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Comparison of reliability and geometrical strength criteria in geodetic networks

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Abstract. The proper and optimal design and subsequent assessment of geodetic networks is an integral part of most surveying engineering projects. Optimization and design are carried out before the measurements are actually made. A geodetic network is designed and optimized in terms of high reliability and the results are compared with those obtained by the robustness analysis technique. The purpose of an optimal design is to solve for both the network configuration (first-order design) and observations accuracy (second-order design) in order to meet the desired criteria. For this purpose, an analytical method is presented for performing the first-order design, second-order design, and/or the combined design. In order to evaluate the geometrical strength of a geodetic network, the results of robustness analysis are displayed in terms of robustness in rotation, robustness in shear, and robustness in scale. Results showed that the robustness parameters were affected by redundancy numbers. The largest robustness parameters were due to the observations with minimum redundancy numbers.

Key words: First-order Design – Second-order Design – Combined Design – Reliability Criterion – Robustness Analysis

1 Introduction

This section deals with the types of problems most widely encountered in the design of a geodetic network. Optimization means minimizing or maximizing an objective function which represents the criteria adopted to define the 'quality of a network'. Generally, the quality of a control geodetic network is characterized by its precision, reliability and strength, and economy. The reliability of a geodetic network, which was proposed by Baarda (1968), can be understood as the ability of the network to detect gross errors in the observations and to be resistant against undetectable errors. The geometrical strength analysis (robustness analysis) of a geodetic network which was proposed by Vaníček et al. (1990) deals with the strain and is another aspect of reliability criteria. The main purpose of the present paper is to design and optimize a geodetic network in the sense of high reliability and compare the results with the robustness analysis. Different optimization problems are usually classified into different orders; Grafarend (1974) identifies four orders of design:

(a) zero-order design (ZOD): design of a reference system
(b) first-order design (FOD): design of the network configuration
(c) second-order design (SOD): selection of the weights for the network observations
(d) third-order design (THOD): addition of observations to improve an existing network.

When the FOD and SOD design problems are solved simultaneously, it is called a combined design (COMD) problem (Vaníček and Krakiwsky 1986).

There are two methods that can be used to solve the design problems, namely, analytical method and trial and error method. In the trial and error method, a solution to the design problem is postulated upon which the design criteria are computed. Should either of these criteria be not fulfilled, a new solution (usually by slightly altering the original postulate) is postulated and the criteria functions are recomputed. The procedure is repeated until a satisfactory network is found. For more information, the interested reader is referred to Cross (1985). In contrast, the so-called analytical method offer specific algorithms for the solution of particular design problems which do not require human intervention. The term analytical design is used to describe a method that solves a particular design problem by a unique series of mathematical steps. In fact, such an algorithm will automatically produce a network that will satisfy the user quality requirements and will be optimum in some mathematical sense.

Kuang (1991) developed an effective analytical method for optimization and design of deformation monitoring schemes which can be used for FOD, SOD, and/or COMD. In this method, the Taylor series expansion is introduced to linearize non-linear matrix functions related to the design criteria of the deformation monitoring network. Then, it is possible to perform either separate (e.g. FOD or SOD) or simultaneous (e.g. COMD) fully analytical optimal solution of the network configuration and the observational plan using the methodology of operations research (e.g. linear programming). This method was used in the optimization and design of geodetic networks in the sense of high reliability by Amiri Seemkooei (1998). More details are presented in Sect. 4. The main objectives of the present paper are as follows:

(a) to define the optimality criteria for the reliability and geometrical strength of a geodetic network;
(b) to formulate optimization mathematical models in the sense of high reliability for FOD, SOD, and/or COMD;
(c) to optimize a simulated geodetic network in the sense of high reliability and compare the results with robustness analysis (geometrical strength analysis).

2 Measures and criteria for reliability

Generally, the reliability of a network can be understood as the ability of the network to detect and resist against gross errors in the observations. In this respect, the internal reliability and external reliability are usually distinguished. The former refers to the ability of the network to allow for the detection of blunders by hypothesis testing, while the latter is related to the effect of the undetected gross errors on the estimated parameters. Reliability of a network depends on the geometry of the network and the accuracy of the observations. At the design stage, we look for an optimal network in the sense of high reliability and strength, which minimizes the magnitude of undetectable gross errors in the observations, and consequently minimizes the effects of the undetected errors on the estimated parameters.

