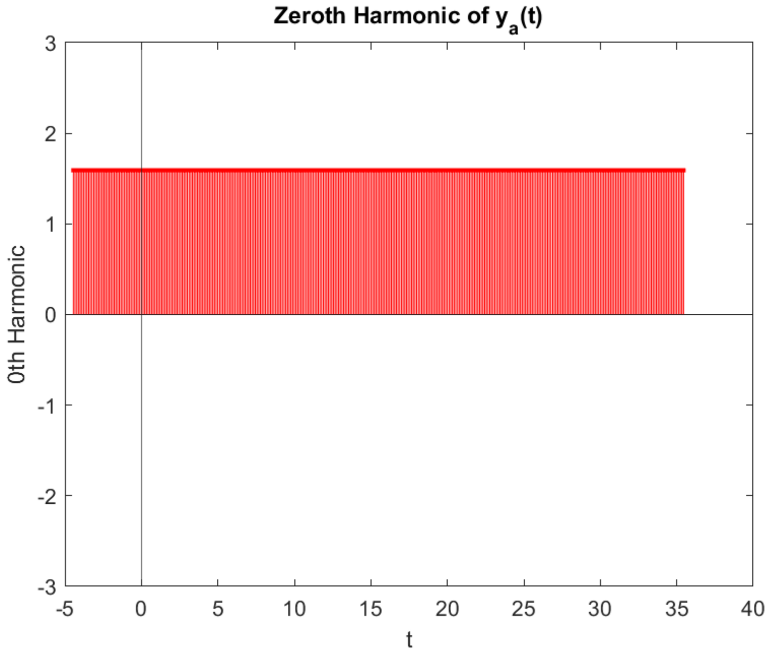


[Figure 45: Plot of zN[n] when N =1]

This question can also be analyzed similarly to the preceding two. When we reduced the number of harmonics we consider when approximating, the quality of the approximation drastically declined. The estimated function was more heavily influenced by the harmonics with higher coefficients than by those with lower coefficients.

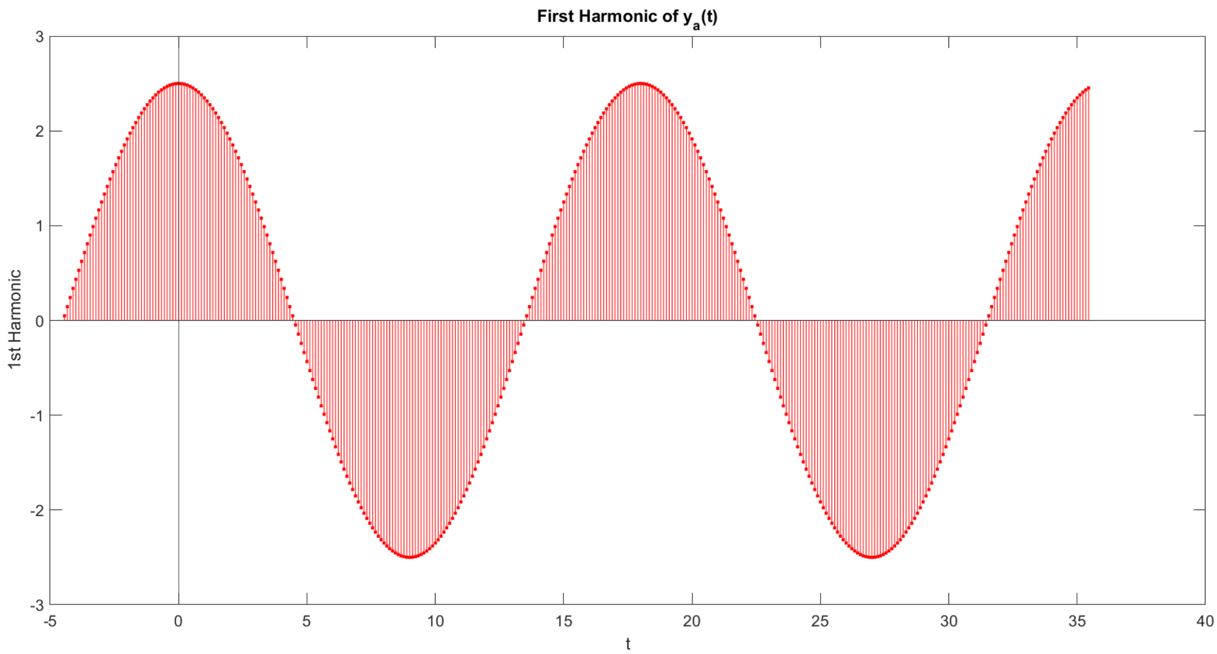
j) The original function ya(t) has four distinct harmonics.

