# CHAPTER 11

## PROBLEM 11.1

A snowboarder starts from rest at the top of a double black diamond hill. As he rides down the slope, GPS coordinates are used to determine his displacement as a function of time: $x = 0.5t^3 + t^2 + 2t$ where x and t are expressed in ft and seconds, respectively. Determine the position, velocity, and acceleration of the boarder when $t = 5$ seconds.

## SOLUTION

Position: $x = 0.5t^3 + t^2 + 2t$

Velocity: $v = \dfrac{dx}{dt} = 1.5t^2 + 2t + 2$

Acceleration: $a = \dfrac{dv}{dt} = 3t + 2$

At $t = 5$ s, $x = 0.5(5)^3 + 5^2 + 2(5)$ $\qquad x = 97.5 \text{ ft}$ ◀

$v = 1.5(5)^2 + 2(5) + 2$ $\qquad v = 49.5 \text{ ft/s}$ ◀

$a = 3(5) + 2$ $\qquad a = 17 \text{ ft/s}^2$ ◀

## PROBLEM 11.2

The motion of a particle is defined by the relation $x = t^3 - 12t^2 + 36t + 30$, where $x$ and $t$ are expressed in feet and seconds, respectively. Determine the time, the position, and the acceleration of the particle when $v = 0$.

## SOLUTION

$$x = t^3 - 12t^2 + 36t + 30$$

Differentiating,
$$v = \frac{dx}{dt} = 3t^2 - 24t + 36 = 3(t^2 - 8t + 12)$$
$$= 3(t-6)(t-2)$$

$$a = \frac{dv}{dt} = 6t - 24$$

So $v = 0$ at $t = 2\text{ s}$ and $t = 6\text{ s}$.

At $t = 2\text{ s}$,
$$x_1 = (2)^3 - 12(2)^2 + 36(2) + 30 = 62$$

$t = 2.00\text{ s}$ ◀

$$a_1 = 6(2) - 24 = -12$$

$x_1 = 62.00\text{ ft}$ ◀

$a_1 = -12.00\text{ ft/s}^2$ ◀

At $t = 6\text{ s}$,

$$x_2 = (6)^3 - 12(6)^2 + 36(6) + 30 = 30$$

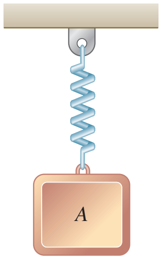
$t = 6.00\text{ s}$ ◀

$x_2 = 30.00\text{ ft}$ ◀

$$a_2 = (6)(6) - 24 = 12$$

$a_2 = 12.00\text{ ft/s}^2$ ◀

## PROBLEM 11.3




The vertical motion of mass $A$ is defined by the relation $x = \cos(10t) - 0.1\sin(10t)$, where $x$ and $t$ are expressed in mm and seconds, respectively. Determine ($a$) the position, velocity and acceleration of $A$ when $t = 0.4$ s, ($b$) the maximum velocity and acceleration of $A$.

## SOLUTION

$$x = \cos(10t) - 0.1\sin(10t)$$

$$v = \frac{dx}{dt} = -10\sin 10t - 0.1(10)\cos(10t)$$

$$a = \frac{dv}{dt} = -100\cos 10t + 0.1(100)\sin 10t$$

For trigonometric functions set calculator to radians:

($a$) At $t = 0.4\,\text{s}$

$$x_1 = \cos 4 - 0.1\sin(4) = 0.578 \qquad x_1 = 0.578 \text{ mm} \blacktriangleleft$$

$$v_1 = -10\sin(4) - \cos(4) = 8.222 \qquad v_1 = 8.22 \text{ mm/s} \blacktriangleleft$$

$$a_1 = -100\cos(4) + 10\sin(4) = 57.80 \qquad a_1 = 57.80 \text{ mm/s}^2 \blacktriangleleft$$

($b$) Maximum velocity occurs when $a = 0$.

$$-100\cos(10t) + 10\sin(10t) = 0$$

$$\tan 10t = 10,\ 10t = \tan^{-1}(10),\ t = 0.147 \text{ s}$$

So $t = 0.147\,\text{s}$ and $0.147+\pi$ s for $v_{\max}$

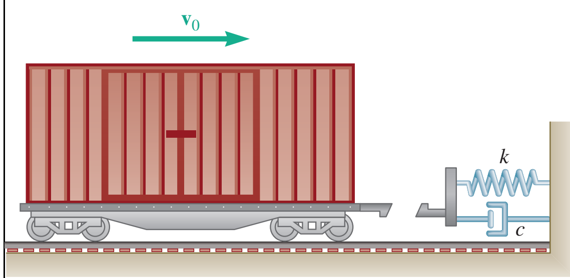
$$v_{\max} = \left|-10\sin(10*0.147) - \cos(10*0.147)\right|$$
$$= 10.05 \qquad v_{\max} = 10.05 \text{ mm/s} \blacktriangleleft$$

Note that we could have also used $v_{\max} = \sqrt{10^2 + 1^2} = 10.05$

by combining the sine and cosine terms. For $a_{\max}$ we can take the derivative and set equal to zero or just combine the sine and cosine terms.

$$a_{\max} = \sqrt{100^2 + 10^2} = 100.5 \text{ mm/s}^2 \qquad a_{\max} = 100.5 \text{ mm/s}^2 \blacktriangleleft$$

## PROBLEM 11.4




A loaded railroad car is rolling at a constant velocity when it couples with a spring and dashpot bumper system. After the coupling, the motion of the car is defined by the relation $x = 60e^{-4.8t}\sin 16t$ where $x$ and $t$ are expressed in mm and seconds, respectively. Determine the position, the velocity and the acceleration of the railroad car when (*a*) $t = 0$, (*b*) $t = 0.3$ s.

## SOLUTION

$$x = 60e^{-4.8t}\sin 16t$$

$$v = \frac{dx}{dt} = 60(-4.8)e^{-4.8t}\sin 16t + 60(16)e^{-4.8t}\cos 16t$$

$$v = -288e^{-4.8t}\sin 16t + 960e^{-4.8t}\cos 16t$$

$$a = \frac{dv}{dt} = 1382.4e^{-4.8t}\sin 16t - 4608e^{-4.8t}\cos 16t$$
$$- 4608e^{-4.8t}\cos 16t - 15360e^{-4.8t}\sin 16t$$

$$a = -13977.6e^{-4.8t}\sin 16t - 9216e^{-4.8}\cos 16t$$

(*a*) At $t = 0$,

$x_0 = 0$ $\qquad x_0 = 0$ mm ◀

$v_0 = 960$ mm/s $\qquad v_0 = 960$ mm/s $\longrightarrow$ ◀

$a_0 = -9216$ mm/s$^2$ $\qquad a_0 = 9220$ mm/s$^2$ $\longleftarrow$ ◀

(*b*) At $t = 0.3$ s,

$$e^{-4.8t} = e^{-1.44} = 0.23692$$
$$\sin 16t = \sin 4.8 = -0.99616$$
$$\cos 16t = \cos 4.8 = 0.08750$$

$x_{0.3} = (60)(0.23692)(-0.99616) = -14.16$ $\qquad x_{0.3} = 14.16$ mm $\longleftarrow$ ◀

$v_{0.3} = -(288)(0.23692)(-0.99616)$
$\quad + (960)(0.23692)(0.08750) = 87.9$ $\qquad v_{0.3} = 87.9$ mm/s $\longrightarrow$ ◀

$a_{0.3} = -(13977.6)(0.23692)(-0.99616)$
$\quad - (9216)(0.23692)(0.08750) = 3108$ $\qquad a_{0.3} = 3110$ mm/s$^2$ ◀

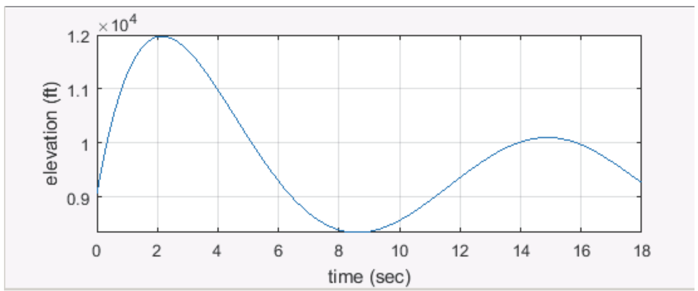
or $3.11$ m/s$^2$ $\longrightarrow$ ◀

## PROBLEM 11.5

A group of hikers uses a GPS while doing a 40 mile trek in Colorado. A curve fit to the data shows that their altitude can be approximated by the function, $y(t) = 0.12t^5 - 6.75t^4 + 135t^3 - 1120t^2 + 3200t + 9070$ where $y$ and $t$ are expressed in feet and hours, respectively. During the 18 hour hike, determine (*a*) the maximum altitude that the hikers reach, (*b*) the total feet they ascend, (*c*) the total feet they descend. *Hint*: You will need to use a calculator or computer to solve for the roots of a fourth order polynomial.

## SOLUTION

You can graph the function to get the elevation profile.




Differentiate y(t) and set to zero to find when the hikers are ascending and descending.

$$\dot{x} = 0.12(5)t^4 - 6.75(4)t^3 + 135(3)t^2 - 1200(2)t + 3200 = 0$$

Use a your calculator to find the roots of this polynomial to get

$$t = 2.151, \ 8.606 \text{ and } 14.884 \text{ hours}$$

Next, evaluate y(t) at start, end and calculated times to find the elevations

$$x(0) = 9070 \text{ ft (begin to ascend)}$$

x(2.151) = 11,976 ft (begin to descend) $y = 11{,}976$ ft at $t = 2.151$ hours ◀

x(8.606) = 8,344 ft (begin to ascend)

x(14.884) = 10,103 ft (begin to descend)

x(18) = 9,270 ft

Add the accents to get the total ascent = 4,664 ft total ascent $= 4{,}660$ ft ◀

Add the descents to get the total descent = 4,464 ft total descent $= -4{,}460$ ft ◀

## PROBLEM 11.6

The motion of a particle is defined by the relation $x = t^3 - 6t^2 + 9t + 5$, where $x$ is expressed in feet and $t$ in seconds. Determine ($a$) when the velocity is zero, ($b$) the position, acceleration, and total distance traveled when $t = 5$ s.

## SOLUTION

Given: $x = t^3 - 6t^2 + 9t + 5$

Differentiate twice. $v = \dfrac{dx}{dt} = 3t^2 - 12t + 9$ and $a = \dfrac{dv}{dt} = 6t - 12$

($a$) When velocity is zero. $v = 0$, $3t^2 - 12t + 9 = 3(t-1)(t-3) = 0$, $t = 1\text{ s}$ and $t = 3\text{ s}$ ◀

($b$) Position at $t = 5$ s. $x_5 = (5)^3 - (6)(5)^2 + (9)(5) + 5$ $x_5 = 25\text{ ft}$ ◀

Acceleration at $t = 5$ s. $a_5 = (6)(5) - 12$ $a_5 = 18\text{ ft/s}^2$ ◀

Position at $t = 0$. $x_0 = 5\text{ ft}$

Over $0 \leq t < 1$ s $x$ is increasing.

Over $1\text{ s} < t < 3$ s $x$ is decreasing.

Over $3\text{ s} < t \leq 5$ s $x$ is increasing.

Position at $t = 1$ s. $x_1 = (1)^3 - (6)(1)^2 + (9)(1) + 5 = 9\text{ ft}$

Position at $t = 3$ s. $x_3 = (3)^3 - (6)(3)^2 + (9)(3) + 5 = 5\text{ ft}$

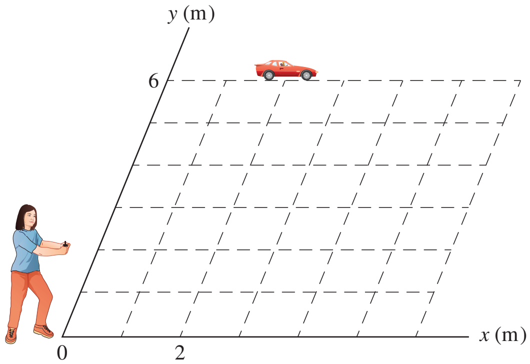
Distance traveled.

