von Neumann-Morgenstern and Savage Theorems for Causal Decision Making

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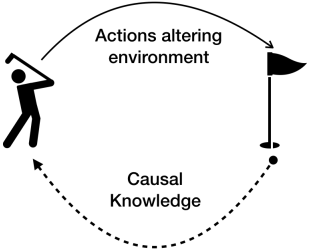
Introduction

Our premises

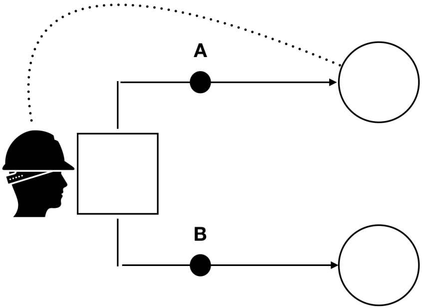
- We live in a causally structured world.
- Our actions *cause* changes in the environment.
- We learn from these changes in order to plan our actions when facing similar situations.
- Causal reasoning can, and should, be incorporated into interactive learning frameworks.



Decision Making and Human use of Causal Relations

Human beings:

- Are constantly asking *why?*
- Consider actions as *interventions* upon the world (Hagmayer and Sloman (2009)).
- Use and modify causal information in sequential decision making processes (Hagmayer and Meder (2013)).



Why is Decision Making important?

In terms of AI:

- Making decisions under uncertain conditions is very important to intelligent reasoning (Lake et al. (2017)).
- Several algorithms and procedures rely on this framework.
- For example, optimal policies in Reinforcement Learning satisfy the Maximum Expected Utility criterion (Sutton and Barto (1998); Webb (2007); Shoham and Leyton-Brown (2008)).
- Even though the original motivation for RL has to do with interactive learning, it is based only on associative information.

Causation in Machine Learning

- "All the impressive achievements of deep learning amount to just curve fitting," J. Pearl (Almeida (2018))
- Data can tell that people who took a medicine recovered faster than those who did not, but it can't tell why.
- Can it be explained from data *why* a certain decision, or parameter, was taken?
- Interpretability and transparency.

Pearl's ladder of Causal Reasoning

According to Pearl and Mackenzie (2018) there are three levels of Causal Reasoning:

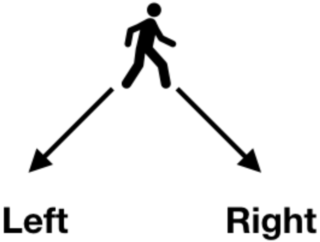
- Level 1: Association: What if I see....
- Level 2: Intervention What would Y be if I do X.
- Level 3: Counterfactual reasoning:What if I had instead done...

Practically all of AI's algorithms work at the Associative level.

Main Idea

Main idea

How to make good choices (interventions) in an uncertain but causally structured world?



Some questions

Several questions arise:

- What is a *good choice*?
- What is *uncertainty*?
- What is *causation*?

Classical Decision Theory

A Classical Decision Problem

A classical decision problem consists of:

- A set $\mathcal{A}$ of available actions
- An algebra of events $\mathcal{E}$, the uncertain events.
- A set $\mathcal{C}$ of consequences.
- A preference relation $\succeq$ defined over actions $\mathcal{A}$.

Classical Decision Theory: von Neumann-Morgenstern Theorem

The Von Neumann and Morgenstern (1944) Theorem:

Theorem

Rationally choosing in an uncertain environment with **known** *probabilities is equivalent to maximizing expected utility.*

Classical Decision Theory: Savage's Theorem

Savage (1954) relaxes the assumption of knowing the probabilities of events.

Theorem

Rationally choosing in an uncertain environment with **unknown** *probabilities is equivalent to maximizing expected utility with respect to a subjective probability measure.*

The subjective probability measure is the degree of certainty the decision maker has on the ocurrence of an uncertain event.

Causation

The manipulationist notion of Causation

If correlation is not causation, then what is causation?

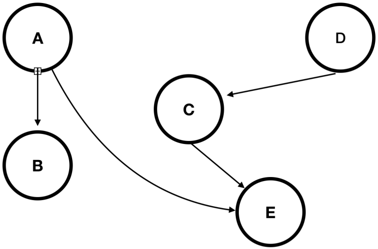
- We consider *probabilistic* theories of Causation.
- In particular, the **manipulationist** notion.
- We say that A causes B if manipulating A results in a change in B.
- Further details in Woodward (2003); Pearl (2009).

