5.1] $\alpha = 3.5^\circ \quad V = 30.5\,m/s \quad c = 3\,ft = 0.9\,m$

$C_m = 0.8 \quad C_l = 1.0$

$L' = C_l \rho V^2 c = (1)(1.2\,kg/m^3)(30.5\,m/s)^2(0.9\,m)$

$\boxed{L' = 1004.7\,N/m}$

$M' = \frac{1}{2}\rho u_\infty^2 c^2 C_m$

$= \frac{1}{2}(1.2\,kg/m^3)(30.5\,m/s)^2(0.9\,m)^2(0.8)$

$\boxed{M' = 361.68\,N}$

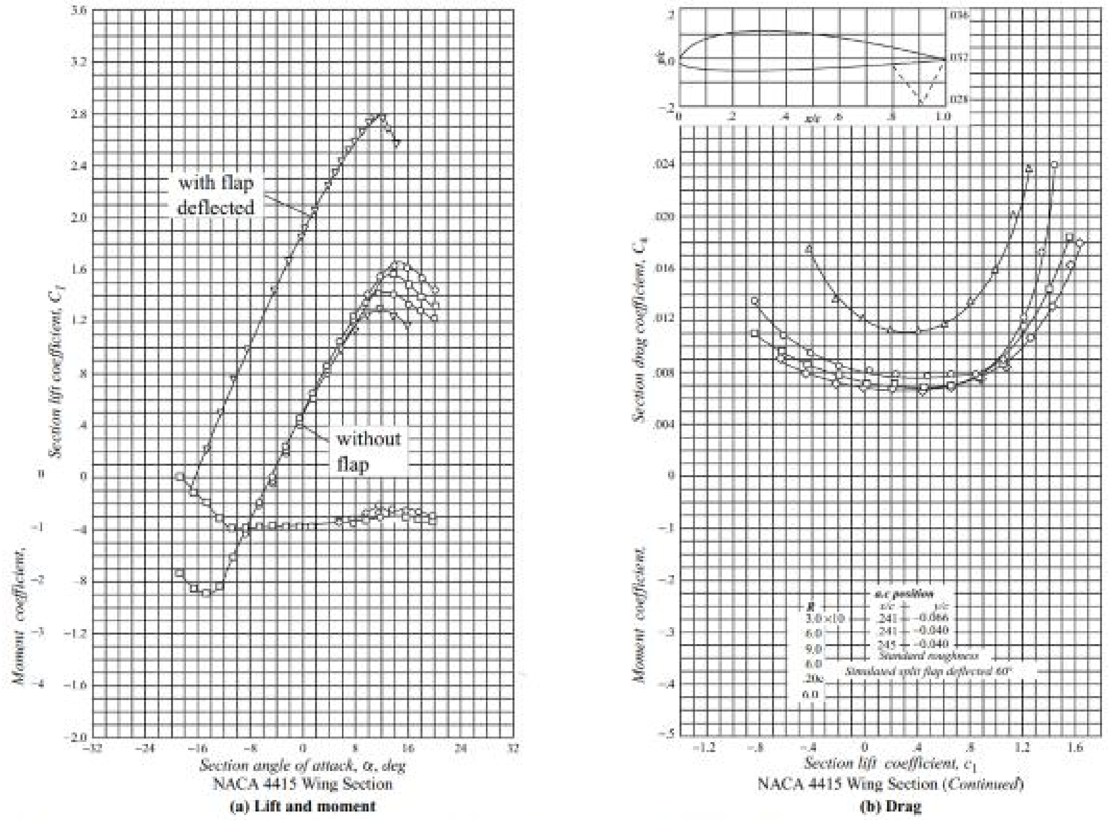

Figure 5.3. Two-dimensional airfoil behavior (NACA 4415, (Abbott and Van Doenhoff, 1959)).