At $t = 1$ s $d_1 = |x_1 - x_0| = |9 - 5| = 4\text{ ft}$

At $t = 3$ s $d_3 = d_1 + |x_3 - x_1| = 4 + |5 - 9| = 8\text{ ft}$

At $t = 5$ s $d_5 = d_3 + |x_5 - x_3| = 8 + |25 - 5| = 28\text{ ft}$

$d_5 = 28\text{ ft}$ ◀




## PROBLEM 11.7

A girl operates a radio-controlled model car in a vacant parking lot. The girl's position is at the origin of the $xy$ coordinate axes, and the surface of the parking lot lies in the x-y plane. She drives the car in a straight line so that the $x$ coordinate is defined by the relation $x(\mathrm{t}) = 0.5t^3 - 3t^2 + 3t + 2$, where $x$ and $t$ are expressed in meters and seconds, respectively. Determine (*a*) when the velocity is zero, (*b*) the position and total distance travelled when the acceleration is zero.

## SOLUTION

Position: $x(t) = 0.5t^3 - 3t^2 + 3t + 2$

Velocity: $v(t) = \dfrac{dx}{dt}$

$$v(t) = 1.5t^2 - 6t + 3$$

(*a*) Time when v = 0 $\quad 0 = 1.5t^2 - 6t + 3$

$$t = \frac{6 \pm \sqrt{6^2 - 4(1.5)(3)}}{2 * 1.5}$$

$t = 0.586$ s and $t = 3.414$ s ◀

Acceleration: $a(t) = \dfrac{dv}{dt}$

$$a(t) = 3t - 6$$

Time when a = 0 $\quad 0 = 3t - 6$ so $t = 2$ s

(*b*) Position at t = 2 s $\quad x(2) = 0.5(2)^3 - 3(2)^2 + 3*2 + 2$

$x(2) = 0$ m ◀

To find total distance note that car changes direction at t = 0.586 s

Position at t = 0 s $\quad x(0) = 0.5(0)^3 - 3(0)^2 + 3*0 + 2$

$$x(0) = 2$$

Position at t = 0.586 s $\quad x(0.586) = 0.5(0.586)^3 - 3(0.586)^2 + 3*0.586 + 2$

$$x(0.586) = 2.828 \text{ m}$$

Distances traveled:

From t = 0 to t = 0.586 s: $\quad |x(0.586) - x(0)| = 0.828$ m

From t = 0.586 to t = 2 s: $\quad |x(2) - x(0.586)| = 2.828$ m

Total distance traveled $= 0.828$ m $+ 2.828$ m

Total distance $= 3.656$ m ◀

## PROBLEM 11.8

The motion of a particle is defined by the relation $x = t^2 - (t-2)^3$, where $x$ and $t$ are expressed in feet and seconds, respectively. Determine ($a$) the two positions at which the velocity is zero, ($b$) the total distance traveled by the particle from $t = 0$ to $t = 4$ s.

## SOLUTION

Position: $x(t) = t^2 - (t-2)^3$

Velocity: $v(t) = \dfrac{dx}{dt}$

$$v(t) = 2t - 3(t-2)^2$$

$$v(t) = 2t - 3(t^2 - 4t + 4)$$

$$= -3t^2 + 14t - 12$$

Time when v(t) = 0 $\quad 0 = -3t^2 + 14t - 12$

$$t = \frac{-14 \pm \sqrt{14^2 - 4(-3)(-12)}}{2(-3)}$$

$$t = 1.131 \text{ s} \quad \text{and} \quad t = 3.535 \text{ s}$$

($a$) Position at t = 1.131 s $\quad x(1.131) = (1.131)^2 - (1.1.31 - 2)^3 \qquad x(1.131) = 1.935$ ft. ◀

Position at t = 3.535 s $\quad x(3.535) = (3.535)^2 - (3.535 - 2)^3$

$x(3.531) = 8.879$ ft. ◀

To find total distance traveled note that the particle changes direction at t = 1.131 s and again at t = 3.535 s.

Position at t = 0 s $\quad x(0) = (0)^2 - (0-2)^3$

$$x(0) = 8 \text{ ft}$$

Position at t = 4 s $\quad x(4) = (4)^2 - (4-2)^3$

$$x(4) = 8 \text{ ft}$$

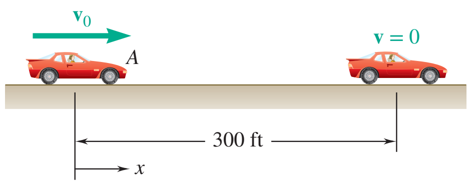
($b$) Distances traveled:

From t = 0 to t = 1.131 s: $\quad |x(1.131) - x(0)| = 6.065$ ft.

From t = 1.131 to t = 3.531 s: $\quad |x(3.535) - x(1.131)| = 6.944$ ft.

From t = 3.531 to t = 4 s: $\quad |x(4) - x(3.535)| = 0.879$ ft.

Total distance traveled = 6.065 ft + 6.944 ft + 0.879 ft $\qquad$ Total distance = 13.888 ft. ◀




## PROBLEM 11.9

The brakes of a car are applied, causing it to slow down at a rate of 10 ft/s$^2$. Knowing that the car stops in 100 ft, determine (*a*) how fast the car was traveling immediately before the brakes were applied, (*b*) the time required for the car to stop.

## SOLUTION

$$a = -10 \text{ ft/s}^2$$

(*a*) Velocity at $x = 0$.

$$v\frac{dv}{dx} = a = -10$$

$$\int_{v_0}^{0} v dv = -\int_0^{x_f} (-10) dx$$

$$0 - \frac{v_0^2}{2} = -10x_f = -(10)(300)$$

$$v_0^2 = 6000$$

$v_0 = 77.5$ ft/s ◀

(*b*) Time to stop.

$$\frac{dv}{dx} = a = -10$$

$$\int_{v_0}^{0} dv = -\int_0^{t_f} -10 dt$$

$$0 - v_0 = -10t_f$$

$$t_f = \frac{v_0}{10} = \frac{77.5}{10}$$

$t_f = 7.75$ s ◀

## PROBLEM 11.10

The acceleration of a particle is defined by the relation $a = 3e^{-0.2t}$, where $a$ and $t$ are expressed in $\text{ft/s}^2$ and seconds, respectively. Knowing that $x = 0$ and $v = 0$ at $t = 0$, determine the velocity and position of the particle when $t = 0.5$ s.

## SOLUTION

Acceleration: $a = 3e^{-0.2t}$ ft/s$^2$

Given: $v_0 = 0$ ft/s, $x_0 = 0$ ft

Velocity: $a = \dfrac{dv}{dt} \Rightarrow dv = adt$

$$\int_{v_0}^{v} dv = \int_0^t adt$$

$$v - v_o = \int_0^t 3e^{-0.2t}dt \Rightarrow v - 0 = -15e^{-0.2t}\Big|_0^t$$

$$v = 15\left(1 - e^{-0.2t}\right) \text{ ft/s}$$

Position: $v = \dfrac{dx}{dt} \Rightarrow dx = vdt$

$$\int_{x_0}^{x} dx = \int_0^t vdt$$

$$x - x_o = \int_0^t 15\left(1 - e^{-0.2t}\right)dt \Rightarrow \text{x} - 0 = 15\left(t + 5e^{-0.2t}\right)\Big|_0^t$$

$$x = 15\left(t + 5e^{-0.2t}\right) - 75 \text{ ft}$$

Velocity at t = 0.5 s $v = 15\left(1 - e^{-0.2*0.5}\right)$ ft/s

$v(0.5) = 1.427$ ft/s ◀

Position at t = 0.5 s $x = 15\left(0.5 + 5e^{-0.2*0.5}\right) - 75$ ft

$x(0.5) = 0.363$ ft. ◀

## PROBLEM 11.11

The acceleration of a particle is defined by the relation $a = 9 - 3t^2$, where $a$ and $t$ are expressed in ft/s$^2$ and seconds, respectively. The particle starts at $t = 0$ with $v = 0$ and x = 5 ft. Determine (*a*) the time when the velocity is again zero, (*b*) the position and velocity when $t = 4$ s, (*c*) the total distance traveled by the particle from $t = 0$ to $t = 4$ s.

## SOLUTION

$$a = 9 - 3t^2$$

Separate variables and integrate. $\int_0^v dv = \int a\, dt = \int_0^t \left(9 - 3t^2\right) dt = 9$

$$v - 0 = 9t - t^3,\ v = t\left(9 - t^2\right)$$

(*a*)When $v$ is zero. $t(9 - t^2) = 0$ so $t = 0$ and $t = 3$ s (2 roots) $t = 3$ s ◀

(*b*)Position and velocity at $t = 4$ s.

$$\int_5^x dx = \int_0^t v\, dt = \int_0^t \left(9t - t^3\right) dt$$

$$x - 5 = \frac{9}{2}t^2 - \frac{1}{4}t^4,\ x = 5 + \frac{9}{2}t^2 - \frac{1}{4}t^4$$

At $t = 4$ s, $x_4 = 5 + \left(\frac{9}{2}\right)(4)^2 - \left(\frac{1}{4}\right)(4)^4$ $x_4 = 13$ ft ◀

$v_4 = (4)\left(9 - 4^2\right)$ $v_4 = -28$ ft/s ◀

(*c*)Distance traveled.

Over $0 < t < 3$ s, $v$ is positive, so $x$ is increasing.

Over $3\text{ s} < t \leq 4$ s, $v$ is negative, so $x$ is decreasing.

At $t = 3$ s, $x_3 = 5 + \left(\frac{9}{2}\right)(3)^2 - \left(\frac{1}{4}\right)(3)^4 = 25.25$ ft

At $t = 3$ s $d_3 = \left|x_3 - x_0\right| = \left|25.25 - 5\right| = 20.25$ ft

At $t = 4$ s $d_4 = d_3 + \left|x_4 - x_3\right| = 20.25 + \left|13 - 25.25\right| = 32.5$ ft $d_4 = 32.5$ ft ◀

## PROBLEM 11.12

Many car companies are performing research on collision avoidance systems. A small prototype applies engine braking that decelerates the vehicle according to the relationship $a = -k\sqrt{t}$, where $a$ and $t$ are expressed in m/s$^2$ and seconds, respectively. The vehicle is travelling 20 m/s when its radar sensors detect a stationary obstacle. Knowing that it takes the prototype vehicle 4 seconds to stop, determine ($a$) expressions for its velocity and position as a function of time, ($b$) how far the vehicle travelled before it stopped.

## SOLUTION

Starting with the given function for acceleration,

$$a = -k\sqrt{t}$$

Differentiate with respect to time to get

$$v = -\frac{2k}{3}t^{3/2} + v_0$$

Let t = 4 and v0 = 20 to find k when v = 0

$$0 = -\frac{2k}{3}4^{3/2} + 20$$
$$k = 3.75$$

$$v = -\frac{2(3.75)}{3}t^{3/2} + 20$$
$$v = -2.5t^{3/2} + 20$$

$v = -2.5t^{3/2} + 20$ ◀

Integrate velocity with respect to time to get position

$$x(t) = -\frac{4}{15}(3.75)t^{5/2} + 20t$$
$$x(t) = -t^{5/2} + 20t$$

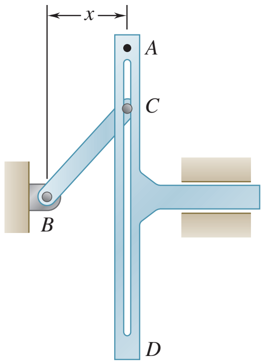
$x(t) = -t^{5/2} + 20t$ ◀

Find the distance when t = 4 seconds to find when it stops,

$$x(t) = -4^{5/2} + 20(4) = 48$$

$x = 48$ m ◀

## PROBLEM 11.13




A Scotch yoke is a mechanism that transforms the circular motion of a crank into the reciprocating motion of a shaft (or vice versa). It has been used in a number of different internal combustion engines and in control valves. In the Scotch yoke shown, the acceleration of Point $A$ is defined by the relation $a = -1.8\sin kt$, where $a$ and $t$ are expressed in m/s² and seconds, respectively, and $k = 3$ rad/s. Knowing that $x = 0$ and $v = 0.6$ m/s when $t = 0$, determine the velocity and position of Point $A$ when $t = 0.5$ s.

