COSC 444/541: Automata

Lecture 4: DFAs, NFAs

Today’s Agenda

- Continue DFA (Chapter 2.1)
- Break at 1:50 PM
- Start NFA (Chapter 2.2)
- Early Semester Check-In Survey

DFA Definition

DEFINITION 2.1

A **deterministic finite accepter** or **dfa** is defined by the quintuple $M = (Q, \Sigma, \delta, q_0, F)$,

where

Q is a finite set of **internal states**, Σ is a finite set of symbols called the **input alphabet**, $\delta : Q \times \Sigma \to Q$ is a total function called the **transition function**, $q_0 \in Q$ is the **initial state**, $F \subseteq Q$ is a set of **final states**.

Q is the set of states. There are only finitely many.

Σ is the input alphabet. The set of symbols the machine can read.

δ is the transition function.

q_0 is the initial state.

F is the set of final or accepting states.

DFAs

In-class Exercise: 3(d) left from last lecture

[Textbook page 48, exercise 3]

3. For $\Sigma = \{a, b\}$, construct dfa's that accept the sets consisting of

   (a) all strings of even length.

   (b) all strings of length greater than 5.

   (c) all strings with an even number of a's.

   (d) all strings with an even number of a's and an odd number of b's.

Regular Language

- Every finite automaton accepts some language.
- The set of all such languages forms a family of languages.
- We call this family regular languages.
- To show that any language is regular, we find a dfa for it.

DEFINITION 2.3

A language L is called **regular** if and only if there exists some deterministic finite accepter M such that

$$L = L(M).$$

Regular Language

Example 2.5 in textbook:

Show that the language $L = \{awa : w \in \{a,b\}^*\}$ is regular.

Regular Language

Example 2.5:

Show that the language

$L = \{awa: w \in \{a,b\}^*\}$

is regular.

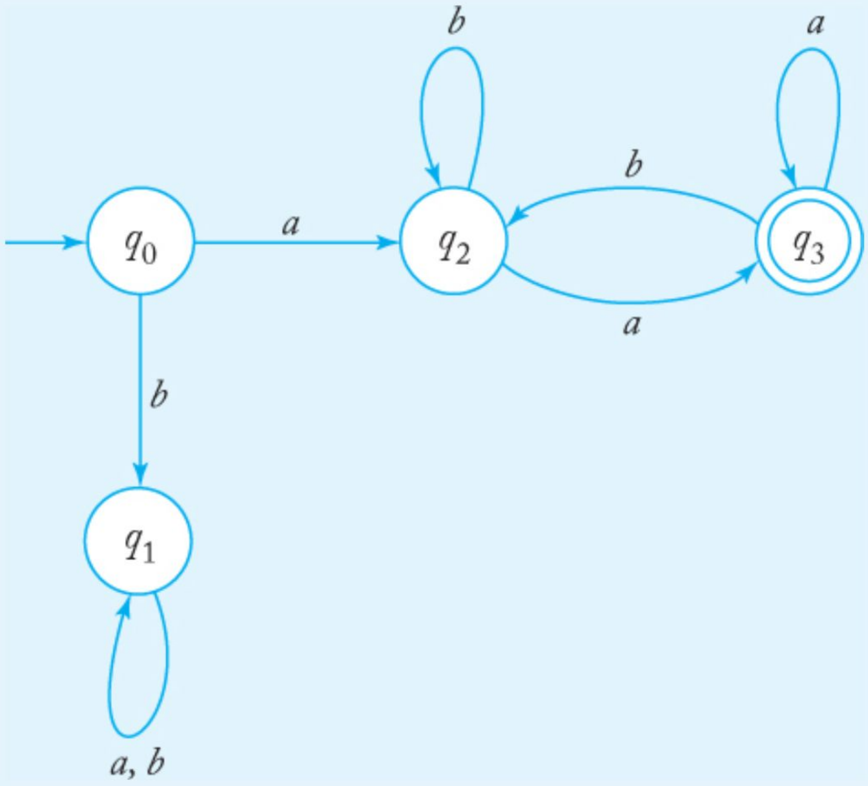


FIGURE 2.6

Regular Language

Example 2.5:

Show that the language

$L = \{awa: w \in \{a,b\}^*\}$

is regular.

