FE 5217: Seminar in Risk Management and Alternative Investment: Algorithmic Trading and Quantitative Strategies–Assignment 2, Solutions

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1 Solutions of Question 1

1.1 Question 1 (a)

From Table 1 we can see p-value of intercept for SML, BIGL, BIG2 and BIGH are not significant and p-value for SM2 and SMH are significant. Therefore, we reject CAMP model for SM2 and SMH.

Figures 1 to 6 provide linear regression analysis for predicted value $\hat{Y}_i$ over absolute value of residuals $|e_i|$. This is a quick way to check non-constant variance (Faraway (2005)). we can see for each portfolio, $|e_i|$ has linear relationships with the predicted value $\hat{Y}_i$. This point can also be seen from Figure 7 to 12 that the residuals change dramatically as the fitted value of response variable. In addition, QQ-plots show that the residuals are not normal distributed.

```
$PF6_SML

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
   Min     1Q Median     3Q    Max 
-3.585 -1.628 -0.597  0.890 34.515 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.38333    0.07885  30.228  < 2e-16 ***
fit_y        0.06799    0.01131   6.009 2.58e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.525 on 1034 degrees of freedom
Multiple R-squared:  0.03375,	Adjusted R-squared:  0.03281 
F-statistic: 36.11 on 1 and 1034 DF,  p-value: 2.58e-09
```

Figure 1: SML: $\hat{Y}_i$ vs $|e_i|$

```
$PF6_SM2

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
   Min     1Q Median     3Q    Max 
-2.685 -1.283 -0.521  0.675 35.316 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.92859    0.06857  28.125  < 2e-16 ***
fit_y        0.06030    0.01052   5.734 1.29e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.182 on 1034 degrees of freedom
Multiple R-squared:  0.03081,	Adjusted R-squared:  0.02988 
F-statistic: 32.87 on 1 and 1034 DF,  p-value: 1.29e-08
```

Figure 2: SM2: $\hat{Y}_i$ vs $|e_i|$

```
$PF6_SMH

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
   Min     1Q Median     3Q    Max 
-4.045 -1.738 -0.737  0.865 38.381 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.57469    0.09583  26.868  < 2e-16 ***
fit_y        0.08672    0.01308   6.629 5.45e-11 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.044 on 1034 degrees of freedom
Multiple R-squared:  0.04076,	Adjusted R-squared:  0.03983 
F-statistic: 43.94 on 1 and 1034 DF,  p-value: 5.449e-11
```

Figure 3: SMH: $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIGL

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.0119 -0.5706 -0.2127  0.3493  4.7313 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.864178   0.024251  35.634  < 2e-16 ***
fit_y       0.018644   0.004595   4.058 5.33e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.7755 on 1034 degrees of freedom
Multiple R-squared:  0.01567,	Adjusted R-squared:  0.01472 
F-statistic: 16.46 on 1 and 1034 DF,  p-value: 5.332e-05
```

Figure 4: BIGL: $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIG2

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max
-1.3198 -0.7521 -0.3731  0.2664 12.2895

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.06264    0.04165  25.512  < 2e-16 ***
fit_y        0.02773    0.00746   3.718 0.000212 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.331 on 1034 degrees of freedom
Multiple R-squared:  0.01319,   Adjusted R-squared:  0.01224
F-statistic: 13.82 on 1 and 1034 DF,  p-value: 0.0002116
```

Figure 5: BIG2: $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIGH

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max
-2.4195 -1.4410 -0.6099  0.5920 23.8199

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.01156    0.07312  27.510   <2e-16 ***
fit_y        0.04133    0.01107   3.733    2e-04 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.332 on 1034 degrees of freedom
Multiple R-squared:  0.01329,   Adjusted R-squared:  0.01234
F-statistic: 13.93 on 1 and 1034 DF,  p-value: 0.0001999
```

Figure 6: BIGH: $\hat{Y}_i$ vs $|e_i|$

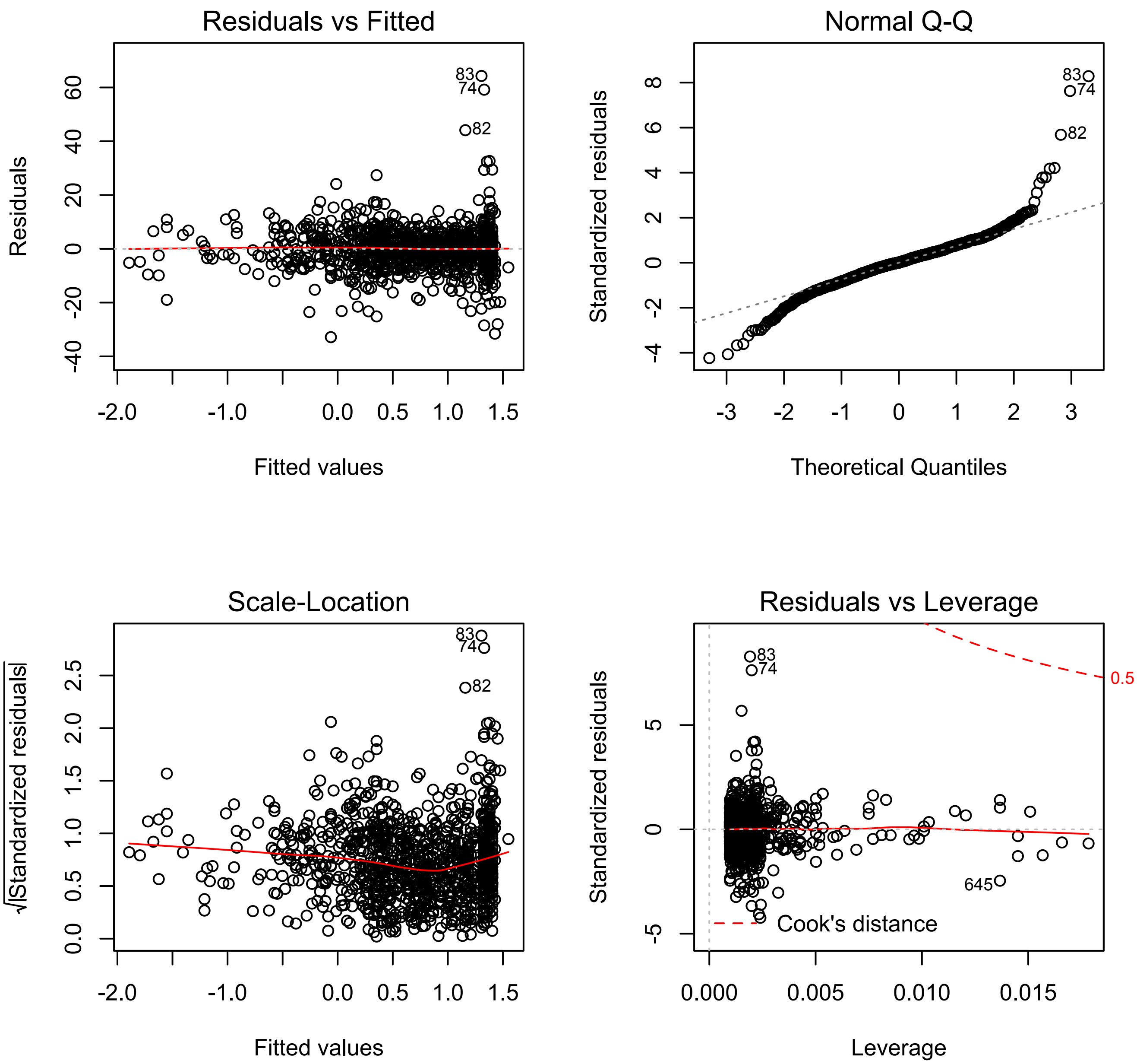

Figure 7: Regression diagnostics figures for SML

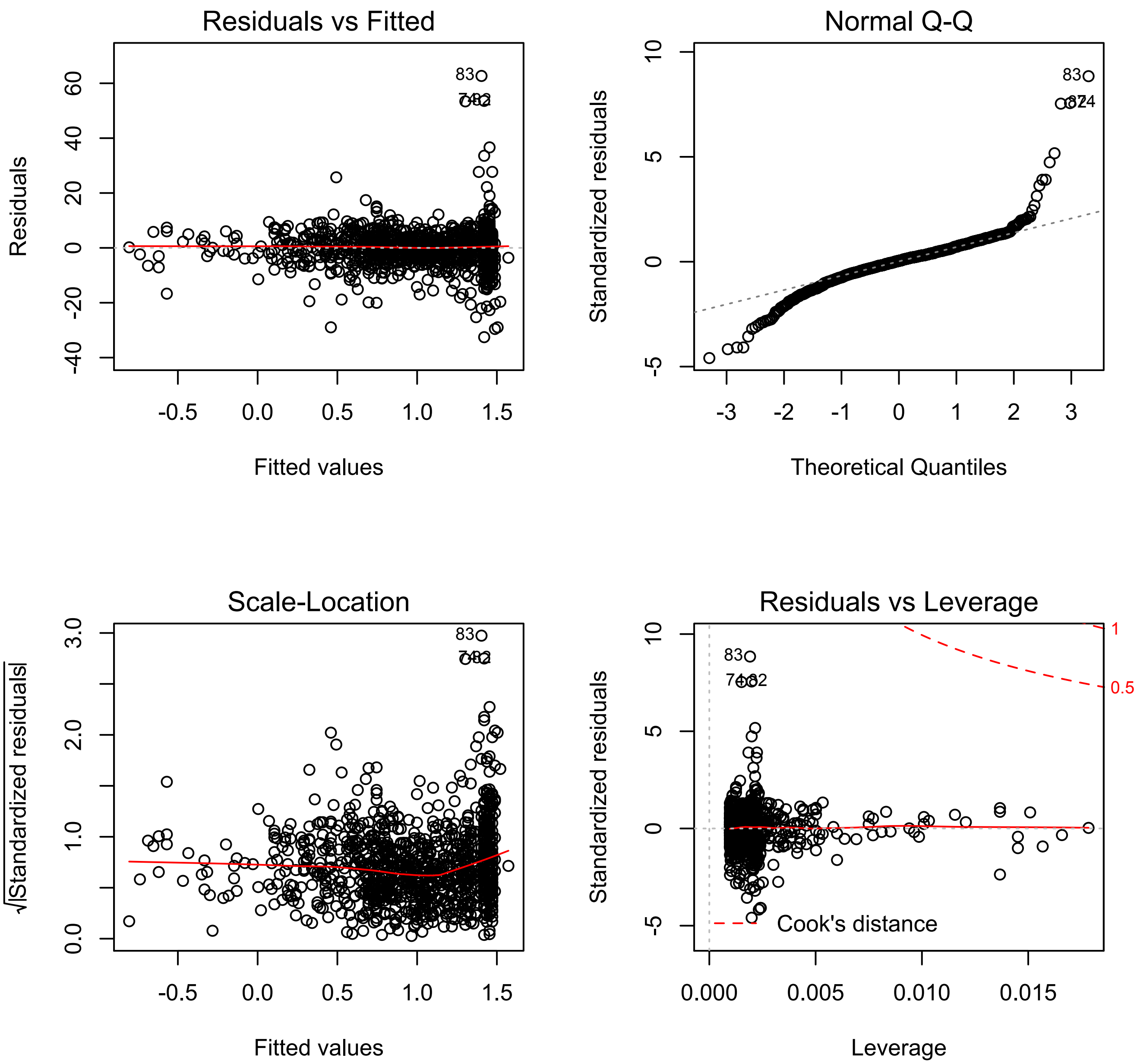

Figure 8: Regression diagnostics figures for SM2

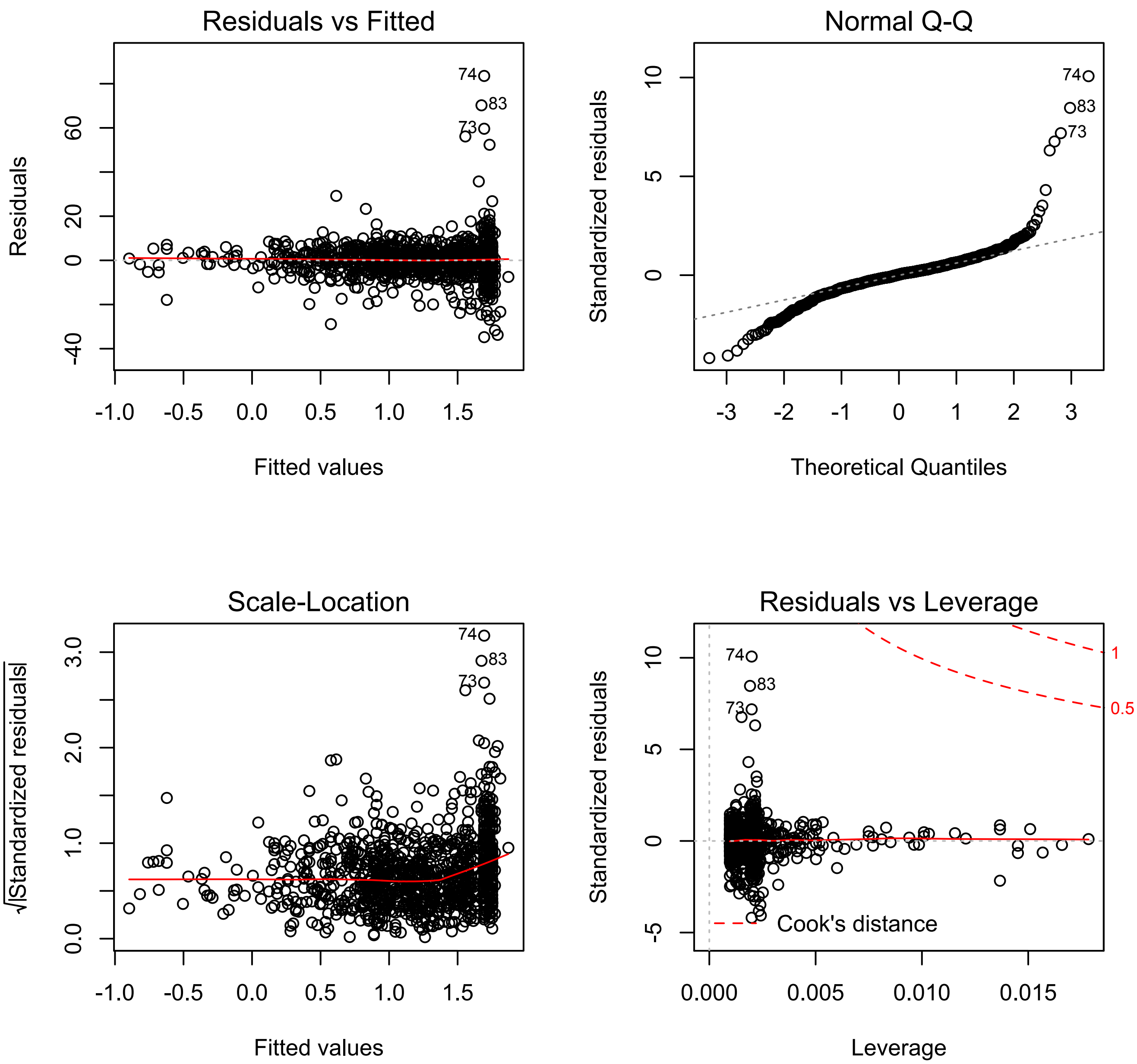

Figure 9: Regression diagnostics figures for SMH

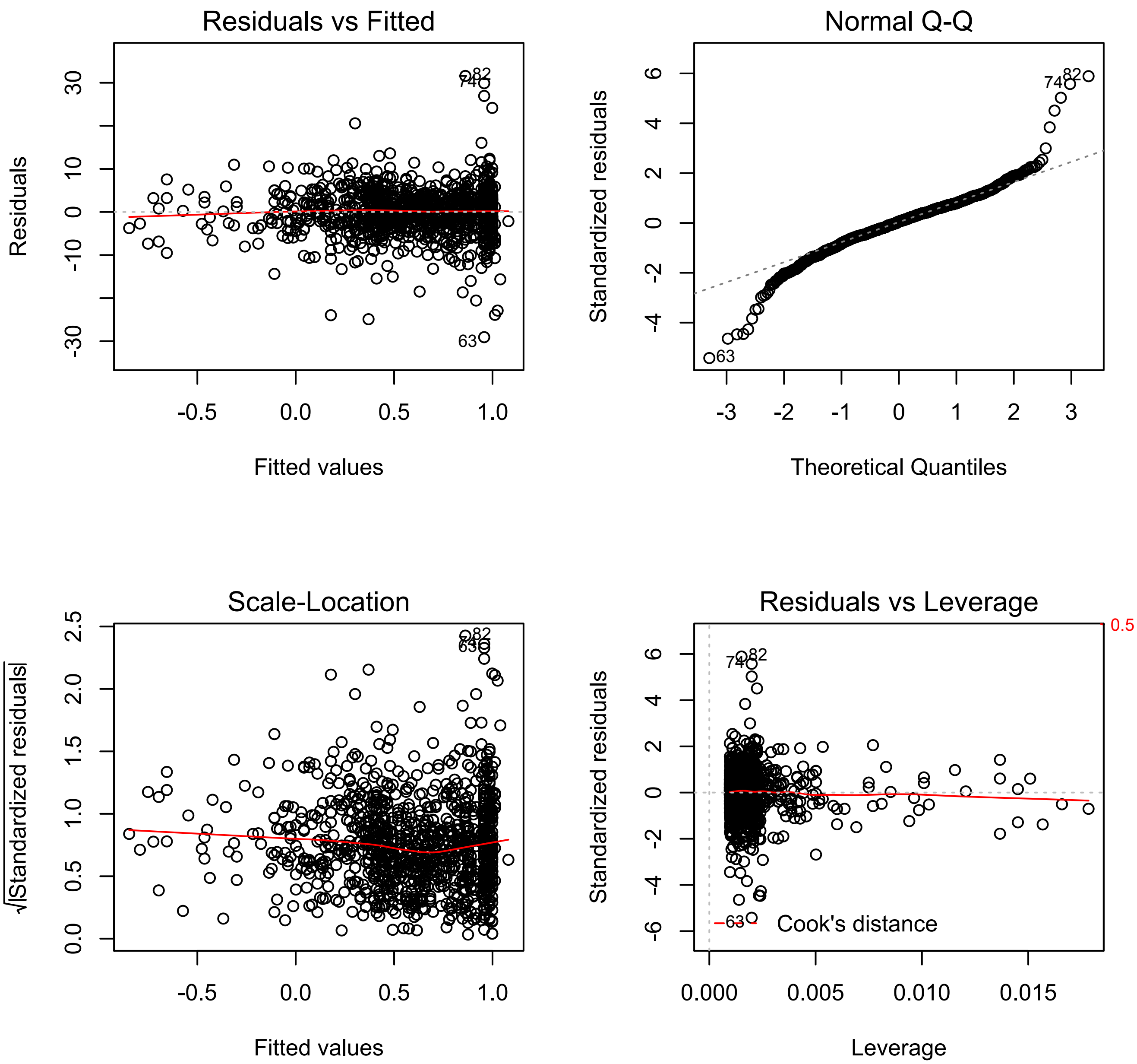

Figure 10: Regression diagnostics figures for BIGL

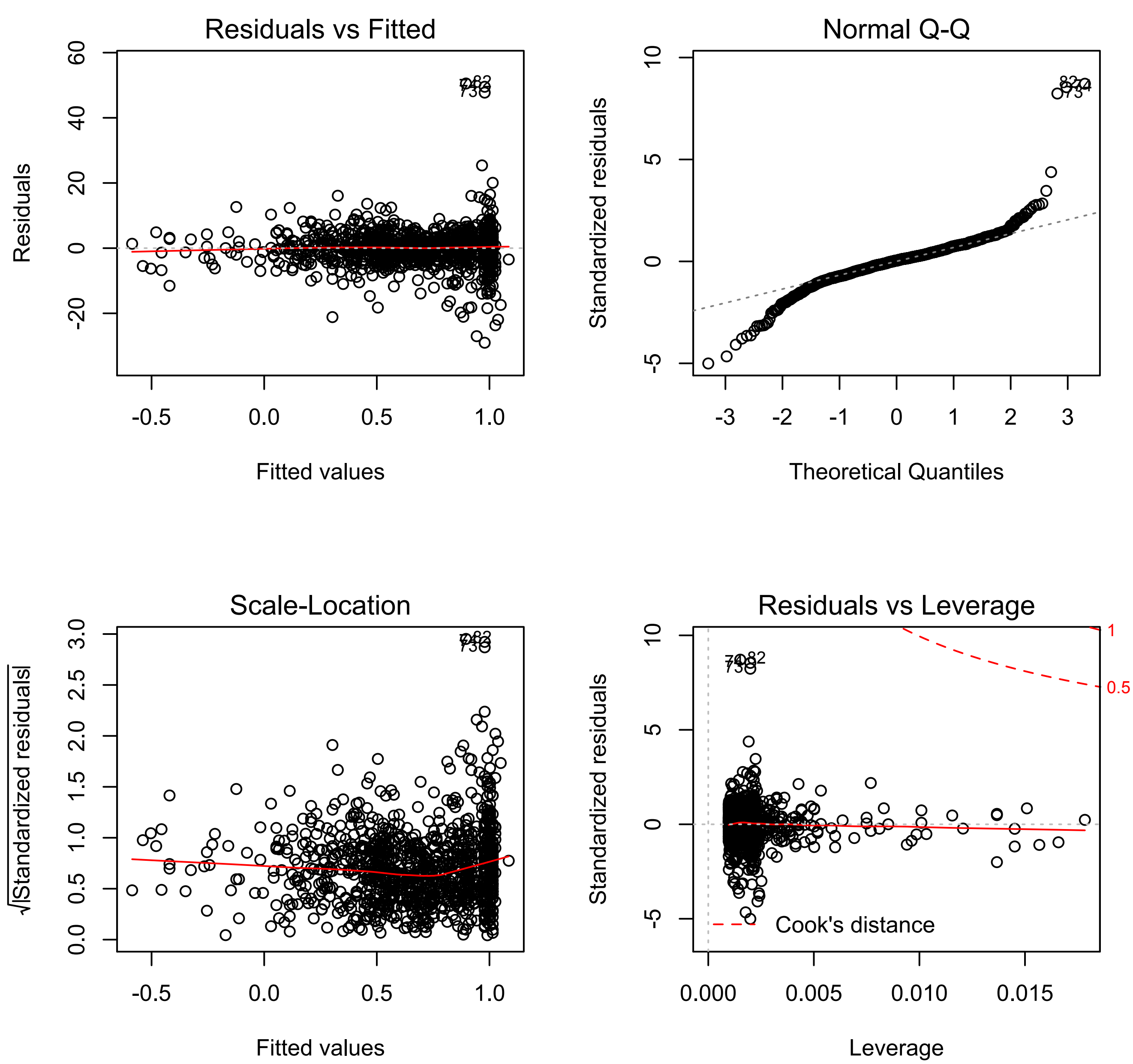

Figure 11: Regression diagnostics figures for BIG2

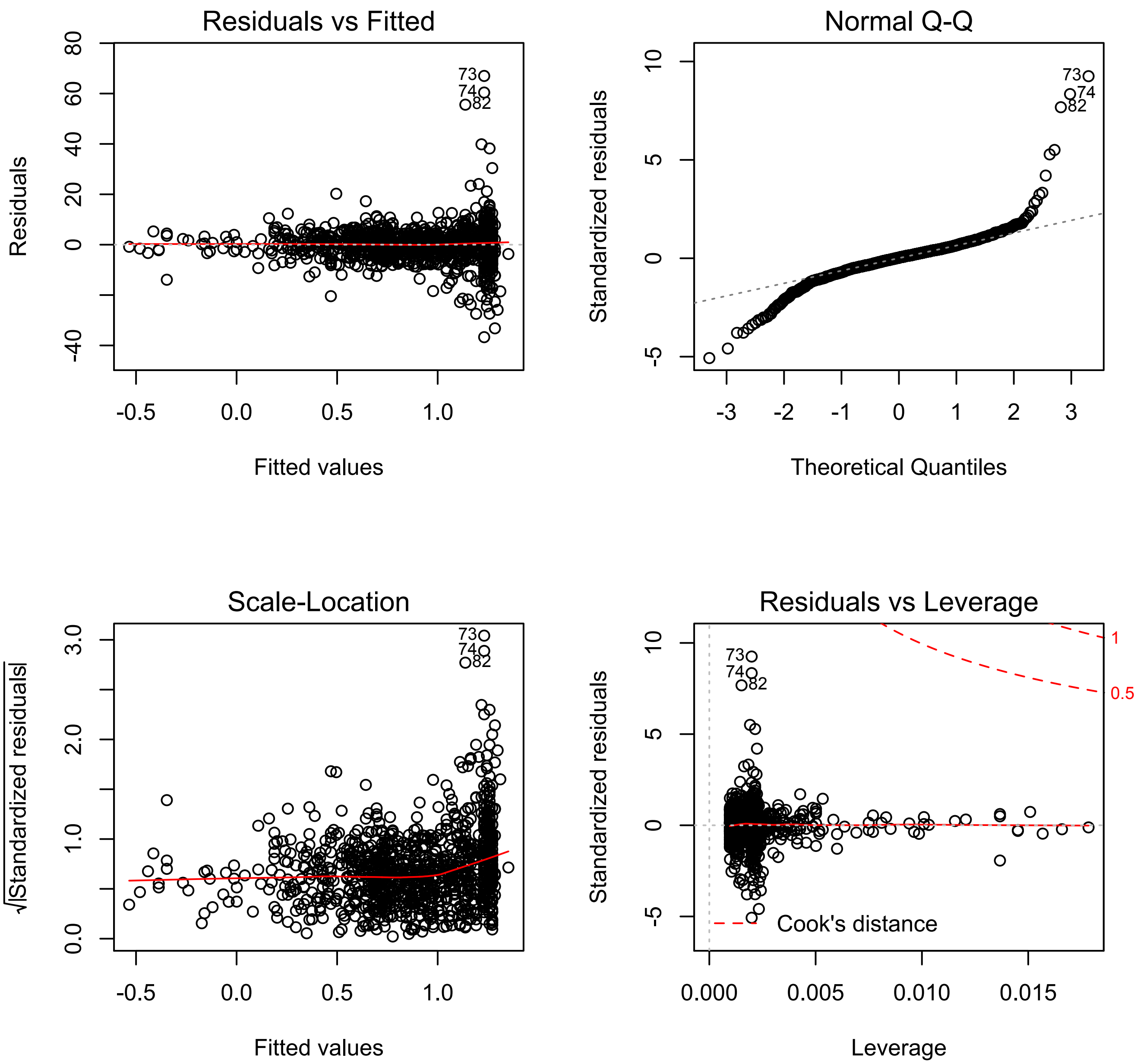

Figure 12: Regression diagnostics figures for BIGH

Portfolios	Ind. variable	Estimate	Std. Error	t value	p value
SML	Intercept	-0.1039	0.1106	-0.9391	0.3479
	Mkt-RF	1.2751	0.0202	63.0866	0.0000
SM2	Intercept	0.2399	0.0931	2.5758	0.0101
	Mkt-RF	1.1853	0.0170	69.6595	0.0000
SMH	Intercept	0.3513	0.1283	2.7375	0.0063
	Mkt-RF	1.3293	0.0234	56.7001	0.0000
BIGL	Intercept	-0.0013	0.0367	-0.0347	0.9723
	Mkt-RF	0.9642	0.0067	143.7338	0.0000
BIG2	Intercept	0.0328	0.0539	0.6081	0.5433
	Mkt-RF	1.0193	0.0098	103.5946	0.0000
BIGH	Intercept	0.1306	0.0975	1.3402	0.1805
	Mkt-RF	1.2033	0.0178	67.5747	0.0000

Table 1: Estimates, standard error, t value and p-value for CAMP

1.2 Question 1 (b)

Table 2 show the results of regression analysis of first part of equal-partitioned sub-samples. From the last column of Table 2, we know we fail to reject CAPM for all portfolios because their intercepts are not significant.

By doing similar regression diagnostics for residuals we know for each portfolio in its first sub-sample, we find that $|e_i|$ depends on $\hat{Y}_i$ and the residuals are not normally distributed.

Table 3 show the results of regression analysis of the second part of equal-partitioned sub-samples. From the last column of Table 2, we know we fail to reject CAPM for SM2, SMH and BIGHall portfolios because their intercepts are significant.

By doing similar regression diagnostics for residuals we know for each portfolio in its second sub-sample, we find that $|e_i|$ does not depend on $\hat{Y}_i$ (p-value is greater thatn 0.05) and some of the residuals are close tp normal distribution.

Portfolios	Ind. variable	Estimate	Std. Error	t value	p value
SML	Intercept	0.0341	0.1620	0.2105	0.8334
	Mkt-RF	1.2402	0.0263	47.1336	0.0000
SM2	Intercept	0.1625	0.1418	1.1460	0.2523
	Mkt-RF	1.2646	0.0230	54.9188	0.0000
SMH	Intercept	0.2298	0.2050	1.1207	0.2630
	Mkt-RF	1.5014	0.0333	45.0854	0.0000
BIGL	Intercept	0.0213	0.0472	0.4522	0.6513
	Mkt-RF	0.9394	0.0077	122.6457	0.0000
BIG2	Intercept	-0.0513	0.0738	-0.6958	0.4868
	Mkt-RF	1.0857	0.0120	90.5769	0.0000
BIGH	Intercept	0.0388	0.1447	0.2679	0.7889
	Mkt-RF	1.3820	0.0235	58.7961	0.0000

Table 2: Estimates, standard error, t value and p-value for CAMP for the first sub-sample