## SOLUTION

Acceleration: $a = -1.8\sin kt \text{ m/s}^2$

Given: $v_0 = 0.6 \text{ m/s}, \quad x_0 = 0, \quad k = 3 \text{ rad/s}$

Velocity: $a = \dfrac{dv}{dt} \Rightarrow \quad dv = adt \quad \Rightarrow \int_{v_0}^{v} dv = \int_0^t adt$

$$v - v_0 = \int_0^t a\,dt = -1.8\int_0^t \sin kt\,dt = \frac{1.8}{k}\cos kt\Big|_0^t$$

$$v - 0.6 = \frac{1.8}{3}(\cos kt - 1) = 0.6\cos kt - 0.6$$

$$v = 0.6\cos kt \text{ m/s}$$

Position: $v = \dfrac{dx}{dt} \Rightarrow \quad dx = vdt \quad \Rightarrow \int_{x_0}^{x} dx = \int_0^t vdt$

$$x - x_0 = \int_0^t v\,dt = 0.6\int_0^t \cos kt\,dt = \frac{0.6}{k}\sin kt\Big|_0^t$$

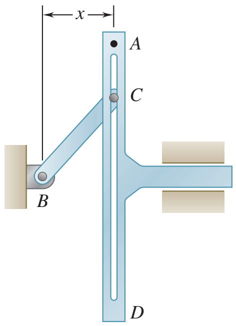
$$x - 0 = \frac{0.6}{3}(\sin kt - 0) = 0.2\sin kt$$

$$x = 0.2\sin kt \text{ m}$$

When t = 0.5 s, $kt = (3)(0.5) = 1.5$ rad

$v = 0.6\cos 1.5 = 0.0424$ m/s $\qquad v = 42.4$ mm/s ◀

$x = 0.2\sin 1.5 = 0.1995$ m $\qquad x = 199.5$ mm ◀




## PROBLEM 11.14

For the scotch yoke mechanism shown, the acceleration of Point $A$ is defined by the relation $a = -1.08\sin kt - 1.44\cos kt$, where $a$ and $t$ are expressed in m/s$^2$ and seconds, respectively, and $k = 3$ rad/s. Knowing that $x = 0.16$ m and $v = 0.36$ m/s when $t = 0$, determine the velocity and position of Point $A$ when $t = 0.5$ s.

## SOLUTION

Acceleration: $a = -1.08\sin kt - 1.44\cos kt \text{ m/s}^2$

Given: $v_0 = 0.36 \text{ m/s}, \quad x_0 = 0.16, \quad k = 3 \text{ rad/s}$

Velocity: $a = \dfrac{dv}{dt} \Rightarrow \ dv = adt \ \Rightarrow \int_{v_0}^{v} dv = \int_0^t adt$

Integrate:

$$v - v_0 = -1.08\int_0^t \sin kt\,dt - 1.44\int_0^t \cos kt\,dt$$

$$v - 0.36 = \frac{1.08}{k}\cos kt\Big|_0^t - \frac{1.44}{k}\sin kt\Big|_0^t$$

$$= \frac{1.08}{3}(\cos 3t - 1) - \frac{1.44}{3}(\sin 3t - 0)$$

$$= 0.36\cos 3t - 0.36 - 0.48\sin 3t$$

$$v = 0.36\cos 3t - 0.48\sin 3t \text{ m/s}$$

Evaluate at $t = 0.5$ s $\quad = 0.36\cos 1.5 - 0.36 - 0.48\sin 1.5 \qquad v = -453$ mm/s ◀

Position: $v = \dfrac{dx}{dt} \Rightarrow \ dx = vdt \ \Rightarrow \int_{x_0}^{x} dx = \int_0^t vdt$

$$x - x_0 = \int_0^t v\,dt = 0.36\int_0^t \cos kt\,dt - 0.48\int_0^t \sin kt\,dt$$

$$x - 0.16 = \frac{0.36}{k}\sin kt\Big|_0^t + \frac{0.48}{k}\cos kt\Big|_0^t$$

$$= \frac{0.36}{3}(\sin 3t - 0) + \frac{0.48}{3}(\cos 3t - 1)$$

$$= 0.12\sin 3t + 0.16\cos 3t - 0.16$$

$$x = 0.12\sin 3t + 0.16\cos 3t \text{ m}$$

Evaluate at $t = 0.5$ s $\quad x = 0.12\sin 1.5 + 0.16\cos 1.5 = 0.1310$ m $\qquad x = 131.0$ mm ◀


## PROBLEM 11.15

A piece of electronic equipment that is surrounded by packing material is dropped so that it hits the ground with a speed of 4 m/s. After contact the equipment experiences an acceleration of $a = -kx,$ where $k$ is a constant and $x$ is the compression of the packing material. If the packing material experiences a maximum compression of 15 mm, determine the maximum acceleration of the equipment.

## SOLUTION

$$a = \frac{v\,dv}{dx} = -kx$$

Separate and integrate.

$$\int_{v_0}^{v_f} v\,dv = -\int_0^{x_f} kx\,dx$$

$$\frac{1}{2}v_f^2 - \frac{1}{2}v_0^2 = -\frac{1}{2}kx^2\Big|_0^{x_f} = -\frac{1}{2}kx_f^2$$

Use $v_0 = 4$ m/s, $x_f = 0.015$ m, and $v_f = 0.$ Solve for $k$.

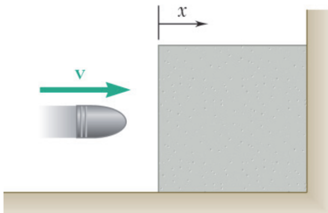
$$0 - \frac{1}{2}(4)^2 = -\frac{1}{2}k(0.015)^2 \qquad k = 71{,}111\ \text{s}^{-2}$$

Maximum acceleration.

$$a_{max} = -kx_{max}: \quad (-71{,}111)(0.015) = -1{,}067\ \text{m/s}^2$$

$$a = 1{,}067\ \text{m/s}^2 \uparrow \blacktriangleleft$$

## PROBLEM 11.16




A projectile enters a resisting medium at $x = 0$ with an initial velocity $v_0 = 1000$ ft/s and travels 3 in. before coming to rest. Assuming that the velocity of the projectile is defined by the relation $v = v_0 - kx$, where $v$ is expressed in ft/s and $x$ is in feet, determine (*a*) the initial acceleration of the projectile, (*b*) the time required for the projectile to penetrate 2.5 in. into the resisting medium.

## SOLUTION

When $x = \frac{3}{12}$ ft, $v = 0$:
$$0 = (1000 \text{ ft/s}) - k\left(\frac{3}{12} \text{ ft}\right)$$

Or
$$k = 4000 \frac{1}{\text{s}}$$

(a) We have
$$v = v_0 - kx, \quad a = \frac{dv}{dt} = \frac{d}{dt}(v_0 - kx) = -kv$$

or
$$a = -k(v_0 - kx)$$

At $t = 0$: $\quad a = 4000 \frac{1}{\text{s}}(1000 \text{ ft/s} - 0)$

$$a_0 = -4.00 \times 10^6 \text{ ft/s}^2 \blacktriangleleft$$

(*b*) We have
$$\frac{dx}{dt} = v = v_0 - kx$$

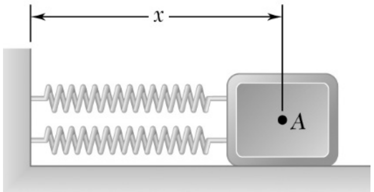
At $t = 0, x = 0$:
$$\int_0^x \frac{dx}{v_0 - kx} = \int_0^t dt$$

or
$$-\frac{1}{k}[\ln(v_0 - kx)]_0^x = t$$

or
$$t = \frac{1}{k}\ln\left(\frac{v_0}{v_0 - kx}\right) = \frac{1}{k}\ln\left(\frac{1}{1 - \frac{k}{v_0}x}\right)$$

When $x = 2.5$ in.:
$$t = \frac{1}{4000 \frac{1}{\text{s}}}\ln\left[\frac{1}{1 - \frac{4000 \text{ 1/s}}{1000 \text{ ft/s}}\left(\frac{2.5}{12} \text{ ft}\right)}\right]$$

Or
$$t = 4.48 \times 10^{-4} \text{ s} \blacktriangleleft$$




## PROBLEM 11.17

Point $A$ oscillates with an acceleration $a = 100(0.25 - x)$, where $a$ and $x$ are expressed in $\mathrm{m/s^2}$ and meters, respectively. Knowing that the system starts at time $t = 0$ with $v = 0$ and $x = 0.2$ m, determine the position and the velocity of $A$ when $t = 0.2$ s.

## SOLUTION

$a$ is function of $x$: $$a = 100(0.25 - x)\,\mathrm{m/s^2}$$

Use $v\,dv = a\,dx = 100(0.25 - x)\,dx$ with limits $v = 0$ when $x = 0.2$ m

$$\int_0^v v\,dv = \int_{0.2}^x 100(0.25 - x)\,dx$$

$$\frac{1}{2}v^2 - 0 = -\frac{1}{2}(100)(0.25 - x)^2\Big|_{0.2}^{x}$$

$$= -50\,(0.25 - x)^2 + 0.125$$

So $$v^2 = 0.25 - 100(0.25 - x)^2 \quad \text{or} \quad v = \pm 0.5\sqrt{1 - 400(0.25 - x)^2}$$

Use $$dx = v\,dt \quad \text{or} \quad dt = \frac{dx}{v} = \frac{dx}{\pm 0.5\sqrt{1 - 400(0.25 - x)^2}}$$

Integrate: $$\int_0^t dt = \pm\int_{0.2}^x \frac{dx}{0.5\sqrt{1 - 400(0.25 - x)^2}}$$

Let $u = 20(0.25 - x)$; when $x = 0.2$ $u = 1$ and $du = -20dx$

So $$t = \mp\int_1^u \frac{du}{10\sqrt{1 - u^2}} = \mp\frac{1}{10}\sin^{-1} u\Big|_1^u = \mp\frac{1}{10}\left(\sin^{-1} u - \frac{\pi}{2}\right)$$

Solve for $u$. $$\sin^{-1} u = \frac{\pi}{2} \mp 10t$$

$$u = \sin\left(\frac{\pi}{2} \mp 10t\right) = \cos(\pm 10t) = \cos 10t$$

## PROBLEM 11.17 (CONTINUED)

$$u = \cos 10t = 20\left(0.25 - x\right)$$

Solve for $x$ and $v$.

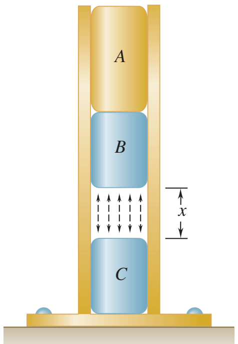
$$x = 0.25 - \frac{1}{20}\cos 10t$$

$$v = \frac{1}{2}\sin 10t$$

Evaluate at $t = 0.2$ s.

$$x = 0.25 - \frac{1}{20}\cos\left(\left(10\right)\left(0.2\right)\right) \qquad x = 0.271 \text{ m} \blacktriangleleft$$

$$v = \frac{1}{2}\sin\left(\left(10\right)\left(0.2\right)\right) \qquad v = 0.455 \text{ m/s} \blacktriangleleft$$




## PROBLEM 11.18

A brass (nonmagnetic) block $A$ and a steel magnet $B$ are in equilibrium in a brass tube under the magnetic repelling force of another steel magnet $C$ located at a distance $x = 0.004$ m from $B$. The force is inversely proportional to the square of the distance between $B$ and $C$. If block $A$ is suddenly removed, the acceleration of block $B$ is $a = -9.81 + k/x^2$, where $a$ and $x$ are expressed in m/s$^2$ and m, respectively, and $k = 4\times10^{-4}\,\text{m}^3/\text{s}^2$. Determine the maximum velocity and acceleration of $B$.