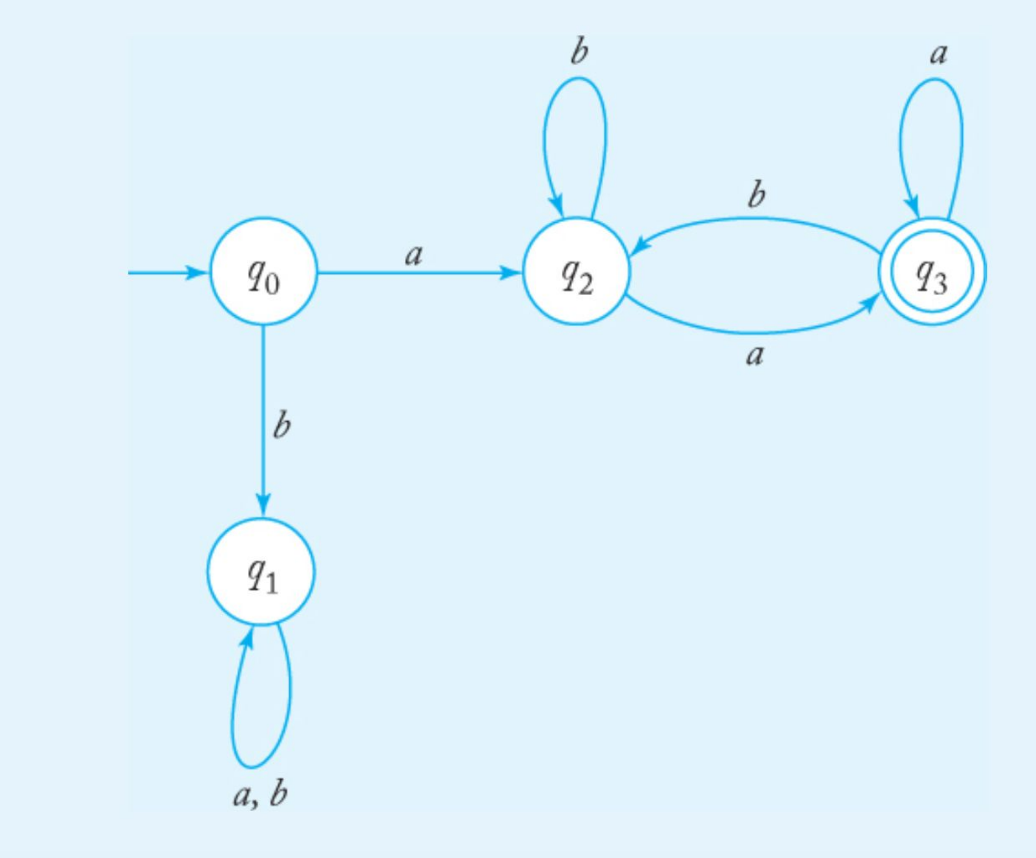


FIGURE 2.6

Trace:

aba

abaaaaba

abbab

abb

Regular Language

Example 2.6 in textbook:

Let $L = \{awa : w \in \{a,b\}^*\}$. Show that L^2 is regular.

Regular Language

Example 2.6 in textbook:

Let $L = \{awa : w \in \{a,b\}^*\}$. Show that L^2 is regular.

$$L^2 = \{aw_1aaw_2a : w_1, w_2 \in \{a,b\}^*\}$$

Regular Language

Example 2.6 in textbook: Show that $L^2=\{aw_1aaw_2a : w_1,w_2\in\{a,b\}^*\}$ is regular.