Zero Harmonic:



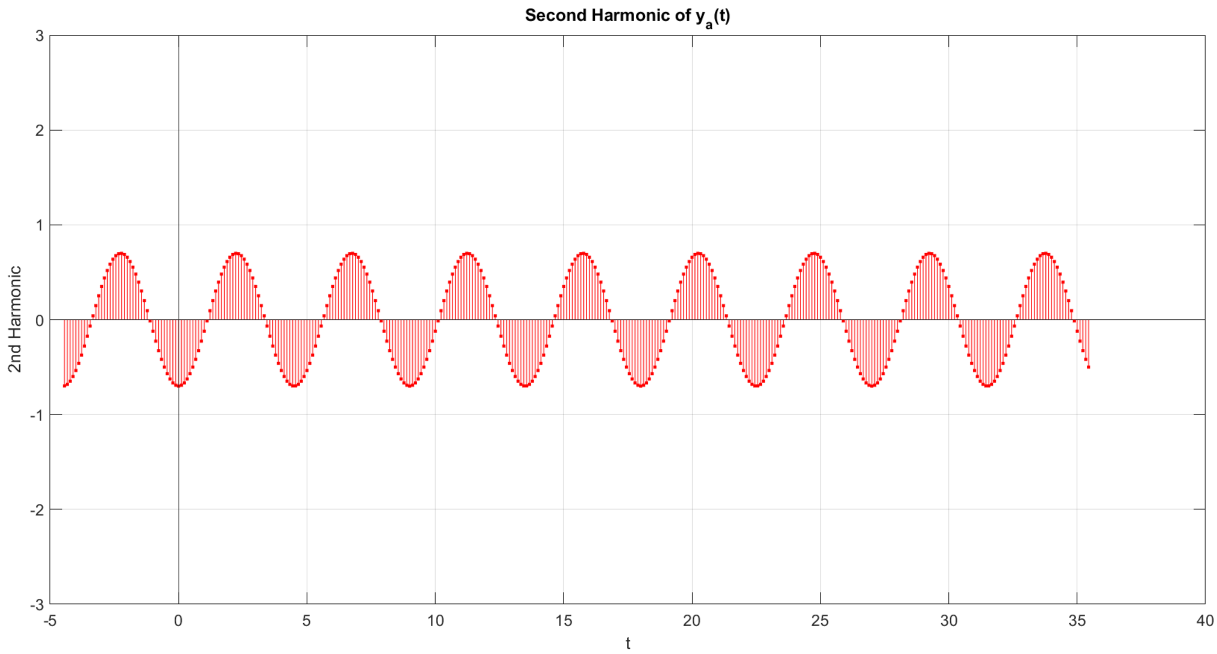
[Figure 46: Zeroth Harmonic]

First Harmonic:



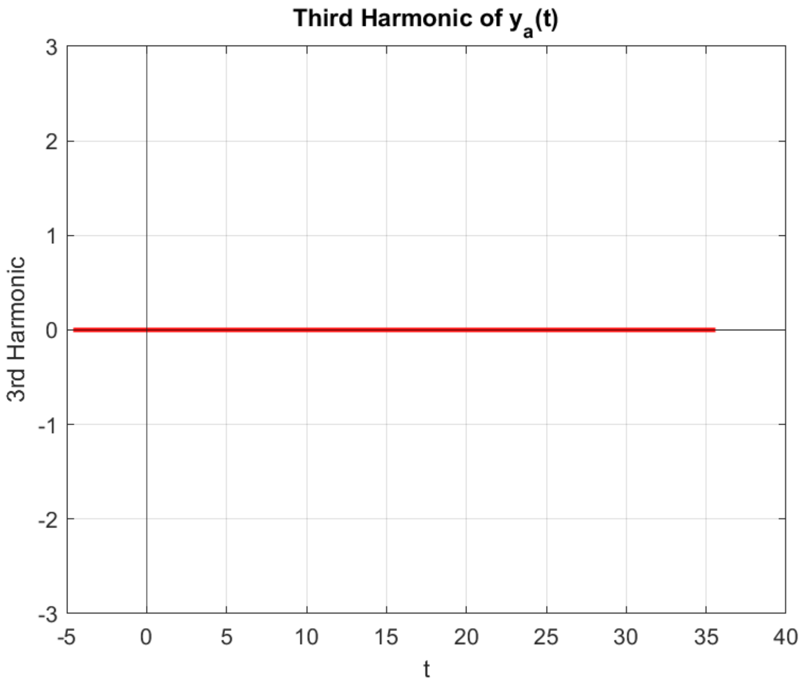
[Figure 47: 1st Harmonic]