As mentioned above, internal reliability is a measure of a network quality in detecting outliers in observations by the one-dimensional outlier test. It refers to the lower bounds of detectable gross errors which can be derived by the following equation (Baarda 1968):

$$\nabla_0 l_i = \frac{\delta_0 \sigma_{l_i}}{\sqrt{r_i}} \tag{1}$$

where δ_0 is the lower bound for the non-centrality parameter and is a function of type I and II errors, and σ_{l_i} and r_i are the standard deviation and redundancy number of the ith observation, respectively. The redundancy numbers of the observations are the diagonal elements of the matrix $\boldsymbol{R}$

$$\boldsymbol{R} = \boldsymbol{I} - \boldsymbol{A}(\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{P} \tag{2}$$

where $\boldsymbol{I}$ is the identity matrix, $\boldsymbol{A}$ is the design (configuration) matrix, and $\boldsymbol{P}$ is the weight matrix of the observations.

Another reliability measure that Baarda developed in 1968 is called the external reliability. It refers to the maximum effect of the undetectable gross error ($\nabla_0 l_i$) on the estimates of unknown parameters. The influence of the maximum undetectable error $\nabla_0 l_i$ on coordinates (i.e. $\nabla_{0,i}\hat{\boldsymbol{x}}$) is given by

$$\nabla_{0,i}\hat{\boldsymbol{x}} = (\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{P}\nabla_{0,i}l \tag{3}$$

From the above discussion, it can be seen that a geodetic network should be designed such that:

(a) gross errors should be detected and eliminated as completely as possible. An undetectable gross error in an observation should be small in comparison with its standard deviation; and
(b) the effect of an undetected error on the coordinates should be as small as possible.

From Eqs. (1) and (3) we can see that the larger the redundancy number r_i, the smaller the size of the undetectable gross error as well as its influence on the estimated coordinates. Baarda (1968) argues that it is desirable to have an approximately constant value for all r_i so that the ability of detecting gross errors will be the same in every part of the network. With this in mind, a special reliability criterion can be of the type

$$\min(r_i) \rightarrow \max \tag{4}$$

This is known as the general criterion for the internal and external reliability.

3 Robustness analysis

The effect of errors on the network is better handled as a virtual deformation and thus depicted by a more appropriate technique than the external reliability. This method, which is known as 'robustness analysis', is based on the concept of strain. The use of strain to analyze the strength of a geodetic network was first proposed by Vaníček et al. (1981), later developed by Dare (1983), and finally culminated in 1990 with Vaníček et al. (1990).

Strain is a purely geometric approach to the analysis of the deformation of a physical body. It is defined as the rate of change (i.e. gradient) of an object's displacement field with respect to position. Given a two-dimensional displacement field $\mathbf{u}(x,y) = (u,v)^T$, as a function of position $\mathbf{x} = (x,y)^T$, the strain matrix $\boldsymbol{E}$ consists of four linear displacement gradients given by (Vaníček et al. 1990)

$$\boldsymbol{E} = \nabla(\mathbf{u}) = \frac{\partial \mathbf{u}(x,y)}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix} = \begin{pmatrix} e_{ux} & e_{uy} \\ e_{vx} & e_{vy} \end{pmatrix} \tag{5}$$

where the derivatives are evaluated at the point of interest. It contains all of the strain information about the displacement field. However, it cannot be easily interpreted. Various scalar parameters can be derived from the strain matrix in order to make the interpretation of strain more convenient and illustrative. These parameters are called deformation primitives.

The strain matrix $\boldsymbol{E}$ can be decomposed into its symmetric and anti-symmetric parts. The symmetric strain tensor describes the expansion and contraction as well as the shearing deformation at a point, while the anti-symmetric strain matrix describes the twisting deformation at a point. Some convenient scalar deformation primitives for robustness analysis can be derived from the strain matrices. Dilation σ describes the average extension or contraction at a point and is defined as follows (Schneider 1982):

$$\sigma = \frac{e_{ux} + e_{vy}}{2} \tag{6}$$

Quantity ω_z is known as the average differential rotation. It describes the twisting about the local vertical axis at a point and is defined as

$$\omega_z = \frac{e_{uy} - e_{vx}}{2} \tag{7}$$

There is another scalar deformation primitive for robustness analysis which is known as total shear γ. This is the geometric mean of the components of pure (τ) and simple (v) shear, i.e.