5.4] $C_l = 2\pi\left[\alpha + \frac{1}{\pi}\int_0^{\pi}\frac{dZ}{dx}(\cos\theta - 1)\,d\theta\right]$

$$\frac{dZ}{dx} = \begin{cases} 0.2 - \frac{x}{2c} & \text{for } 0 \le \frac{x}{c} \le 0.4 \\ 0.09 - 0.22\frac{x}{c} & \text{for } 0.4 \le \frac{x}{c} \le 1.0 \end{cases}$$

go from $x \to \theta$: $x = \frac{c}{2}(1-\cos\theta)$

$$\frac{dZ}{dx} = \begin{cases} 0.25\cos(\theta) - 0.05 & 0 \le \theta \le 1.37 \\ 0.11\cos(\theta) - 0.02 & 1.37 \le \theta \le \pi \end{cases}$$

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi}\frac{dz}{dx}(\cos\theta - 1)\,d\theta$$

$$= -\frac{1}{\pi}\int_0^{1.37}[0.25\cos(\theta) - 0.05](\cos\theta - 1)\,d\theta - \frac{1}{\pi}\int_{1.37}^{\pi}[0.11\cos(\theta) - 0.02](\cos\theta - 1)\,d\theta$$

(((−1) / pi) × integrate(((0.25 × cos(x radians)) − 0.05) × (cos(x radians) − 1), 0, 1.37)) − ((1 / pi) × (integrate(((0.11 × cos(x radians)) − 0.02) × (cos(x radians) − 1), 1.37, pi) × radians))

#9

≈ −4.007 140 169°

$C_l = 2\pi(\alpha - \alpha_{L=0})$

$C_l = 2\pi(\alpha + 4.007)$

$$\boxed{C_l = 6.28\alpha + 25.18}$$

5.5] $C_{m_{c/4}} = \frac{\pi}{4}(A_2 - A_1)$

$$A_2 = \frac{2}{\pi}\int_0^{\pi} \frac{d\bar{z}}{dx}\cos(2\theta)\,d\theta$$

$$A_1 = \frac{2}{\pi}\int_0^{\pi} \frac{d\bar{z}}{dx}\cos(\theta)\,d\theta$$

$$\frac{d\bar{z}}{dx} = \begin{cases} 0.25\cos(\theta) - 0.05 & 0 \le \theta \le 1.37 \\ 0.11\cos(\theta) - 0.02 & 1.37 \le \theta \le \pi \end{cases}$$

$$C_{m_{c/4}} = \frac{1}{2}\Big[\int_0^{1.37}(0.25\cos\theta - 0.05)\cos(2\theta)\,d\theta + \int_{1.37}^{\pi}(0.11\cos\theta - 0.02)\cos(2\theta)\,d\theta$$
$$- \int_0^{1.37}(0.25\cos\theta - 0.05)\cos(\theta)\,d\theta - \int_{1.37}^{\pi}(0.11\cos\theta - 0.02)\cos\theta\,d\theta\Big]$$