```
$PF6_SML

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
   Min     1Q Median     3Q    Max 
-5.094 -1.540 -0.563  0.847 34.260 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.27419    0.11743  19.366  < 2e-16 ***
fit_y        0.10626    0.01537   6.913 1.41e-11 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.649 on 516 degrees of freedom
Multiple R-squared:  0.08477,	Adjusted R-squared:  0.08299 
F-statistic: 47.79 on 1 and 516 DF,  p-value: 1.405e-11
```

Figure 13: SML (first sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_SM2

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
   Min     1Q Median     3Q    Max 
-3.307 -1.322 -0.526  0.605 33.247 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.92878    0.10727   17.98  < 2e-16 ***
fit_y        0.07419    0.01374    5.40 1.02e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.414 on 516 degrees of freedom
Multiple R-squared:  0.05349,	Adjusted R-squared:  0.05166 
F-statistic: 29.16 on 1 and 516 DF,  p-value: 1.018e-07
```


Figure 14: SM2 (first sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_SMH

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
   Min     1Q Median     3Q    Max 
-7.658 -1.973 -0.858  0.899 34.372 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.86566    0.15325  18.699  < 2e-16 ***
fit_y        0.08340    0.01652   5.048  6.2e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.446 on 516 degrees of freedom
Multiple R-squared:  0.04706,	Adjusted R-squared:  0.04521 
F-statistic: 25.48 on 1 and 516 DF,  p-value: 6.201e-07
```

Figure 15: SMH (first sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIGL

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-0.9402 -0.5066 -0.2198  0.2920  5.3583 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.741835   0.032800  22.617  < 2e-16 ***
fit_y       0.018454   0.005669   3.255  0.00121 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.7399 on 516 degrees of freedom
Multiple R-squared:  0.02012,	Adjusted R-squared:  0.01823 
F-statistic:  10.6 on 1 and 516 DF,  p-value: 0.001207
```


Figure 16: BIGL (first sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIG2

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.4443 -0.7087 -0.3592  0.2626  9.8621 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1.056301   0.055009  19.202  < 2e-16 ***
fit_y       0.033724   0.008238   4.094 4.93e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.243 on 516 degrees of freedom
Multiple R-squared:  0.03146,	Adjusted R-squared:  0.02958 
F-statistic: 16.76 on 1 and 516 DF,  p-value: 4.926e-05
```

Figure 17: BIG2 (first sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIGH

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.3488 -1.5291 -0.5960  0.6228 18.2718 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.18213    0.10606  20.575   <2e-16 ***
fit_y        0.02412    0.01246   1.936   0.0534 .  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.392 on 516 degrees of freedom
Multiple R-squared:  0.007213,	Adjusted R-squared:  0.005289 
F-statistic: 3.749 on 1 and 516 DF,  p-value: 0.05338
```


Figure 18: BIGH (first sub-sample): $\hat{Y}_i$ vs $|e_i|$

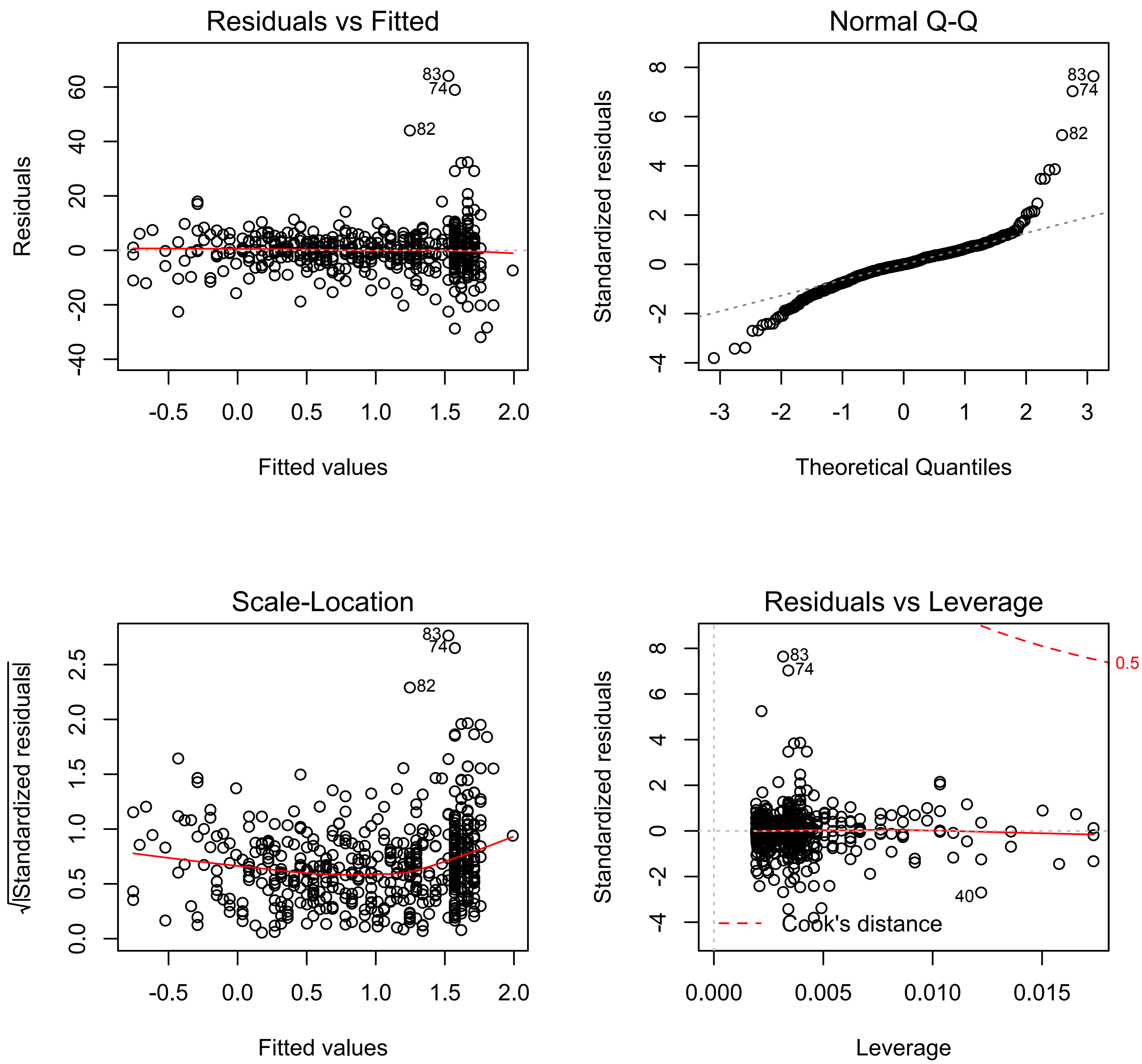

Figure 19: Regression diagnostics figures for SML (first sub-sample)

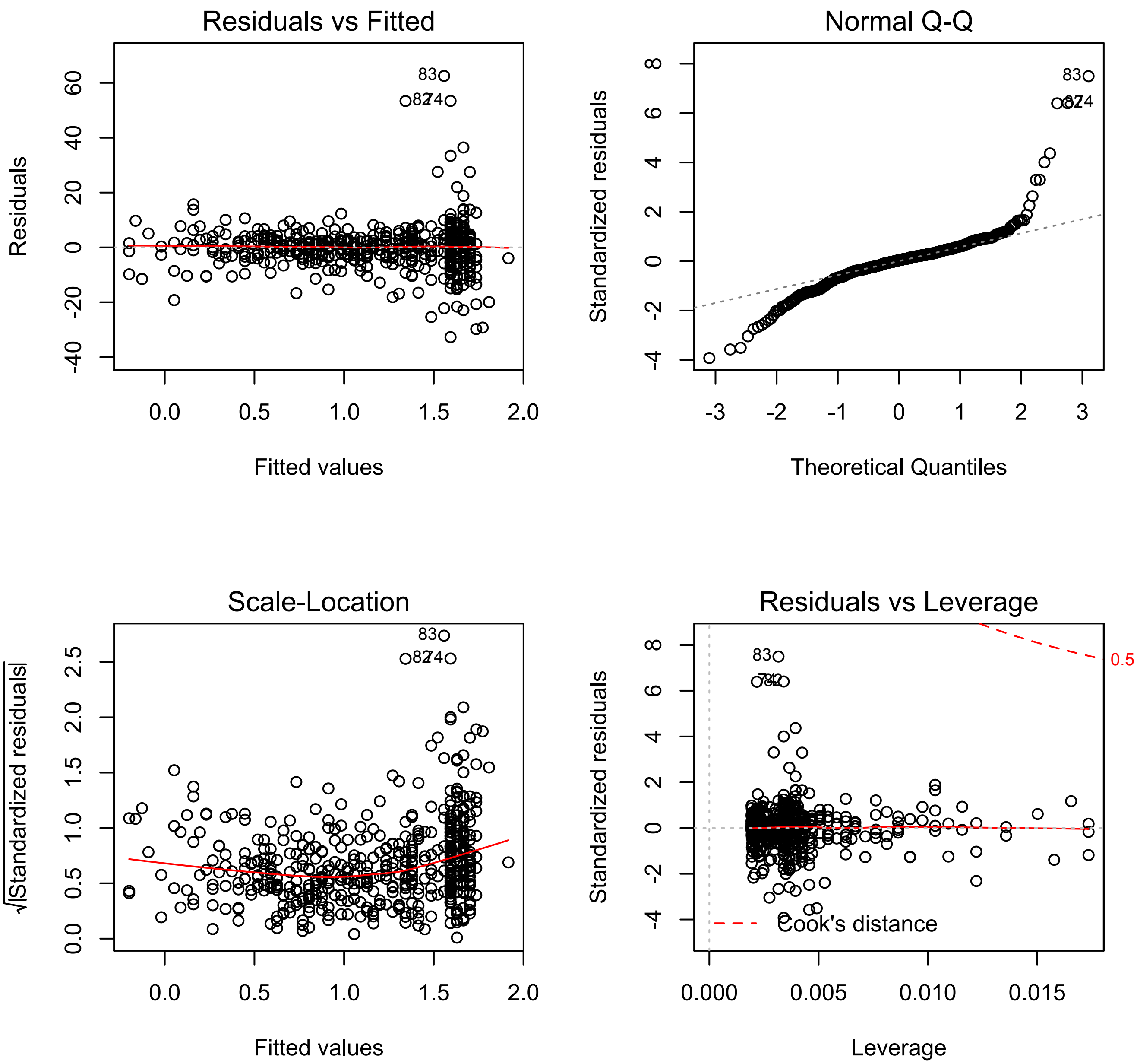

Figure 20: Regression diagnostics figures for SM2 (first sub-sample)

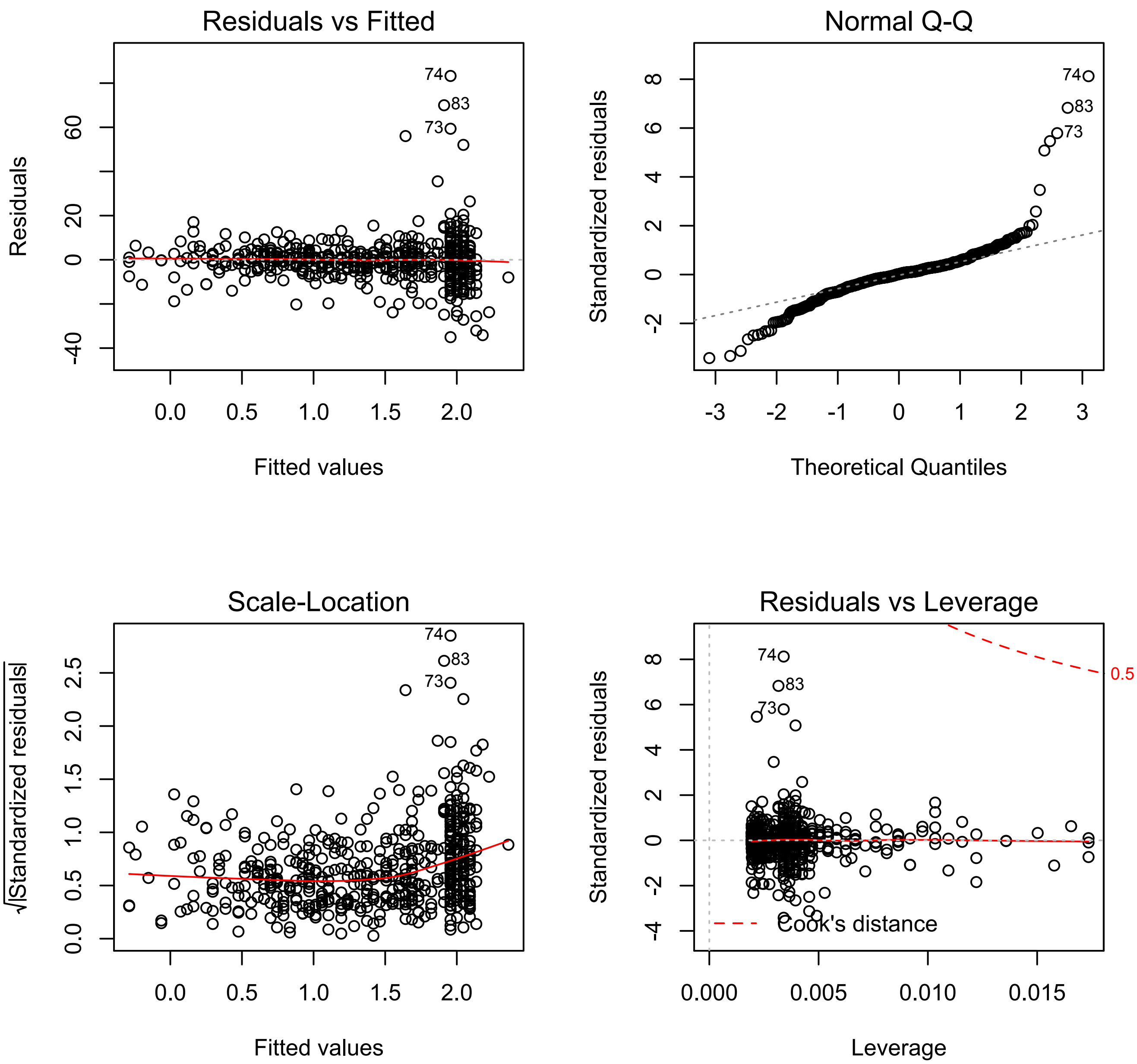

Figure 21: Regression diagnostics figures for SMH (first sub-sample)

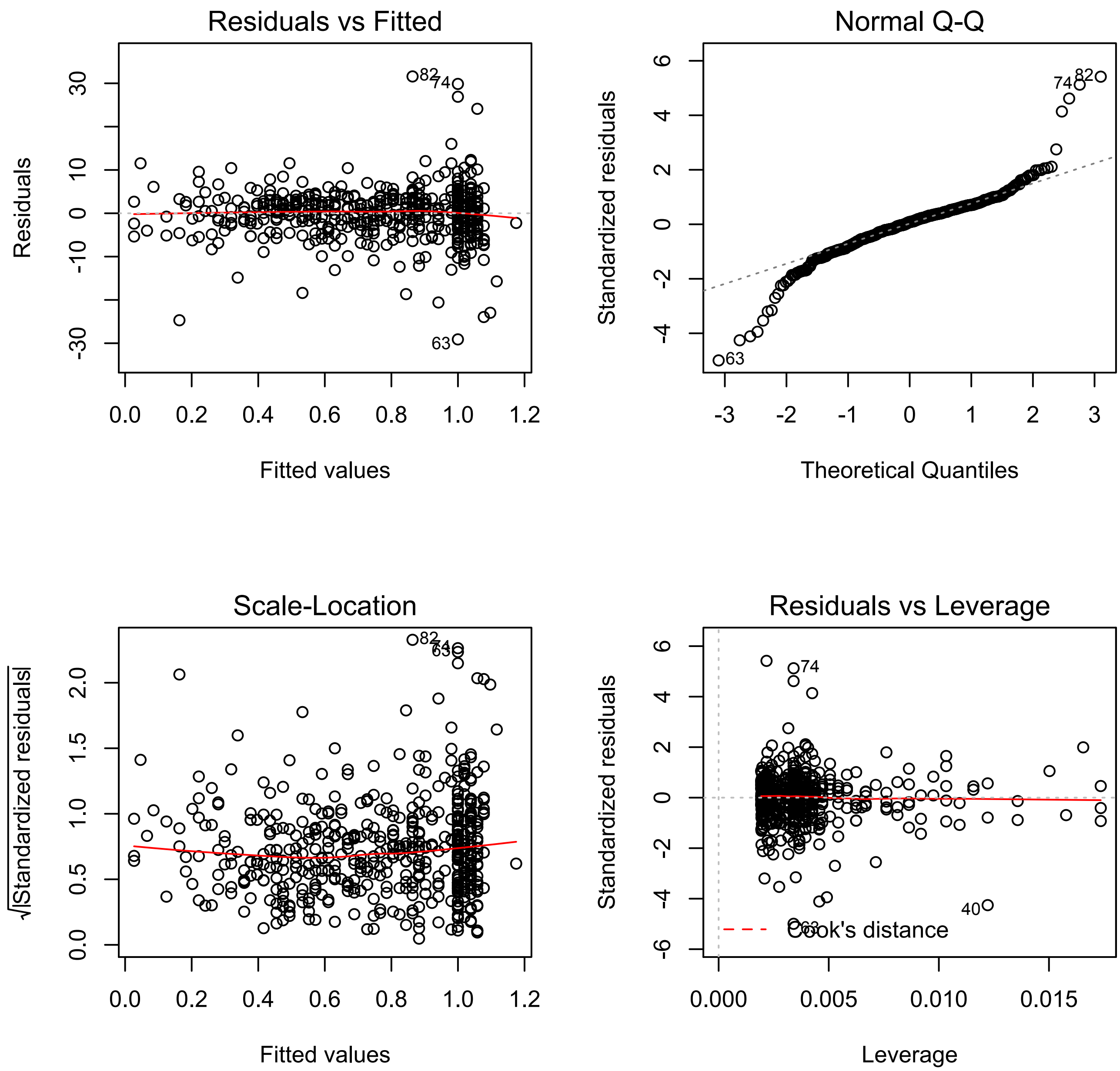

Figure 22: Regression diagnostics figures for BIGL (first sub-sample)