$$\gamma_{xy} = \sqrt{\tau_{xy}^2 + v_{xy}^2} \tag{8}$$

where

$$\tau_{xy} = -\tau_{yx} = \tfrac{1}{2}(e_{ux} - e_{vy}) \tag{9}$$

and

$$v_{xy} = -v_{yx} = \tfrac{1}{2}(e_{uy} + e_{vx}) \tag{10}$$

Robustness analysis uses Baarda's external reliability criterion as the local displacement field. The gradients of the local displacement field are evaluated separately for each of the coordinates x and y. A separate local displacement field is determined for each coordinate component and the gradients along each of the coordinate axes are evaluated to give the components of the strain matrix. Fitting a plane to each displacement field results in a very simple determination of the strain; the strain components are just the slopes of the planes along each of the coordinates axes. The local displacement field components u and v are approximated as follows [Vaníček et al. 1990]:

$$\begin{aligned} u &= a_0 + a_1 x + a_2 y \\ v &= b_0 + b_1 x + b_2 y \end{aligned} \tag{11}$$

where x and y are the coordinate components of the points in the local displacement field, and as and bs are the coefficients defining the planes. For numerical stability, these coordinates are expressed relative to the point of interest. Solving for the coefficients in both sets of equations results in

$$\begin{aligned} [a_0 \quad a_1 \quad a_2]^T &= (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\mathbf{u} \\ [b_0 \quad b_1 \quad b_2]^T &= (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\mathbf{v} \end{aligned} \tag{12}$$

where $\boldsymbol{A} = [\mathbf{1}\,\mathbf{x}\,\mathbf{y}]$ with $\mathbf{1}$ being a column of ones. The strain elements are then

$$\begin{aligned} e_{ux} &= a_1, \quad e_{uy} = a_2 \\ e_{vx} &= b_1, \quad e_{vy} = b_2 \end{aligned} \tag{13}$$

We are now interested in only the largest deformations at each point as measured by the deformation primitives: dilation σ, total shear γ, and differential rotation ω. New deformation primitives are computed one at a time for a change in each observation by its upper bound for undetectable gross errors. Only the largest primitives (in absolute value) at each point are retained as a measure of the weakest link. These maximum values (denoted by $\sigma_{\max}$, $\gamma_{\max}$, $\omega_{\max}$) at each point in the network describe the network strength and are referred to as strength in scale, strength in shear, and strength in rotation (twist), respectively.

4 Optimization of a geodetic network in the sense of high reliability

Traditionally, the internal reliability criterion is used as a measure for the optimal design of geodetic networks in the high-reliability sense. The matrix $\boldsymbol{R}$ which contains the redundancy numbers on its main diagonal has the following form:

$$\boldsymbol{R} = \boldsymbol{I} - \boldsymbol{A}(\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{A} + \boldsymbol{D}\boldsymbol{D}^T)^{-1}\boldsymbol{A}^T\boldsymbol{P} \tag{14}$$

where $\boldsymbol{D}$ is the datum matrix. In the optimization of a geodetic network, the free parameters are the configuration matrix, $\boldsymbol{A}$ and $\boldsymbol{D}$, characterized by the positions of the network points, and the weight matrix $\boldsymbol{P}$, consisting of accuracies of geodetic observables. A solution for these parameters is optimal if it satisfies the optimal criterion adopted to define the quality of the geodetic network and it lies within the range defined by their physical properties, i.e. the parameters should be physically realizable. We can see that the matrix $\boldsymbol{R}$ consists of non-linear functions (because of $\boldsymbol{A}$, $\boldsymbol{D}$, and $\boldsymbol{P}$) of both the locations of points characterized by coordinates (x_i, y_i, $i = 1, 2, \ldots, m$) and observational weights (p_i, $i = 1, 2, \ldots, n$). In order to establish an explicit relation between the preset design criteria and the unknown parameters to be solved, this matrix are expanded to its linear term using Taylor series expansion. It is then possible to perform either separate (e.g. FOD or SOD) or simultaneous (e.g. COMD) fully analytical optimal solution of the network configuration and the observational plan using the methodology of operations research (e.g. linear programming). A general criterion for increasing the internal reliability can be stated as

$$\|r\| = \|\operatorname{vecdiag}(\boldsymbol{R})\| = \|(\boldsymbol{I}_n \otimes \boldsymbol{I}_n)^T \operatorname{vec}(\boldsymbol{R})\| \rightarrow \max \tag{15}$$