Second Harmonic:



[Figure 48: Second Harmonic]

Third Harmonic:



[Figure 49: Third Harmonic]

CONCLUSION AND COMMENTS:

We studied the Fourier series expansion and a few associated approximations in this lab. All of the requirements were satisfied, and the lab was a complete success. Overall, I think the lab was challenging and time consuming because of the variety of the subsections in the questions.

Appendices:

Q1)

```
n = -40:319;
N = 30;
a = zeros(1, 2*N + 1);
a(N + 1) = 4/3;
for k = -N:N
    if k ~= 0
        a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
    end
end
zn = zeros(1, length(n));
for k = -N:N
    zn = zn + a(k + N + 1) * exp(1i * pi * k * n / 81);
end

%plot(n , real(zn), 'LineWidth', 1.5);
%title('Plot of z_N[n] - Approximation of y_a(t) - when N is 30');
%ylabel('z_{30}[n]');
%xlabel('n');
%xline(0);
%yline(0);
%grid on;
n = -40:319;
N = 5;
a = zeros(1, 2*N + 1);
a(N + 1) = 4/3;
for k = -N:N
    if k ~= 0
        a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
    end
end
zn = zeros(1, length(n));
for k = -N:N
    zn = zn + a(k + N + 1) * exp(1i * pi * k * n / 81);
end

%plot(n , real(zn), 'LineWidth', 1.5);
%title('Plot of z_N[n] - Approximation of y_a(t) - when N is 5');
%ylabel('z_{5}[n]');
%xlabel('n');
%xline(0);
%yline(0);
%grid on;
```

```
%stem(n,ones(1,length(n))*3/3,'b.');
%title('Zeroth Harmonic of y_a(t)');
%ylabel('0th Harmonic');xlabel('t');xline(0);yline(0);


n = -40:319;
k = 1;
f_first_harmonic = a(k + N + 1) * exp(1i * pi * k * n / 81);


%stem(n / 9, real(f_first_harmonic), 'b.');
%title('First Harmonic of y_a(t)');
%ylabel('1st Harmonic');
%xlabel('t');
%xline(0);
%yline(0);
%grid on;
n = -40:319;
N = 2;
a = zeros(1, 2 * N + 1);
a(N + 1) = 4/3;


for k = -N:N
    if k ~= 0
        a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
    end
end
k = 2;
f_second_harmonic = a(k + N + 1) * exp(1i * pi * k * n / 81);
%stem(n / 9, real(f_second_harmonic), 'b.');
%title('Second Harmonic of y_a(t)');
%ylabel('2nd Harmonic');
%xlabel('t');
%xline(0);
%yline(0);
%grid on;

n = -40:319;
N = 3;
a = zeros(1, 2 * N + 1);
a(N + 1) = 4 / 3;
```

```
n = -40:1:319;
ya = zeros(size(n));
for i = n
    dum = mod(i/10,18);
    if (dum >= 7)&&(dum < 10)
        ya(i+41) = 8;
    end
end

%stem(n,ya,'.k');
%xlabel('n')
%ylabel('y[n]')
%title('Plot of sampled version of ya(t) with Ts =1/10 ; y[n]')

m = -20:1:20;
a = zeros(size(m));

a(21) = 4/3;

for k = 1:length(m)
    if m(k) ~= 0
        a(k) = (4/(1i*pi*m(k)))*(exp(1i*22*pi*m(k)/18) - exp(1i*16*pi*m(k)/18));