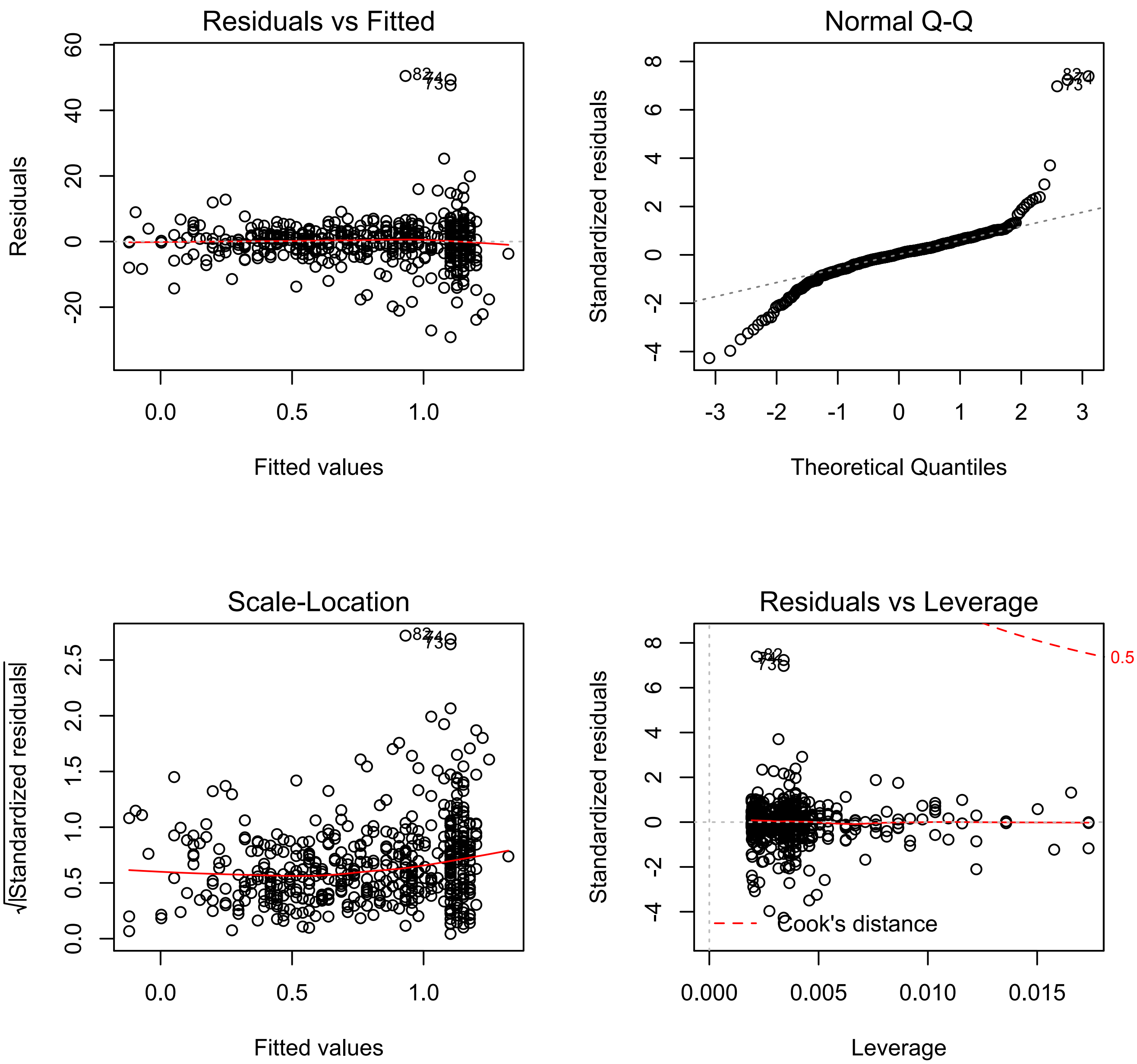

Figure 23: Regression diagnostics figures for BIG2 (first sub-sample)

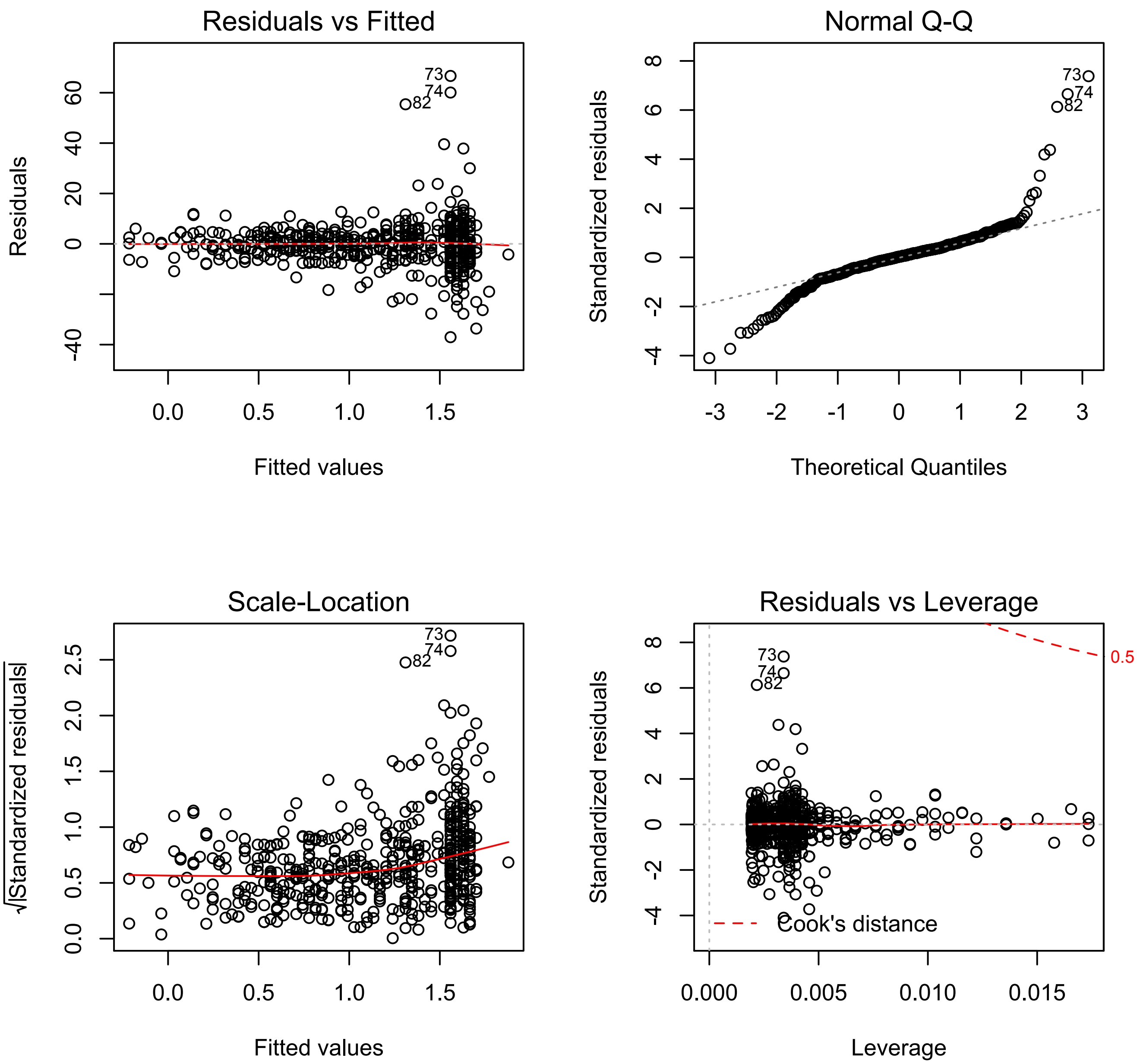

Figure 24: Regression diagnostics figures for BIGH (first sub-sample)

Portfolios	Ind. variable	Estimate	Std. Error	t value	p value
SML	Intercept	-0.2402	0.1502	-1.5995	0.1103
	Mkt-RF	1.3330	0.0320	41.6588	0.0000
SM2	Intercept	0.3152	0.1160	2.7173	0.0068
	Mkt-RF	1.0502	0.0247	42.4901	0.0000
SMH	Intercept	0.4685	0.1349	3.4740	0.0006
	Mkt-RF	1.0352	0.0287	36.0258	0.0000
BIGL	Intercept	-0.0232	0.0553	-0.4199	0.6747
	Mkt-RF	1.0064	0.0118	85.4496	0.0000
BIG2	Intercept	0.1149	0.0729	1.5770	0.1154
	Mkt-RF	0.9064	0.0155	58.3894	0.0000
BIGH	Intercept	0.2182	0.1040	2.0980	0.0364
	Mkt-RF	0.8974	0.0222	40.4917	0.0000

Table 3: Estimates, standard error, t value and p-value for CAMP for the second sub-sample

```
$PF6_SML

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.5228 -1.5978 -0.6065  0.9722 21.7834 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.44523    0.10336  23.657   <2e-16 ***
fit_y        0.02216    0.01657   1.337    0.182    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.348 on 516 degrees of freedom
Multiple R-squared:  0.003454,	Adjusted R-squared:  0.001523 
F-statistic: 1.788 on 1 and 516 DF,  p-value: 0.1817
```

Figure 25: SML (second sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_SM2

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.9257 -1.3089 -0.4924  0.8395 10.0614 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.930059   0.079260  24.351   <2e-16 ***
fit_y       -0.001466   0.015949  -0.092    0.927    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.781 on 516 degrees of freedom
Multiple R-squared:  1.636e-05,	Adjusted R-squared:  -0.001922 
F-statistic: 0.008444 on 1 and 516 DF,  p-value: 0.9268
```

Figure 26: SM2 (second sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_SMH

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max
-2.1960 -1.4921 -0.6551  0.8346 13.1211

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.197467   0.094878  23.161   <2e-16 ***
fit_y       -0.001687   0.019260  -0.088     0.93
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.12 on 516 degrees of freedom
Multiple R-squared:  1.486e-05, Adjusted R-squared:  -0.001923
F-statistic: 0.007668 on 1 and 516 DF,  p-value: 0.9303
```

Figure 27: SMH (second sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIGL

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max
-1.0308 -0.6080 -0.1797  0.4173  3.6353

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.969328   0.034621  27.998   <2e-16 ***
fit_y       0.009058   0.007332   1.235    0.217
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.7847 on 516 degrees of freedom
Multiple R-squared:  0.002948,  Adjusted R-squared:  0.001016
F-statistic: 1.526 on 1 and 516 DF,  p-value: 0.2173
```


Figure 28: BIGL (second sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIG2

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.1275 -0.7437 -0.3381  0.2457  9.6108 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1.101574   0.054258  20.302   <2e-16 ***
fit_y       0.003875   0.012715   0.305    0.761    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.226 on 516 degrees of freedom
Multiple R-squared:  0.00018,	Adjusted R-squared:  -0.001758 
F-statistic: 0.09287 on 1 and 516 DF,  p-value: 0.7607
```

Figure 29: BIG2 (second sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIGH

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.7164 -1.1560 -0.4684  0.4940 10.1487 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.64821    0.07510  21.947   <2e-16 ***
fit_y       -0.01834    0.01772  -1.035    0.301    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.691 on 516 degrees of freedom
Multiple R-squared:  0.002073,	Adjusted R-squared:  0.0001389 
F-statistic: 1.072 on 1 and 516 DF,  p-value: 0.301
```


Figure 30: BIGH (second sub-sample): $\hat{Y}_i$ vs $|e_i|$

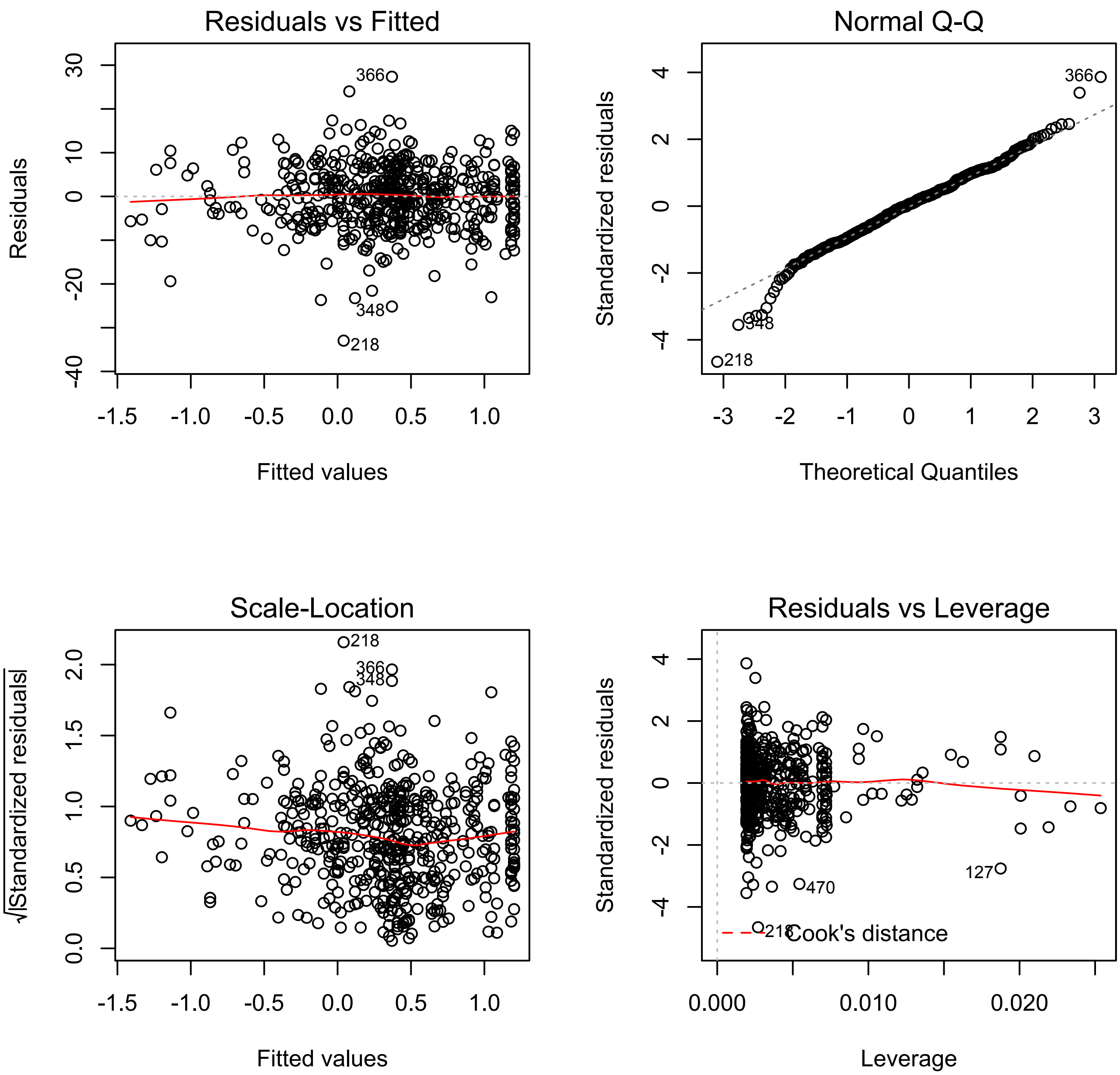

Figure 31: Regression diagnostics figures for SML (second sub-sample)

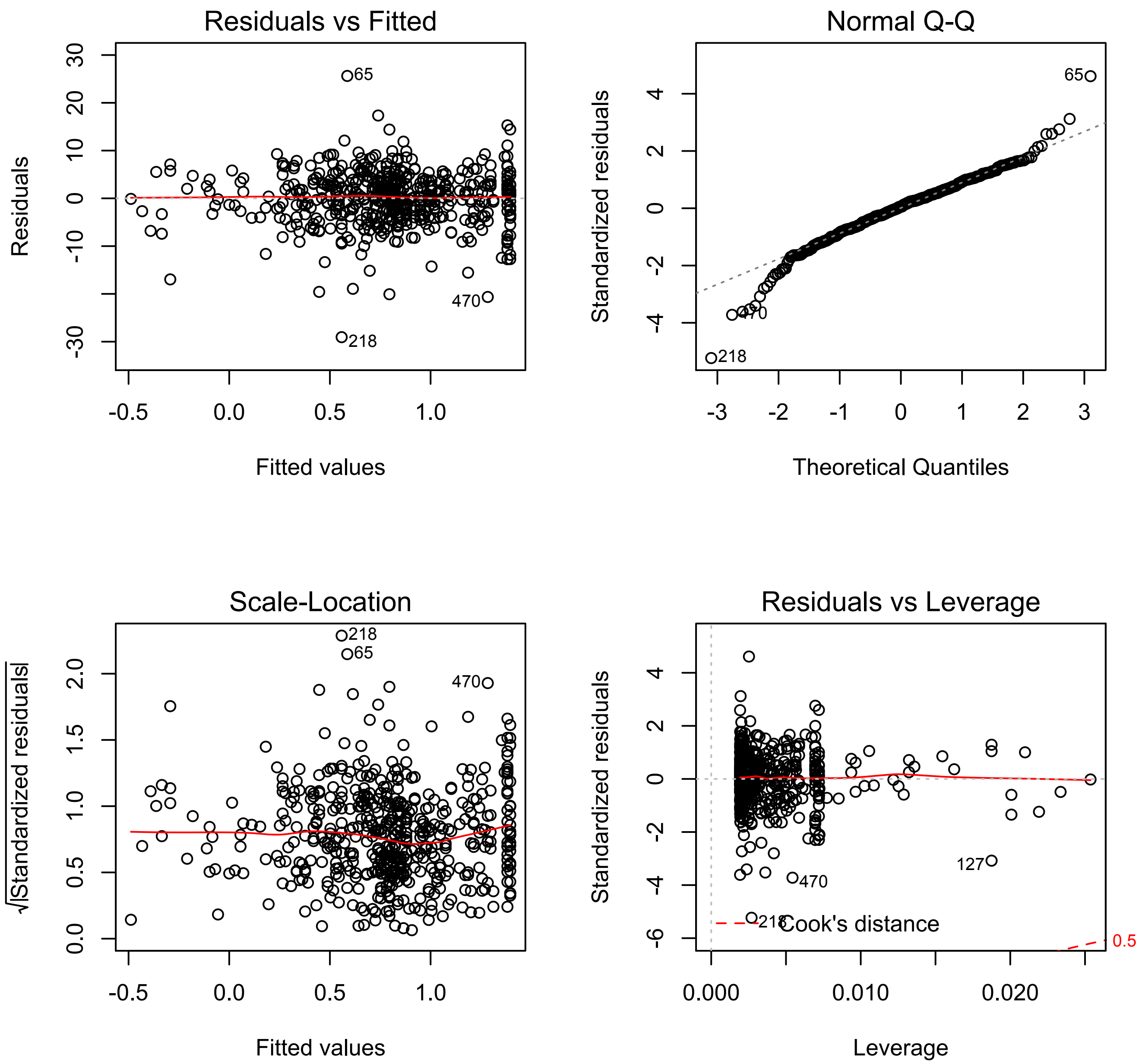

Figure 32: Regression diagnostics figures for SM2 (second sub-sample)

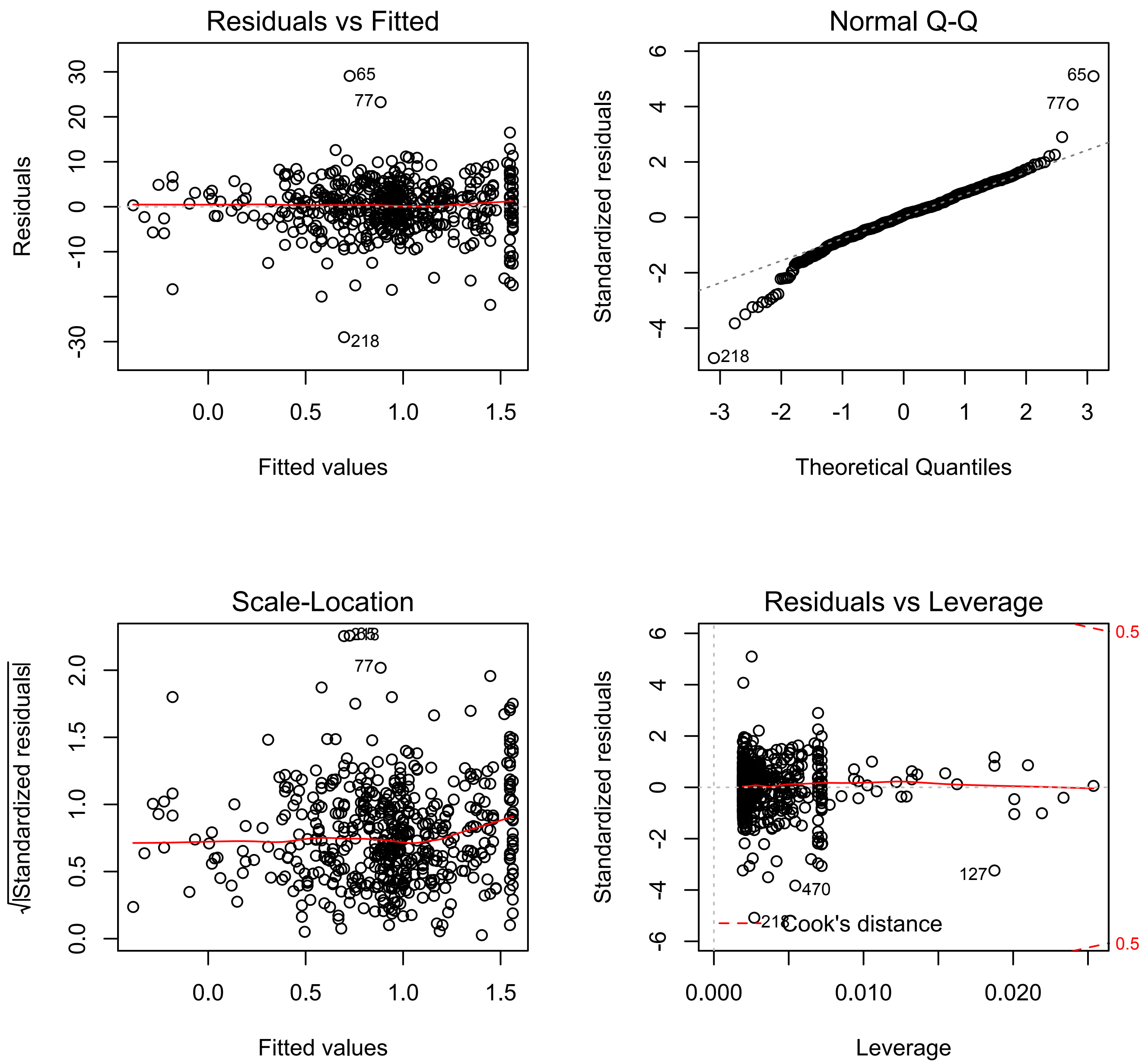

Figure 33: Regression diagnostics figures for SMH (second sub-sample)

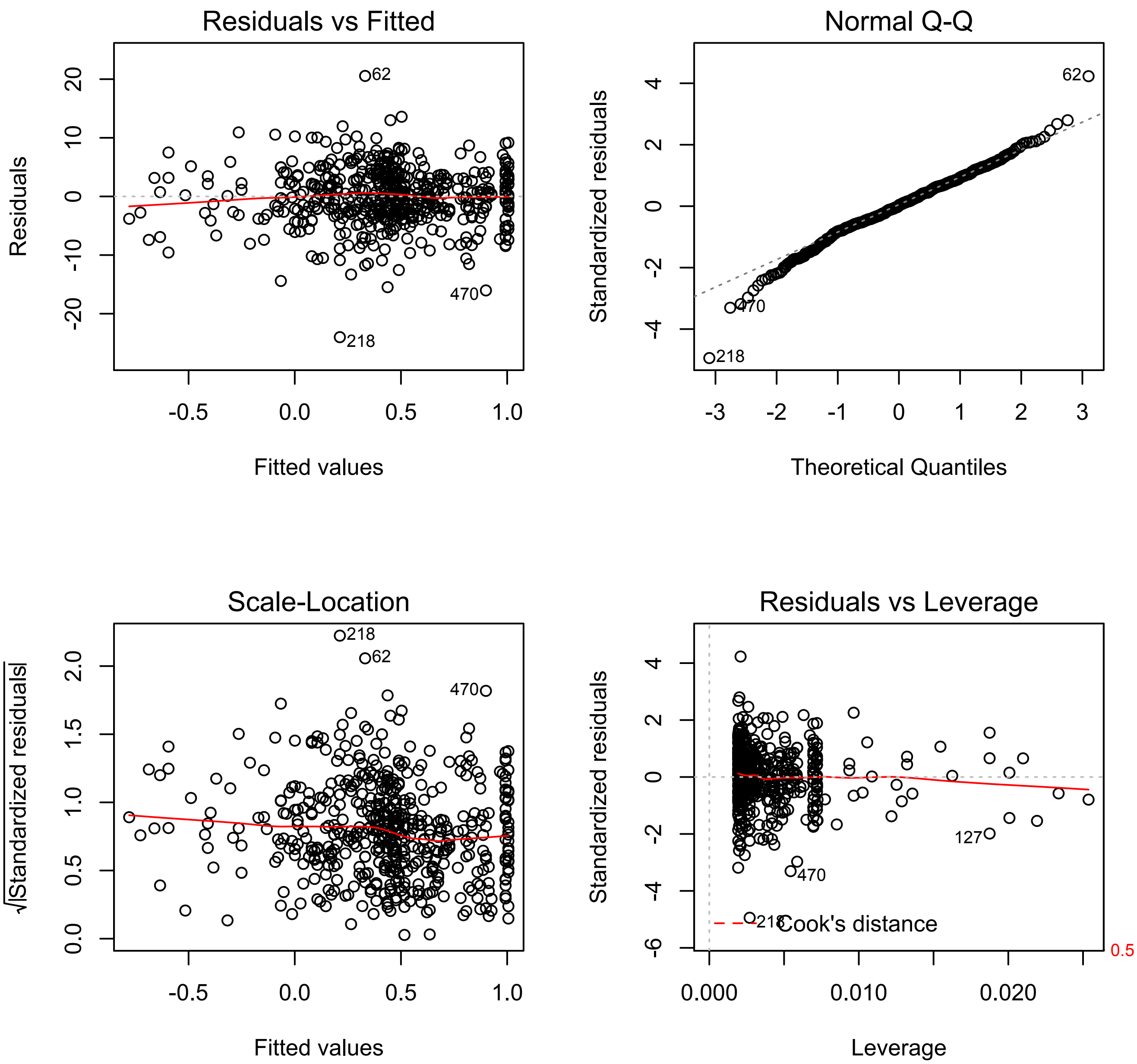

Figure 34: Regression diagnostics figures for BIGL (second sub-sample)

1.3 Question 1 (c)

In the first place, we analyse these 6 portfolios for the whole sample. From Table 4 we know that the Fama-French model should be rejected for portfolio SML, BIGLand BIGH.

From Figures 37 to 42, we can see the absolute value of residuals depend on the predicted response variable.

From Figures 43 to 48, we can see there are some very large residuals and the residuals are not normally distributed.

Second, we divide these 6 portfolio in two equal sub-samples and repeat the analysis in our first step. Table 5 show the estimates, t value and p-value for first sub-sample of each portfolio. We can see we do not reject Fama-French model for any portfolio. If we apply regression diagnosis, we find that for the first sub-sample, $|e_i|$ depends on $\hat{Y}_i$ and and the residuals are not normal (see Figures **??** to 60). Similarly, Table 6 show the estimates, t value and p-value for second sub-sample of each portfolio. We reject the Fama-French model for the second sub-sample of SML, SM2, SMH, BIGL and BIGH. Moreover, we do regression diagnostic. From the results shown in Figures **??** to 67, the absolute value of residuals do not depend on the predicted response variable except SMH, and tey are not normal.

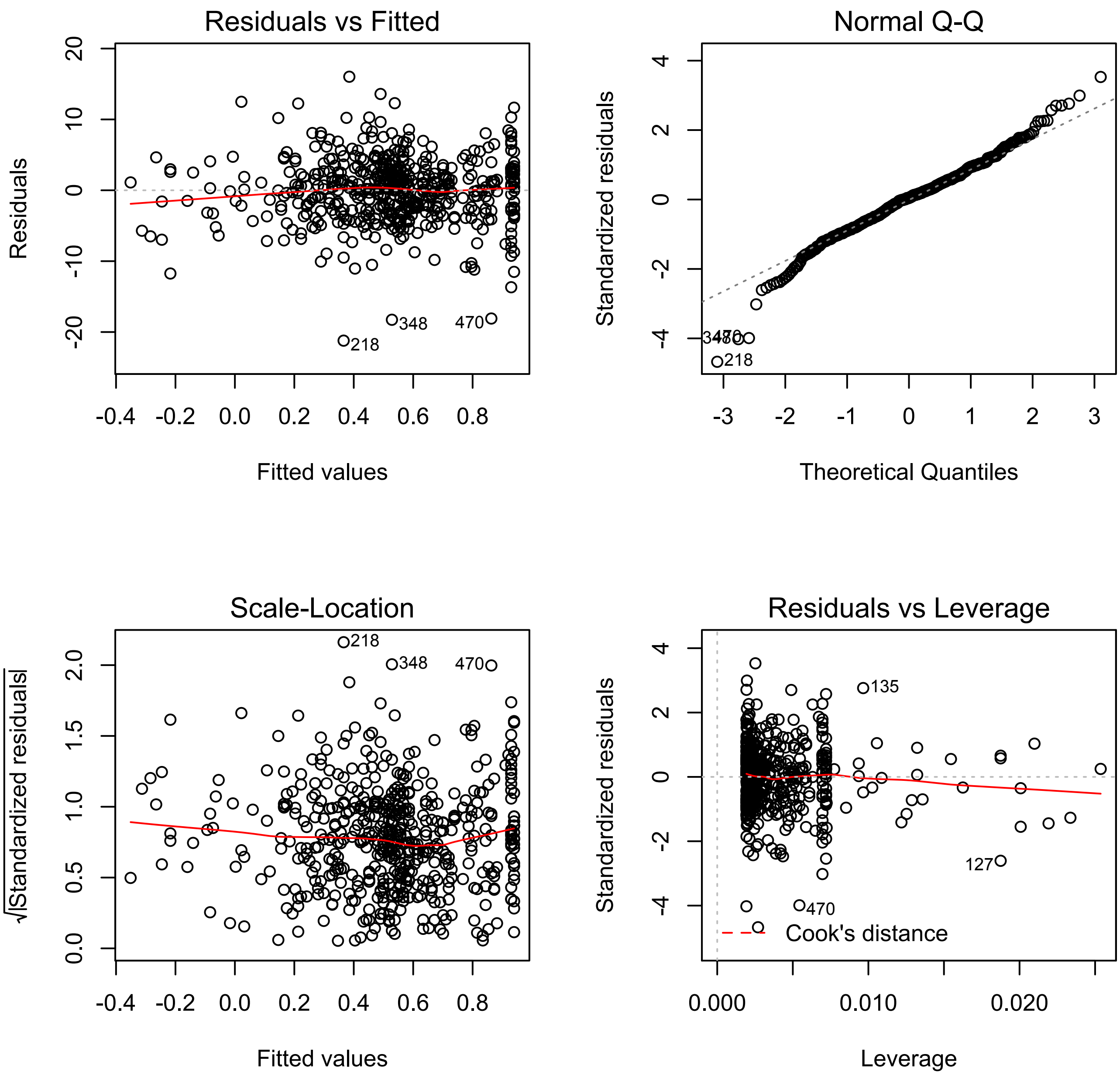

Figure 35: Regression diagnostics figures for BIG2 (second sub-sample)

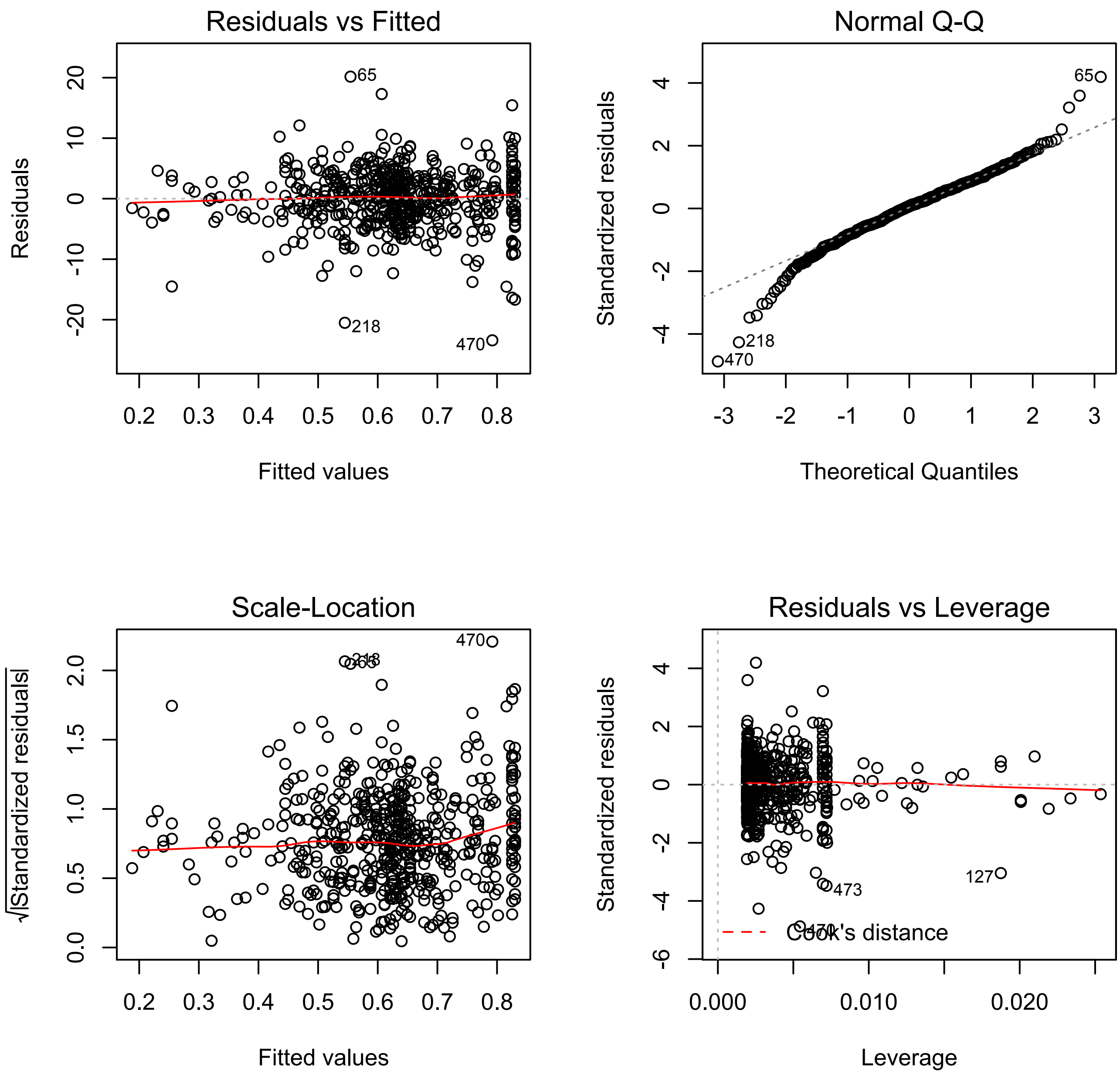

Figure 36: Regression diagnostics figures for BIGH (second sub-sample)

Portfolios	Ind. variable	Estimate	Std. Error	t value	p value
SML	Intercept	-0.1685	0.0393	-4.2927	0
	Mkt-RF	1.0886	0.0077	140.8001	0
	SMB	1.0502	0.0125	84.2046	0
	HML	-0.1674	0.0112	-14.9025	0
SM2	Intercept	0.0625	0.0334	1.8737	0.0613
	Mkt-RF	0.9765	0.0066	148.5603	0
	SMB	0.8183	0.0106	77.1721	0
	HML	0.3021	0.0096	31.6223	0
SMH	Intercept	0.0202	0.0247	0.8188	0.4131
	Mkt-RF	1.0269	0.0049	211.3256	0
	SMB	0.9331	0.0078	119.0325	0
	HML	0.783	0.0071	110.8714	0
BIGL	Intercept	0.077	0.0251	3.0663	0.0022
	Mkt-RF	1.0174	0.0049	205.774	0
	SMB	-0.0965	0.008	-12.1027	0
	HML	-0.2307	0.0072	-32.111	0
BIG2	Intercept	-0.0507	0.0391	-1.2981	0.1945
	Mkt-RF	0.9956	0.0077	129.3624	0
	SMB	-0.1223	0.0124	-9.8531	0
	HML	0.3292	0.0112	29.4372	0
BIGH	Intercept	-0.1121	0.0406	-2.761	0.0059
	Mkt-RF	1.0791	0.008	134.8755	0
	SMB	0.0205	0.0129	1.5862	0.113
	HML	0.8188	0.0116	70.426	0

Table 4: Estimates, standard error, t value and p-value for Fama-French model