where I_n is an $n \times n$ identity matrix; 'vecdiag' denotes an operation which puts the diagonal elements of a quadratic matrix into a single vector; 'vec' is an operator obtained by stacking the columns of a quadratic matrix in a single column; $\otimes$ denotes the Khatri–Rao product, which for any two arbitrary matrices $\boldsymbol{B}_{k\times m}$ and $\boldsymbol{A}_{n\times m}$ (with the same number of columns) yields a matrix $\boldsymbol{C}_{k\times n\times m}$; and the redundancy numbers r_i (the elements of r) are defined as

$$r_i = (\boldsymbol{R})_{ii} = (\boldsymbol{I}_n - \boldsymbol{A}(\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{A} + \boldsymbol{D}\boldsymbol{D}^T)^{-1}\boldsymbol{A}^T\boldsymbol{P})_{ii}, \quad i = 1, 2, \ldots, n \tag{16}$$

As mentioned, the matrix $\boldsymbol{R}$ consists of non-linear functions of both point coordinates and observational weights. The Taylor series of $\boldsymbol{R}$ can be expressed as

$$\boldsymbol{R} = \boldsymbol{R}^0 + \sum_{i=1}^{m} \frac{\partial \boldsymbol{R}}{\partial x_i}\Delta x_i + \sum_{i=1}^{m} \frac{\partial \boldsymbol{R}}{\partial y_i}\Delta y_i + \sum_{i=1}^{n} \frac{\partial \boldsymbol{R}}{\partial p_i}\Delta p_i \tag{17}$$

where

$$\boldsymbol{R}^0 = [\boldsymbol{I}_n - \boldsymbol{A}(\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{A} + \boldsymbol{D}\boldsymbol{D}^T)^{-1}\boldsymbol{A}^T\boldsymbol{P}]|_{x^0,y^0,p^0} \tag{18}$$

All of the above partial derivatives are given in, for example, Amiri Seemkooei (1998), and they can be evaluated analytically. Substituting for $\boldsymbol{R}$ in Eq. (15) from Eq. (17) yields

$$\|r\| = \left\|(\boldsymbol{I}_n \otimes \boldsymbol{I}_n)^T \mathrm{vec}(\boldsymbol{R}^0 + \sum_{i=1}^{m} \frac{\partial \boldsymbol{R}}{\partial x_i}\Delta x_i + \sum_{i=1}^{m} \frac{\partial \boldsymbol{R}}{\partial y_i}\Delta y_i + \sum_{i=1}^{n} \frac{\partial \boldsymbol{R}}{\partial p_i}\Delta p_i)\right\| = \max \tag{19}$$

Denote

$$\mathbf{r_1} = \mathrm{vec}(\boldsymbol{R}^0) \tag{20}$$

and

$$\boldsymbol{R}_1 = \left[\mathrm{vec}\left(\frac{\partial \boldsymbol{R}}{\partial x_1}\right)\mathrm{vec}\left(\frac{\partial \boldsymbol{R}}{\partial y_1}\right)\ldots\left(\frac{\partial \boldsymbol{R}}{\partial x_m}\right)\mathrm{vec}\left(\frac{\partial \boldsymbol{R}}{\partial y_m}\right) \mathrm{vec}\left(\frac{\partial \boldsymbol{R}}{\partial p_1}\right)\ldots\mathrm{vec}\left(\frac{\partial \boldsymbol{R}}{\partial p_n}\right)\right] \tag{21}$$

After a few operations, and using the properties of the Khatri–Rao product, Eq. (19) reduces to

$$\|\boldsymbol{R}_{11}\Delta\mathbf{w} + \mathbf{r_{11}}\| = \max \tag{22}$$

where

$$\boldsymbol{R}_{11} = (\boldsymbol{I}_n \otimes \boldsymbol{I}_n)^T\boldsymbol{R}_1 \tag{23}$$

and

$$\mathbf{r_{11}} = (\boldsymbol{I}_n \otimes \boldsymbol{I}_n)^T\mathbf{r_1} \tag{24}$$

and

$$\Delta\mathbf{w} = (\Delta x_1\Delta y_1 \ldots \Delta x_m\Delta y_m\Delta p_1\Delta p_2 \ldots \Delta p_n)^T \tag{25}$$