Spirtes' definition of Causation

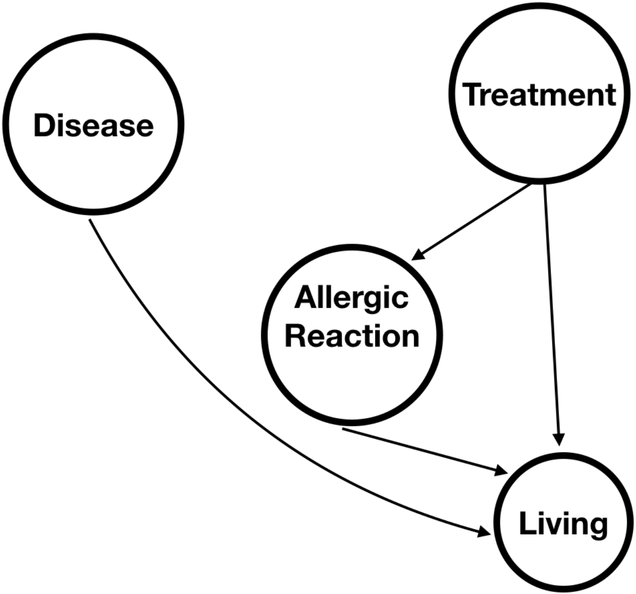
We use Spirtes et al. (2000) definition of Causation as binary relation defined over *events* in a probability space which satisfy:

- An event A does not cause itself.
- If an event A causes an event B, then B can not cause A.
- If A causes B and B causes C, then A causes C.

We can represent the list of causal relations in a Directed Acyclic Graph (DAG).



An example of a Causal Graphical Model



Interventions

- An intervention upon a variable consists of forcing a value upon such variable.
- On Causal Graphical Models we have the *do*() operator described in Pearl (2009).

A basic fact

$$P(Y|X) \neq P(Y|do(X = x)).$$

It is not the same asking the probability of rain last night given the floor is wet, than given that I manually wet the floor with a hose.

Causal Decision Making

We are now going to consider a decision maker in a *causal environment*; this is, actions and consequences will be defined as causally connected.

Causal Decision Problems

A Causal Decision Problem is composed of:

- A set of available actions $\mathcal{A}$.
- A set of uncertain events $\mathcal{E}$.
- A set of possible consequences $\mathcal{C}$.
- A preference relation $\succeq$.
- A Causal Graphical Model $\mathcal{G}$

$\mathcal{G}$ must satisfy some conditions: there exists one variable, with no parents, whose possible values correspond to the elements of $\mathcal{A}$; one variable for $\mathcal{C}$ and one variable for each uncertain event $E \in \mathcal{E}$.

von Neumann-Morgenstern Theorem for Causal Decision Making

Consider a *known* Causal Model G and its associated distribution P_G and let C the set of consequences of interest for a decision maker. Then,

Theorem

(Pearl (2009)) If a rational decision maker faces a Causal Environment and if the causal model is known, then the preference relation $\succeq$ is rational if and only if there exists a function u such that:

$$a \succeq b \textit{ if and only if } \sum_{c \in C} P(c|do(a))u(c) \geq \sum_{cinC} P(c|do(b))u(c). \quad (1)$$

Equivalently, the action that must be chosen is

$$a^* = \operatorname{argmax}_{a \in \mathcal{A}} \sum_{c \in C} P(c|do(a))u(c).$$

When the causal model is not known

Basic idea:

- We want to choose the action that causes a desired consequence with high probability.
- If the causal model is not known, the decision maker will use probabilistic beliefs about causal structures.

Savage Theorem for Causal Decision Making

Theorem

In a Causal Decision Problem $(\mathcal{A}, \mathcal{G}, \mathcal{E}, \mathcal{C}, \succeq)$*, the preferences* $\succeq$ *of a decision maker are rational if and only if there exists a probability distribution* P_C *over a family* $\mathcal{F}$ *of causal structures such that*

$$a \succeq b \text{ iff}$$

$$\sum_{c \in \mathcal{C}} u(c) \left(\sum_{g \in \mathcal{F}} P_g(c|do(a)) P_C(g) \right) \geq \sum_{c \in \mathcal{C}} u(c) \left(\sum_{g \in \mathcal{F}} P_g(c|do(b)) P_C(g) \right)$$

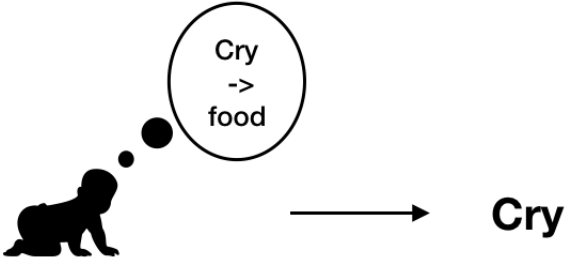
where P_g *a the probability distribution associated with the causal structure* g*.*

Proof.

