Linear Discriminant Analysis:-

C_1 $(X_1, X_2) = \{(4,1), (2,4), (2,3), (3,6), (4,4)\}$

C_2 $(X_1, X_2 = \{(9,10), (\cancel{1}6,8), (9,5), (8,7), (10,8)\}$

Theory - finds the axes that maximizes the separatn between multiple classes.

PCA - maximise the variance in the given dataset.

LDA - aims to find the linear discriminants to represent the axes that maximise separation between diff. classes of data.

Two criteria used by LDA -

1. Maximise the distance between means of the two classes. → include as many similar points as possible in one class
2. Minimize the variation within each class. → mean as far as possible

ep1 :- Compute within-class Scatter Matrix.

(within class)

$$S_W = S_1 + S_2$$

S_2 — class C_2

S_1 — Co-variance Matrix of Class C1

$$S_1 = \sum_{x \in C_1} (x - \mu_1)(x - \mu_1)^T$$

$$\mu_1 = \left\{ \frac{4+2+2+3+4}{5} , \frac{1+4+3+6+4}{5} \right\}$$

$$\mu_1 = \begin{bmatrix} 3 & 3.6 \end{bmatrix}$$

$$\mu_2 = \begin{bmatrix} 8.4 & 7.6 \end{bmatrix}$$

$$\mu_1 = \begin{bmatrix} 3 \\ 3.6 \end{bmatrix} \quad \mu_2 = \begin{bmatrix} 8.4 \\ 7.6 \end{bmatrix}$$

$$(x - \mu_1) = \begin{bmatrix} 4-3 & 2-3 & 2-3 & 3-3 & 4-3 \\ 1-3.6 & 4-3.6 & 3-3.6 & 6-3.6 & 4-3.6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -1 & 0 & 1 \\ -2.6 & 0.4 & -0.6 & 2.4 & 0.4 \end{bmatrix}$$

$$S_1 = (x_1 - \mu_1)(x_1 - \mu_1)^T + (x_2 - \mu_1)(x_2 - \mu_1)^T \ldots\ldots$$

$$\begin{bmatrix} 1 \\ -2.6 \end{bmatrix} \begin{bmatrix} 1 & -2.6 \end{bmatrix} = \begin{bmatrix} 1 & -2.6 \\ -2.6 & 6.76 \end{bmatrix} \quad \text{—— (1)}$$

$$\begin{bmatrix} -1 \\ 0.4 \end{bmatrix} \begin{bmatrix} -1 & 0.4 \end{bmatrix} = \begin{bmatrix} 1 & -0.4 \\ -0.4 & 0.16 \end{bmatrix} \quad \text{— (2)}$$

$$\begin{bmatrix} -1 \\ -0.6 \end{bmatrix} \begin{bmatrix} -1 & -0.6 \end{bmatrix} = \begin{bmatrix} 1 & 0.6 \\ 0.6 & 0.36 \end{bmatrix} \quad \text{— (3)}$$

$$\begin{bmatrix} 0 \\ 2.4 \end{bmatrix} \begin{bmatrix} 0 & 2.4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5.76 \end{bmatrix} \quad \text{— (4)}$$

$$\begin{bmatrix} 1 \\ 0.4 \end{bmatrix} \begin{bmatrix} 1 & 0.4 \end{bmatrix} = \begin{bmatrix} 1 & 0.4 \\ 0.4 & 0.16 \end{bmatrix} \quad \text{— (5)}$$

S_1 = summation of (1), (2), (3), (4), 5 & the average it to get covariance matrix S_1

$4/5 = 0.8$

$$S_1 = \begin{bmatrix} 0.8 & -0.4 \\ -0.4 & 2.6 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 1.84 & -0.04 \\ -0.04 & 2.64 \end{bmatrix}$$

$$S_W = S_1 + S_2 = \begin{bmatrix} 0.8 & -0.4 \\ -0.4 & 26 \end{bmatrix} \begin{bmatrix} 1.84 & -0.04 \\ -0.04 & 2.64 \end{bmatrix}$$

$$S_W = \begin{bmatrix} 2.64 & -0.44 \\ -0.44 & 5.28 \end{bmatrix}$$

(3)

ep 2 :- Compute Between class Scatter matrix → scattered across the class

$$S_B = (\mu_1 - \mu_2)(\mu_1 - \mu_2)^T$$

$$\mu_1 - \mu_2 = \begin{bmatrix} 3 - 8.4 \\ 3.6 - 7.6 \end{bmatrix} = \begin{bmatrix} -5.4 \\ -4 \end{bmatrix}$$

$$S_B = \begin{bmatrix} -5.4 \\ -4 \end{bmatrix} \begin{bmatrix} -5.4 & -4 \end{bmatrix} = \begin{bmatrix} 29.16 & 21.6 \\ 21.6 & 16.00 \end{bmatrix}$$

Step 3 :- Find best LDA projection Vector (eigen vector having largest eigen value)

inverse

$$|S_W^{-1} S_B - \lambda I| = 0$$

$$\begin{vmatrix} 11.89 - \lambda & 8.81 \\ 5.08 & 3.76 - \lambda \end{vmatrix} = 0 \qquad \lambda_1 = 15.65$$

Eigen Vector V $\quad S_W^{-1} S_B V = \lambda V$

$$\begin{bmatrix} 11.89 & 8.81 \\ 5.08 & 3.76 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 15.65 \cancel{\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0.91 \\ 0.39 \end{bmatrix}$$

S_W^{-1} is found by using the foomula -

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$S_W = \begin{bmatrix} 2.64 & -0.44 \\ 0.44 & 5.28 \end{bmatrix}$$

$$S_W^{1} = \frac{1}{13.74}\begin{bmatrix} 5.28 & 0.44 \\ 0.44 & 2.64 \end{bmatrix}$$

$$= \begin{bmatrix} 0.384 & 0.032 \\ 0.032 & 0.192 \end{bmatrix}$$

Step 4 : Dimesion Reduction.

$$Y = W^T X$$

i/p data samples

Projection Vector.