As a typical case, we will use the L^∞-norm for the maximum reliability requirements. Under this consideration, the proposed mathematical model becomes

$$\mathbf{r_{min}} = \max \tag{26}$$

subject to

$$\boldsymbol{R}_{11}\Delta\mathbf{w} - \mathbf{r_{min}} \geq -\mathbf{r_{11}} \tag{27}$$

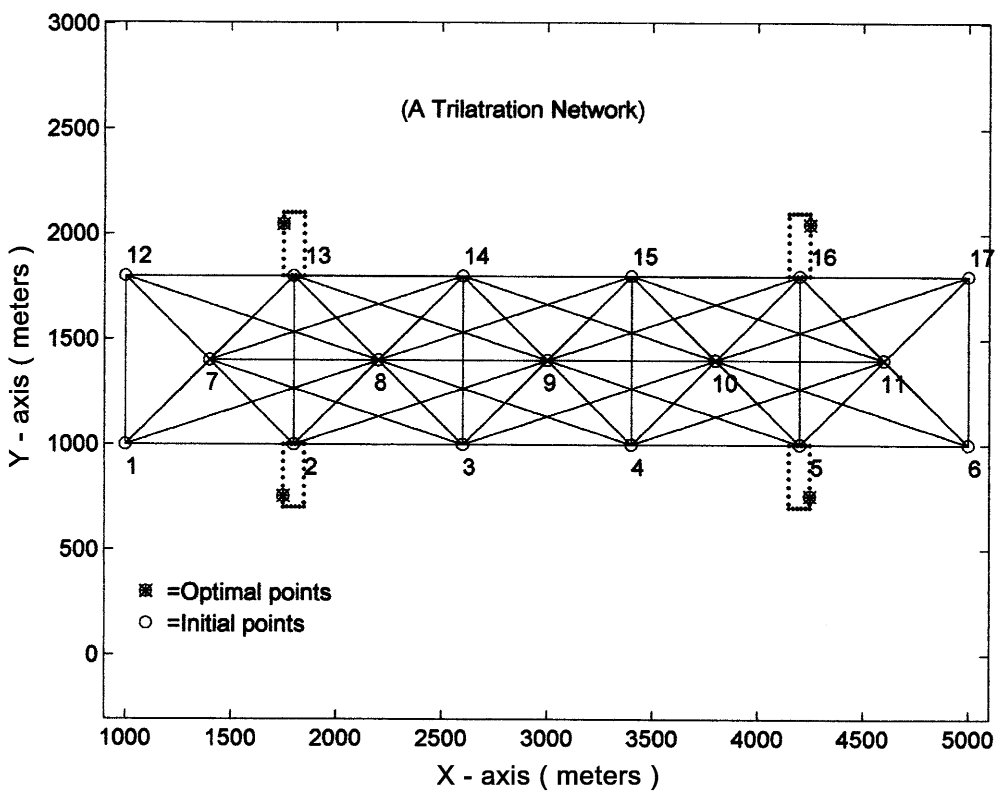

Fig. 1. Initial and optimum locations of the stations and the different types of distance observations for a simulated example

Besides the inequality constraints proposed in Eq. (27), there may be some other constraints due to datum consideration and intervisibility conditions (realizability). The interested reader is referred to, for example, Amiri Seemkooei (1998).

As is clear from the above discussion, the above mathematical model can be solved by linear programming. A computer program has been written to design a geodetic network to meet reliability criterion (i.e. a linear programming problem).

5 Numerical results

For the evaluation of reliability criteria and robustness analysis, only a trilatration network is assumed. Testing more than one example is beyond the scope of this paper. However, the obtained results were tested on several simulated and real geodetic networks by the author (see Amiri Seemkooei 1998). The present network consists of 17 points with 66 distance observations. Figure 1 illustrates the initial locations of the network stations. Table 1 gives the list of distance observations. The approximate accuracy of the distance observations is 5 mm, and it is assumed that the datum of the network is provided by inner constraints.

In order to compare the reliability criteria and robustness analysis, the geodetic network is evaluated before any optimization, after FOD, and after SOD. In this investigation, it is assumed that the parameters to be optimized are the positions of the netpoints 2, 5, 13, and 16, which can be changed in a 100×300 m rectangle (see Fig. 1). In the SOD problem, we assume that the parameters to be optimized are the standard deviation of the observations ranging from 3 to 5 mm for each observation. After implementing the computer program, the optimum positions of the netpoints and the optimum observational accuracy can be derived (see Fig. 1 and Table 1 respectively).