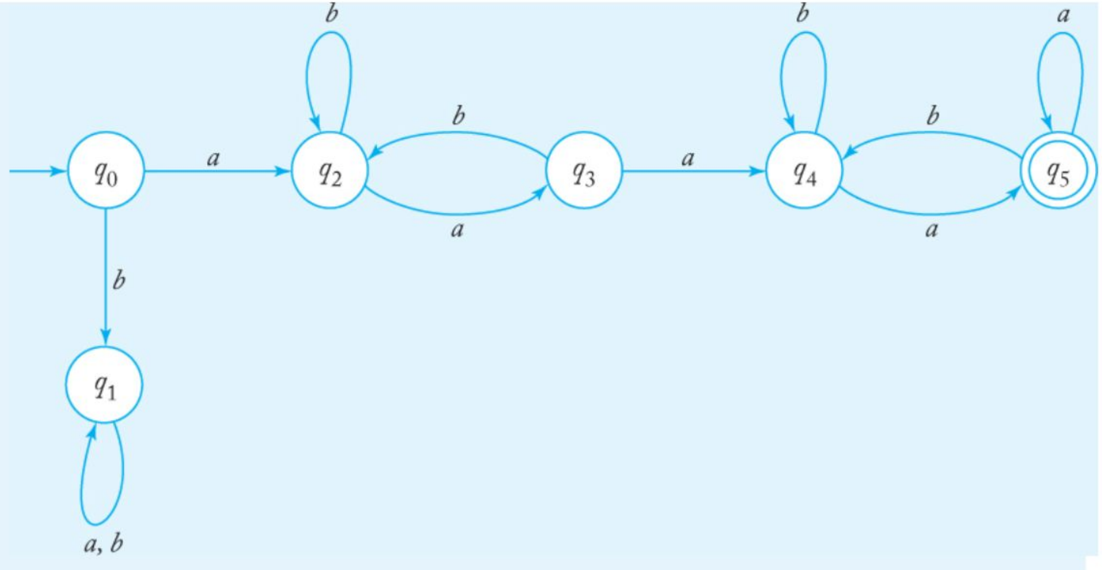


FIGURE 2.7

Regular Language

In-Class Example 1:

[Textbook page 50, exercise 13]

13. Show that the language $L = \{vwv : v, w \in \{a, b\}^*, |v| = 3\}$ is regular.

Regular Language

In-Class Example 2:

[Textbook page 50, exercise 14]

14. Show that $L = \{a^n : n \geq 3\}$ is regular.

Regular Language

In-Class Example 3:

[Textbook page 50, exercise 16]

16. Show that the language $L = \{a^n : n \text{ is either a multiple of three or a multiple of } 5\}$ is regular.

Regular Language

In-Class Exercise 1:

[Textbook page 50, exercise 15]

15. Show that the language $L = \{a^n : n \geq 0, n \neq 3\}$ is regular.

Regular Language

In-Class Exercise 2:

[Textbook page 50, exercise 17]

17. Show that the language $L = \{a^n : n \text{ is a multiple of three, but not a multiple of } 5\}$ is regular.

NFAs

What is Nondeterminism?

- Deterministic FA: one move in each situation.
- Nondeterministic FA (NFA): may have multiple possible moves.
  - Transition function returns a set of states.

NFAs

Why Nondeterminism?

- Simpler and more concise representation
    - Fewer states
    - Repetitions are easy to express

NFAs and DFAs recognize the same class of languages (regular).

NFA Definition

DEFINITION 2.4

A **nondeterministic finite accepter** or **nfa** is defined by the quintuple

$$M = (Q, \Sigma, \delta, q_0, F),$$

where Q, Σ, q_0, F are defined as for deterministic finite accepters, but

$$\delta : Q \times (\Sigma \cup \{\lambda\}) \to 2^Q.$$

Range of function is a subset of Q.

λ: nfa can make a transition without consuming an input.