## SOLUTION

The maximum velocity occurs when $a = 0$. $\qquad 0 = -9.81 + \dfrac{k}{x_m^2}$

$$x_m^2 = \frac{k}{9.81} = \frac{4\times10^{-4}}{9.81} = 40.775\times10^{-6}\,\text{m}^2 \qquad x_m = 0.0063855\text{ m}$$

The acceleration is given as a function of $x$.

$$v\frac{dv}{dx} = a = -9.81 + \frac{k}{x^2}$$

Separate variables and integrate:

$$vdv = -9.81dx + \frac{k\,dx}{x^2}$$

$$\int_0^v vdv = -9.81\int_{x_0}^x dx + k\int_{x_0}^x \frac{dx}{x^2}$$

$$\frac{1}{2}v^2 = -9.81(x - x_0) - k\left(\frac{1}{x} - \frac{1}{x_0}\right)$$

$$\frac{1}{2}v_m^2 = -9.81(x_m - x_0) - k\left(\frac{1}{x_m} - \frac{1}{x_0}\right)$$

$$= -9.81(0.0063855 - 0.004) - (4\times10^{-4})\left(\frac{1}{0.0063855} - \frac{1}{0.004}\right)$$

$$= -0.023402 + 0.037358 = 0.013956\text{ m}^2/\text{s}^2$$

Maximum velocity: $\qquad v_m = 0.1671\text{ m/s} \qquad\qquad v_m = 167.1\text{ mm/s} \uparrow$ ◀

The maximum acceleration occurs when $x$ is smallest, that is, $x = 0.004$ m.

$$a_m = -9.81 + \frac{4\times10^{-4}}{(0.004)^2} \qquad\qquad a_m = 15.19\text{ m/s}^2 \uparrow$$ ◀

## PROBLEM 11.19

Based on experimental observations, the acceleration of a particle is defined by the relation $a = -(0.1 + \sin x/b)$, where $a$ and $x$ are expressed in m/s$^2$ and meters, respectively. Knowing that $b = 0.8$ m and that $v = 1$ m/s when $x = 0$, determine (*a*) the velocity of the particle when $x = -1$ m, (*b*) the position where the velocity is maximum, (*c*) the maximum velocity.

## SOLUTION

We have
$$v\frac{dv}{dx} = a = -\left(0.1 + \sin\frac{x}{0.8}\right)$$

When $x = 0,\ v = 1$ m/s:
$$\int_1^v v\,dv = \int_0^x -\left(0.1 + \sin\frac{x}{0.8}\right)dx$$

or
$$\frac{1}{2}(v^2 - 1) = -\left[0.1x - 0.8\cos\frac{x}{0.8}\right]_0^x$$

or
$$\frac{1}{2}v^2 = -0.1x + 0.8\cos\frac{x}{0.8} - 0.3$$

(*a*) When $x = -1$ m:
$$\frac{1}{2}v^2 = -0.1(-1) + 0.8\cos\frac{-1}{0.8} - 0.3$$

or $v = \pm 0.323$ m/s ◀

(*b*) When $v = v_{\max},\ a = 0$: $-\left(0.1 + \sin\frac{x}{0.8}\right) = 0$

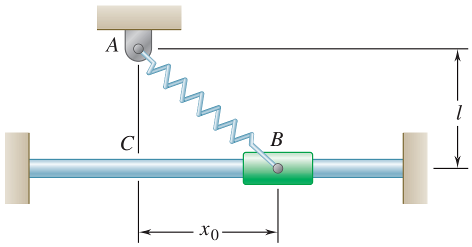
or $x = -0.080\,134$ m $x = -0.0801$ m ◀

(*c*) When $x = -0.080\,134$ m:

$$\frac{1}{2}v_{\max}^2 = -0.1(-0.080\,134) + 0.8\cos\frac{-0.080\,134}{0.8} - 0.3$$
$$= 0.504\ \text{m}^2/\text{s}^2$$

or $v_{\max} = 1.004$ m/s ◀

## PROBLEM 11.20




A spring $AB$ is attached to a support at $A$ and to a collar. The unstretched length of the spring is $l$. Knowing that the collar is released from rest at $x = x_0$ and has an acceleration defined by the relation $a = -100(x - lx/\sqrt{l^2 + x^2})$, determine the velocity of the collar as it passes through Point $C$.

## SOLUTION

Since $a$ is function of $x$,

$$a = v\frac{dv}{dx} = -100\left(x - \frac{lx}{\sqrt{l^2 + x^2}}\right)$$

Separate variables and integrate:

$$\int_{v_0}^{v_f} vdv = -100\int_{x_0}^{0}\left(x - \frac{lx}{\sqrt{l^2 + x^2}}\right)dx$$

$$\frac{1}{2}v_f^2 - \frac{1}{2}v_0^2 = -100\left(\frac{x^2}{2} - l\sqrt{l^2 + x^2}\right)\Bigg|_{x_0}^{0}$$

$$\frac{1}{2}v_f^2 - 0 = -100\left(-\frac{x_0^2}{2} - l^2 + l\sqrt{l^2 + x_0^2}\right)$$

$$\frac{1}{2}v_f^2 = \frac{100}{2}(-l^2 + x_0^2 - l^2 - 2l\sqrt{l^2 + x_0^2})$$

$$= \frac{100}{2}(\sqrt{l^2 + x_0^2} - l)^2$$

$$v_f = 10(\sqrt{l^2 + x_0^2} - l) \blacktriangleleft$$

## PROBLEM 11.21

The acceleration of a particle is defined by the relation $a = k(1 - e^{-x})$, where $k$ is a constant. Knowing that the velocity of the particle is $v = +9$ m/s when $x = -3$ m and that the particle comes to rest at the origin, determine (*a*) the value of $k$, (*b*) the velocity of the particle when $x = -2$ m.

## SOLUTION

Acceleration: $a = k(1 - e^{-x})$

Given: at $x = -3$ m, $v = 9$ m/s

at $x = 0$ m, $v = 0$ m/s

$$adx = vdv$$

$$\mathrm{k}(1 - e^{-x})dx = vdv$$

Integrate using x = −3 m and v = 9 m/s as the lower limits of the integrals

$$\int_{-3}^{x} \mathrm{k}(1 - e^{-x})dx = \int_{9}^{v} vdv$$

$$k(x + e^{-x})\Big|_{-3}^{x} = \frac{1}{2}v^2\Big|_{9}^{v}$$

Velocity: $$k\left(x + e^{-x} - \left(-3 + e^{3}\right)\right) = \frac{1}{2}v^2 - \frac{1}{2}(9)^2 \qquad (1)$$

(*a*) Now substitute v = 0 m/s and x = 0 m into (1) and solve for k

$$k\left(0 + e^{-0} - \left(-3 + e^{3}\right)\right) = \frac{1}{2}0^2 - \frac{1}{2}(9)^2$$

$k = 2.52\ \mathrm{m^2/s^2}$ ◀

(*b*) Find velocity when x = −2 m using the equation (1) and the value of k

$$2.518\left(-2 + e^{2} - \left(-3 + e^{3}\right)\right) = \frac{1}{2}v^2 - \frac{1}{2}(9)^2$$

$v = 4.70$ m/s ◀

## PROBLEM 11.22

Starting from $x = 0$ with no initial velocity, a particle is given an acceleration, $a = 0.8\sqrt{v^2 + 49}$ where a and $v$ are expressed in $\text{ft/s}^2$ and ft/s, respectively. Determine (*a*) the position of the particle when $v = 24$ ft/s, (*b*) the speed of the particle when $x = 40$ ft.

## SOLUTION

$$a = 0.8\sqrt{v^2 + 49}$$

$$v\,dv = a\,dx \qquad dx = \frac{v\,dv}{a} = \frac{v\,dv}{0.8\sqrt{v^2 + 49}}$$

Integrating using $x = 0$ when $v = 0$,

$$\int_0^x dx = \frac{1}{0.8}\int_0^v \frac{v\,dv}{\sqrt{v^2 + 49}} = \frac{1}{0.8}\sqrt{v^2 + 49}\Big]_0^v$$

$$x = 1.25\left(\sqrt{v^2 + 49} - 7\right) \qquad (1)$$

(a) When $v = 24$ ft/s,

$$x = 1.25\left(\sqrt{24^2 + 49} - 7\right)$$

$x = 22.5$ ft ◀

(*b*) Solving equation (1) for $v^2$,

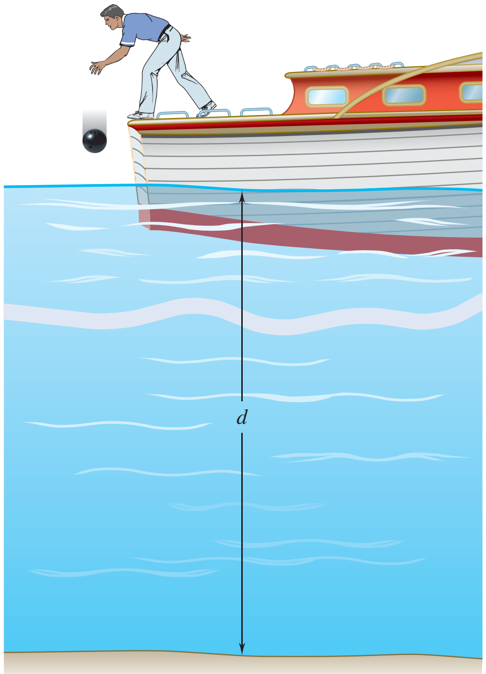
$$\sqrt{v^2 + 49} = 7 + 0.8x$$

$$v^2 = (7 + 0.8x)^2 - 49$$

When $x = 40$ ft,

$$v^2 = \left[7 + (0.8)(40)\right]^2 - 49 = 1472 \text{ ft}^2/\text{s}^2$$

$v = 38.4$ ft/s ◀




## PROBLEM 11.23

A ball is dropped from a boat so that it strikes the surface of a lake with a speed of 16.5 ft/s. While in the water the ball experiences an acceleration of $a = 10 - 0.8v$, where $a$ and $v$ are expressed in ft/s$^2$ and ft/s, respectively. Knowing the ball takes 3 s to reach the bottom of the lake, determine (*a*) the depth of the lake, (*b*) the speed of the ball when it hits the bottom of the lake.

## SOLUTION

$$a = \frac{dv}{dt} = 10 - 0.8v$$

Separate and integrate:

$$\int_{v_0}^{v} \frac{dv}{10 - 0.8v} = \int_0^t dt$$

$$-\frac{1}{0.8}\ln(10 - 0.8v)\Big|_{v_0}^{v} = t$$

$$\ln\left(\frac{10 - 0.8v}{10 - 0.8v_0}\right) = -0.8t$$

$$10 - 0.8v = (10 - 0.8v_0)e^{-0.8t}$$

or

$$0.8v = 10 - (10 - 0.8v_0)e^{-0.8t}$$

$$v = 12.5 - (12.5 - v_0)e^{-0.8t}$$

With $v_0 = 16.5\,\text{ft/s}$

$$v = 12.5 + 4e^{-0.8t}$$

Integrate to determine $x$ as a function of $t$.

$$v = \frac{dx}{dt} = 12.5 + 4e^{-0.8t}$$

$$\int_0^x dx = \int_0^t (12.5 + 4e^{-0.8t})dt$$

## PROBLEM 11.23 (CONTINUED)

$$x = 12.5t - 5e^{-0.8t}\Big|_0^t = 12.5t - 5e^{-0.8t} + 5$$

(*a*) At $t = 35$ s,

$$x = 12.5(3) - 5e^{-2.4} + 5 = 42.046 \text{ ft}$$

$x = 42.0$ ft ◀

(*b*) $v = 12.5 + 4e^{-2.4} = 12.863$ ft/s

$v = 12.86$ ft/s ◀

## PROBLEM 11.24

The acceleration of a particle is defined by the relation $a = -k\sqrt{v}$, where $k$ is a constant. Knowing that $x = 0$ and $v = 81$ m/s at $t = 0$ and that $v = 36$ m/s when $x = 18$ m, determine (*a*) the velocity of the particle when $x = 20$ m, (*b*) the time required for the particle to come to rest.