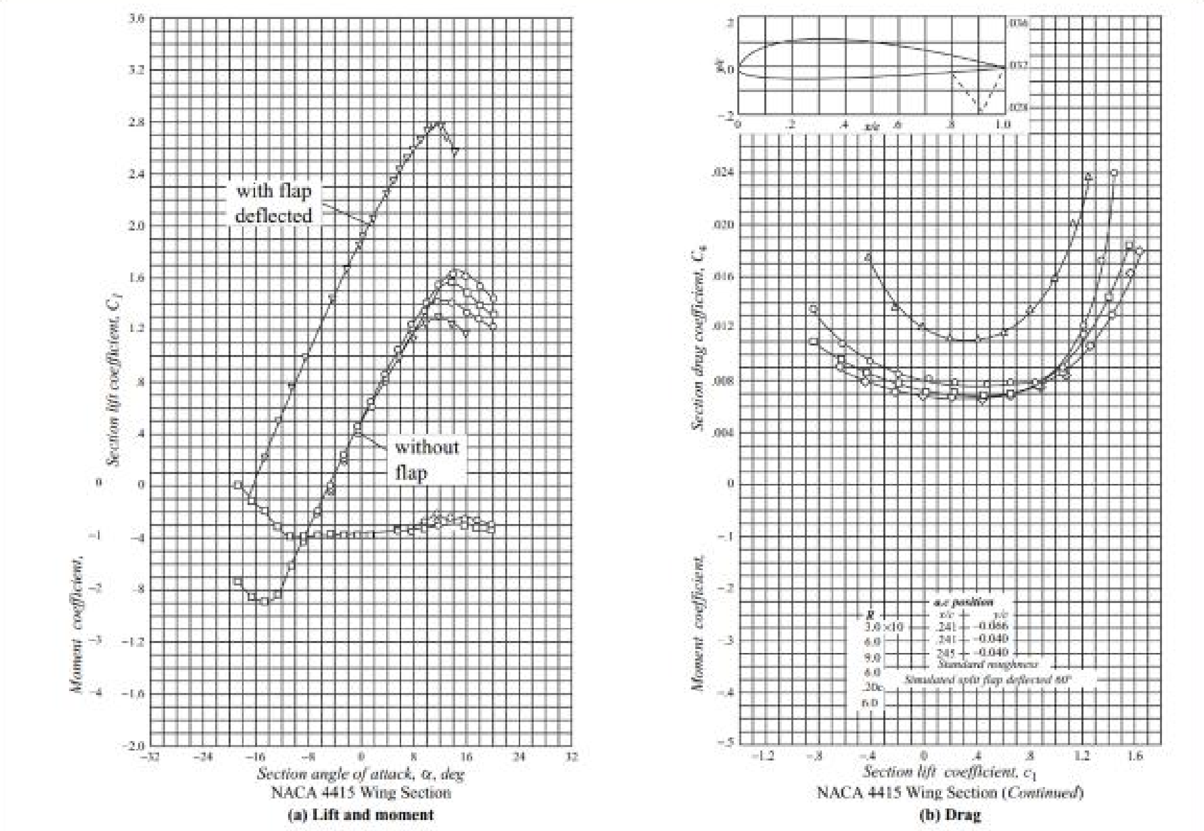

Figure 5.3. Two-dimensional airfoil behavior (NACA 4415, (Abbott and Van Doenhoff, 1959)).