NFA Example

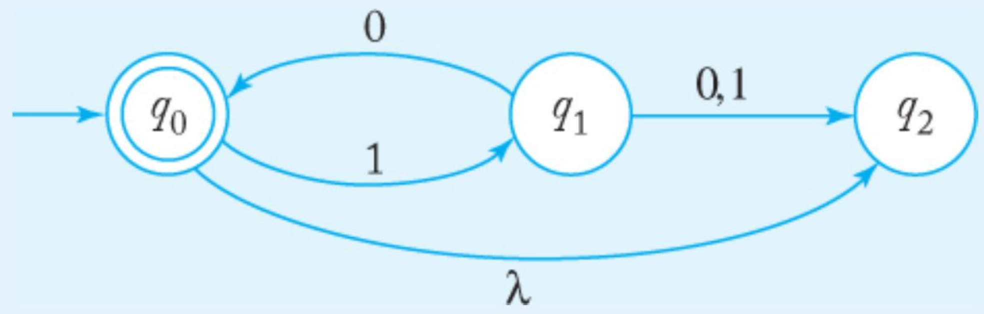


FIGURE 2.9

If any walk on the graph labeled with the input ends in a final state, the string is accepted.

The string 10 is accepted, but 101 is not.

NFA Language Definition

DEFINITION 2.6

The language L accepted by an nfa $M = (Q, \Sigma, \delta, q_0, F)$ is defined as the set of all strings accepted in the above sense. Formally,

$$L(M) = \{w \in \Sigma^* : \delta^*(q_0, w) \cap F \neq \varnothing\}.$$

In words, the language consists of all strings w for which there is a walk labeled w from the initial vertex of the transition graph to some final vertex.

NFA Example

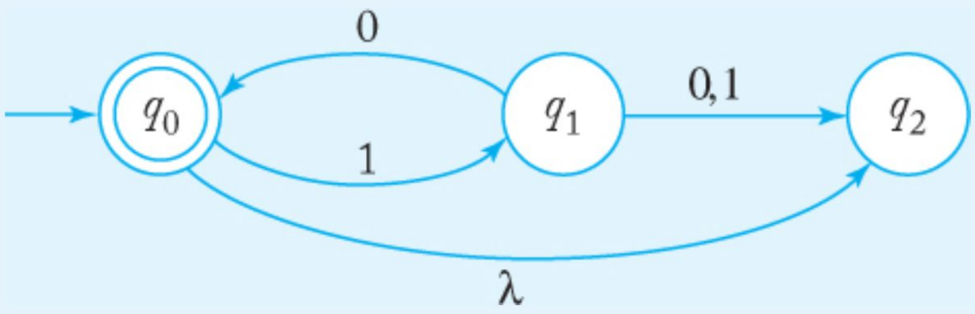


FIGURE 2.9

$$L = \{(10)^n : n \geq 0\}$$

NFA

In-Class Example 1: [Textbook page 57, exercise 3]

3. Find a dfa that accepts the language defined by the nfa in Figure 2.8.

EXAMPLE 2.7

Consider the transition graph in Figure 2.8. It describes a nondeterministic accepter since there are two transitions labeled a out of q_0.

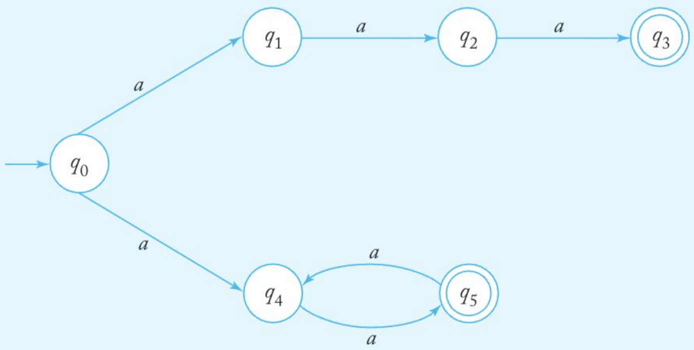


FIGURE 2.8

NFA

In-Class Example 2: [Textbook page 57, exercise 6]