## SOLUTION

(*a*) We have $$v\frac{dv}{dx} = a = -k\sqrt{v}$$

so that $$\sqrt{v}\, dv = -k\,dx$$

When $x = 0, v = 81\,\text{m/s}$: $$\int_{81}^{v} \sqrt{v}\, dv = \int_0^x -k\,dx$$

or $$\frac{2}{3}[v^{3/2}]_{81}^{v} = -kx$$

or $$\frac{2}{3}[v^{3/2} - 729] = -kx$$

When $x = 18$ m, $v = 36$ m/s: $$\frac{2}{3}(36^{3/2} - 729) = -k(18)$$

or $$k = 19\sqrt{\text{m/s}^2}$$

Finally

When $x = 20$ m: $$\frac{2}{3}(v^{3/2} - 729) = -19(20)$$

or $$v^{3/2} = 159$$

$v = 29.3$ m/s ◀

(*b*) We have $$\frac{dv}{dt} = a = -19\sqrt{v}$$

At $t = 0, v = 81$ m/s: $$\int_{81}^{v} \frac{dv}{\sqrt{v}} = \int_0^t -19dt$$

or $$2[\sqrt{v}]_{81}^{v} = -19t$$

or $$2(\sqrt{v} - 9) = -19t$$

When $v = 0$: $$2(-9) = -19t$$

or

$t = 0.947$ s ◀

## PROBLEM 11.25

The acceleration of a particle is defined by the relation $a = -kv^{2.5}$, where $k$ is a constant. The particle starts at $x = 0$ with a velocity of 16 mm/s, and when $x = 6$ mm the velocity is observed to be 4 mm/s. Determine (*a*) the velocity of the particle when $x = 5$ mm, (*b*) the time at which the velocity of the particle is 9 mm/s.

## SOLUTION

Acceleration: $a = -kv^{2.5}$

Given: at t = 0, $x = 0$ mm, $v = 16$ mm/s

at $x = 6$ mm, $v = 4$ mm/s

$$adx = vdv$$

$$-kv^{2.5}dx = vdv$$

Separate variables $-kdx = v^{-3/2}dv$

Integrate using x = −0 m and v = 16 mm/s as the lower limits of the integrals

$$\int_0^x -\mathrm{k}dx = \int_{16}^v v^{-3/2}dv$$

$$-kx\Big|_0^x = -2v^{-1/2}\Big|_{16}^v$$

Velocity and position: $kx = 2v^{-1/2} - \frac{1}{2}$ (1)

Now substitute v = 4 mm/s and x = 6 mm into (1) and solve for k

$$k(6) = 4^{-1/2} - \frac{1}{2}$$

$$k = 0.0833 \text{ mm}^{-3/2}s^{1/2}$$

(*a*) Find velocity when x = 5 mm using the equation (1) and the value of k

$$k(5) = 2v^{-1/2} - \frac{1}{2}$$

$v = 4.76$ mm/s ◀

*(b)* $a = \frac{dv}{dt}$ or $adt = dv$

$$a = \frac{dv}{dt} \text{ or } adt = dv$$

Separate variables $-kdt = v^{-2.5}dv$

## PROBLEM 11.25 (CONTINUED)

Integrate using t = 0 and v = 16 mm/s as the lower limits of the integrals

$$\int_0^t -\mathrm{k}dt = \int_{16}^v v^{-5/2} dv$$

$$-kt\Big|_0^t = -\frac{2}{3} v^{-3/2}\Big|_{16}^v$$

Velocity and time: $$kt = \frac{2}{3} v^{-3/2} - \frac{1}{96} \qquad (2)$$

Find time when v = 9 mm/s using the equation (2) and the value of k

$$kt = \frac{2}{3}(9)^{-3/2} - \frac{1}{96}$$

$t = 0.171$ s ◀


**Problem 11.26**

A human powered vehicle (HPV) team wants to model the acceleration during the 260 *m* sprint race (the first 60 *m* is called a flying start) using $a = A - Cv^2$, where $a$ is acceleration in m/s$^2$ and $v$ is the velocity in m/s. From wind tunnel testing, they found that C = 0.0012 m$^{-1}$. Knowing that the cyclist is going 100 km/h at the 260 meter mark, what is the value of $A$?

**SOLUTION**

Acceleration: $a = A - Cv^2$ m/s$^2$

Given: $C = 0.0012\ \text{m}^{-1},\ v_f = 100$ km/hr when $x_f = 260$ m

Note: 100 km/hr = 27.78 m/s

$$adx = vdv$$

$$\left(A - Cv^2\right)dx = vdv$$

Separate variables
$$dx = \frac{vdv}{A - Cv^2}$$

Integrate starting from rest and traveling a distance $x_f$ with a final velocity $v_f$.

$$\int_0^{x_f} dx = \int_0^{v_f} \frac{vdv}{A - Cv^2}$$

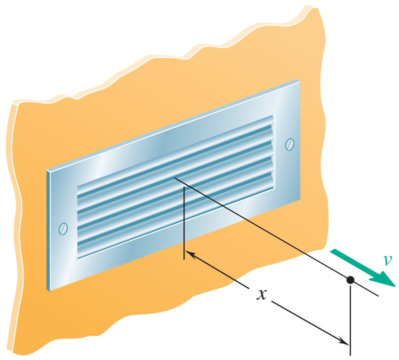
$$x\big|_0^{x_f} = -\frac{1}{2C}\ln\left(A - Cv^2\right)\Big|_0^{v_f}$$

$$x_f = -\frac{1}{2C}\ln\left(A - v_f^2\right) + \frac{1}{2C}\ln(A)$$

$$x_f = \frac{1}{2C}\ln\left(\frac{A}{A - Cv_f^2}\right)$$

Next solve for A
$$A = -\frac{Cv_f^2 e^{2Cx_f}}{1 - e^{2Cx_f}}$$

Now substitute values of C, $x_f$ and $v_f$ and solve for A $\quad A = 1.995$ m/s$^2$ ◀




## PROBLEM 11.27

Experimental data indicate that in a region downstream of a given louvered supply vent the velocity of the emitted air is defined by $v = 0.18v_0/x$, where $v$ and $x$ are expressed in m/s and meters, respectively, and $v_0$ is the initial discharge velocity of the air. For $v_0 = 3.6$ m/s, determine (*a*) the acceleration of the air at $x = 2$ m, (*b*) the time required for the air to flow from $x = 1$ to $x = 3$ m.

## SOLUTION

(*a*) We have

$$a = v\frac{dv}{dx}$$

$$= \frac{0.18v_0}{x}\frac{d}{dx}\left(\frac{0.18v_0}{x}\right)$$

$$= -\frac{0.0324v_0^2}{x^3}$$

When $x = 2$ m:

$$a = -\frac{0.0324(3.6)^2}{(2)^3}$$

or $a = -0.0525 \text{ m/s}^2$ ◀

(*b*) We have

$$\frac{dx}{dt} = v = \frac{0.18v_0}{x}$$

From $x = 1$ m to $x = 3$ m:

$$\int_1^3 x dx = \int_{t_1}^{t_3} 0.18v_0\, dt$$

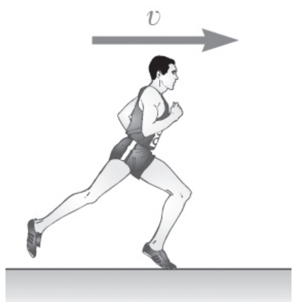
or

$$\left[\frac{1}{2}x^2\right]_1^3 = 0.18v_0(t_3 - t_1)$$

or

$$(t_3 - t_1) = \frac{\frac{1}{2}(9-1)}{0.18(3.6)}$$

or $t_3 - t_1 = 6.17$ s ◀




## PROBLEM 11.28

Based on observations, the speed of a jogger can be approximated by the relation $v = 7.5(1-0.04x)^{0.3}$, where $v$ and $x$ are expressed in km/h and kilometers, respectively. Knowing that $x = 0$ at $t = 0$, determine ($a$) the distance the jogger has run when $t = 1$ h, ($b$) the jogger's acceleration in m/s$^2$ at $t = 0$, ($c$) the time required for the jogger to run 6 km.

## SOLUTION

Given: $v = 7.5\left(1-0.04x\right)^{0.3}$ with units km and km/h

(a) Distance at $t = 1$ hr.

$$\text{Using } dx = v\,dt, \text{ we get } dt = \frac{dx}{v} = \frac{dx}{7.5(1-0.04x)^{0.3}}$$

Integrating, using $t = 0$ when $x = 0$,

$$\int_0^t dt = \frac{1}{7.5}\int_0^x \frac{dx}{\left(1-0.04\right)^{0.3}} \quad \text{or} \quad [t]_0^t = \frac{1}{(7.5)}\cdot\frac{-1}{(0.7)(0.04)}\left\{1-0.04x^{0.7}\right\}\Big|_0^x$$

$$t = 4.7619\left\{1-\left(1-0.04x\right)^{0.7}\right\} \tag{1}$$

Solving for $x$, $$x = 25\left\{1-\left(1-0.210t\right)^{1/0.7}\right\}$$

When t = 1 h $$x = 25\left\{1-\left[1-\left(0.210\right)\left(1\right)\right]^{1/0.7}\right\}$$ $x = 7.15$ km ◀

(b) Acceleration when $t = 0$.

$$\frac{dv}{dx} = (7.5)(0.3)(-0.04)(1-0.04x)^{-0.7} = -0.0900(1-0.04x)^{-0.7}$$

When $t = 0$ and $x = 0$, $\quad v = 7.5$ km/h, $\quad \dfrac{dv}{dx} - 0.0900\ \text{h}^{-1}$

$$a = v\frac{dv}{dx} = (7.5)(-0.0900) = -0.675\ \text{km/h}^2$$

## PROBLEM 11.28 (CONTINUED)

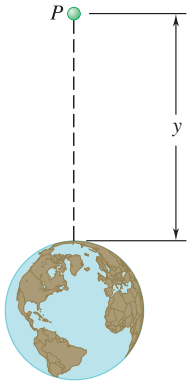
$$= -\frac{(0.675)(1000)}{(3600)^2}\ \text{m/s}^2 \qquad a = -52.1 \times 10^{-6}\ \text{m/s}^2 \blacktriangleleft$$

*(c)* Time to run 6 km.

Using $x = 6\ \text{km}$ in equation (1),

$$t = 4.7619\left\{1 - \left[1 - (0.04)(6)\right]^{0.7}\right\} = 0.8323\ \text{h}$$

$$t = 49.9\ \text{min} \blacktriangleleft$$




## PROBLEM 11.29

The acceleration due to gravity at an altitude $y$ above the surface of the earth can be expressed as

$$a = \frac{-32.2}{[1 + (y/20.9 \times 10^6)]^2}$$

where $a$ and $y$ are expressed in ft/s$^2$ and feet, respectively. Using this expression, compute the height reached by a projectile fired vertically upward from the surface of the earth if its initial velocity is ($a$) 1800 ft/s, ($b$) 3000 ft/s, ($c$) 36,700 ft/s.