    end
end

%stem(m*2*pi/18, a , 'filled', 'k.');
%xlabel('rad/s');
%ylabel('ak');
%title('Spectrum plot of the real part of ya(t)');
%grid on;

%a_imag = imag(a);

%figure;
%stem(m * 2 * pi / 18, a_imag, 'filled', 'r.');
%xlabel('Frequency (rad/s)');
%ylabel('Im(a_k)');
%title('Imaginary Part of the Spectrum of y_a(t)');
%grid on;
```

```
n = -40:319;
N = 150;
a = zeros(1, 2*N + 1);
a(N + 1) = 4/3;

for k = -N:N
    if k ~= 0
        a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
    end
end
zn = zeros(1, length(n));
for k = -N:N
    zn = zn + a(k + N + 1) * exp(1i * pi * k * n / 81);
end
%plot(n , real(zn), 'LineWidth', 1.5);
%title('Plot of z_N[n] - Approximation of y_a(t) - when N is 150');
%ylabel('z_{150}[n]');
%xlabel('n');
%xline(0);
%yline(0);
%grid on;
n = -40:319;
N = 75;
a = zeros(1, 2*N + 1);
a(N + 1) = 4/3;
for k = -N:N
    if k ~= 0
        a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
    end
end

zn = zeros(1, length(n));
for k = -N:N
    zn = zn + a(k + N + 1) * exp(1i * pi * k * n / 81);
end
%plot(n , real(zn), 'LineWidth', 1.5);
%title('Plot of z_N[n] - Approximation of y_a(t) - when N is 75');
%ylabel('z_{75}[n]');
%xlabel('n');
%xline(0);
%yline(0);
%grid on;
```

```
n = -40:319;
N = 3;
a = zeros(1, 2*N + 1);
a(N + 1) = 4/3;
for k = -N:N
    if k ~= 0
        a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
    end
end
zn = zeros(1, length(n));
for k = -N:N
    zn = zn + a(k + N + 1) * exp(1i * pi * k * n / 81);
end

%plot(n, real(zn), 'LineWidth', 1.5);
%title('Plot of z_N[n] - Approximation of y_a(t) - when N is 3');
%ylabel('z_{3}[n]');
%xlabel('n');
%xline(0);
%yline(0);
%grid on;
n = -40:319;
N = 1;
a = zeros(1, 2*N + 1);
a(N + 1) = 4/3;

for k = -N:N
    if k ~= 0
        a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
    end
end
zn = zeros(1, length(n));
for k = -N:N
    zn = zn + a(k + N + 1) * exp(1i * pi * k * n / 81);
end
%plot(n , real(zn), 'LineWidth', 1.5);
%title('Plot of z_N[n] - Approximation of y_a(t) - when N is 1');
%ylabel('z_{1}[n]');
%xlabel('n');
%xline(0);
%yline(0);
%grid on;
```

```
n = -40:319;
N = 3;
a = zeros(1, 2 * N + 1);
a(N + 1) = 4 / 3;


for k = -N:N
    if k ~= 0
        a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
    end
end


k = 3;
f_third_harmonic = a(k + N + 1) * exp(1i * pi * k * n / 81);


%stem(n / 9, real(f_third_harmonic), 'b.');
%title('Third Harmonic of y_a(t)');
%ylabel('3rd Harmonic');
%xlabel('t');
%xline(0);
%yline(0);
%grid on;
```