```
$PF6_SML

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-0.9737 -0.6050 -0.2383  0.3003  8.8573 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.872945   0.027513  31.728  < 2e-16 ***
fit_y       0.010041   0.003567   2.815  0.00497 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.882 on 1034 degrees of freedom
Multiple R-squared:  0.007607,	Adjusted R-squared:  0.006647 
F-statistic: 7.926 on 1 and 1034 DF,  p-value: 0.004966
```

Figure 37: SML: $\hat{Y}_i$ vs $|e_i|$

```
$PF6_SM2

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.5992 -0.4607 -0.2082  0.1974  5.8873 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.667873   0.024535  27.221  < 2e-16 ***
fit_y       0.026139   0.003461   7.551 9.44e-14 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.7821 on 1034 degrees of freedom
Multiple R-squared:  0.05227,	Adjusted R-squared:  0.05135 
F-statistic: 57.02 on 1 and 1034 DF,  p-value: 9.435e-14
```

Figure 38: SM2: $\hat{Y}_i$ vs $|e_i|$

```
$PF6_SMH

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-0.7113 -0.3590 -0.1446  0.1900  6.2273 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.52999    0.01713  30.932  < 2e-16 ***
fit_y        0.01560    0.00205   7.609 6.22e-14 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.546 on 1034 degrees of freedom
Multiple R-squared:  0.05302,	Adjusted R-squared:  0.0521 
F-statistic: 57.89 on 1 and 1034 DF,  p-value: 6.218e-14
```

Figure 39: SMH: $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIGL

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-0.6639 -0.3865 -0.1575  0.2421  3.1958 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.583825   0.016790  34.772  < 2e-16 ***
fit_y       0.008393   0.003140   2.673  0.00763 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.537 on 1034 degrees of freedom
Multiple R-squared:  0.006863,	Adjusted R-squared:  0.005902 
F-statistic: 7.145 on 1 and 1034 DF,  p-value: 0.007635
```


Figure 40: BIGL: $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIG2

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.0131 -0.5740 -0.2452  0.3184  6.5739 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.877394   0.027142  32.326  < 2e-16 ***
fit_y       0.013259   0.004754   2.789  0.00539 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8676 on 1034 degrees of freedom
Multiple R-squared:  0.007466,	Adjusted R-squared:  0.006506 
F-statistic: 7.778 on 1 and 1034 DF,  p-value: 0.005386
```

Figure 41: BIG2: $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIGH

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.0282 -0.6171 -0.2445  0.3115  9.0844 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.905771   0.028499   31.78   <2e-16 ***
fit_y       0.009282   0.003966    2.34   0.0195 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9104 on 1034 degrees of freedom
Multiple R-squared:  0.005269,	Adjusted R-squared:  0.004307 
F-statistic: 5.477 on 1 and 1034 DF,  p-value: 0.01946
```