6. In Figure 2.10, find $\delta^*(q_0, a)$ and $\delta^*(q_1, \lambda)$.

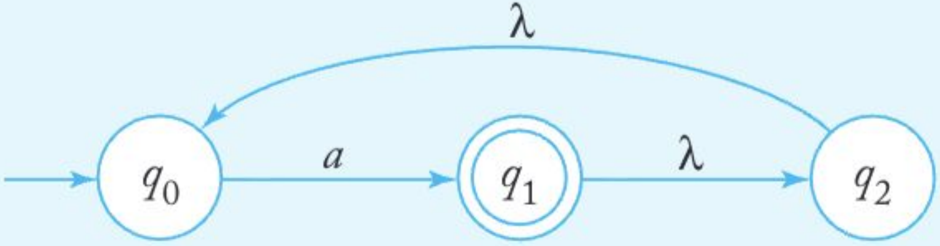
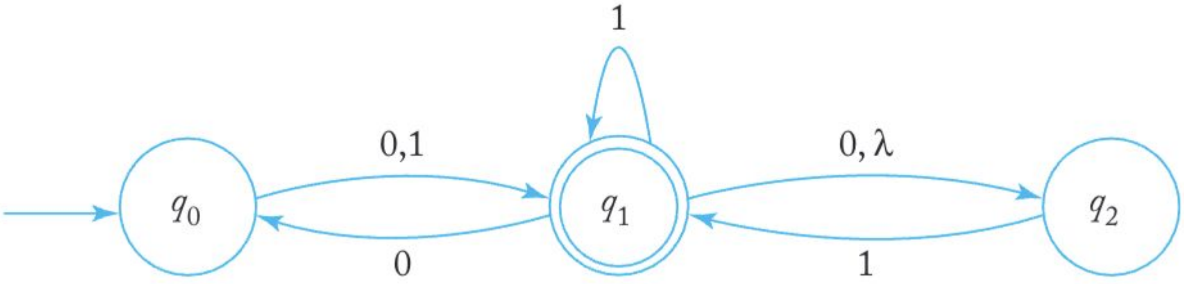


FIGURE 2.10

NFA

In-Class Example 3: [Textbook page 57, exercise 13]

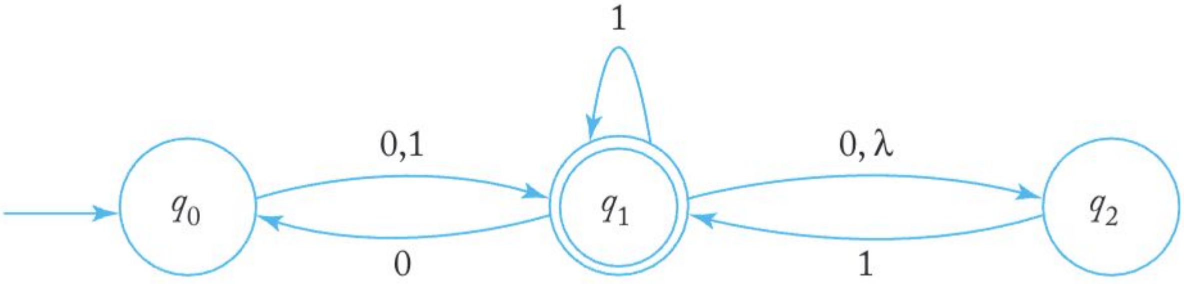
13. Which of the strings 00, 01001, 10010, 000, 0000 are accepted by the following nfa?



NFA

In-Class Example 3: [Textbook page 57, exercise 13]

13. Which of the strings 00, 01001, 10010, 000, 0000 are accepted by the following nfa?



01001 and 000

Today’s Takeaways

DFAs

NFAs

Regular language

HW 1 posted on Canvas

Due Tuesday 9/16/2025 at the start of class.

Answer on paper and submit before class.

Early Semester Check-In Survey

Complete the anonymous survey at:

https://forms.gle/zyP5DSsbP5BCET86A