Q2)

```
n = -40:1:319;
ya = zeros(size(n));
for i = n
    dum = mod(i/10,9);
    ya(i+41) = abs(5*cos(pi/9*dum));

end

%stem(n,ya,'.r');
%xlabel('n/10')
%ylabel('y[n]')
%title('Plot of sampled version of ya(t) with Ts =1/10 ; y[n]')

m = -20:1:20;
a = zeros(size(m));

a(21) = 10/pi ;
a = 5*((sin(pi/2*(1-2*m))./(pi.*(1-2*m)))+(sin(pi/2*(1+2*m))./(pi.*(1+2*m))));


%stem(m.*pi/4,a,'filled','m.');
%xlabel('rad/s');
%ylabel('ak');
%title('Spectrum plot of a(t)');


n = -40:319;
a_values = zeros(1, length(n));

syms k

a_sym(k) = 5 * (sin(pi/2 * (1 - 2*k)) / (pi * (1 - 2*k))) + (sin(pi/2 * (1 + 2*k)) / (pi * (1 + 2*k)));


for i = 1:length(n)
    a_values(i) = double(a_sym(i));
end

f_sym(k, n_sym) = a_sym(k) * cos(2 * pi / 9 * k * n_sym / 9);
syms a
zn_sym(a, n_sym) = 10 / pi + 2 * symsum(f_sym(k, n_sym), k, 1, a);
```

```
n = -40:319;
a_values = zeros(1, length(n));

syms k

a_sym(k) = 5 * (sin(pi/2 * (1 - 2*k)) / (pi * (1 - 2*k))) + (sin(pi/2 * (1 + 2*k)) / (pi * (1 + 2*k)));


for i = 1:length(n)
    a_values(i) = double(a_sym(i));
end

f_sym(k, n_sym) = a_sym(k) * cos(2 * pi / 9 * k * n_sym / 9);
syms a
zn_sym(a, n_sym) = 10 / pi + 2 * symsum(f_sym(k, n_sym), k, 1, a);

zn_values = double(zn_sym(30, n));

%plot(n, zn_values, 'LineWidth', 1.5);
%title('Plot of z_N[n] - Approximation of y_a(t) - when N is 30');
%ylabel('z_{30}[n]');
%xlabel('n');
%xline(0);
%yline(0);

n = -40:319;
a_values = zeros(1, length(n));

syms k

a_sym(k) = 5 * (sin(pi/2 * (1 - 2*k)) / (pi * (1 - 2*k))) + (sin(pi/2 * (1 + 2*k)) / (pi * (1 + 2*k)));


for i = 1:length(n)
    a_values(i) = double(a_sym(i));
end

f_sym(k, n_sym) = a_sym(k) * cos(2 * pi / 9 * k * n_sym / 9);
syms a
zn_sym(a, n_sym) = 10 / pi + 2 * symsum(f_sym(k, n_sym), k, 1, a);
```

```
zn_values = double(zn_sym(150, n));

%plot(n, zn_values,'m', 'LineWidth', 1.5);
%title('Plot of z_N[n] - Approximation of y_a(t) - when N is 150');
%ylabel('z_{150}[n]');
%xlabel('n');
%xline(0);
%yline(0);



% Define n as an array and create an empty array for 'a'
n = -40:319;
a_values = zeros(1, length(n)); % Initialize a_values for storing coefficients

% Define symbolic variable k for symbolic operations
syms k

% Define a(k) as a symbolic expression for general k
a_sym(k) = 5 * (sin(pi/2 * (1 - 2*k)) / (pi * (1 - 2*k))) + (sin(pi/2 * (1 + 2*k)) / (pi * (1 + 2*k)));

% Evaluate a_values using the symbolic expression and store results in a numeric array
for i = 1:length(n)
    a_values(i) = double(a_sym(i)); % Evaluate a(k) at integer values of k and store as double
end

% Define f(k) as a symbolic function of both k and n
f_sym(k, n_sym) = a_sym(k) * cos(2 * pi / 9 * k * n_sym / 9);

% Define symbolic variable for summation limit 'a'
syms a

% Calculate zn(a) as a symbolic summation for different values of 'a'
zn_sym(a, n_sym) = 10 / pi + 2 * symsum(f_sym(k, n_sym), k, 1, a);

% Choose N = 150 and evaluate zn(150, n) for each value of n
zn_values = double(zn_sym(150, n));

% Plot the result