Figure 42: BIGH: $\hat{Y}_i$ vs $|e_i|$

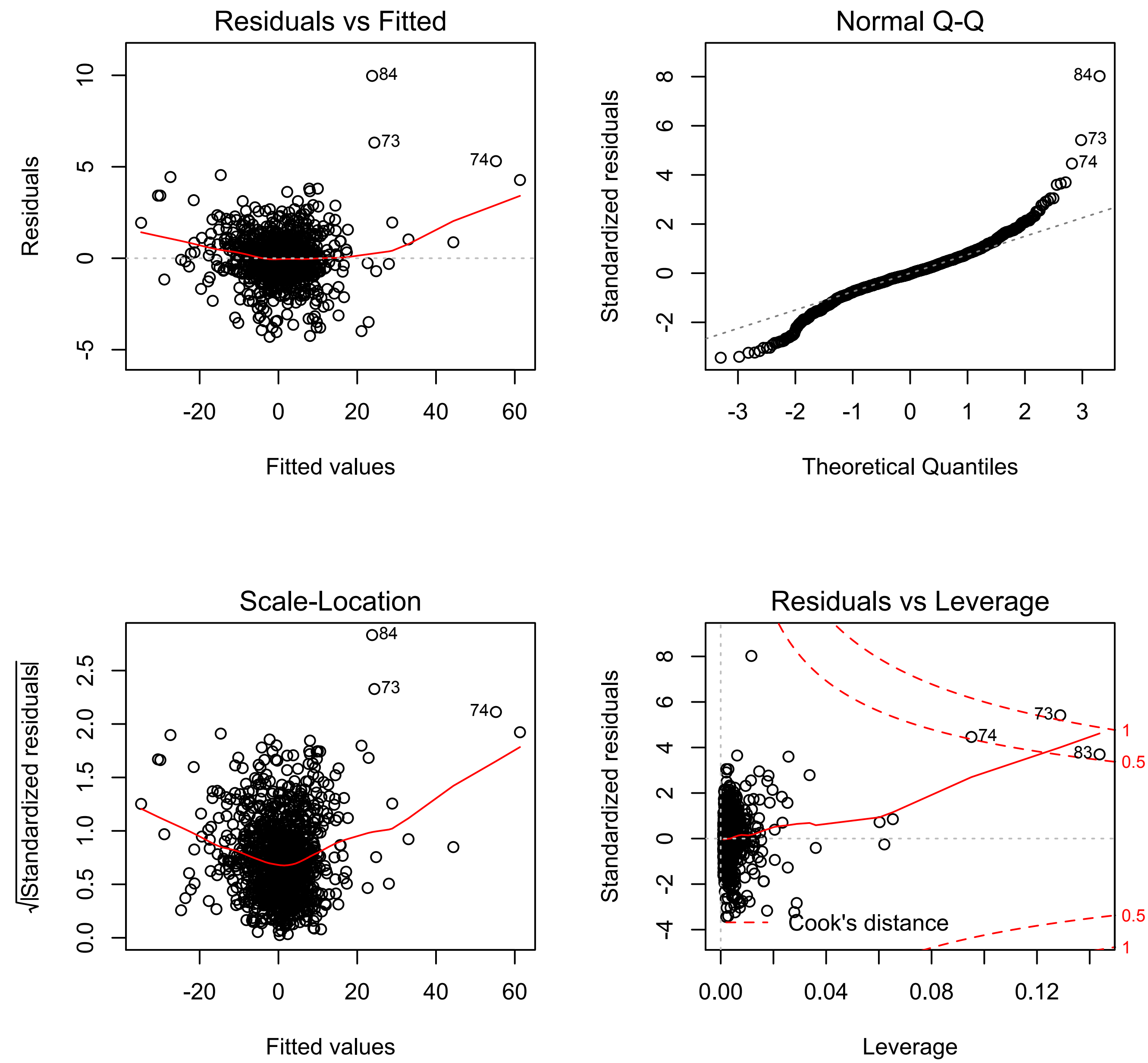

Figure 43: Regression diagnostics figures for SML

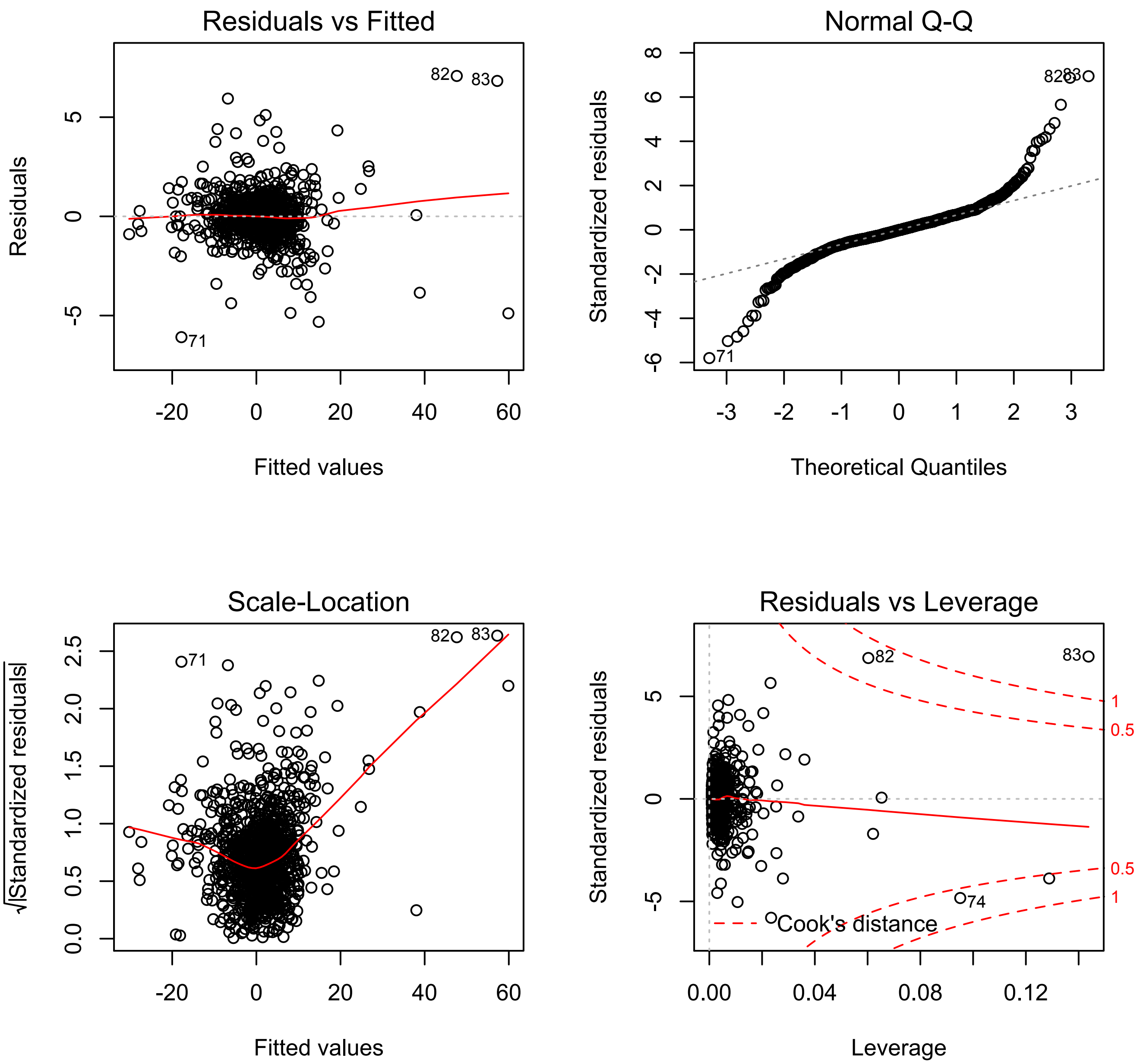

Figure 44: Regression diagnostics figures for SM2

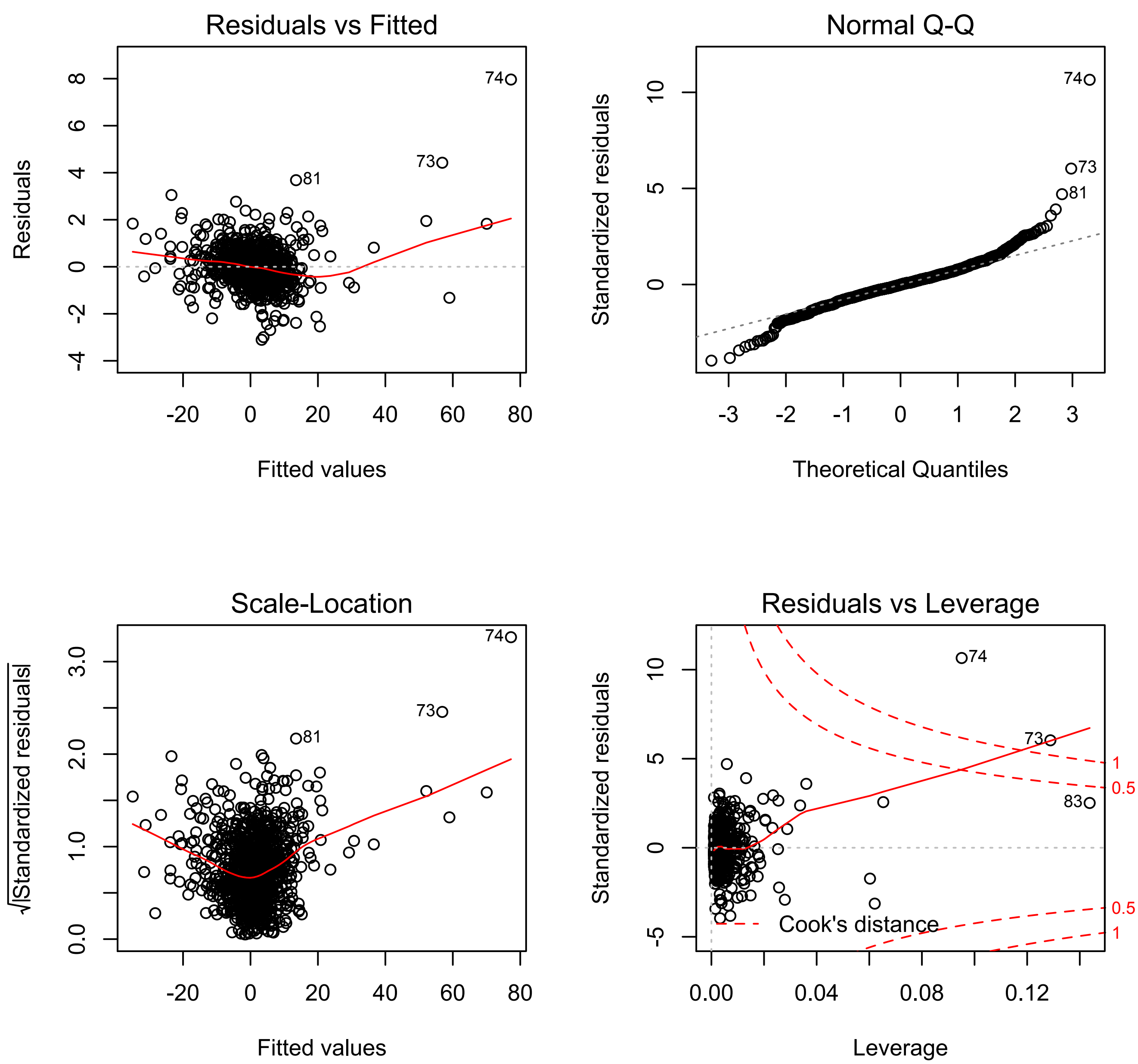

Figure 45: Regression diagnostics figures for SMH

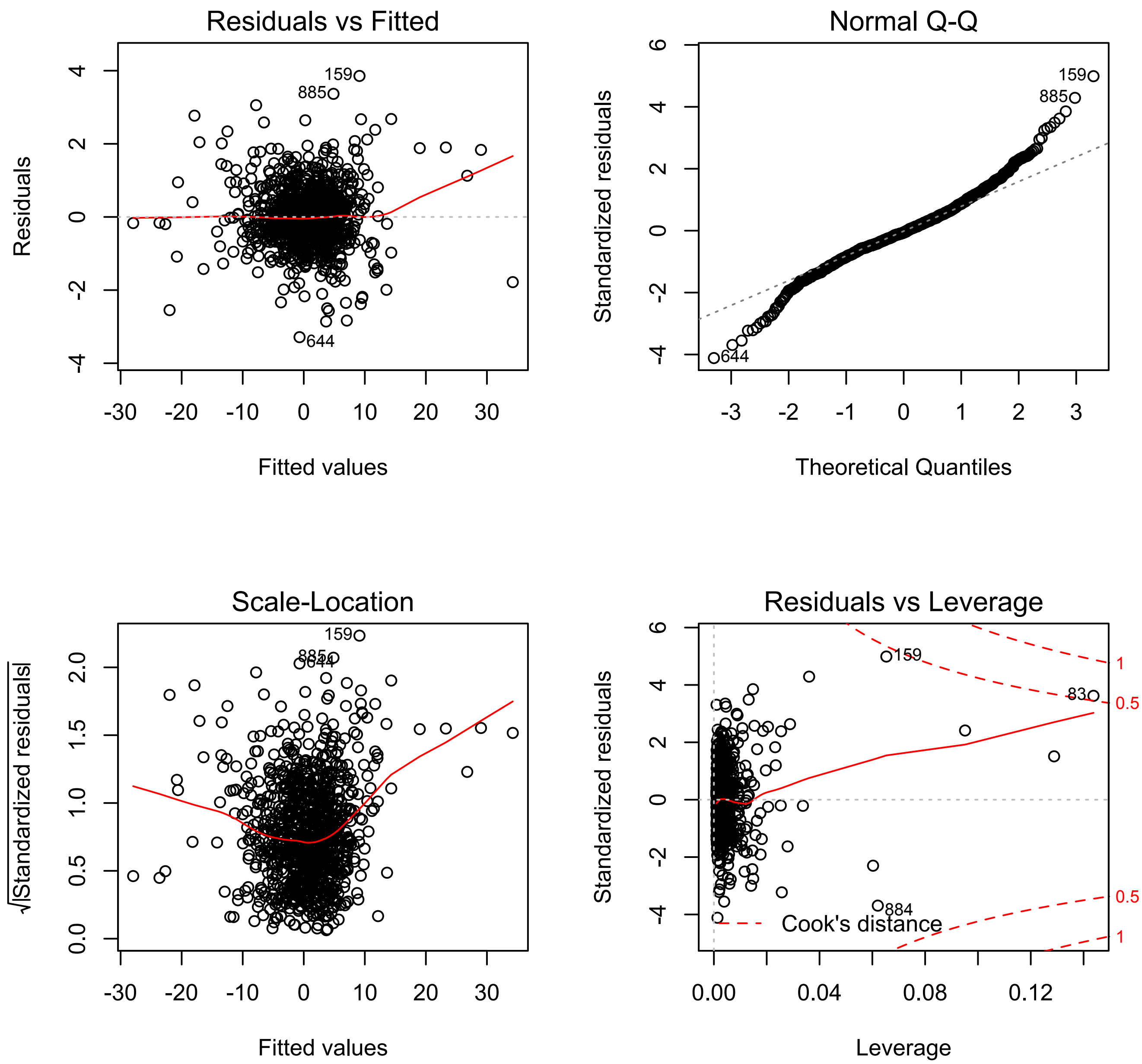

Figure 46: Regression diagnostics figures for BIGL

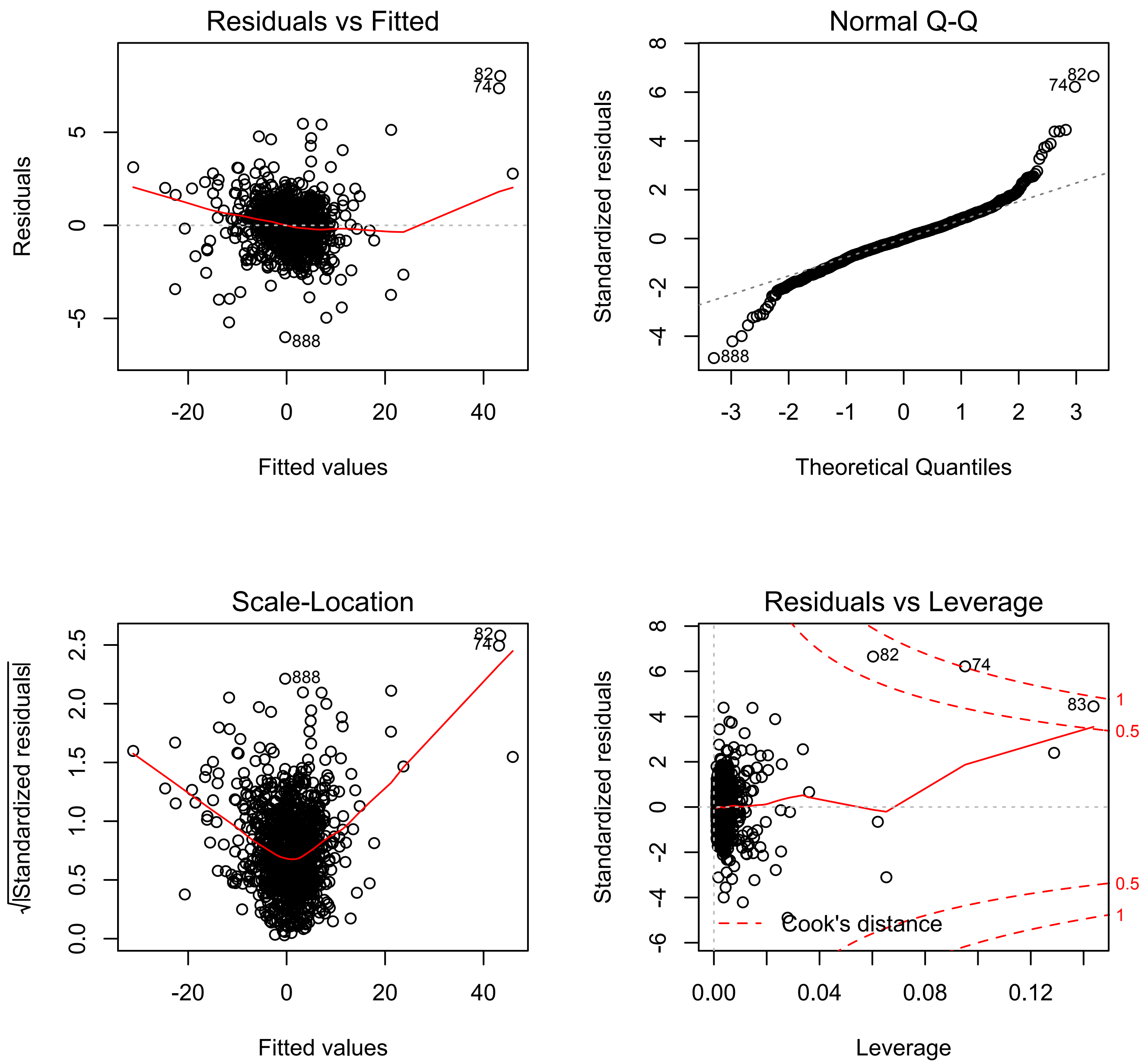

Figure 47: Regression diagnostics figures for BIG2

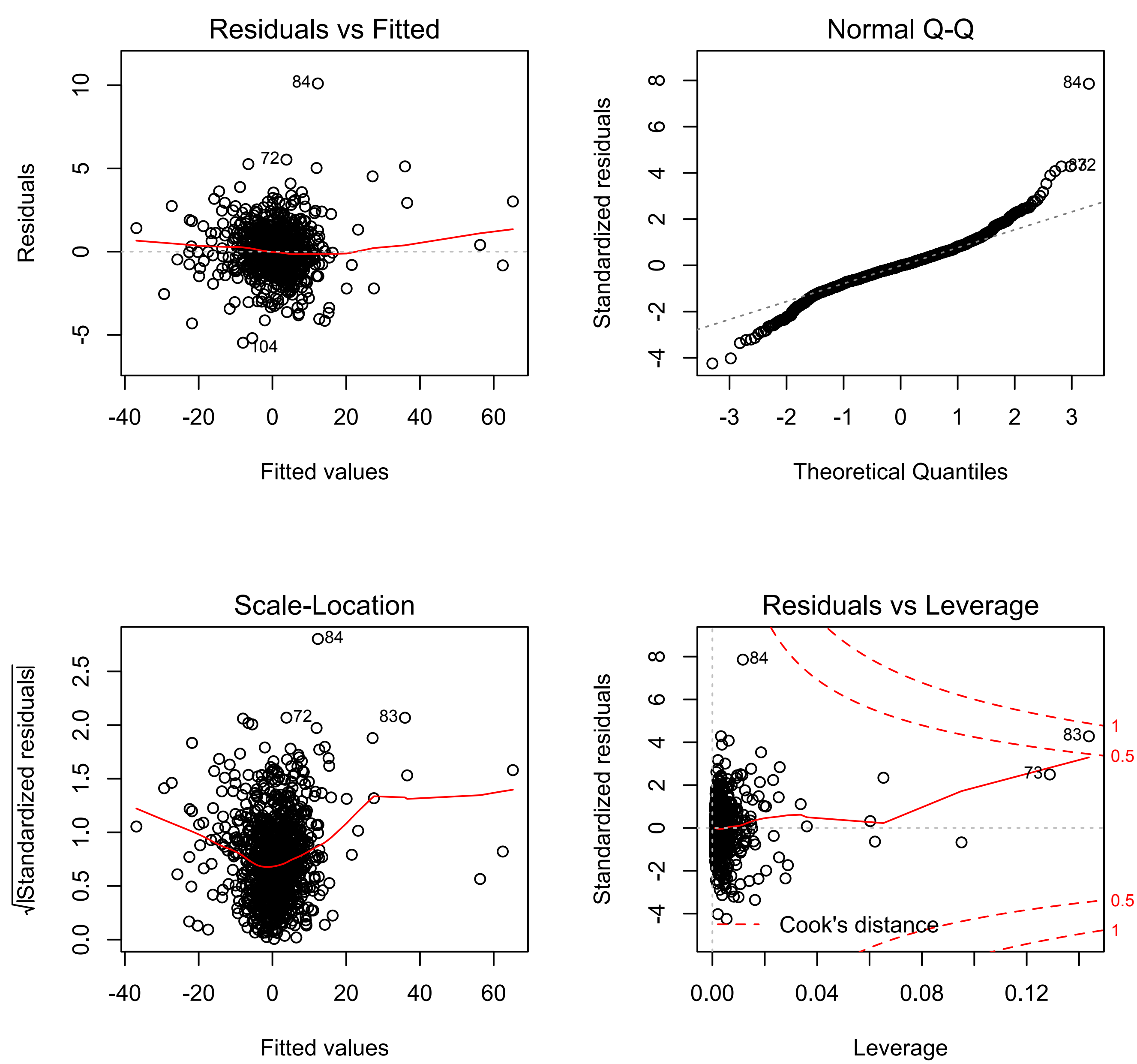

Figure 48: Regression diagnostics figures for BIGH

```
$PF6_SML

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.2408 -0.6497 -0.2801  0.3245  8.9176 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.949346   0.043592  21.778  < 2e-16 ***
fit_y       0.016271   0.005219   3.118  0.00192 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9847 on 516 degrees of freedom
Multiple R-squared:  0.01849,	Adjusted R-squared:  0.01659 
F-statistic:  9.72 on 1 and 516 DF,  p-value: 0.001925
```

Figure 49: SML (first sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_SM2

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.2227 -0.5249 -0.2360  0.2334  5.5210 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.730333   0.038276   19.08  < 2e-16 ***
fit_y       0.029785   0.004582    6.50  1.9e-10 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8625 on 516 degrees of freedom
Multiple R-squared:  0.07568,	Adjusted R-squared:  0.07389 
F-statistic: 42.25 on 1 and 516 DF,  p-value: 1.9e-10
```

Figure 50: SM2 (first sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_SMH

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.0241 -0.3578 -0.1599  0.1730  4.8853 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.515580   0.025333  20.352  < 2e-16 ***
fit_y       0.013822   0.002452   5.637 2.85e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.571 on 516 degrees of freedom
Multiple R-squared:  0.05801,	Adjusted R-squared:  0.05618 
F-statistic: 31.77 on 1 and 516 DF,  p-value: 2.853e-08
```

Figure 51: SMH (first sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIGL

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-0.5232 -0.3099 -0.1163  0.1531  2.5993 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.475967   0.020031  23.761  < 2e-16 ***
fit_y       0.010670   0.003427   3.113  0.00195 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4519 on 516 degrees of freedom
Multiple R-squared:  0.01844,	Adjusted R-squared:  0.01654 
F-statistic: 9.694 on 1 and 516 DF,  p-value: 0.001952
```

Figure 52: BIGL (first sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIG2

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.0170 -0.5874 -0.2516  0.3221  6.3426 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.893099   0.039941  22.360  < 2e-16 ***
fit_y       0.023062   0.005909   3.903 0.000108 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9024 on 516 degrees of freedom
Multiple R-squared:  0.02867,	Adjusted R-squared:  0.02679 
F-statistic: 15.23 on 1 and 516 DF,  p-value: 0.0001077
```