Table 2 summarizes the results for the reliability criterion and geometrical strength criterion (robustness parameters). This table shows the minimum redundancy numbers as well as the robustness parameters in different-order designs. As can be seen from Table 2, the minimum redundancy number increases from 0.1842 (initial value) to 0.3127 (after FOD), and then to 0.4541 (after SOD). All of these minimum redundancy numbers are belong to the distances 1–12 and 6–17 which are located on the perimeter of the network. This means that the perimeter observations are usually weak; this is because of the bad configuration of the network for outer observations. The same situation is valid for robustness parameters. Table 2 also shows the largest absolute values, the average, and the standard deviation of each robustness parameter of the network points. For example, the robustness parameter in rotation (maximum value) decreases from 10.95 (initial value) to 9.12 (after FOD), and then to 5.44 (after SOD). The same situation is valid for robustness parameters in shear and scale. These maximum robustness parameters are belong to the points 1, 6, 12, and 17. This indicates that, again, the

Table 1. Initial and optimal accuracy of the observations for a simulated network

Observation no.	Distances		Initial standard deviation (mm)	Optimal standard deviation (mm)
	From	To		
1	1	2	5	4.73
2	1	7	5	3.00
3	1	8	5	3.13
4	1	12	5	5.00
5	1	13	5	3.04
6	2	3	5	3.00
7	2	7	5	3.00
8	2	8	5	3.00
9	2	9	5	3.00
10	2	12	5	3.04
11	2	13	5	3.00
12	2	14	5	3.00
13	3	4	5	3.00
14	3	7	5	3.00
15	3	8	5	3.00
16	3	9	5	3.00
17	3	10	5	3.00
18	3	13	5	3.00
19	3	14	5	3.00
20	3	15	5	3.00
21	4	5	5	3.00
22	4	8	5	3.00
23	4	9	5	3.00
24	4	10	5	3.00
25	4	11	5	3.00
26	4	14	5	3.00
27	4	15	5	3.00
28	4	16	5	3.00
29	5	6	5	4.73
30	5	9	5	3.00
31	5	10	5	3.00
32	5	11	5	3.00
33	5	15	5	3.00
34	5	16	5	3.00
35	5	17	5	3.04
36	6	10	5	3.13
37	6	11	5	3.00
38	6	16	5	3.04
39	6	17	5	5.00
40	7	8	5	3.00
41	7	12	5	3.00
42	7	13	5	3.00
43	7	14	5	3.00
44	8	9	5	3.00
45	8	12	5	3.13
46	8	13	5	3.00
47	8	14	5	3.00
48	8	15	5	3.00
49	9	10	5	3.00
50	9	13	5	3.00
51	9	14	5	3.00
52	9	15	5	3.00
53	9	16	5	3.00
54	10	11	5	3.00
55	10	14	5	3.00
56	10	15	5	3.00
57	10	16	5	3.00
58	10	17	5	3.13
59	11	15	5	3.00
60	11	16	5	3.00
61	11	17	5	3.00
62	12	13	5	4.73
63	13	14	5	3.00
64	14	15	5	3.00
65	15	16	5	3.00
66	16	17	5	4.73

Table 2. Summary of the redundancy numbers and the robustness parameters in different-order designs

Robustness parameter	Design order	Max	r_i^{a}	r_{min}	Mean	Standard deviation
ω (μrad)	Before design	10.9550	0.5288	0.1842	6.719	3.002
	After FOD	9.1200	0.5062	0.3127	6.185	2.400
	After SOD	5.4417	0.5015	0.4541	3.718	1.430
γ (ppm)	Before design	8.2887	0.1842	0.1842	6.083	2.223
	After FOD	6.5786	0.3127	0.3127	3.864	1.717
	After SOD	4.2787	0.4541	0.4541	2.616	1.125
σ (ppm)	Before design	5.8112	0.1842	0.1842	4.236	1.546
	After FOD	4.4985	0.3127	0.3127	2.570	1.255
	After SOD	2.7175	0.4541	0.4541	1.608	0.746

[a] Maximum robustness parameters belong to the observation with redundancy number r_i

perimeter of the network is relatively weaker than the middle. The smallest absolute value of the robustness parameters (not included in the table) was obtained for the point 9, which is located at the center of the simulated network and has many reliable observation ties. As a result, it can be concluded that both techniques are used to evaluate the strength (or weakness) of the network: the reliability criterion in the observations, and robustness analysis in the configuration of the network. It can be also concluded that the perimeter of a geodetic network is relatively weaker than the middle.