## SOLUTION

We have
$$v\frac{dv}{dy} = a = -\frac{32.2}{\left(1 + \frac{y}{20.9\times 10^6}\right)^2}$$

When
$$y = 0, \qquad v = v_0$$

provided that $v$ does reduce to zero,
$$y = y_{\max}, \quad v = 0$$

Then
$$\int_{v_0}^{0} v\,dv = \int_0^{y_{\max}} \frac{-32.2}{\left(1 + \frac{y}{20.9\times 10^6}\right)^2}\,dy$$

or
$$-\frac{1}{2}v_0^2 = -32.2\left[-20.9\times 10^6 \frac{1}{1 + \frac{y}{20.9\times 10^6}}\right]_0^{y_{\max}}$$

or
$$v_0^2 = 1345.96\times 10^6\left(1 - \frac{1}{1 + \frac{y_{\max}}{20.9\times 10^6}}\right)$$

or
$$y_{\max} = \frac{v_0^2}{64.4 - \frac{v_0^2}{20.9\times 10^6}}$$

($a$) $v_0 = 1800$ ft/s:
$$y_{\max} = \frac{(1800)^2}{64.4 - \frac{(1800)^2}{20.9\times 10^6}}$$

or
$$y_{\max} = 50.4\times 10^3 \text{ ft} \blacktriangleleft$$

## PROBLEM 11.29 (CONTINUED)

(*b*) $v_0 = 3000$ ft/s: $$y_{\max} = \frac{(3000)^2}{64.4 - \frac{(3000)^2}{20.9\times 10^6}}$$

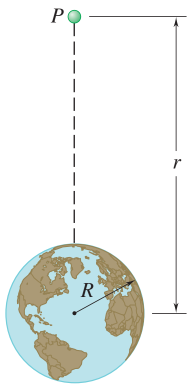
or $$y_{\max} = 140.7\times 10^3 \text{ ft} \blacktriangleleft$$

(*c*) $v_0 = 36,700$ ft/s: $$y_{\max} = \frac{(36,700)^2}{64.4 - \frac{(36,700)^2}{20.9\times 10^6}} = -3.03\times 10^{10} \text{ ft}$$

This solution is invalid since the velocity does not reduce to zero. The velocity 36,700 ft/s is above the escape velocity $v_R$ from the earth. For $v_R$ and above.

$$y_{\max} \longrightarrow \infty \blacktriangleleft$$

## PROBLEM 11.30




The acceleration due to gravity of a particle falling toward the earth is $a = -gR^2/r^2$, where $r$ is the distance from the *center* of the earth to the particle, $R$ is the radius of the earth, and $g$ is the acceleration due to gravity at the surface of the earth. If $R = 3960$ mi, calculate the *escape velocity,* that is, the minimum velocity with which a particle must be projected vertically upward from the surface of the earth if it is not to return to the earth. (*Hint:* $v = 0$ for $r = \infty$.)

## SOLUTION

We have
$$v\frac{dv}{dr} = a = -\frac{gR^2}{r^2}$$

When
$$r = R, \quad v = v_e$$
$$r = \infty, \quad v = 0$$

then
$$\int_{v_e}^{0} v\,dv = \int_{R}^{\infty} -\frac{gR^2}{r^2}\,dr$$

or
$$-\frac{1}{2}v_e^2 = gR^2\left[\frac{1}{r}\right]_R^{\infty}$$

or
$$v_e = \sqrt{2gR}$$
$$= \left(2 \times 32.2 \text{ ft/s}^2 \times 3960 \text{ mi} \times \frac{5280 \text{ ft}}{1 \text{ mi}}\right)^{1/2}$$

or
$$v_e = 36.7 \times 10^3 \text{ ft/s} \blacktriangleleft$$

## PROBLEM 11.31

The velocity of a particle is $v = v_0[1 - \sin(\pi t/T)]$. Knowing that the particle starts from the origin with an initial velocity $v_0$, determine (*a*) its position and its acceleration at $t = 3T$, (*b*) its average velocity during the interval $t = 0$ to $t = T$.

## SOLUTION

(*a*) We have

$$\frac{dx}{dt} = v = v_0\left[1 - \sin\left(\frac{\pi t}{T}\right)\right]$$

At $t = 0, x = 0$:

$$\int_0^x dx = \int_0^t v_0\left[1 - \sin\left(\frac{\pi t}{T}\right)\right]dt$$

$$x = v_0\left[t + \frac{T}{\pi}\cos\left(\frac{\pi t}{T}\right)\right]_0^t = v_0\left[t + \frac{T}{\pi}\cos\left(\frac{\pi t}{T}\right) - \frac{T}{\pi}\right] \qquad (1)$$

At $t = 3T$:

$$x_{3T} = v_0\left[3T + \frac{T}{\pi}\cos\left(\frac{\pi \times 3T}{T}\right) - \frac{T}{\pi}\right] = v_0\left(3T - \frac{2T}{\pi}\right) \qquad x_{3T} = 2.36\, v_0 T \blacktriangleleft$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left\{v_0\left[1 - \sin\left(\frac{\pi t}{T}\right)\right]\right\} = -v_0\frac{\pi}{T}\cos\frac{\pi t}{T}$$

At $t = 3T$:

$$a_{3T} = -v_0\frac{\pi}{T}\cos\frac{\pi \times 3T}{T} \qquad a_{3T} = \frac{\pi v_0}{T} \blacktriangleleft$$

(*b*) Using Eq. (1)

At $t = 0$:

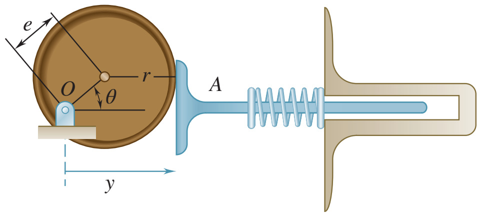
$$x_0 = v_0\left[0 + \frac{T}{\pi}\cos(0) - \frac{T}{\pi}\right] = 0$$

At $t = T$:

$$x_T = v_0\left[T + \frac{T}{\pi}\cos\left(\frac{\pi T}{T}\right) - \frac{T}{\pi}\right] = v_0\left(T - \frac{2T}{\pi}\right) = 0.363 v_0 T$$

Now

$$v_{\text{ave}} = \frac{x_T - x_0}{\Delta t} = \frac{0.363 v_0 T - 0}{T - 0} \qquad v_{\text{ave}} = 0.363 v_0 \blacktriangleleft$$




## Problem 11.32

An eccentric circular cam, which serves a similar function as the Scotch yoke mechanism in Problem 11.13, is used in conjunction with a flat face follower to control motion in pumps and in steam engine valves. Knowing that the eccentricity is denoted by $e$, the maximum range of the displacement of the follower is $d_{max}$, and the maximum velocity of the follower is $v_{max}$, determine the displacement, velocity, and acceleration of the follower.

## SOLUTION

Constraint: $$y = r + e\cos\theta \qquad (1)$$

Differentiate: $$\dot{y} = -e\dot{\theta}\sin\theta \qquad (2)$$

Differentiate again: $$\ddot{y} = -e\ddot{\theta}\sin\theta - e\dot{\theta}^2\cos\theta \qquad (3)$$

$y_{max}$ occurs when $\cos\theta = 1$ and $y_{min}$ occurs when $\cos\theta = -1$

$$d_{max} = y_{max} - y_{min}$$
$$d_{max} = r + e - (r - e)$$
$$d_{max} = 2e$$
$$e = \frac{d_{max}}{2} \qquad (4)$$

Substitute (4) into (1) to get Position $$y = r + \frac{d_{max}}{2}\cos\theta \blacktriangleleft$$

Max Velocity occurs when $\sin\theta = \pm 1$

$$v_{max} = \mp e\dot{\theta}$$

$$\dot{\theta} = \frac{v_{max}}{\mp e} \qquad (5)$$

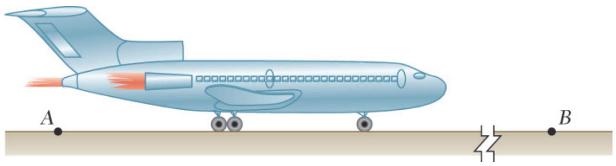
Substitute (5) into (2) to get velocity and assume cw rotation. $$\dot{y} = -v_{max}\sin\theta$$

Substitute (4) and (5) into (3)

$$\ddot{y} = -\frac{d_{max}}{2}\ddot{\theta}\sin\theta - \frac{d_{max}}{2}\left(\frac{4v_{max}^2}{d_{max}^2}\right)\cos\theta$$

Acceleration: $$\ddot{y} = -\frac{d_{max}}{2}\ddot{\theta}\sin\theta - \frac{2v_{max}^2}{d_{max}}\cos\theta \blacktriangleleft$$

## PROBLEM 11.33




An airplane begins its take-off run at $A$ with zero velocity and a constant acceleration $a$. Knowing that it becomes airborne 30 s later at $B$ with a take-off velocity of 270 km/h, determine ($a$) the acceleration $a$, ($b$) distance $AB$.

## SOLUTION

Since it is constant acceleration you can find the acceleration from the distance over time

$$a = dv / dt$$

$$270\,\text{km} = 75\,\text{m/s}$$

$$a = \frac{75\,\text{m/s}}{30\,\text{s}} = 2.5\,\text{m/s}^2$$

$$a = 2.50 \ \text{m/s}^2 \blacktriangleleft$$

Next find the distance

$$x = \frac{1}{2}at^2 + v_o t + x_0$$

$$x = \frac{1}{2}2.5(30)^2 + 0t + 0 = 1125$$

$$x = 1125 \text{ m} \blacktriangleleft$$

## PROBLEM 11.34

A minivan is tested for acceleration and braking. In the street-start acceleration test, elapsed time is 8.2 s for a velocity increase from 10 km/h to 100 km/h. In the braking test, the distance traveled is 44 m during braking to a stop from 100 km/h. Assuming constant values of acceleration and deceleration, determine (*a*) the acceleration during the street-start test, (*b*) the deceleration during the braking test.

## SOLUTION

$$10 \text{ km/h} = 2.7778 \text{ m/s} \qquad 100 \text{ km/h} = 27.7778 \text{ m/s}$$

*(a)* Acceleration during start test.

$$a = \frac{dv}{dt}$$

$$\int_0^{8.2} a\,dt = \int_{2.7778}^{27.7778} v\,dt$$

$$8.2a = 27.7778 - 2.7778 \qquad a = 3.05 \text{ m/s}^2 \blacktriangleleft$$

*(b)* Deceleration during braking.

$$a = v\frac{dv}{dx} =$$

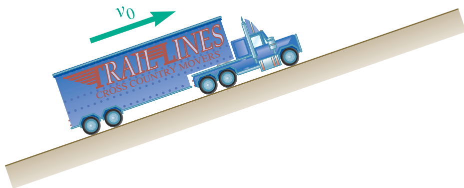
$$\int_0^{44} a\ dx = \int_{27.7778}^{0} v\ dv =$$

$$a(x)\Big|_0^{44} = \frac{1}{2}\left(v^2\right)\Big|_{27.7778}^{0}$$

$$44\ a = -\frac{1}{2}(27.7778)^2$$

$$a = -8.77 \text{ m/s}^2 \qquad \text{deceleration} = -a = 8.77 \text{ m/s}^2 \blacktriangleleft$$

## Problem 11.35




Steep safety ramps are built beside mountain highways to enable vehicles with defective brakes to stop safely. A truck enters a 750-ft ramp at a high speed $v_0$ and travels 540 ft in 6 s at constant deceleration before its speed is reduced to $v_0/2$. Assuming the same constant deceleration, determine (*a*) the additional time required for the truck to stop, (*b*) the additional distance traveled by the truck.