Figure 53: BIG2 (first sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIGH

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.4165 -0.6836 -0.3183  0.3520  8.8854 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1.010425   0.046435  21.760   <2e-16 ***
fit_y       0.010280   0.005161   1.992   0.0469 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.048 on 516 degrees of freedom
Multiple R-squared:  0.007631,	Adjusted R-squared:  0.005707 
F-statistic: 3.968 on 1 and 516 DF,  p-value: 0.04691
```

Figure 54: BIGH (first sub-sample): $\hat{Y}_i$ vs $|e_i|$

Portfolios	Ind. variable	Estimate	Std. Error	t value	p value
SML	Intercept	-0.1036	0.0617	-1.6793	0.0937
	Mkt-RF	1.0633	0.0121	87.533	0
	SMB	1.0699	0.0194	55.0751	0
	HML	-0.1043	0.0184	-5.6748	0
SM2	Intercept	0.0197	0.0525	0.3753	0.7076
	Mkt-RF	1.0148	0.0103	98.2084	0
	SMB	0.8525	0.0165	51.5878	0
	HML	0.226	0.0156	14.4615	0
SMH	Intercept	0.0059	0.0354	0.1674	0.8671
	Mkt-RF	1.0198	0.007	146.2873	0
	SMB	0.9867	0.0111	88.5096	0
	HML	0.8088	0.0105	76.7196	0
BIGL	Intercept	0.0521	0.0296	1.7586	0.0792
	Mkt-RF	1.0301	0.0058	176.6643	0
	SMB	-0.0402	0.0093	-4.309	0
	HML	-0.2349	0.0088	-26.6342	0
BIG2	Intercept	-0.0719	0.0575	-1.2497	0.212
	Mkt-RF	0.9941	0.0113	87.773	0
	SMB	-0.0938	0.0181	-5.1774	0
	HML	0.3134	0.0171	18.2976	0
BIGH	Intercept	-0.0584	0.0653	-0.8946	0.3714
	Mkt-RF	1.0737	0.0129	83.5053	0
	SMB	0.0429	0.0206	2.0874	0.0373
	HML	0.8518	0.0194	43.8054	0

Table 5: Estimates, standard error, t value and p-value for Fama-French model for first sub-sample

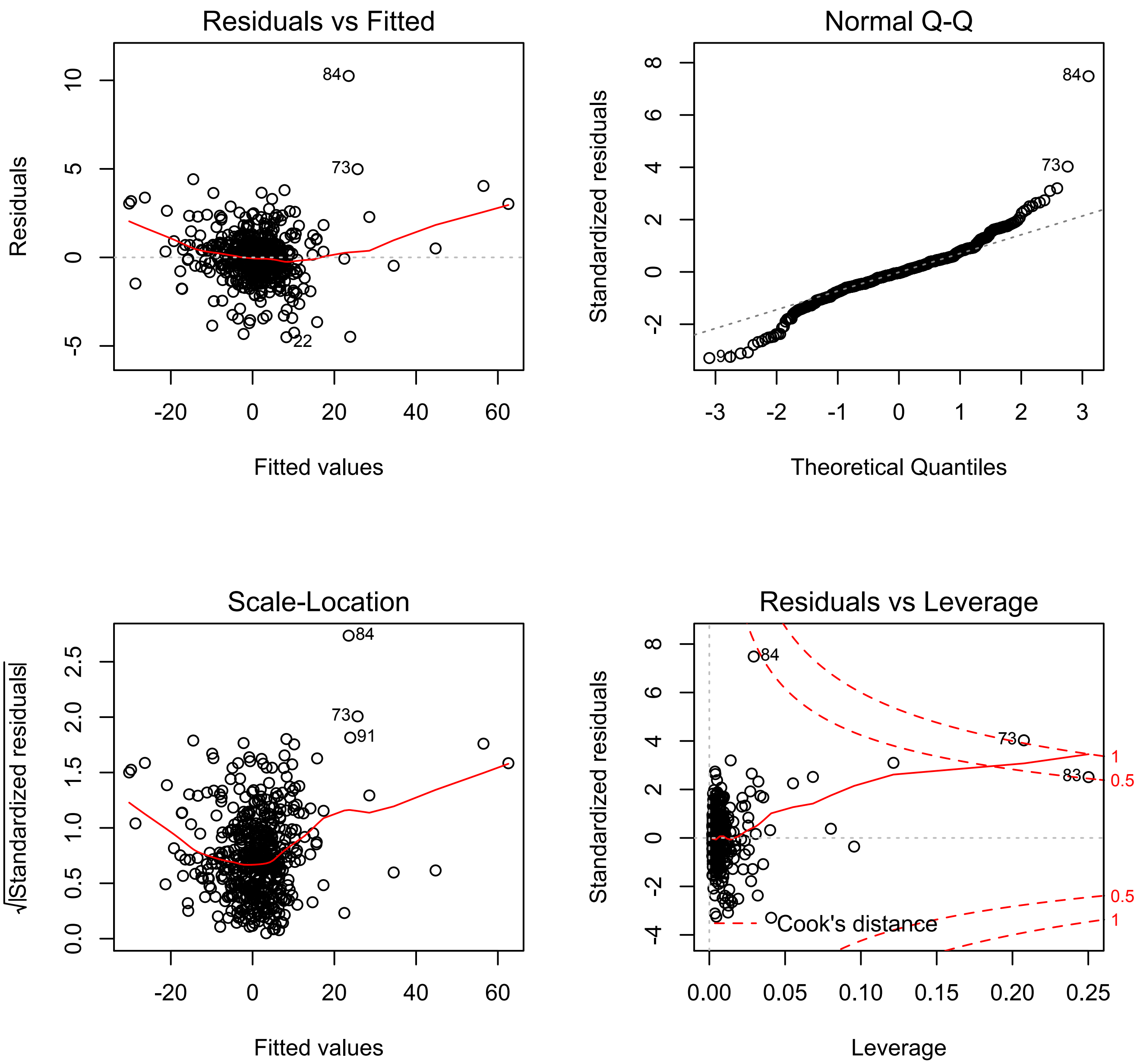

Figure 55: Regression diagnostics figures for SML (first sub-sample)

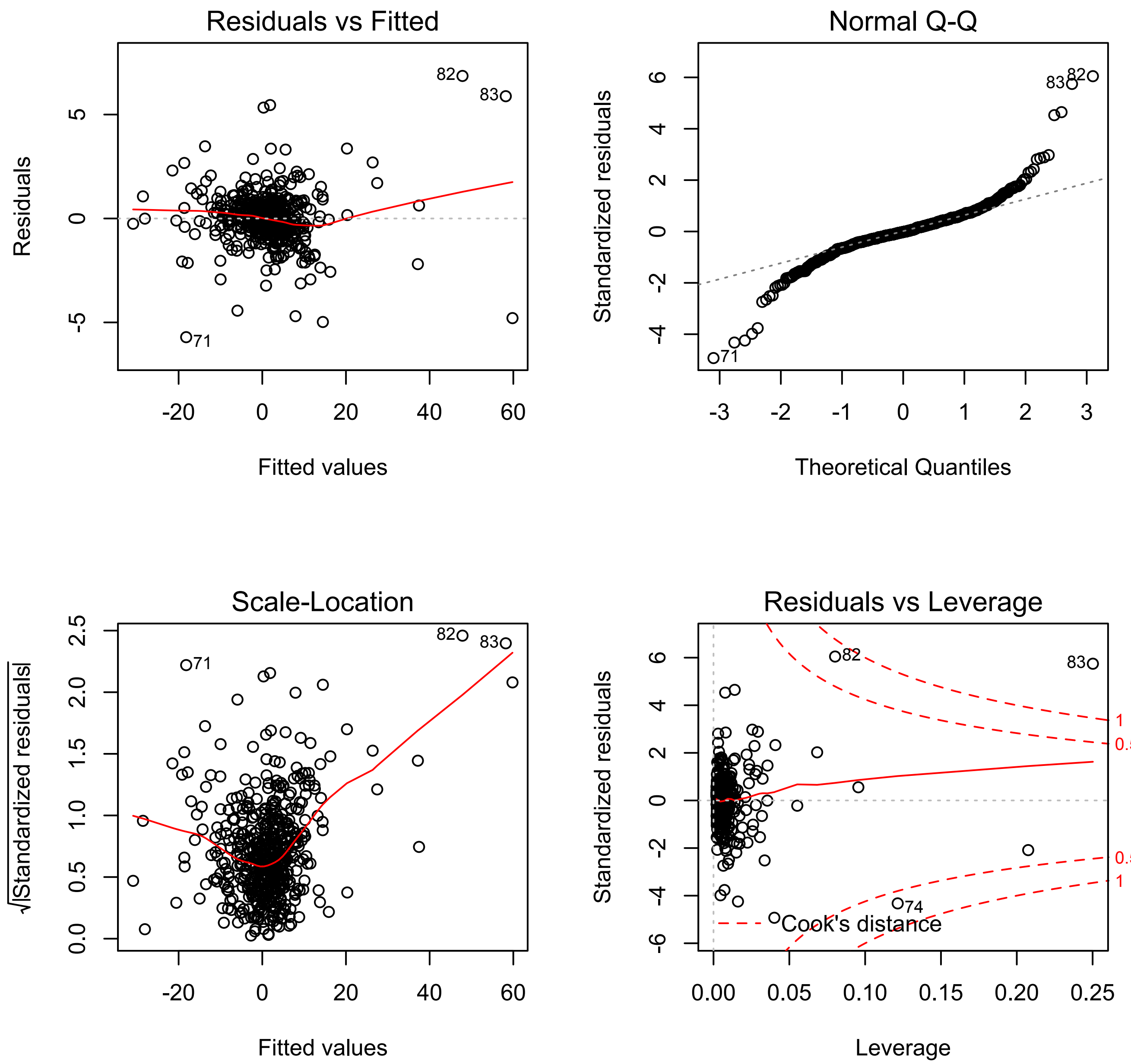

Figure 56: Regression diagnostics figures for SM2 (first sub-sample)

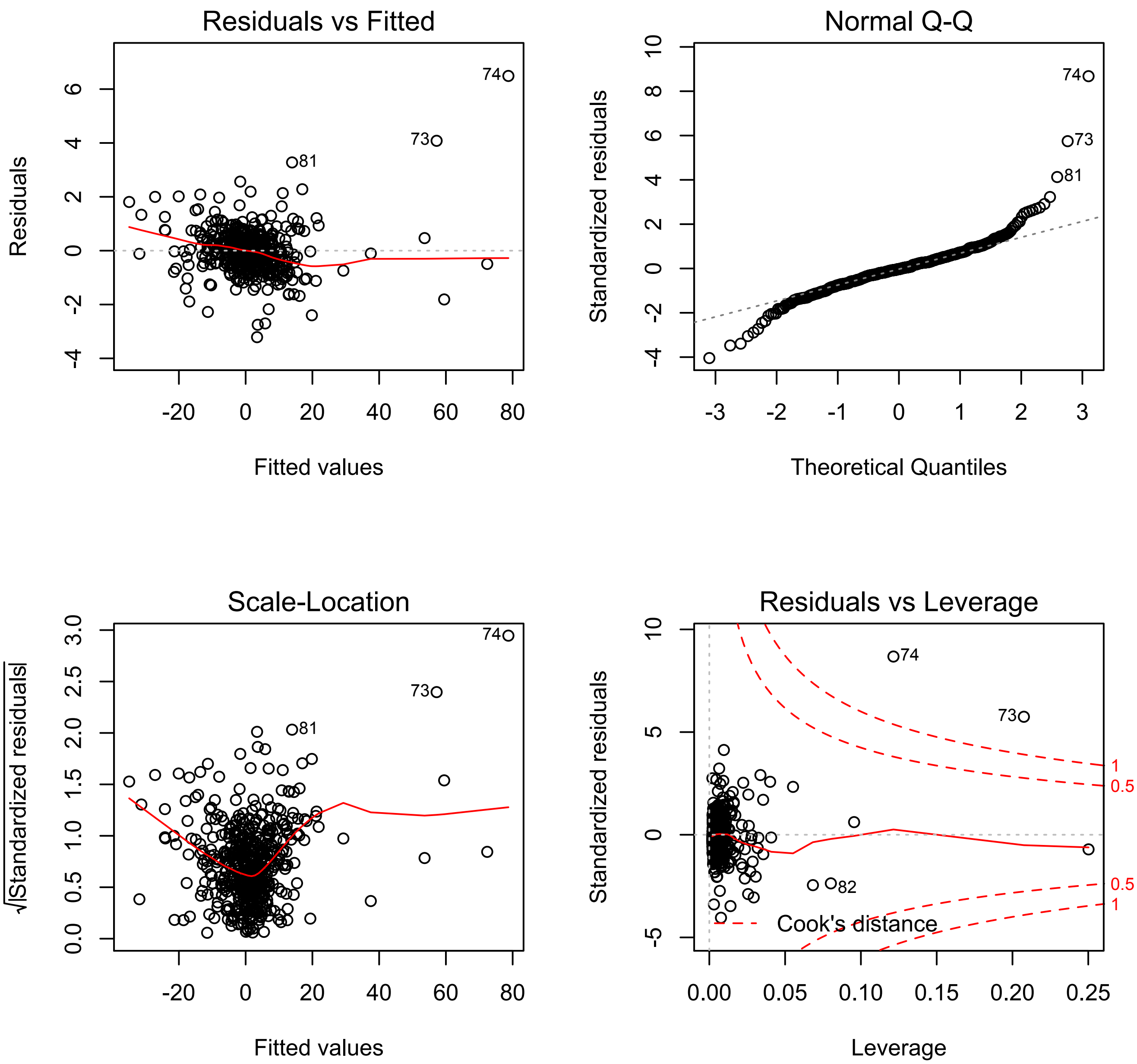

Figure 57: Regression diagnostics figures for SMH (first sub-sample)

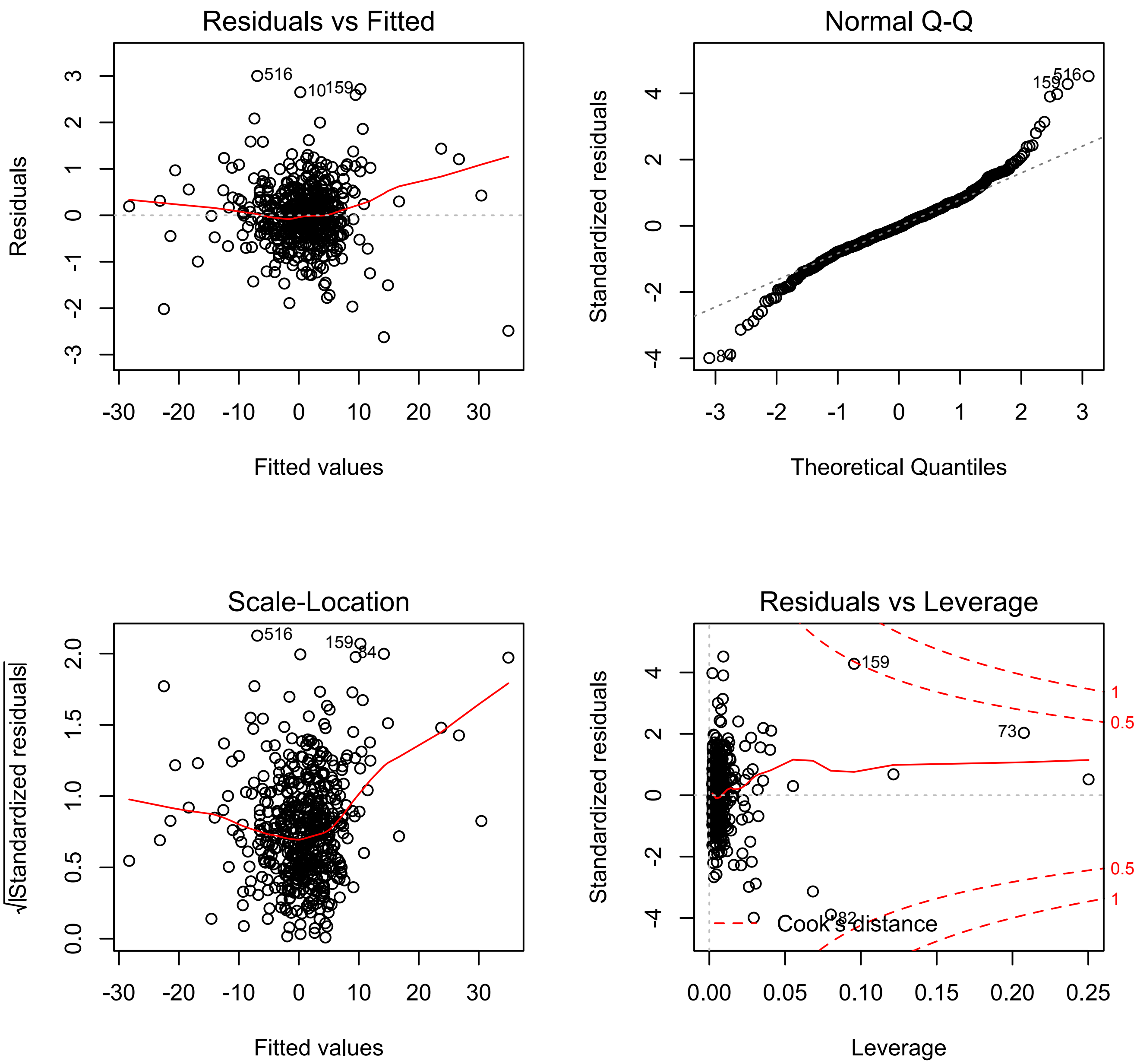

Figure 58: Regression diagnostics figures for BIGL (first sub-sample)

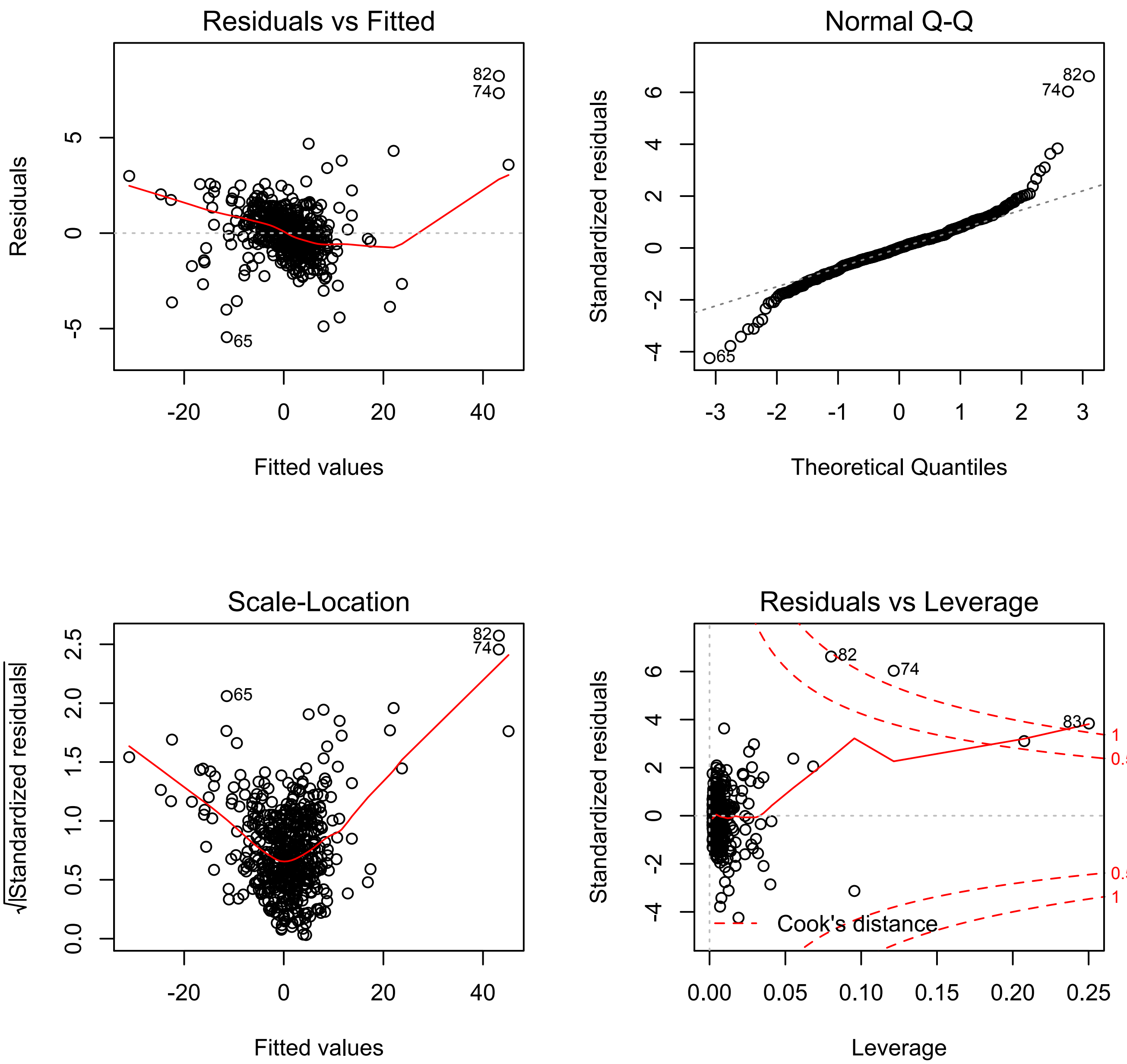

Figure 59: Regression diagnostics figures for BIG2 (first sub-sample)

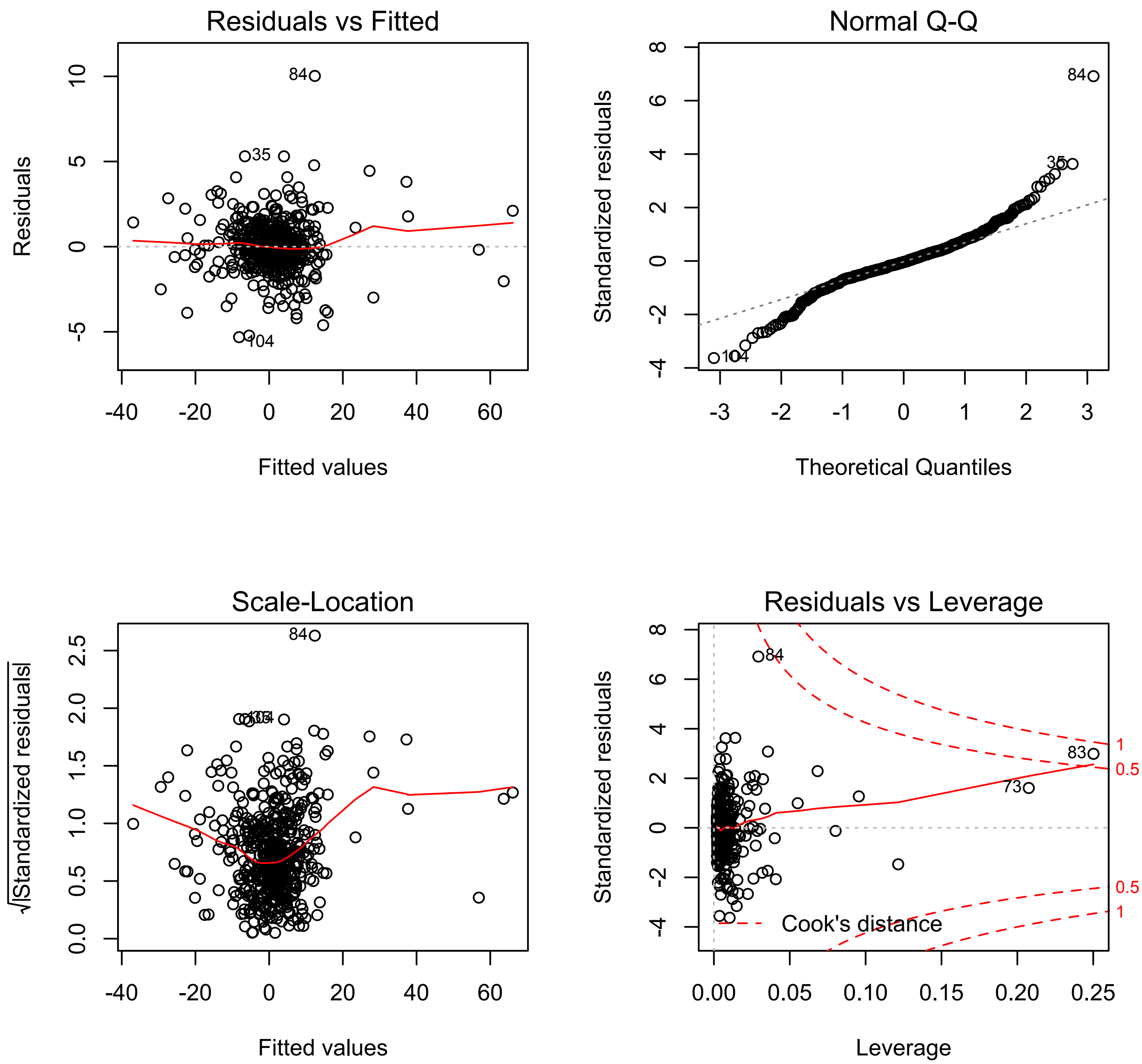

Figure 60: Regression diagnostics figures for BIGH (first sub-sample)

Portfolios	Ind. variable	Estimate	Std. Error	t value	p value
SML	Intercept	-0.1885	0.0456	-4.1295	0
	Mkt-RF	1.0753	0.0104	103.3947	0
	SMB	0.9975	0.015	66.5329	0
	HML	-0.2724	0.0158	-17.2031	0
SM2	Intercept	0.0856	0.0386	2.2185	0.027
	Mkt-RF	0.9584	0.0088	108.9705	0
	SMB	0.8051	0.0127	63.498	0
	HML	0.3515	0.0134	26.2486	0
SMH	Intercept	0.0754	0.0294	2.5651	0.0106
	Mkt-RF	1.0022	0.0067	149.5204	0
	SMB	0.8528	0.0097	88.2617	0
	HML	0.6878	0.0102	67.4062	0
BIGL	Intercept	0.1365	0.0367	3.7206	2e-04
	Mkt-RF	0.9782	0.0084	116.9937	0
	SMB	-0.1704	0.0121	-14.1402	0
	HML	-0.3083	0.0127	-24.2217	0
BIG2	Intercept	-0.0366	0.0532	-0.688	0.4918
	Mkt-RF	1.0064	0.0121	83.0221	0
	SMB	-0.1485	0.0175	-8.4962	0
	HML	0.3432	0.0185	18.6014	0
BIGH	Intercept	-0.1274	0.0466	-2.7341	0.0065
	Mkt-RF	1.0515	0.0106	99.046	0
	SMB	-0.0258	0.0153	-1.6863	0.0923
	HML	0.7318	0.0162	45.2798	0

Table 6: Estimates, standard error, t value and p-value for Fama-French model for second sub-sample

```
$PF6_SML

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-0.7904 -0.4871 -0.2099  0.2477  3.5525 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.740825   0.030605  24.206   <2e-16 ***
fit_y       -0.002983   0.004354  -0.685    0.494    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6956 on 516 degrees of freedom
Multiple R-squared:  0.0009088,	Adjusted R-squared:  -0.001027 
F-statistic: 0.4694 on 1 and 516 DF,  p-value: 0.4936
```

Figure 61: SML (second sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_SM2

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-0.6717 -0.3946 -0.1524  0.1878  4.3712 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.624986   0.026328   23.74   <2e-16 ***
fit_y       -0.005639   0.004741   -1.19    0.235    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.5931 on 516 degrees of freedom
Multiple R-squared:  0.002735,	Adjusted R-squared:  0.000802 
F-statistic: 1.415 on 1 and 516 DF,  p-value: 0.2348
```

Figure 62: SM2 (second sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_SMH

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max
-0.5656 -0.3027 -0.1153  0.2310  1.9567

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.495324   0.018593  26.641   <2e-16 ***
fit_y       0.007392   0.003230   2.289   0.0225 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4175 on 516 degrees of freedom
Multiple R-squared:  0.01005,   Adjusted R-squared:  0.00813
F-statistic: 5.238 on 1 and 516 DF,  p-value: 0.02251
```

Figure 63: SMH (second sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIGL

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max
-0.6096 -0.3997 -0.1427  0.2771  2.8507

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.614221   0.023698  25.919   <2e-16 ***
fit_y       0.001485   0.004921   0.302    0.763
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.5372 on 516 degrees of freedom
Multiple R-squared:  0.0001765, Adjusted R-squared:  -0.001761
F-statistic: 0.0911 on 1 and 516 DF,  p-value: 0.7629
```

Figure 64: BIGL (second sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIG2

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-0.8843 -0.5709 -0.2475  0.3045  4.7048 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.860335   0.036196  23.769   <2e-16 ***
fit_y       -0.011695   0.008191  -1.428    0.154    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.818 on 516 degrees of freedom
Multiple R-squared:  0.003935,	Adjusted R-squared:  0.002005 
F-statistic: 2.038 on 1 and 516 DF,  p-value: 0.154
```

Figure 65: BIG2 (second sub-sample): $\hat{Y}_i$ vs $|e_i|$

```
$PF6_BIGH

Call:
lm(formula = abs(resid) ~ fit_y)