Proof in `https://arxiv.org/abs/1907.11752` □

Interpretation

The previous result tells us that a decision maker who does not know the causal model which controls her environment must use a subjective probability distribution over causal structures and then use Pearl's result *within* each structure *as if* it were the true one.



Applications

Applications

- Optimal action learning in causal contexts.
- Causal Games and Causal Nash Equilibrium.

Optimal action learning in causal contexts

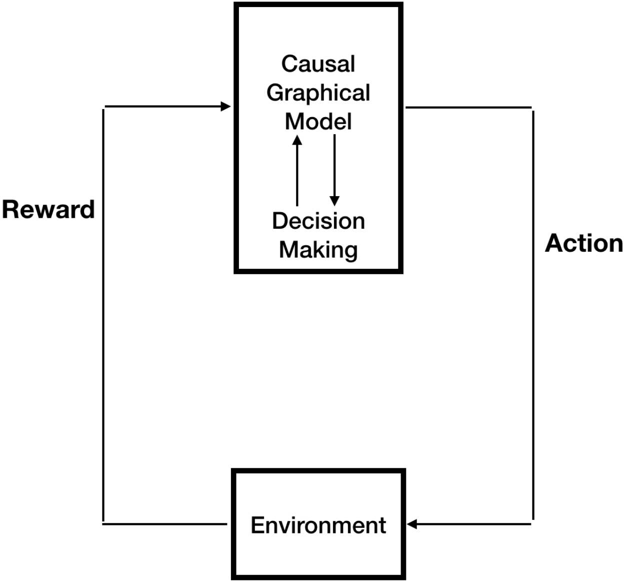
Consider the problem of finding the optimal action a^* when actions *cause* consequences. This problem is considered by:

- Ortega and Braun (2014) who use Thompson Sampling for Causal Inference
- Lattimore et al. (2016) who require to know the conditionals probabilities of causal model; this work minimizes regret which is equivalent to maximizing expected utility.
- Sen et al. (2017) allows partially known causal model.
- Gonzalez-Soto et al. (2018) considers as given the *structure* of causal model; i.e., P_C assigns probability mass only to models with a given structure.

Theorem 4 provides a unified framework in which the previous works fit.

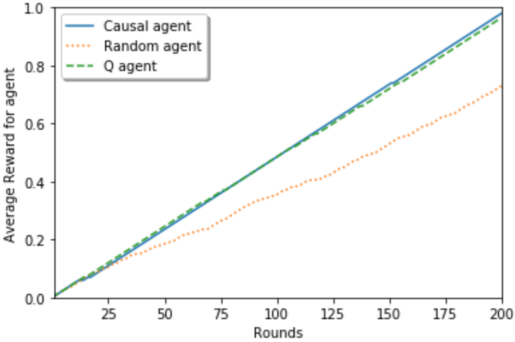
Our learning procedure

- In our work, an agent draws a probability vector from a Dirichlet distribution.
- Uses such vector to form a CGM with given structure.
- Uses it *as if* it was the true one, as in our Theorem, and selects best action.
- Updates probabilities.


Causal Graphical Model
Decision Making
Reward
Action
Environment

Experimental Results

We achieved similar performance to the classical, non-causal, Q-Learning:



Causal Games and Causal Nash Equilibrium

Causal Games and Causal Nash Equilibrium

Consider a *strategic game* in which every player is located at a causal environment.

- Using the notion of a Bayesian Game in Gonzalez-Soto et al. (2019) we defined a probability updating for a player who doesn't know the causal model the controls her environment.
- In a Bayesian Game, players do not know neither the actions made by other players nor the *information* that made them take an action.
- We consider an unknown causal model to be such information.
- Using such probability updating, we were able to define a Causal Nash Equilibrium for one-shot games in causal environments.

Causal Nash Equilibrium

- For each player $i \in N$ in the strategic game, we define the following probability distribution over consequences:

$$p_i^a(c) = p_i^\omega(c|do(a_i), a_{-i})p_i(\omega) \text{ for } a \in A = A_1 \times \cdots \times A_N. \tag{2}$$

- We now define:

$$u_i^C(a) = \sum_{c \in C} u_i(c)p_i^a(c) \text{ for } a \in A = A_1 \times \cdots \times A_N. \tag{3}$$

Definition

An an action profile $a^* \in A$ is a Nash equilibrium for this *causal strategic game* if and only if

$$u_i^C(a^*) \geq u_i^C(a_i, a_{-i}^*) \text{ for any other } a_i \in A_i. \tag{4}$$

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