### SOLUTION

Given: $x_o = 0,\ x_A = 540\text{ m},\ t_A = 6\text{ s},\ v_A = \frac{1}{2}v_o,\ L_{ramp} = 750 ft$

Uniform Acceleration: $v_A = v_o + at_A$

Substitute known values: $\frac{1}{2}v_o = v_o + (a)(6)$

$$a = -\frac{1}{12}v_o$$

Uniform Acceleration: $x_A = x_o + v_o t_A + \frac{1}{2}at_A^2$

Substitute known values: $540 = 0 + v_o(6) - \frac{1}{24}v_o(6)^2$

$$v_o = 120\text{ ft/s and } a = -10\text{ ft/s}^2$$

(*a*) $v_B = v_o + at_B$

Substitute known values: $0 = 120 - 10t_B$

Solve for $t_B$: $t_B = 12\text{ s}$

Additional time to stop: $t_B - t_A = 6.0\text{ s}$ ◀

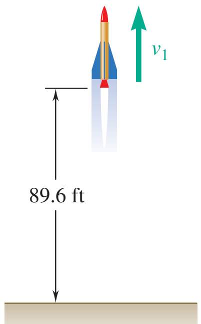
(*b*) $x_B = x_o + v_o t_B + \frac{1}{2}at_B^2$

Substitute known values: $x_B = 0 + 120(12) - \frac{1}{2}10(12)^2$

Solve for $x_B$: $x_B = 720\text{ ft}$

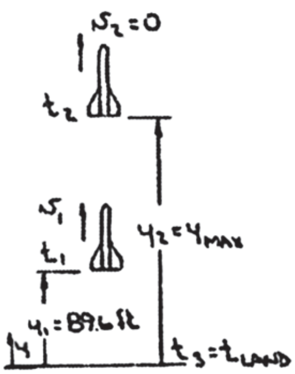
Additional time to stop: $x_B - x_A = 180.0\text{ ft}$ ◀

## PROBLEM 11.36




A group of students launches a model rocket in the vertical direction. Based on tracking data, they determine that the altitude of the rocket was 89.6 ft at the end of the powered portion of the flight and that the rocket landed 16 s later. Knowing that the descent parachute failed to deploy so that the rocket fell freely to the ground after reaching its maximum altitude and assuming that $g = 32.2\ \text{ft/s}^2$, determine (*a*) the speed $v_1$ of the rocket at the end of powered flight, (*b*) the maximum altitude reached by the rocket.

## SOLUTION




(*a*) We have $\quad y = y_1 + v_1 t + \frac{1}{2} a t^2$

At $t_{\text{land}}$, $\quad y = 0$

Then
$$0 = 89.6\ \text{ft} + v_1(16\ \text{s}) + \frac{1}{2}(-32.2\ \text{ft/s}^2)(16\ \text{s})^2$$

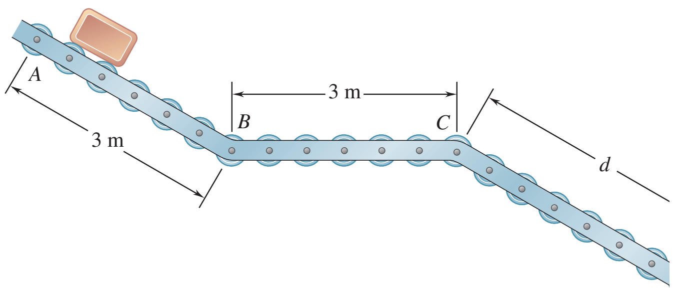
or $\quad v_1 = 252\ \text{ft/s}$ ◀

(*b*) We have $\quad v^2 = v_1^2 + 2a(y - y_1)$

At $\quad y = y_{\max}, \quad v = 0$

Then
$$0 = (252\ \text{ft/s})^2 + 2(-32.2\ \text{ft/s}^2)(y_{\max} - 89.6)\ \text{ft}$$

or $\quad y_{\max} = 1076\ \text{ft}$ ◀




## PROBLEM 11.37

A small package is released from rest at $A$ and moves along the skate wheel conveyor $ABCD$. The package has a uniform acceleration of $4.8\ \text{m/s}^2$ as it moves down sections $AB$ and $CD$, and its velocity is constant between $B$ and $C$. If the velocity of the package at $D$ is 7.2 m/s, determine (*a*) the distance $d$ between $C$ and $D$, (*b*) the time required for the package to reach $D$.

## SOLUTION

(*a*) For $A \longrightarrow B$ and $C \longrightarrow D$ we have

$$v^2 = v_0^2 + 2a(x - x_0)$$

Then, at $B$

$$v_{BC}^2 = 0 + 2(4.8\ \text{m/s}^2)(3 - 0)\ \text{m}$$

$$= 28.8\ \text{m}^2/\text{s}^2 \qquad (v_{BC} = 5.3666\ \text{m/s})$$

and at $D$

$$v_D^2 = v_{BC}^2 + 2a_{CD}(x_D - x_C) \qquad d = x_D - x_C$$

or

$$(7.2\ \text{m/s})^2 = (28.8\ \text{m}^2/\text{s}^2) + 2(4.8\ \text{m/s}^2)d$$

or

$$d = 2.40\ \text{m} \blacktriangleleft$$

(*b*) For $A \longrightarrow B$ and $C \longrightarrow D$ we have

$$v = v_0 + at$$

Then $A \longrightarrow B$

$$5.3666\ \text{m/s} = 0 + (4.8\ \text{m/s}^2)t_{AB}$$

or

$$t_{AB} = 1.11804\ \text{s}$$

and $C \longrightarrow D$

$$7.2\ \text{m/s} = 5.3666\ \text{m/s} + (4.8\ \text{m/s}^2)t_{CD}$$

or

$$t_{CD} = 0.38196\ \text{s}$$

Now, for $B \longrightarrow C$, we have

$$x_C = x_B + v_{BC}t_{BC}$$

or

$$3\ \text{m} = (5.3666\ \text{m/s})t_{BC}$$

or

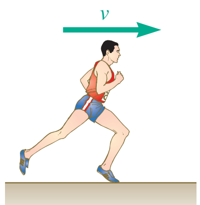
$$t_{BC} = 0.55901\ \text{s}$$

Finally,

$$t_D = t_{AB} + t_{BC} + t_{CD} = (1.11804 + 0.55901 + 0.38196)\ \text{s}$$

or

$$t_D = 2.06\ \text{s} \blacktriangleleft$$




## PROBLEM 11.38

A sprinter in a 100-m race accelerates uniformly for the first 35 m and then runs with constant velocity. If the sprinter's time for the first 35 m is 5.4 s, determine (*a*) his acceleration, (*b*) his final velocity, (*c*) his time for the race.

## SOLUTION

<u>Given</u>: $0 \le x \le 35$ m, $a = \text{constant}$

$35 \text{ m} < x \le 100$ m, $v = \text{constant}$

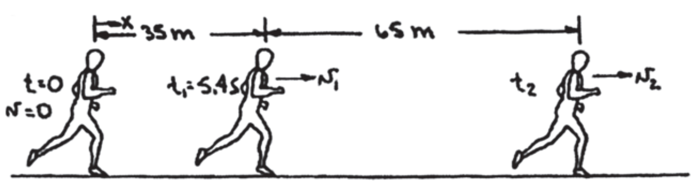
At $t = 0$, $v = 0$ when $x = 35$ m, $t = 5.4$ s

Find:

(*a*) $a$

(*b*) $v$ when $x = 100$ m

(*c*) $t$ when $x = 100$ m




(*a*) We have $$x = 0 + 0t + \frac{1}{2}at^2 \quad \text{for} \quad 0 \le x \le 35 \text{ m}$$

At $t = 5.4$ s: $$35 \text{ m} = \frac{1}{2}a(5.4 \text{ s})^2$$

or $$a = 2.4005 \text{ m/s}^2$$

$a = 2.40 \text{ m/s}^2$ ◀

(*b*) First note that $v = v_{max}$ for $35 \text{ m} \le x \le 100$ m.

Now $$v^2 = 0 + 2a(x - 0) \quad \text{for} \quad 0 \le x \le 35 \text{ m}$$

When $x = 35$ m: $$v_{max}^2 = 2(2.4005 \text{ m/s}^2)(35 \text{ m})$$

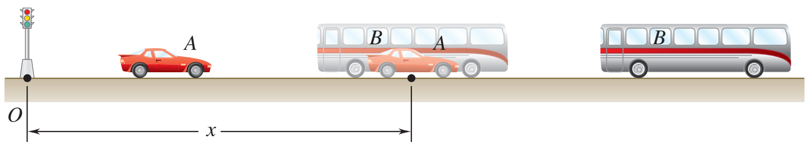
or $$v_{max} = 12.9628 \text{ m/s}$$

$v_{max} = 12.96$ m/s ◀

(*c*) We have $$x = x_1 + v_0(t - t_1) \quad \text{for} \quad 35 \text{ m} < x \le 100 \text{ m}$$

When $x = 100$ m: $$100 \text{ m} = 35 \text{ m} + (12.9628 \text{ m/s})(t_2 - 5.4) \text{ s}$$

or $t_2 = 10.41$ s ◀




## PROBLEM 11.39

Automobile $A$ starts from $O$ and accelerates at the constant rate of 0.75 m/s$^2$. A short time later it is passed by bus $B$ which is traveling in the opposite direction at a constant speed of 6 m/s. Knowing that bus $B$ passes point $O$ 20 s after automobile $A$ started from there, determine when and where the vehicles passed each other.

## SOLUTION

Place origin at 0.

Motion of auto. $\quad (x_A)_0 = 0, \quad (v_A)_0 = 0, \quad a_A = 0.75 \text{ m/s}^2$

$$x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2}a_A t^2 = 0 + 0 + \left(\frac{1}{2}\right)(0.75)t^2$$

$$x_A = 0.375t^2 \text{ m}$$

Motion of bus. $\quad (x_B)_0 = ?, \quad (v_B)_0 = -6 \text{ m/s}, \quad a_B = 0$

$$x_B = (x_B)_0 - (v_B)_0 t = (x_B)_0 - 6t \text{ m}$$

At $t = 20\,s, \quad x_B = 0.$

$$0 = (x_B)_0 - (6)(20) \qquad (x_B)_0 = 120 \text{ m}$$

Hence, $\quad x_B = 120 - 6t$

When the vehicles pass each other, $x_B = x_A$.

$$120 - 6t = 0.375\, t^2$$

$$0.375\, t^2 + 6t - 120 = 0$$

$$t = \frac{-6 \pm \sqrt{(6)^2 - (4)(0.375)(-120)}}{(2)(0.375)}$$

$$t = \frac{-6 \pm 14.697}{0.75} = 11.596 \text{ s} \quad \text{and} \quad -27.6 \text{ s}$$

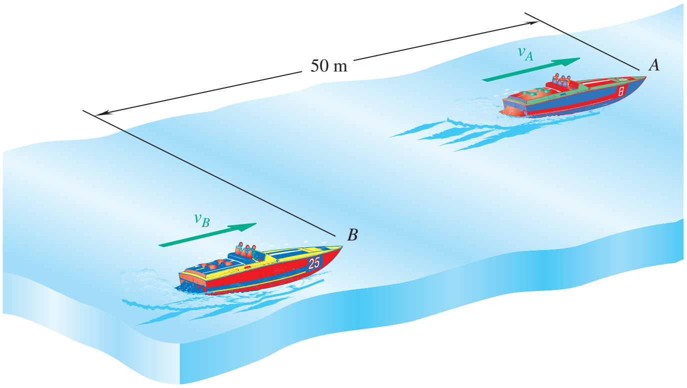
Reject the negative root. $\qquad t = 11.60 \text{ s}$ ◀

Corresponding values of $x_A$ and $x_B$.

$$x_A = (0.375)(11.596)^2 = 50.4 \text{ m}$$

$$x_B = 120 - (6)(11.596) = 50.4 \text{ m}$$

$x = 50.4$ m ◀




## PROBLEM 11.40

In a boat race, boat $A$ is leading boat $B$ by 50 m and both boats are traveling at a constant speed of 180 km/h. At $t = 0$, the boats accelerate at constant rates. Knowing that when $B$ passes $A$, $t = 8$ s and $v_A = 225$ km/h, determine ($a$) the acceleration of $A$, ($b$) the acceleration of $B$.