Residuals:
    Min      1Q  Median      3Q     Max 
-0.8010 -0.5300 -0.1932  0.2946  3.0624 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.762891   0.031257  24.407   <2e-16 ***
fit_y       -0.005435   0.006600  -0.823    0.411    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.7052 on 516 degrees of freedom
Multiple R-squared:  0.001312,	Adjusted R-squared:  -0.000623 
F-statistic: 0.6781 on 1 and 516 DF,  p-value: 0.4106
```


Figure 66: BIGH (second sub-sample): $\hat{Y}_i$ vs $|e_i|$

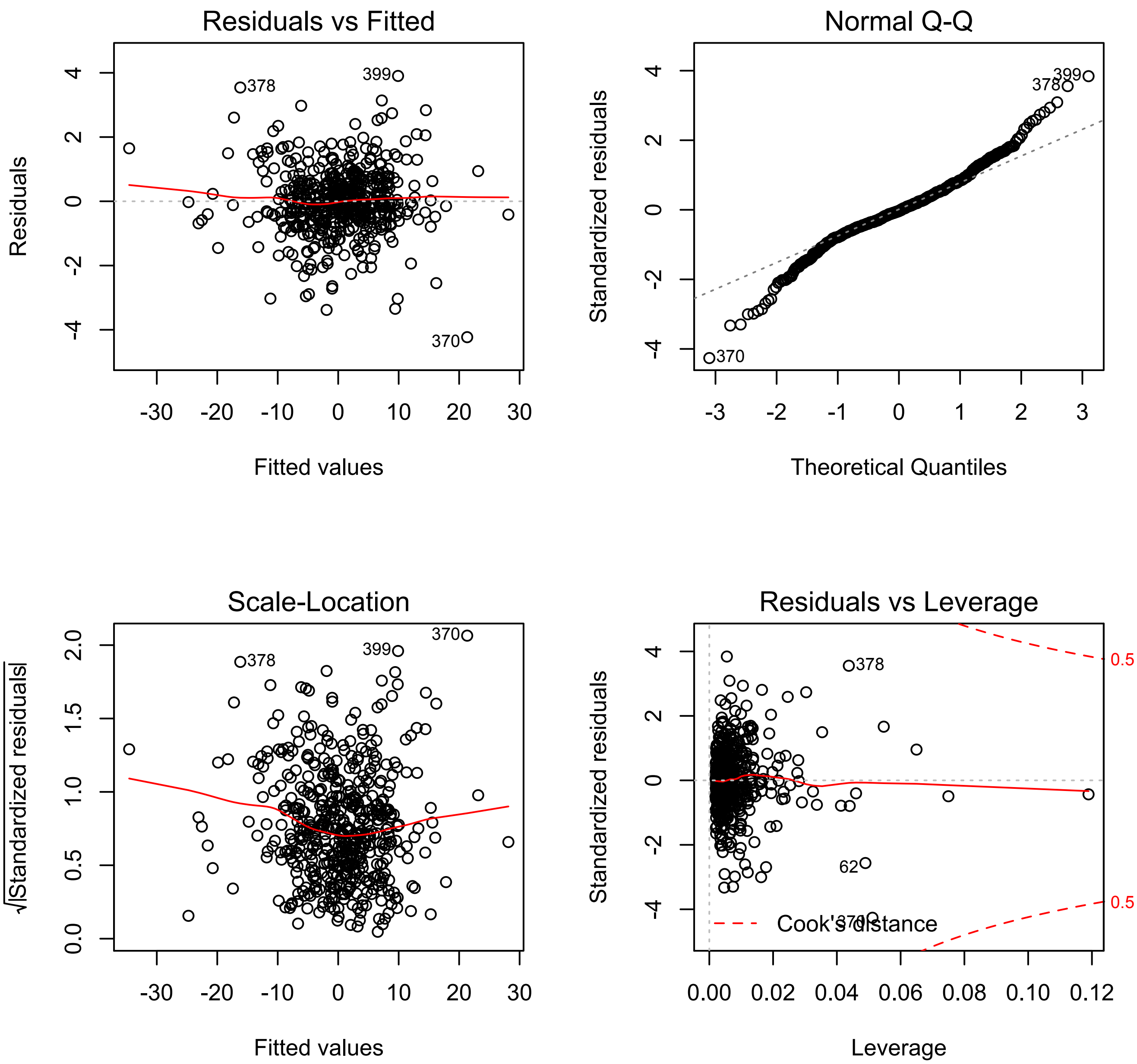

Figure 67: Regression diagnostics figures for SML (second sub-sample)

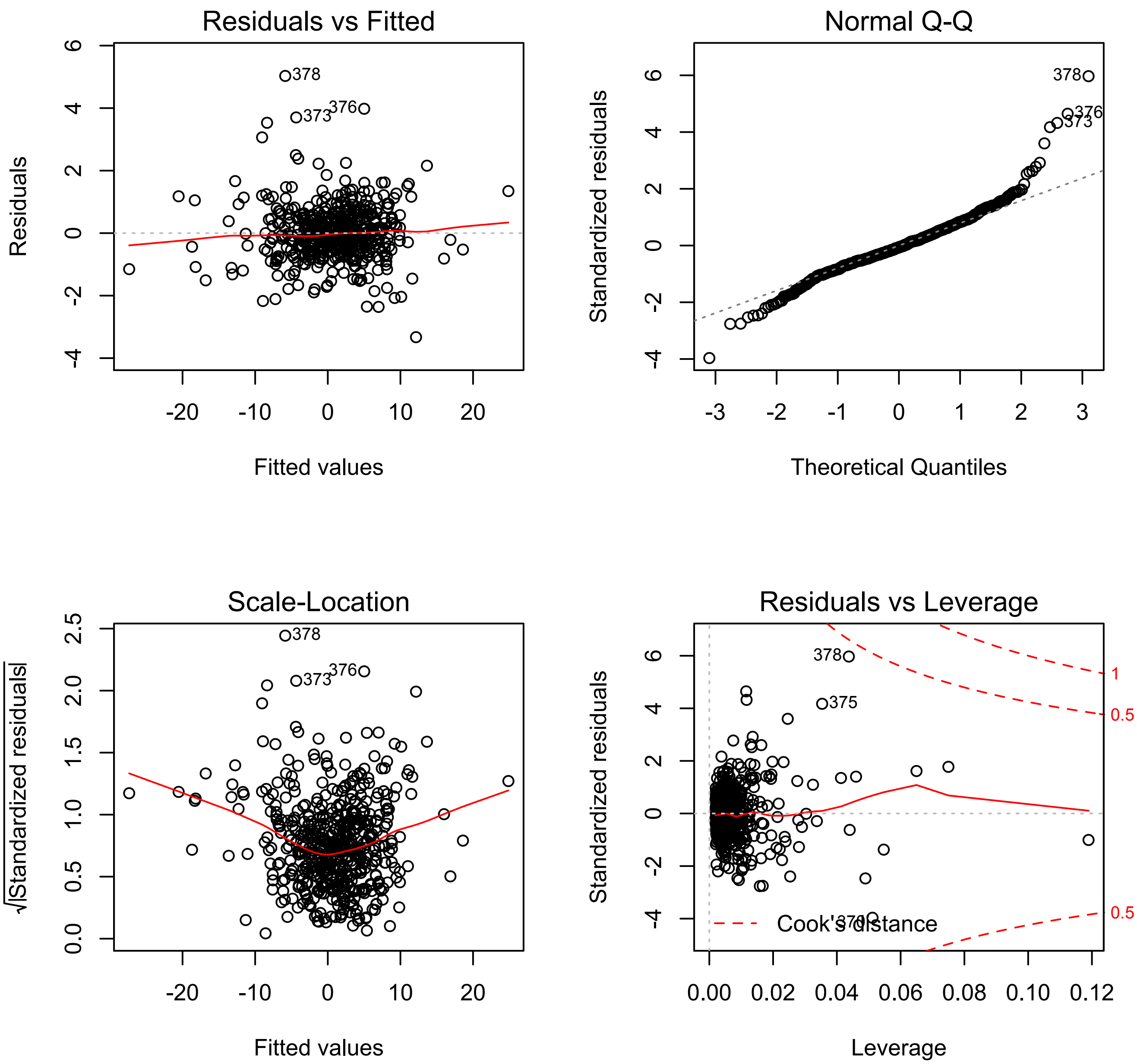

Figure 68: Regression diagnostics figures for SM2 (second sub-sample)

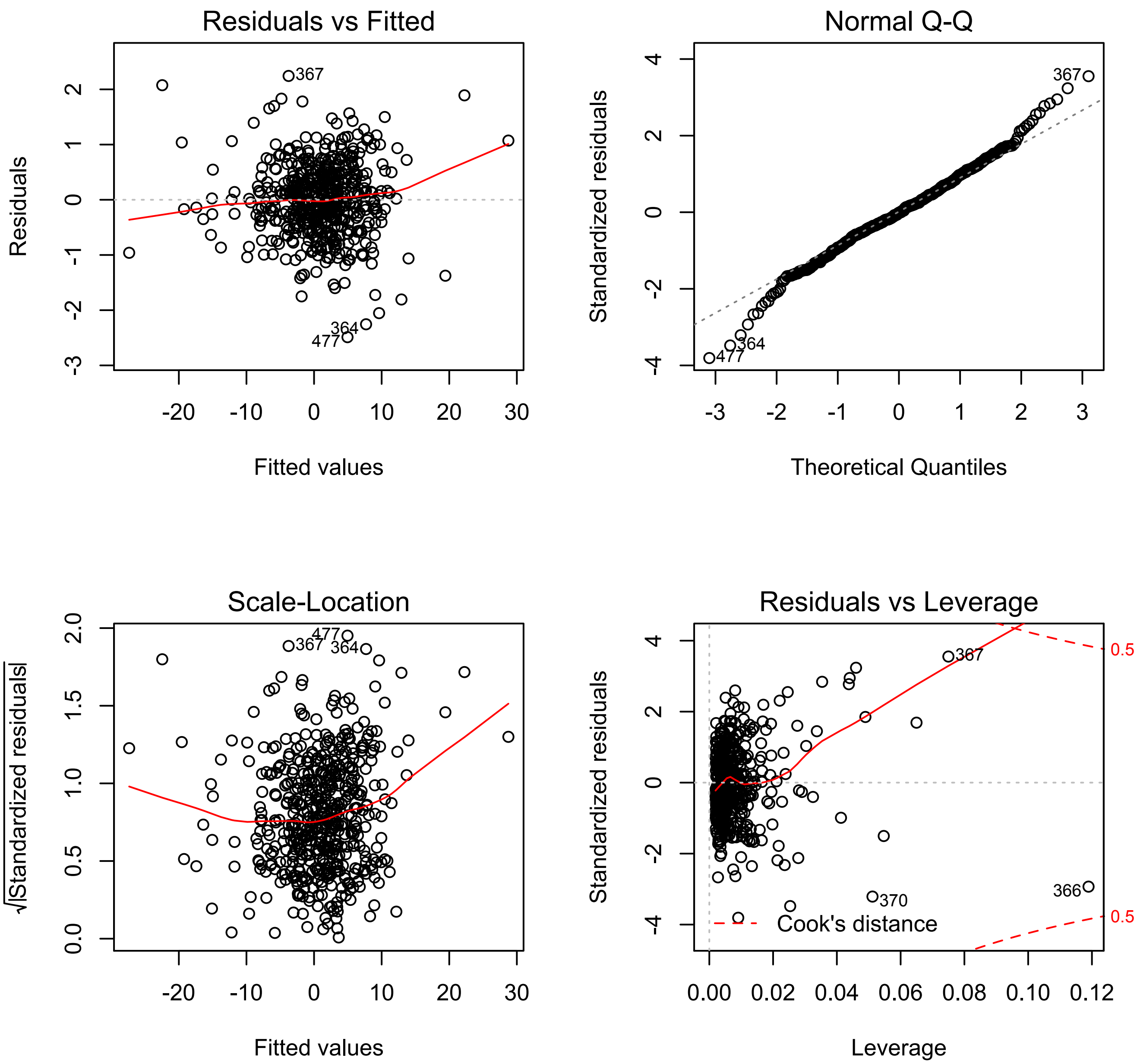

Figure 69: Regression diagnostics figures for SMH (second sub-sample)

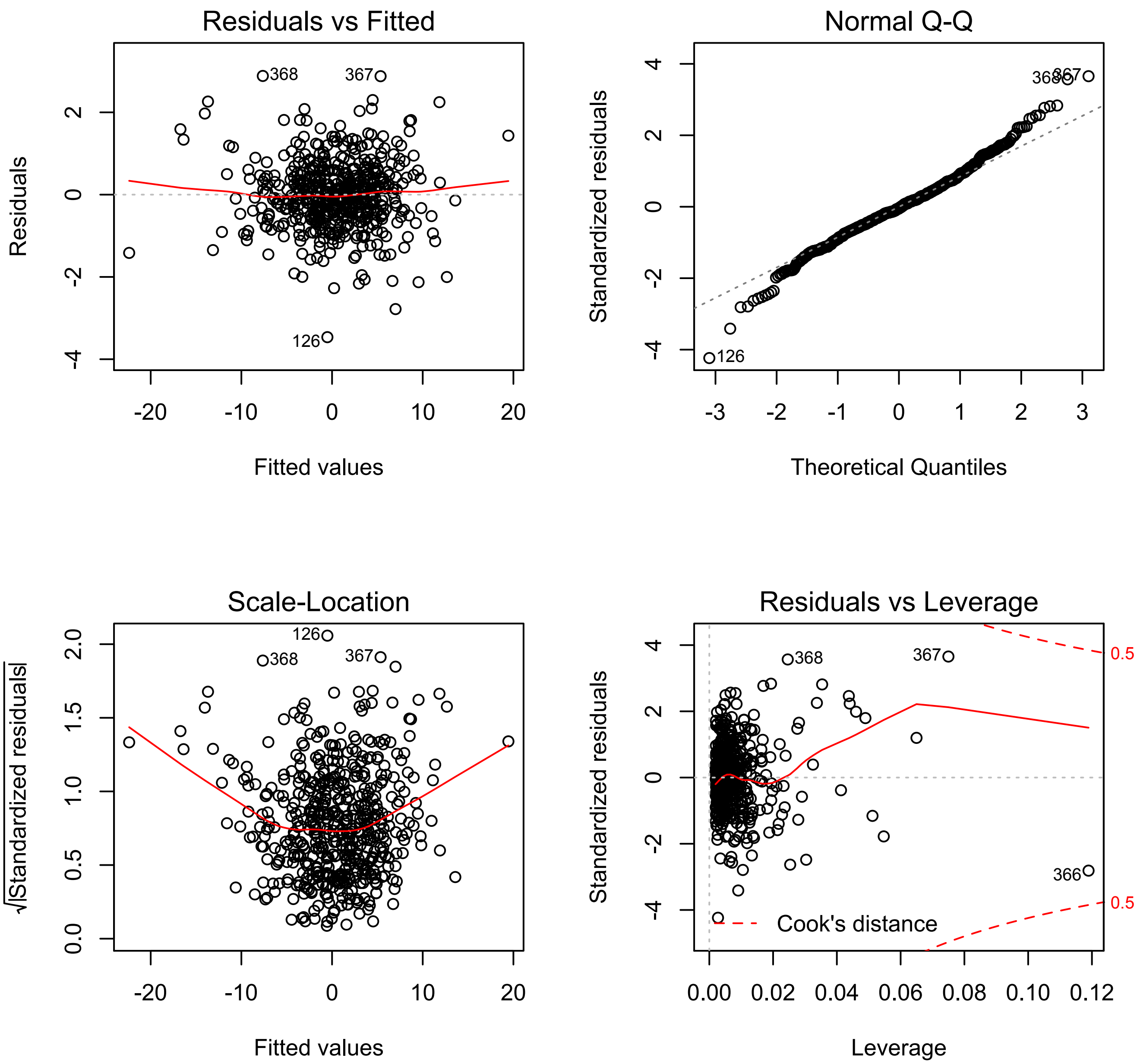

Figure 70: Regression diagnostics figures for BIGL (second sub-sample)

1.4 Question 1(d)

Table 7 show the results of $Hotelling's\ T^2$ test. We can see the p-value for intercept is close to 0, therefore we reject the hull hypothesis and believe at least a intercept for these 6 portfolio is significant.

	DF	Hotelling-Lawley	approx F	num DF	den DF	p-valye
Intercept	1	13575	2323509	6	1027	$< 2.2e-16$
Mkr-RF	1	128255	21953034	6	1027	$< 2.2e-16$
SMB	1	687169	117620447	6	1027	$< 2.2e-16$
HML	1	752609	128821544	6	1027	$< 2.2e-16$

Table 7: Hotelling's Test for the whole sample

Tables **??** and **??** show the$Hotelling's\ T^2$ for the two sub-samples. We can see both of them reject the null hypothesis and imply that there exist at least an intercept to be non-zero.

	DF	Hotelling-Lawley	approx F	num DF	den DF	p-valye
Intercept	1	16218	1375832	6	509	$< 2.2e-16$
Mkr-RF	1	407200	34544137	6	509	$< 2.2e-16$
SMB	1	778970	66082660	6	509	$< 2.2e-16$
HML	1	708331	60090093	6	509	$< 2.2e-16$

Table 8: Hotelling's Test for the first sub-sample sample

1.5 Question 1 (e)

Figure 73 provides the results of Johansen's test. We know from it that the null hypothesis test $r \leq 5$ is rejected. Therefore we know the number of cointegrating relationships is at least 6.

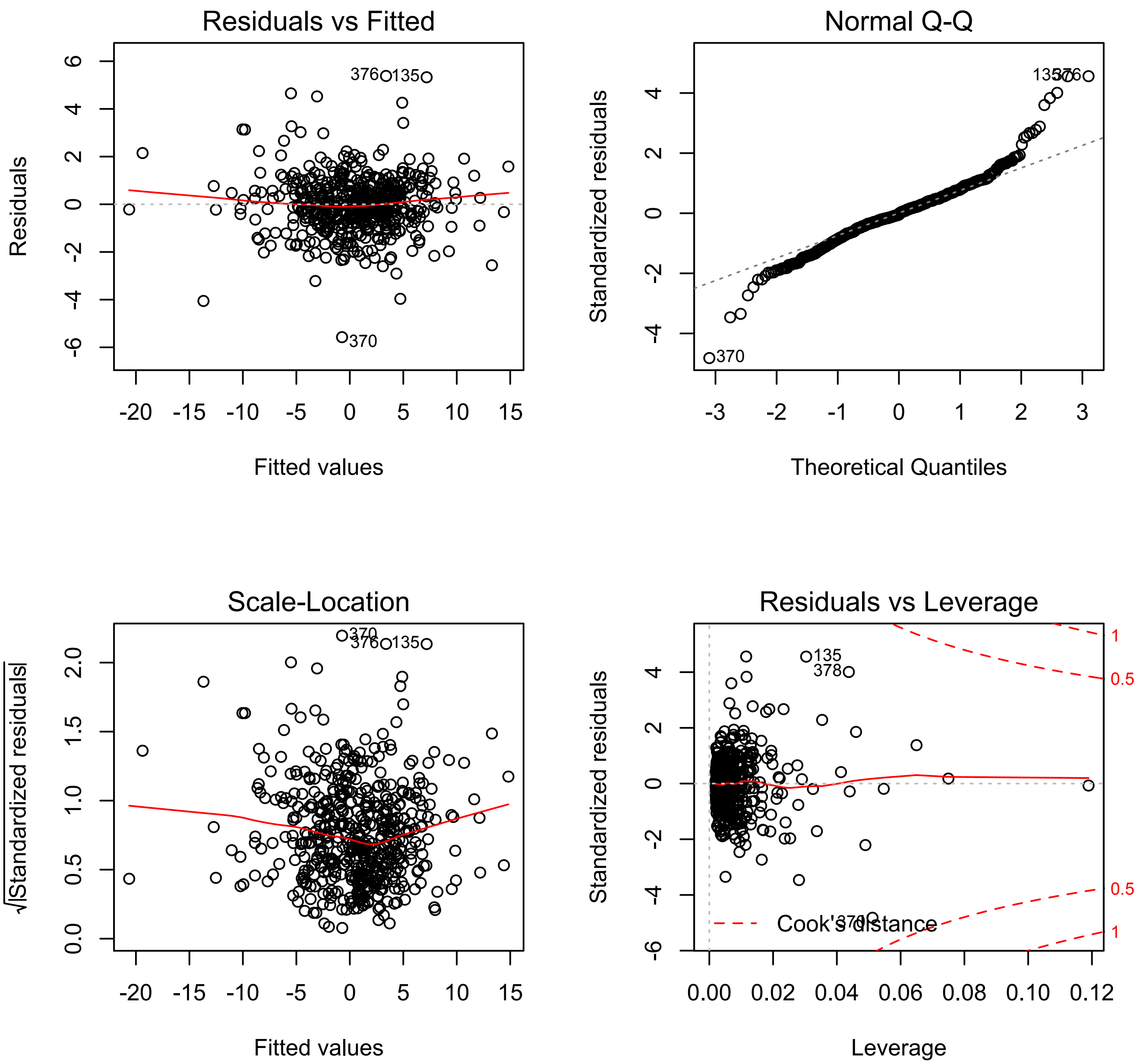

Figure 71: Regression diagnostics figures for BIG2 (second sub-sample)

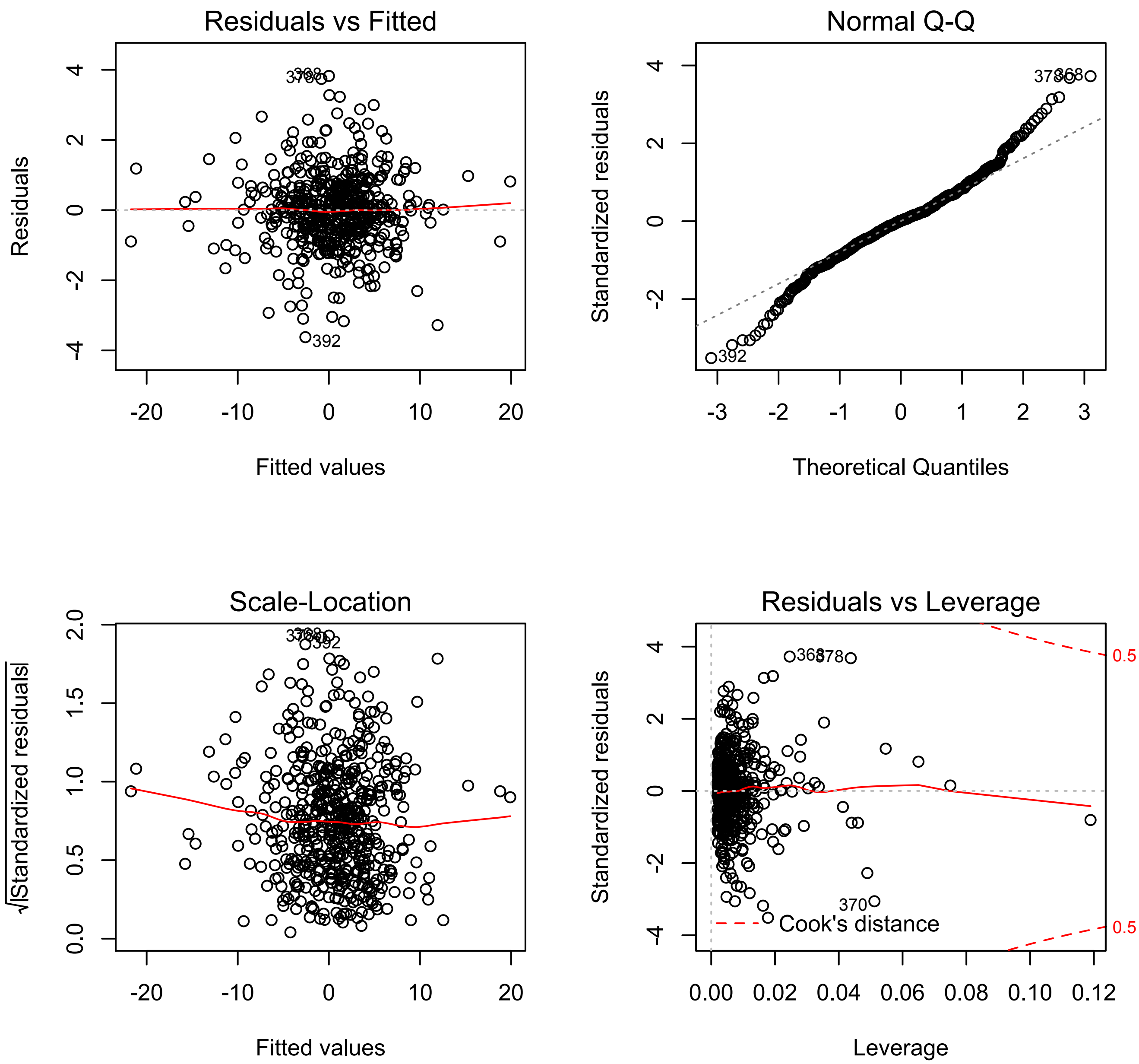

Figure 72: Regression diagnostics figures for BIGH (second sub-sample)

```
######################
# Johansen-Procedure #
######################

Test type: maximal eigenvalue statistic (lambda max) , with linear trend

Eigenvalues (lambda):
[1] 0.3979819 0.3508500 0.3195214 0.3043049 0.2850447 0.2691609

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 5 | 324.22  6.50  8.18 11.65
r <= 4 | 346.94 12.91 14.90 19.19
r <= 3 | 375.18 18.90 21.07 25.75
r <= 2 | 398.05 24.78 27.14 32.14
r <= 1 | 446.78 30.84 33.32 38.78
r = 0  | 524.72 36.25 39.43 44.59

Eigenvectors, normalised to first column:
(These are the cointegration relations)

            PF6_SML.l2 PF6_SM2.l2 PF6_SMH.l2 PF6_BIGL.l2 PF6_BIG2.l2 PF6_BIGH.l2
PF6_SML.l2    1.000000  1.0000000  1.0000000  1.00000000   1.0000000   1.0000000
PF6_SM2.l2    7.612158 -1.7478037 -3.7721521 -0.95147273  -1.9710022   0.2534857
PF6_SMH.l2   -8.560977  1.0895383  0.1062858 -0.60134787  -0.2491604  -0.7735319
PF6_BIGL.l2  -4.599279 -0.4805226 -1.4609204 -0.06511014   3.7178486  -1.2243454
PF6_BIG2.l2  -2.964420 -0.2212149  1.4602933  0.82338264  -6.4715549   0.5070909
PF6_BIGH.l2   5.403634 -0.2141485  2.4322887 -0.07105352   4.2722978   0.5012981

Weights W:
(This is the loading matrix)