%plot(n, zn_values, 'LineWidth', 1.5);
%title('Plot of z_N[n] - Approximation of y_a(t) - when N is 150');
%ylabel('z_{150}[n]');
```

```
zn_values = double(zn_sym(5, n));

%plot(n, zn_values, 'LineWidth', 1.5);
%title('Plot of z_N[n] - Approximation of y_a(t) - when N is 5');
%ylabel('z_{5}[n]');
%xlabel('n');
%xline(0);
%yline(0);

n = -40:319;
a_values = zeros(1, length(n));

% a_values hesaplandı
for k = 1:length(n)
    a_values(k) = 5 * (sin(pi/2 * (1 - 2*k)) / (pi * (1 - 2*k))) + (sin(pi/2 * (1 + 2*k)) / (pi * (1 + 2*k)));
end

% Zn hesaplama fonksiyonu
zn_values = zeros(1, length(n));
for i = 1:length(n)
    zn_values(i) = 10 / pi + 2 * sum(a_values(1:75) .* cos(2 * pi / 9 * (1:75) * n(i) / 9));
end

% Grafiği çizdirme
plot(n, zn_values, 'm', 'LineWidth', 1.5);
title('Plot of z_N[n] - Approximation of y_a(t) - when N is 75');
ylabel('z_{75}[n]');
xlabel('n');
xline(0);
yline(0);


n = -40:319;
a_values = zeros(1, length(n));

syms k

a_sym(k) = 5 * (sin(pi/2 * (1 - 2*k)) / (pi * (1 - 2*k))) + (sin(pi/2 * (1 + 2*k)) / (pi * (1 + 2*k)));
```

```
for i = 1:length(n)
    a_values(i) = double(a_sym(i));
end

f_sym(k, n_sym) = a_sym(k) * cos(2 * pi / 9 * k * n_sym / 9);
syms a
zn_sym(a, n_sym) = 10 / pi + 2 * symsum(f_sym(k, n_sym), k, 1, a);

zn_values = double(zn_sym(1, n));

%plot(n, zn_values, 'LineWidth', 1.5);
%title('Plot of z_N[n] - Approximation of y_a(t) - when N is 1');
%ylabel('z_{1}[n]');
%xlabel('n');
%xline(0);
%yline(0);

stem(n,ones(1,length(n))*10/pi,'b.');
title('Zeroth Harmonic of y_a(t)');
ylabel('0th Harmonic');xlabel('t');xline(0);yline(0);


n = -40:319;
ya = zeros(size(n));
for i = n
    dum = mod(i/10,9);
    ya(i+41) = abs(5*cos(pi/9*dum));
end

m = -20:1:20;
a = zeros(size(m));
a(21) = 10/pi ;
a = 5*((sin(pi/2*(1-2*m))./(pi.*(1-2*m)))+(sin(pi/2*(1+2*m))./(pi.*(1+2*m))));

% First Harmonic
k = 1;
f_first_harmonic = (3/2) * a(k + 21) * cos(2 * pi / 9 * k * n / 9);

%figure()
%stem(n / 9, f_first_harmonic, 'b.');
%title('First Harmonic of y_a(t)');
```

```
n = -40:319;
ya = zeros(size(n));
for i = n
    dum = mod(i/10,9);
    ya(i+41) = abs(5*cos(pi/9*dum));
end

m = -20:1:20;
a = zeros(size(m));
a(21) = 10/pi ;
a = 5*((sin(pi/2*(1-2*m))./(pi.*(1-2*m)))+(sin(pi/2*(1+2*m))./(pi.*(1+2*m))));

% Second Harmonic
k = 2;
f_second_harmonic = 2 * a(k + 21) * cos(2 * pi / 9 * k * n / 9);

%figure()
%stem(n / 9, f_second_harmonic, 'b.');
%title('Second Harmonic of y_a(t)');
%ylabel('2nd Harmonic');
%xlabel('t');
%xline(0);
%yline(0);
%grid on;


n = -40:319;
ya = zeros(size(n));
for i = n
    dum = mod(i/10,9);
    ya(i+41) = abs(5*cos(pi/9*dum));
end

m = -20:1:20;
a = zeros(size(m));
a(21) = 10/pi ;
a = 5*((sin(pi/2*(1-2*m))./(pi.*(1-2*m)))+(sin(pi/2*(1+2*m))./(pi.*(1+2*m))));

% Third Harmonic
k = 3;
f_second_harmonic = 3 * a(k + 21) * cos(2 * pi / 9 * k * n / 9);

%figure()
%stem(n / 9, f_second_harmonic, 'b.');
%title('Third Harmonic of y_a(t)');
%ylabel('3rd Harmonic');
%xlabel('t');
%xline(0);
%yline(0);
%grid on;
```