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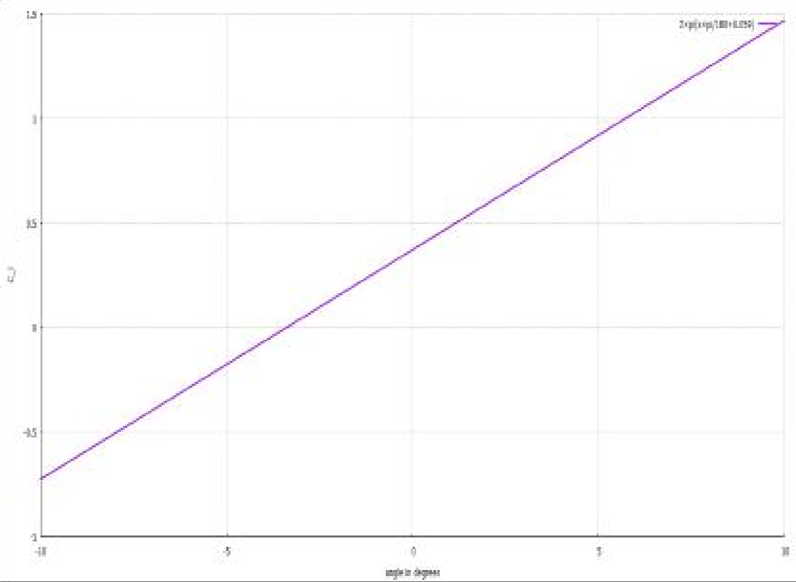
(1 / 2) × (integrate(((0.25 × cos(x radians)) − 0.05) × cos((2 × x) radians), 0, 1.37) + integrate(((0.11 × cos(x radians)) − 0.02) × cos((2 × x) radians), 1.37, pi) −
integrate(((0.25 × cos(x radians)) − 0.05) × cos(x radians), 0, 1.37) − integrate(((0.11 × cos(x radians)) − 0.02) × cos(x radians), 1.37, pi))
#1
≈ −0.104 733 665 0
```

$$\boxed{C_{m_{c/4}} = -0.104}$$

$$\frac{x_{cp}}{c} = \frac{1}{4} - \frac{\pi}{4}\frac{(A_2 - A_1)}{C_\ell} = \frac{1}{4} - \frac{C_{mac}}{C_\ell}.$$

$$\left[C_\ell = 6.28\alpha + 25.18 \right] \longrightarrow$$

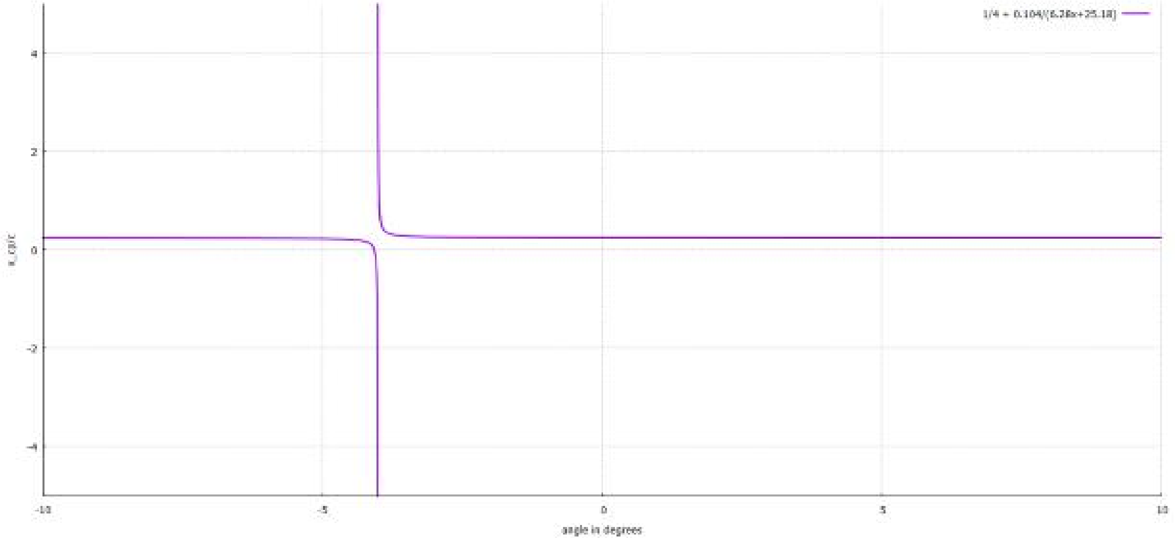


$$\frac{x_{cp}}{c} = \frac{1}{4} - \frac{\pi}{4C_\ell}\cdot\frac{2}{\pi}\left[\int_0^{1.37} (0.25\cos\theta - 0.05)\cos(2\theta)\,d\theta + \int_{1.37}^{\pi} (0.11\cos\theta - 0.02)\cos(2\theta)\,d\theta \right.$$
$$\left. - \int_0^{1.37} (0.25\cos\theta - 0.05)\cos(2\theta)\,d\theta - \int_{1.37}^{\pi} (0.11\cos\theta - 0.02)\cos\theta\,d\theta \right]$$

$\frac{x_{cp}}{c} = \frac{1}{4} - \frac{C_{m_{c/4}}}{C_\ell}$ oh... so that's what C_{mac} means haha

$$\frac{x_{cp}}{c} = \frac{1}{4} + \frac{0.104}{(6.28\alpha + 25.18)}$$

they're almost identical which demonstrates some drawbacks of thin airfoil theory!


1/4 + 0.104/(6.28x+25.18)
4
2
0
-2
-4
-10
-5
0
5
10
angle in degrees