           PF6_SML.l2  PF6_SM2.l2  PF6_SMH.l2 PF6_BIGL.l2 PF6_BIG2.l2 PF6_BIGH.l2
PF6_SML.d   0.1612068 -0.03224714  0.25386785  0.03382524 -0.16616109  -1.0809039
PF6_SM2.d   0.1466789  0.31467598  0.20286631  0.51157246 -0.11251332  -0.9675190
PF6_SMH.d   0.2300348  0.06174478  0.10518280  0.83189342 -0.12205509  -1.0402461
PF6_BIGL.d  0.1694964  0.59822942  0.11723078 -0.12447841 -0.14851987  -0.5113920
PF6_BIG2.d  0.1726984  0.63688443 -0.01296543  0.11942915 -0.04546706  -0.7496461
PF6_BIGH.d  0.1624547  0.58317818 -0.09728070  0.56873883 -0.15847220  -0.9426054
```

Figure 73: Johansen's test

	DF	Hotelling-Lawley	approx F	num DF	den DF	p-valye
Intercept	1	11812	1002034	6	509	$< 2.2e-16$
Mkr-RF	1	112220	9519990	6	509	$< 2.2e-16$
SMB	1	635060	53874216	6	509	$< 2.2e-16$
HML	1	506390	42958768	6	509	$< 2.2e-16$

Table 9: Hotelling's Test for the second sub-sample sample

2 Solution of Question 2

2.1 Question 2 (a)

Consider invest 1 Rupee for each foreign currency (USD, GBP and EUR). Total portfolio return on each day should be computed as a weighted sum of returns from each active trade (weighted by current value of each trade). The risk free rate is assumed to be 3%. The daily risk free rate is $\frac{0.003}{252}$. Annualized Sharpe ratio is

$$\text{Sharpe ratio} = \frac{E(R) - r_f}{\sigma} \times \sqrt{252}$$

where r_f denotes the risk free rate and $E(R)$ and σ are expectation and standard deviation of returns. They will be estimated by sample mean and sample standard deviation of returns.

The portfolio PNL is calculated as total profit or loss weighted by total value of each foreign currency.

2.1.1 Rule 1: Moving average trading rules

Table 10 present Sharpe ratio and PNL without transaction cost by moving average trading rules for different L. From it we know the optimal L is 240.

Assume the traded cost is 0.0015 for each dollar and each transaction. If we take transaction cost into consideration, Sharpe ratio and PNL get smaller. Table

11 presents Sharpe ratio and PNL with traded cost by moving average trading rules for different L. It can be seen that the optimal L is still 240.

2.1.2 Rule 2: Bollinger band

Table 12 presents Sharpe ratio and PNL without traded cost by Bollinger band trading rules for different L. We can see that the optimal L is 300 if we use Sharpe ratio to evaluate them and the optimal L is 170 if we use PNL.

If we take transaction costs into consideration, the optimal Ls are 300 and 170 by Sharpe ratio and PNL.

2.1.3 Rule 3: Resistance-Support

The momentum based Resistance-Support rule is

$$\begin{aligned} \text{Buy} \quad \text{if} \quad & p_t > \max_{0<i\leq L} p_{t-i} \\ \text{Sell} \quad \text{if} \quad & p_t < \min_{0<i\leq L} p_{t-i} \end{aligned}$$

The contrarian one is

$$\begin{aligned} \text{Sell} \quad \text{if} \quad & p_t > \max_{0<i\leq L} p_{t-i} \\ \text{Buy} \quad \text{if} \quad & p_t < \min_{0<i\leq L} p_{t-i} \end{aligned}$$

We try both momentum based Resistance -Support trading rules and contrarian one. Table 14 presents Sharpe ratio and PNL without traded cost by momentum based Resistance-Support band trading rules for different L. After L=210, no trade will happen if we use this rule. It can be seen that the optimal L is 70 by Sharpe ratio and PNL. The maximum Sharpe ratio is 9.3148 and maximum PNL is 0.0852.

If we take the transaction cost into consideration, the optimal L is sill 70 by both Sharpe ratio and PNL (See Table 15). The maximum Sharpe ratio is 8.9654 and maximum PNL is 0.0840.

Table 16 presents Sharpe ratio and PNL with traded cost by momentum based Resistance-Support band trading rules for different L. The optimal L is 160 by Sharpe ratio and 20 by PNL. The maximum Sharpe ratio is 4.0199 and maximum PNL is 0.048.

If we take transaction costs into consideration, the optimal L is 160 by Shape ratio and 20 by PNL. the maximum Sharpe ratio is 3.9904 and maximum PNL is 0.0394 (See Table 17).

Compare momentum based Resistance-Support and contrarian Resistance -Support rules, we know that we may have higher profit and Sharpe ratio by using the former one. In other words, the former one is more profitable.

2.1.4 Rule 4: Momentum

In order to find out the optimal n and m, we try different combinations of n and m and find out the maximum Sharpe ratio and PNL. We choose n from 50 to 300 and step size is 50. Within each n, m starts from 20 and stops at $n - 10$. Table 18 present Sharpe ratio and PNL without traded cost by mommentum trading rules for different n and m. It can be seen that when $n = 150$ and $n = 60$, we have maximum Sharpe ratio, which is 11.2263; when $n = 100$ and $m = 70$, we have maximum PNL, which is 0.1354.

If we take transaction cost into consideration, when $n = 150$ and $n = 60$, we have maximum Sharpe ratio, which is 10.7455 and $n = 100$ and $m = 70$, we have maximum PNL, which is 0.1279 (see Table 19).

n	m	Sharpe Ratio	PNL
50	20	1.5364	0.0484
50	30	-0.2655	-0.0153
50	40	-2.0290	-0.0735
100	20	1.0822	0.0107
100	30	1.0448	0.0093

100	40	3.7536	0.0573
100	50	5.3653	0.0716
100	60	7.8864	0.1099
100	70	8.2284	0.1354
100	80	7.8399	0.1264
100	90	4.1028	0.0957
150	20	-0.4437	-0.0092
150	30	-3.4022	-0.0505
150	40	0.2454	-0.0027
150	50	10.1229	0.1295
150	60	11.2263	0.0953
150	70	8.0280	0.0602
150	80	4.7527	0.0409
150	90	-1.3210	-0.0146
150	100	-1.8817	-0.0177
150	110	-4.8096	-0.0406
150	120	-3.7682	-0.0345
150	130	-2.6243	-0.0293
150	140	-2.7130	-0.0284
200	20	-3.5009	-0.0467
200	30	-5.7140	-0.0540
200	40	-0.7551	-0.0072
200	50	1.1353	0.0062
200	60	0.5498	0.0026
200	70	-1.7540	-0.0160
200	80	-2.4286	-0.0186
200	90	-3.2309	-0.0257

200	100	-5.5230	-0.0377
200	110	-7.1806	-0.0457
200	120	-0.8103	-0.0103
200	130	-3.1002	-0.0286
200	140	-3.5976	-0.0323
200	150	-7.2386	-0.0608
200	160	-12.2753	-0.0918
200	170	-10.1912	-0.0807
200	180	-5.1297	-0.0782
200	190	-3.5909	-0.0655
250	20	-7.9770	-0.0747
250	30	-7.7369	-0.0566
250	40	-9.3183	-0.0602
250	50	-5.1252	-0.0285
250	60	-4.0150	-0.0166
250	70	-5.2783	-0.0437
250	80	-3.6593	-0.0298
250	90	-3.4741	-0.0336
250	100	-5.3651	-0.0385
250	110	-6.8599	-0.0576
250	120	-4.7295	-0.0386
250	130	-5.2856	-0.0521
250	140	-15.6684	-0.0799
250	150	-14.4523	-0.0810
250	160	-11.9718	-0.0691
250	170	-9.4326	-0.0544
250	180	-9.0317	-0.0682

250	190	-12.6987	-0.0574
250	200	-11.1140	-0.0524
250	210	-17.3612	-0.0809
250	220	-5.8557	-0.0587
250	230	-2.1931	-0.0316
250	240	-0.5611	-0.0096
300	20	-21.0153	-0.0959
300	30	-15.3757	-0.1095
300	40	-21.4949	-0.0879
300	50	-20.2495	-0.0857
300	60	-25.9661	-0.0795
300	70	-26.6696	-0.0735
300	80	-23.8088	-0.0755
300	90	-22.4488	-0.0716
300	100	-12.0100	-0.0507
300	110	-72.6691	-0.0761
300	120	-24.2938	-0.0869
300	130	-22.1292	-0.0921
300	140	-29.8219	-0.0931
300	150	-19.5291	-0.0845
300	160	-27.9434	-0.0767
300	170	-26.8193	-0.0809
300	180	-43.2211	-0.1223
300	190	-50.2584	-0.1329
300	200	-55.9479	-0.1220
300	210	-45.9631	-0.1153
300	220	-38.6809	-0.0968

300	230	-19.4599	-0.0787
300	240	-8.0795	-0.0848
300	250	-18.3195	-0.1164
300	260	-6.4151	-0.0742
300	270	-5.6704	-0.0648
300	280	-11.9127	-0.1140
300	290	-6.1151	-0.0929

Table 18: Sharpe ratio and PNL without traded cost by momentum trading rules for different m and n

n	m	Sharpe Ratio	PNL
50	20	1.0319	0.0315
50	30	-0.7910	-0.0301
50	40	-2.5815	-0.0902
100	20	0.6223	0.0030
100	30	0.5677	0.0022
100	40	3.2352	0.0494
100	50	4.9465	0.0658
100	60	7.4660	0.1039
100	70	7.8243	0.1279
100	80	7.3211	0.1173
100	90	3.6373	0.0844
150	20	-0.9761	-0.0159
150	30	-3.8737	-0.0564
150	40	-0.2306	-0.0089
150	50	9.7554	0.1243

150	60	10.7455	0.0908
150	70	7.4277	0.0553
150	80	4.2667	0.0361
150	90	-1.7712	-0.0188
150	100	-2.3970	-0.0220
150	110	-5.3313	-0.0446
150	120	-4.2962	-0.0390
150	130	-3.2023	-0.0356
150	140	-3.4505	-0.0364
200	20	-3.9901	-0.0522
200	30	-6.2492	-0.0585
200	40	-1.3581	-0.0116
200	50	0.6257	0.0022
200	60	-0.0049	-0.0013
200	70	-2.2554	-0.0199
200	80	-2.9544	-0.0225
200	90	-3.7785	-0.0295
200	100	-6.1300	-0.0415
200	110	-7.7501	-0.0491
200	120	-1.2267	-0.0137
200	130	-3.5073	-0.0319
200	140	-4.0005	-0.0355
200	150	-7.5960	-0.0636
200	160	-12.8102	-0.0954
200	170	-10.7112	-0.0843
200	180	-5.4894	-0.0828
200	190	-4.0154	-0.0718

250	20	-8.5957	-0.0807
250	30	-8.2635	-0.0606
250	40	-9.8610	-0.0634
250	50	-5.7575	-0.0314
250	60	-4.6876	-0.0195
250	70	-5.7902	-0.0475
250	80	-4.1906	-0.0336
250	90	-3.9430	-0.0374
250	100	-5.8852	-0.0418
250	110	-7.2334	-0.0605
250	120	-5.1144	-0.0415
250	130	-5.5953	-0.0549
250	140	-16.1696	-0.0822
250	150	-14.9133	-0.0833
250	160	-12.4262	-0.0714
250	170	-9.8856	-0.0568
250	180	-9.4501	-0.0710
250	190	-13.3625	-0.0602
250	200	-11.7638	-0.0553
250	210	-18.0202	-0.0835
250	220	-6.3329	-0.0629
250	230	-2.6170	-0.0374
250	240	-1.0111	-0.0167
300	20	-21.5611	-0.0991
300	30	-15.7874	-0.1132
300	40	-21.9069	-0.0901
300	50	-20.6119	-0.0877

300	60	-26.3586	-0.0811
300	70	-27.1058	-0.0751
300	80	-24.2000	-0.0771
300	90	-22.8491	-0.0732
300	100	-12.3263	-0.0525
300	110	-73.6810	-0.0773
300	120	-24.6618	-0.0885
300	130	-22.4529	-0.0938
300	140	-30.2538	-0.0947
300	150	-20.1084	-0.0868
300	160	-28.8638	-0.0790
300	170	-27.6695	-0.0832
300	180	-44.0777	-0.1244
300	190	-51.1453	-0.1350
300	200	-57.0173	-0.1241
300	210	-46.8909	-0.1175
300	220	-39.7322	-0.0990
300	230	-20.1026	-0.0810
300	240	-8.5217	-0.0891
300	250	-18.8545	-0.1193
300	260	-6.7529	-0.0776
300	270	-6.0028	-0.0683
300	280	-12.2956	-0.1171
300	290	-6.5232	-0.0983

Table 19: Sharpe ratio and PNL with traded cost by momentum trading rules for different m and n

2.2 Question 2 (b)

Table 20 presents Sharpe ratio and PNL without traded cost by Oscillator trading rules for different L. From it we can see the optimal L is 220 by both Sharpe ratio and PNL. The maximum Sharpe ratio is 3.4372 and maximum PNL is 0.0994.

If we take transaction costs into consideration, the optimal L is 220 by Sharpe ratio and 170 by PNL (see Table 21).

3 Solution of Question 3

In this question, we want to develop a pairs trading strategy and check if it does any better than the strategies in Problem 2. We follow the strategy described on page 2-4 in Engleberg et al (2009).

Following Engleberg et al (2009), we identify the universe of eligible pairs using return data for all three currencies during the immediately preceding 3 month "estimation period" at the end of each calendar month, m. We assume invest 1 Rupee for each foreign currency and normalized each foreign exchange rate. The normalized foreign exchange rate for currency i is

$$FX_t^i = \prod_{i=1}^{t}(1 + r_t^i)$$

where r_t^i the ratio of current foreign exchange rate and the one of preceding day.

For each calendar month, we compute the average squared normalized exchange rate difference measure $FXD_{i,j,m}$ between exchange rate i and j, for each pair of exchange rate we have

$$FXD_{i,j,m} = \frac{\sum_{t=1}^{T_m}(FX_t^i - FX_t^j)^2}{T_m}$$

where T_m is the total number of trading days in the estimation period for calender month m, FX_t^i and FX_t^j are the normalized exchange rate for foreign currency i and j

respectively on trading day t in the estimation period. The standard deviation of the wquared normalized exchange rate differences is given by

$$StdFX_{i,j,m} = \sqrt{\frac{1}{T_m - 1} \sum_{t=1} T_m \left[(FX_t^i - FX_t^j)^2 - FX_{i,j,m} \right]^2}$$

For each calender month m, we identify 1 pair that have the smallest average normalized exchange rate difference during its estimation period. This pair will be "eligible" for investment during the following 3 months "eligibility period" immediately following calendar month m. Our trading rule is: if the exchange rate in the eligible pair diverge by more than $2 \times StdFX_{i,j,m}$ then we buy the "cheap" currency and sell the "expensive" one. If the pair later convergence, i.e., when their normalized prices cross for the first time after divergence, we unwind the position and wait for the pair to diverge again. If the pair diverges but does not converge within 2 month, we close the position and call this "no convergence". In summary, each cycle contains three periods (8 months): estimation (3 months), eligibility (3 months) and convergence (2 months). Furthermore, for our convenience we assume there are 21 trading days in each month.

Table 22 presents the trading results with and without transaction cost. From it we know we have Sharpe ratio 3.5013 if we do not consider transaction cost and PNL is 0.0682. However, if we take the trading cost into consideration, the Sharpe ratio is 2.3654 and PNL is 0.0523.

Furthermore, Figure 74 shows the foreign exchange divergence and threshold. During the 48 months period, we traded 10 times. We unwind the position for 6 times and close the position 4 times because of not converging.

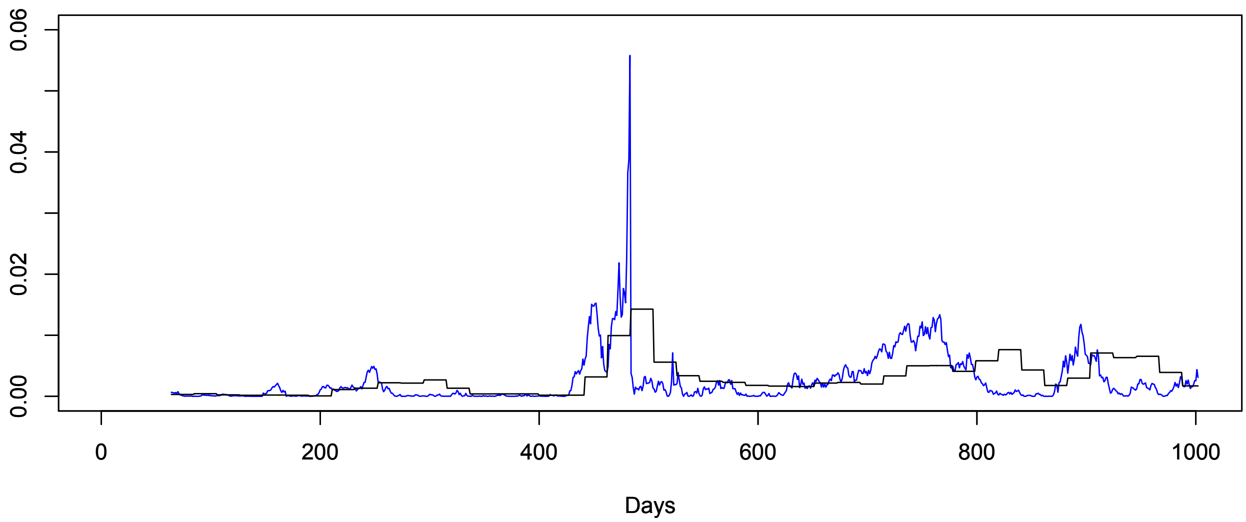

Figure 74: Foreign exchange divergence and threshold

L	Sharpe Ratio	PNL
20	0.1116	0.0134
30	0.5984	0.0240
40	-0.0640	-0.0049
50	0.6583	0.0249
60	0.6574	0.0204
70	1.0205	0.0181
80	-0.3709	-0.0290
90	-1.1743	-0.0581
100	-1.0006	-0.0439
110	-1.1057	-0.0515
120	-0.7838	-0.0370
130	-0.8369	-0.0341
140	0.9164	0.0191
150	0.9035	0.0056
160	0.2948	-0.0073
170	-0.4305	-0.0147
180	-0.3062	-0.0095
190	0.7776	0.0153
200	0.4753	0.0057
210	0.5082	0.0115
220	0.2258	0.0077
230	1.0789	0.0289
240	2.0719	0.0558
250	1.4772	0.0384
260	-0.0604	0.0035
270	0.2389	0.0088
280	-0.0323	0.0065
290	-0.3129	-0.0012
300	0.4374	0.0108

Table 10: Sharpe ratio and PNL without traded cost by moving average trading rules

L	Sharpe Ratio	PNL
20	-1.2781	-0.0694
30	-0.8339	-0.0486
40	-1.4089	-0.0612
50	-0.4178	-0.0280
60	-0.4012	-0.0278
70	-0.0524	-0.0251
80	-1.3855	-0.0626
90	-1.9707	-0.0844
100	-1.7383	-0.0707
110	-1.8627	-0.0749
120	-1.5175	-0.0577
130	-1.5777	-0.0556
140	0.0775	-0.0077
150	-0.0806	-0.0210
160	-0.7590	-0.0292
170	-1.5540	-0.0356
180	-1.3205	-0.0279
190	-0.2684	-0.0055
200	-0.4333	-0.0150
210	-0.4626	-0.0128
220	-0.7083	-0.0149
230	0.1341	0.0053
240	1.0427	0.0295
250	0.3743	0.0137
260	-1.0112	-0.0158
270	-0.6714	-0.0116
280	-0.9372	-0.0109
290	-1.2051	-0.0182
300	-0.4010	-0.0060

Table 11: Sharpe ratio and PNL with traded cost by moving average trading rules for

L	Sharpe Ratio	PNL
20	-1.8656	-0.0519
30	-3.4376	-0.0617
40	0.5175	-0.0116
50	-3.9879	-0.0601
60	-3.2344	-0.0766
70	-1.2513	-0.0459
80	-0.7048	-0.0353
90	-1.0659	-0.0180
100	-0.5882	-0.0136
110	0.8013	-0.0002
120	-1.5621	-0.0209
130	-1.3795	-0.0196
140	2.4326	0.0287
150	2.2655	0.0237
160	2.7861	0.0311
170	3.7938	0.0433
180	0.6632	0.0047
190	-3.7767	-0.0228
200	-4.8494	-0.0319
210	-5.0328	-0.0336
220	-3.5816	-0.0256
230	-3.2860	-0.0229
240	-4.5120	-0.0289
250	-4.1469	-0.0259
260	-3.4462	-0.0205
270	-2.9081	-0.0167
280	-2.3767	-0.0127
290	0.2460	0.0025
300	4.6106	0.0225

Table 12: Sharpe ratio and PNL without traded cost by Bollinger band trading rules

L	Sharpe Ratio	PNL
20	-2.3021	-0.0637
30	-4.0134	-0.0692
40	0.0909	-0.0186
50	-4.4093	-0.0650
60	-3.5989	-0.0804
70	-1.6344	-0.0498
80	-1.0623	-0.0393
90	-1.3500	-0.0209
100	-0.8797	-0.0165
110	0.5059	-0.0032
120	-1.8103	-0.0232
130	-1.6291	-0.0219
140	2.1819	0.0256
150	1.9969	0.0206
160	2.5375	0.0280
170	3.5474	0.0402
180	0.3661	0.0027
190	-4.0132	-0.0243
200	-5.0706	-0.0333
210	-5.2512	-0.0351
220	-3.7784	-0.0271
230	-3.4864	-0.0244
240	-4.7379	-0.0303
250	-4.3774	-0.0273
260	-3.6838	-0.0220
270	-3.1507	-0.0182
280	-2.6310	-0.0142
290	-0.0410	0.0010
300	4.4004	0.0214

Table 13: Sharpe ratio and PNL with traded cost by Bollinger band trading rules for

L	Sharpe Ratio	PNL
20	1.7529	0.0320
30	1.9483	0.0430
40	2.3878	0.0486
50	4.2326	0.0483
60	7.3989	0.0588
70	9.3148	0.0852
80	7.0462	0.0775
90	5.2266	0.0692
100	0.7631	0.0537
110	-1.7926	0.0463
120	-3.1405	0.0446
130	-5.0072	0.0428
140	-7.4831	0.0352
150	-7.9710	0.0352
160	-10.8886	0.0349
170	-12.5160	0.0385
180	-12.6565	0.0385
190	-41.1730	0.0385
200	-59.2593	0.0356
210	-82.1055	0.0335

Table 14: Sharpe ratio and PNL without traded cost by momentum based Resistance-Support band trading rules for different L

1	2	3
20	1.1859	0.0246
30	1.4378	0.0382
40	1.9945	0.0454
50	3.8894	0.0455
60	7.0419	0.0568
70	8.9654	0.0840
80	6.6805	0.0764
90	4.8266	0.0681
100	0.3304	0.0527
110	-2.2444	0.0454
120	-3.5821	0.0437
130	-5.4631	0.0419
140	-7.9348	0.0344
150	-8.4463	0.0344
160	-11.4056	0.0340
170	-12.9067	0.0381
180	-13.2791	0.0381
190	-42.5340	0.0381
200	-61.0939	0.0352
210	-84.5901	0.0331

Table 15: Sharpe ratio and PNL with traded cost by momentum based Resistance-Support band trading rules for different L

L	Sharpe ratio	PNL
20	0.3372	0.0480
30	-0.0188	0.0373
40	0.9496	0.0330
50	0.1938	0.0325
60	-3.8661	0.0219
70	-4.6292	0.0065
80	-3.1827	0.0072
90	-2.2832	0.0077
100	-0.6554	0.0127
110	1.0793	0.0208
120	1.4004	0.0223
130	2.7693	0.0237
140	4.0199	0.0305
150	3.2423	0.0305
160	4.2566	0.0305
170	3.6921	0.0129
180	0.7118	0.0129
190	1.7854	0.0129
200	1.8313	0.0129
210	1.8313	0.0129
220	-3.4340	0.0129
230	-3.3554	0.0129
240	-3.0124	0.0129
250	-2.1544	0.0129
260	-1.6660	0.0154
270	-0.3887	0.0222
280	0.6677	0.0259
290	1.8501	0.0283
300	1.8501	0.0283

Table 16: Sharpe ratio and PNL without traded cost by contrarian Resistance-Support

L	Sharpe ratio	PNL
20	-0.2749	0.0394
30	-0.5742	0.0327
40	0.5008	0.0302
50	-0.2343	0.0300
60	-4.2015	0.0205
70	-4.9832	0.0059
80	-3.5188	0.0065
90	-2.6194	0.0071
100	-0.9805	0.0120
110	0.7475	0.0201
120	1.0641	0.0216
130	2.4411	0.0230
140	3.6962	0.0297
150	2.9585	0.0297
160	3.9904	0.0297
170	3.4344	0.0126
180	0.4996	0.0126
190	1.5928	0.0126
200	1.6396	0.0126
210	1.6396	0.0126
220	-3.6318	0.0126
230	-3.5541	0.0126
240	-3.2143	0.0126
250	-2.3628	0.0126
260	-1.8811	0.0150
270	-0.6224	0.0218
280	0.4295	0.0255
290	1.6281	0.0279
300	1.6281	0.0279

Table 17: Sharpe ratio and PNL with traded cost by contrarian Resistance-Support

L	Sharpe Ratio	PNL
20	-0.8231	-0.0555
30	0.0049	-0.0040
40	1.0455	0.0505
50	1.3107	0.0538
60	0.7123	0.0300
70	-1.2078	-0.0441
80	-0.8570	-0.0321
90	-0.6341	-0.0219
100	-0.2510	-0.0177
110	-0.6694	-0.0420
120	0.4764	0.0070
130	1.4910	0.0425
140	0.5396	0.0080
150	2.5242	0.0583
160	-0.0490	-0.0053
170	3.3663	0.0932
180	2.2760	0.0713
190	1.2524	0.0378
200	2.7928	0.0759
210	2.0144	0.0663
220	3.4372	0.0994
230	2.3383	0.0641
240	-0.5109	-0.0144
250	0.0447	-0.0114
260	1.3115	0.0336
270	1.9253	0.0515
280	1.6127	0.0458
290	-0.4933	-0.0014
300	2.5920	0.0427

Table 20: Sharpe ratio and PNL without traded cost by Oscillator trading rules for

L	Sharpe Ratio	PNL
20	-2.1732	-0.1211
30	-1.1067	-0.0597
40	0.0952	0.0019
50	0.4085	0.0106
60	-0.2640	-0.0136
70	-1.9887	-0.0722
80	-1.7160	-0.0626
90	-1.3670	-0.0460
100	-1.0085	-0.0402
110	-1.3665	-0.0619
120	-0.2806	-0.0127
130	0.6052	0.0162
140	-0.3834	-0.0149
150	1.4010	0.0359
160	-0.8558	-0.0218
170	2.5089	0.0748
180	1.3204	0.0449
190	0.4763	0.0200
200	2.0317	0.0550
210	1.0061	0.0415
220	2.5467	0.0726
230	1.3863	0.0402
240	-1.4893	-0.0321
250	-0.7726	-0.0255
260	0.6143	0.0193
270	1.2251	0.0357
280	1.0064	0.0339
290	-1.2141	-0.0113
300	1.5483	0.0278

Table 21: Sharpe ratio and PNL with traded cost by Oscillator trading rules for different

	Sharpe Ratio	PNL
Without transaction cost	3.5013	0.0682
With transaction cost	2.3654	0.0523

Table 22: Sharpe ratio and PNL without / with traded cost by pair trading rules