A comparison of the results of the first or fourth columns of Table 2 shows that the robustness parameters have significantly decreased in the two different-order designs. It is noticeable that the robustness in shear and scale results is correlated with redundancy numbers. This means that the largest robustness parameters in shear and scale are due to observations with minimum redundancy numbers. Thus, improving the minimum redundancy number helps to decrease the maximum robustness parameters in shear and scale.

As a result, it can be stated that there is a good correlation between reliability criteria and robustness analysis. Thus, we can design a geodetic network in the sense of high reliability by analytical method and test it by robustness analysis in order to obtain a network with high geometrical strength.

6 Conclusions

The proper design and subsequent assessment of geodetic networks is an integral part of most surveying engineering projects. The main goal of the present study was to design and optimize a geodetic network in the sense of high reliability and compare the results with the robustness analysis.

The purpose of an optimal design is to solve for both the network configuration (FOD) and observations accuracy (SOD) in order to meet the desired criteria. For this purpose, a developed methodology was presented for performing the FOD, SOD, and/or COMD.

For evaluating the reliability and strength of a geodetic network, two methods were recommended, namely, reliability criteria and robustness analysis. The main advantage of the reliability criteria is that they can be given in a mathematical form and use the analytical method. The main advantage of the robustness analysis is that the computed robustness parameters are not dependent on the choice of datum (i.e. it is an invariant quantity).

In order to evaluate the geometrical strength of a geodetic network, the results of robustness analysis are displayed in terms of robustness in rotation, robustness in shear, and robustness in scale. The results indicate that the perimeter of a geodetic network is relatively weaker than the middle since the largest robustness parameters are usually located on the edges of the network. This is because there are fewer and/or weaker observations tying these stations to the rest of the network. To counter this problem, it is suggested that, if possible, the outer points of the network (from one side to another) should be observed.

The results showed that the robustness parameters were affected by redundancy numbers. This means that the robustness parameters are correlated with redundancy numbers. In fact, the largest robustness parameters were due to the observations with minimum redundancy numbers. Thus, improving the minimum redundancy number helps to decrease the maximum robustness parameters (to increase the robustness of the network). It can be concluded that reliability criteria and robustness analysis are used to evaluate the strength of the network; one in the observations of the network and the other in the configuration of the network. These two measures are strongly correlated to each other. As a final result, it can be stated that we can design a geodetic network in the sense of high reliability by analytical method and test it by robustness analysis in order to obtain a network with high strength.

References

Amiri Seemkooei A (1998) Analytical methods in optimization and design of geodetic networks. Department of Surveying Engineering, K.N. Toosi University of Technology, Tehran

Baarda W (1968) A testing procedure for use in geodetic networks. Publ Geodesy, New series 2, No. 5 Netherland Geodetic commission, Delft

Cross PA (1985) Numerical methods in network design. In: Grafarend EW, Sanso F (eds) Optimization and design of geodetic networks. Springer, Berlin Heidelberg New York, pp 429–435

Dare P (1983) Strength analysis of horizontal network using strain. Survey Science tech rep 2, University of Toronto, Erindale Campus, Mississauga

Grafarend EW (1974) Optimization of geodetic networks. Boll Geod Sci Aff 33(4): 351–406

Kuang SL (1991) Optimization and design of deformations monitoring schemes. PhD dissertation, Department of Surveying Engineering, Tech rep 157, University of New Branswick, Fredericton

Schneider D (1982) Complex crustal strain approximation. Department of Surveying Engineering, Tech rep 91, University of New Branswick, Fredericton

Vaníček P, Thapa K, Schneider D (1981) The use of strain to identify incompatible observations and constraints in horizontal geodetic networks. Manuscr Geod 6: 257–281

Vaníček P, Krakiwsky EJ (1986) Geodesy: the concepts, 2nd edn. North Holland Amsterdam

Vaníček P, Krakiwsky EJ, Craymer MR, Gao Y, Ong PS (1990) Robustness analysis. Final contract report, Department of Surveying Engineering, Tech rep 156, University of New Branswick, Fredericton