Q3)

```
n = -40:1:319;
ya = zeros(size(n));
for i = n
    dum = mod(i/10+4.5,18);
    if (dum >= 0) && (dum < 9)
        ya(i+41) = abs(5*cos(pi/9*(dum+4.5)));
    end
end

%stem(n, ya, '.k');
%xlabel('n/8')
%ylabel('y[n]')
%title('Plot of sampled version of ya(t) with Ts = 1/8 : y[n]')

m = -20:1:20;
a = 2.5 * ((sin(pi/2 * (1 - m)) ./ (pi * (1 - m))) + (sin(pi/2 * (1 + m)) ./ (pi * (1 + m))));
a(20) = 5/4;
a(21) = 5/pi;
a(22) = 5/4;

%stem(m * pi / 10, a, 'filled', 'g.');
%xlabel('rad/s');
%ylabel('a_k');
%title('Spectrum plot of y_a(t)');
```

```matlab
T = 18;
omega0 = pi/9;
Ts = 0.1;
N = 1;
n = -40:319;


zN = zeros(size(n));


a_coeff = zeros(1, 2*N + 1);
k_values = -N:N;

for idx = 1:length(k_values)
    k = k_values(idx);
    if k == 0
        a_coeff(idx) = 5/pi;
    elseif k == 1 || k == -1
        a_coeff(idx) = 5/4;
    elseif mod(k, 2) == 0

        a_coeff(idx) = (5/pi) * (cos(pi * k / 2)) / (1 - k^2);
    else

        a_coeff(idx) = 0;
    end
end


for i = 1:length(n)
    zN(i) = sum(a_coeff .* exp(1j * omega0 * k_values * n(i) * Ts));
end

% Plot the real part of zN[n]
%figure;
%plot(n, real(zN),'g', 'LineWidth', 1.5);
%xlabel('Sample index n');
%ylabel('Amplitude of z_N[n]');
%title('z_N[n] for N=1');
%grid on;
%xlim([-40, 319]);
```

```matlab
%stem(n/9,ones(1,length(n))*5/pi,'r.');
%title('Zeroth Harmonic of y_a(t)');
%ylabel('0th Harmonic');xlabel('t');xline(0);yline(0);


%stem(n/9,5/2*cos(pi/9*n/9),'r.');
%title('First Harmonic of y_a(t)');
%ylabel('1st Harmonic');xlabel('t');xline(0);yline(0);

% Define the range for n
n = -40:319;

% Initialize the array for ya
ya = zeros(size(n));

% Calculate ya values for each n
for i = n
    dum = mod(i / 10, 9);  % Get the modulo value for the current n
    ya(i + 41) = abs(5 * cos(pi / 9 * dum));  % Calculate the corresponding value of ya
end

% Define the range for m (Fourier coefficients)
m = -20:1:20;
a = zeros(size(m));

a = 5 * ((sin(pi / 2 * (1 - 2 * m)) ./ (pi .* (1 - 2 * m))) + (sin(pi / 2 * (1 + 2 * m)) ./ (pi .* (1 + 2 * m))));

% Second Harmonic (k = 2)
k = 2;
f_second_harmonic = 3.3 * a(k + 21) * cos(2 * pi / 9 * k * n / 9);

% Plot the second harmonic
%figure;
%stem(n / 9, f_second_harmonic, 'r.');
%title('Second Harmonic of y_a(t)');
%ylabel('2nd Harmonic');
%xlabel('t');
%xline(0);  % Adds a vertical line at t = 0
%yline(0);  % Adds a horizontal line at y = 0
%grid on;   % Enables grid for better readability
```

```
% Define the range for n
n = -40:319;

% Initialize the array for ya
ya = zeros(size(n));


for i = n
    dum = mod(i / 10, 9);
    ya(i + 41) = abs(5 * cos(pi / 9 * dum));
end


m = -20:1:20;
a = zeros(size(m));

a = 5 * ((sin(pi / 2 * (1 - 2 * m)) ./ (pi .* (1 - 2 * m))) + (sin(pi / 2 * (1 + 2 * m)) ./ (pi .* (1 + 2 * m))));


k = 3;
f_second_harmonic = 0 * a(k + 21) * cos(2 * pi / 9 * k * n / 9);


figure;
stem(n / 9, f_second_harmonic, 'r.');
title('Third Harmonic of y_a(t)');
ylabel('3rd Harmonic');
xlabel('t');
xline(0);
yline(0);
grid on;
```