## SOLUTION

($a$) We have

$$v_A = (v_A)_0 + a_A t$$

$$(v_A)_0 = 180 \text{ km/h} = 50 \text{ m/s}$$

At $t = 8$ s:

$$v_A = 225 \text{ km/h} = 62.5 \text{ m/s}$$

Then

$$62.5 \text{ m/s} = 50 \text{ m/s} + a_A(8 \text{ s})$$

or

$$a_A = 1.563 \text{ m/s}^2 \blacktriangleleft$$

($b$) We have

$$x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2}a_A t^2 = 50 \text{ m} + (50 \text{ m/s})(8 \text{ s}) + \frac{1}{2}(1.5625 \text{ m/s}^2)(8 \text{ s})^2 = 500 \text{ m}$$

and

$$x_B = 0 + (v_B)_0 t + \frac{1}{2}a_B t^2 \qquad (v_B)_0 = 50 \text{ m/s}$$

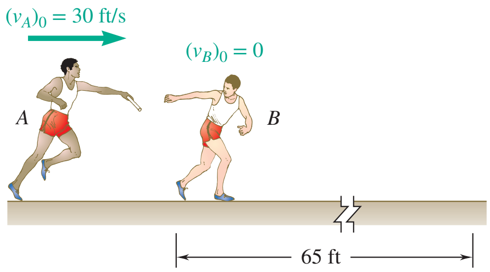
At $t = 8$ s:

$$x_A = x_B$$

$$500 \text{ m} = (50 \text{ m/s})(8 \text{ s}) + \frac{1}{2}a_B(8 \text{ s})^2$$

or

$$a_B = 3.13 \text{ m/s}^2 \blacktriangleleft$$




## PROBLEM 11.41

As relay runner $A$ enters the 65-ft-long exchange zone with a speed of 30 ft/s, he begins to slow down. He hands the baton to runner $B$ 2.5 s later as they leave the exchange zone with the same velocity. Determine ($a$) the uniform acceleration of each of the runners, ($b$) when runner $B$ should begin to run.

## SOLUTION

Let $x = 0$ at the start of the exchange zone, and $t = 0$ as runner $A$ enters the exchange zone.

Then, $$(v_A)_0 = 30 \text{ ft/s}.$$

Motion of runner $A$: $$v_A = (v_A)_0 + a_A t$$

$$x_A = (v_A)_0 t + \frac{1}{2} a_A t^2$$

($a$) Solving for $a_A$, and noting that $x_A = 65$ ft when $t = 2.5$ s

$$a_A = \frac{2\left[x_A - (v_A)_0 t\right]}{t^2} = \frac{2[65 - (30)(2.5)]}{2.5^2} \qquad a_A = -3.20 \text{ ft/s}^2 \blacktriangleleft$$

At $t = 2.5$ s, $$(v_A)_f = 30 + (-3.20)(2.5) = 22.0 \text{ ft/s}$$

Motion of runner $B$: $(v_B)_0 = 0$ and $x_B = 0$ at the starting time $t_B$ of runner $B$.

Then, $$v_B^2 - (v_B)_0^2 = 2a_B x_B$$

Solving for $a_B$, and noting that $v_B = (v_A)_f = 22.0$ ft/s

When $x_B = 65$ ft, $$a_B = \frac{(v_B)^2 - (v_B)_0^2}{2x_B} = \frac{22.0^2 - 0}{(2)(65)} \qquad a_B = 3.723 \text{ ft/s}^2 \blacktriangleleft$$

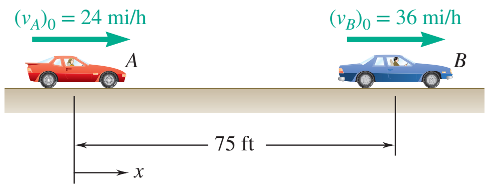
$$v_B = (v_B)_0 + a_B(t - t_B) = a_B(t - t_B)$$

$$t - t_B = \frac{v_B}{a_B}$$

($b$) Solving for $t_B$, and noting that $v_B = 22.0$ ft/s when $t = 2.5$ s

$$t_B = t - \frac{v_B}{a_B} = 2.5 - \frac{22.0}{3.723} = -3.41 \text{ s}$$

Runner $B$ should begin 3.41 s before runner $A$ reaches the exchange zone. ◀




## PROBLEM 11.42

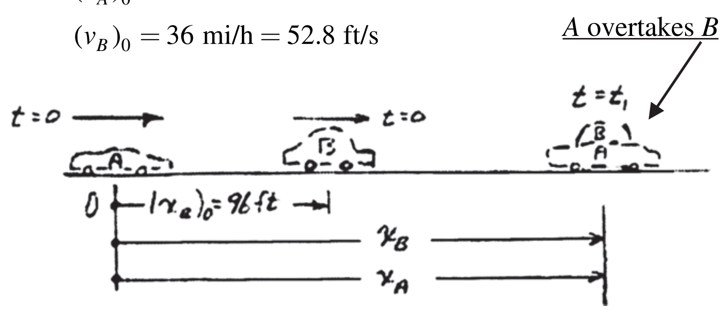
Automobiles $A$ and $B$ are traveling in adjacent highway lanes and at $t = 0$ have the positions and speeds shown. Knowing that automobile $A$ has a constant acceleration of 1.8 ft/s$^2$ and that $B$ has a constant deceleration of 1.2 ft/s$^2$, determine (*a*) when and where $A$ will overtake $B$, (*b*) the speed of each automobile at that time.

## SOLUTION

$$a_A = +1.8 \text{ ft/s}^2 \quad a_B = -1.2 \text{ ft/s}^2$$

$$(v_A)_0 = 24 \text{ mi/h} = 35.2 \text{ ft/s}$$

$$(v_B)_0 = 36 \text{ mi/h} = 52.8 \text{ ft/s}$$




Motion of auto $A$:

$$v_A = (v_A)_0 + a_A t = 35.2 + 1.8t \tag{1}$$

$$x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2 = 0 + 35.2t + \frac{1}{2}(1.8)t^2 \tag{2}$$

Motion of auto $B$:

$$v_B = (v_B)_0 + a_B t = 52.8 - 1.2t \tag{3}$$

$$x_B = (x_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2 = 75 + 52.8t + \frac{1}{2}(-1.2)t^2 \tag{4}$$

(*a*) $A$ overtakes $B$ at $t = t_1$.

$$x_A = x_B: \quad 35.2t + 0.9t_1^2 = 75 + 52.8t_1 - 0.6t_1^2$$

$$1.5t_1^2 - 17.6t_1 - 75 = 0$$

$$t_1 = -3.22 \text{ s} \quad \text{and} \quad t_1 = 15.0546 \qquad t_1 = 15.05 \text{ s} \blacktriangleleft$$

Eq. (2): $$x_A = 35.2(15.05) + 0.9(15.05)^2 \qquad x_A = 734 \text{ ft} \blacktriangleleft$$

## PROBLEM 11.42 (CONTINUED)

(*b*) Velocities when $t_1 = 15.05$ s

Eq. (1): $v_A = 35.2 + 1.8(15.05)$

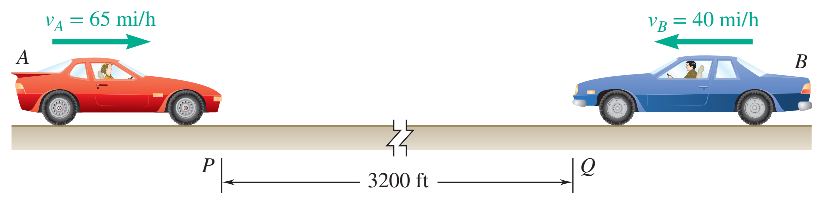
$v_A = 62.29$ ft/s $\qquad v_A = 42.5$ mi/h $\longrightarrow$ ◀

Eq. (3): $v_B = 52.8 - 1.2(15.05)$

$v_B = 34.74$ ft/s $\qquad v_B = 23.7$ mi/h $\longrightarrow$ ◀

## PROBLEM 11.43

Two automobiles $A$ and $B$ are approaching each other in adjacent highway lanes. At $t = 0$, $A$ and $B$ are 3200 ft apart, their speeds are $v_A = 65$ mi/h and $v_B = 40$ mi/h, and they are at Points $P$ and $Q$, respectively. Knowing that $A$ passes Point $Q$ 40 s after $B$ was there and that $B$ passes Point $P$ 42 s after $A$ was there, determine ($a$) the uniform accelerations of $A$ and $B$, ($b$) when the vehicles pass each other, ($c$) the speed of $B$ at that time.




## SOLUTION

($a$) We have $\quad x_A = 0 + (v_A)_0 t + \frac{1}{2} a_A t^2 \quad (v_A)_0 = 65 \text{ mi/h} = 95.33 \text{ ft/s}$

($x$ is positive $\longrightarrow$; origin at $P$.)

At $t = 40$ s: $\quad 3200 \text{ m} = (95.333 \text{ m/s})(40 \text{ s}) + \frac{1}{2} a_A (40 \text{ s})^2 \qquad a_A = -0.767 \text{ ft/s}^2$ ◀

Also, $x_B = 0 + (v_B)_0 t + \frac{1}{2} a_B t^2 \quad (v_B)_0 = 40 \text{ mi/h} = 58.667 \text{ ft/s}$

($x_B$ is positive $\longleftarrow$; origin at $Q$.)

At $t = 42$ s: $\quad 3200 \text{ ft} = (58.667 \text{ ft/s})(42 \text{ s}) + \frac{1}{2} a_B (42 \text{ s})^2$

or $\quad a_B = 0.83447 \text{ ft/s}^2 \qquad a_B = 0.834 \text{ ft/s}^2$ ◀

($b$) When the cars pass each other $\quad x_A + x_B = 3200$ ft

Then $(95.333 \text{ ft/s})t_{AB} + \frac{1}{2}(-0.76667 \text{ ft/s})t_{AB}^2 + (58.667 \text{ ft/s})t_{AB} + \frac{1}{2}(0.83447 \text{ ft/s}^2)t_{AB}^2 = 3200 \text{ ft}$

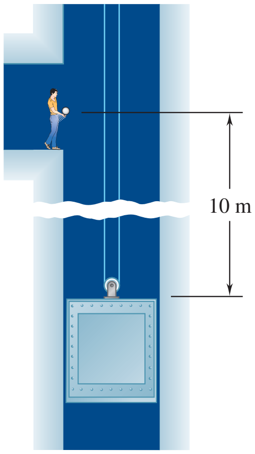
or $\quad 0.03390 t_{AB}^2 + 154 t_{AB} - 3200 = 0$

Solving $\quad t = 20.685$ s and $t = -4563$ s $\qquad t > 0 \Rightarrow \qquad t_{AB} = 20.7$ s ◀

($c$) We have $\quad v_B = (v_B)_0 + a_B t$

At $t = t_{AB}$: $\quad v_B = 58.667 \text{ ft/s} + (0.83447 \text{ ft/s}^2)(20.685 \text{ s})$

$= 75.927$ ft/s $\qquad v_B = 51.8$ mi/h ◀




## PROBLEM 11.44

An elevator is moving upward at a constant speed of 4 m/s. A man standing 10 m above the top of the elevator throws a ball upward with a speed of 3 m/s. Determine (*a*) when the ball will hit the elevator, (*b*) where the ball will hit the elevator with respect to the location of the man.

## SOLUTION

Place the origin of the position coordinate at the level of the standing man, the positive direction being up. The ball undergoes uniformly accelerated motion.

$$y_B = (y_B)_0 + (v_B)_0 t - \frac{1}{2} g t^2$$

with $(y_B)_0 = 0$, $(v_B)_0 = 3$ m/s, and $g = 9.81$ m/s$^2$.

$$y_B = 3t - 4.905t^2$$

The elevator undergoes uniform motion.

$$y_E = (y_E)_0 + v_E t$$

with $(y_E)_0 = -10$ m and $v_E = 4$ m/s.

(*a*) Time of impact. Set $y_B = y_E$

$$3t - 4.905t^2 = -10 + 4t$$

$$4.905t^2 + t - 10 = 0$$

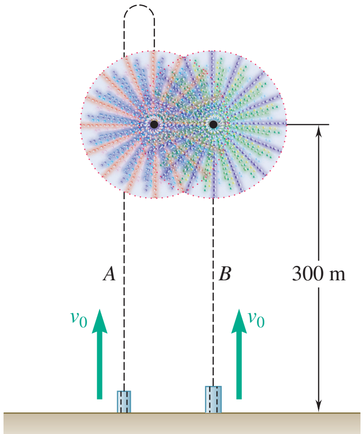
$t = 1.3295$ and $-1.5334$ $\qquad t = 1.330$ s ◀

(*b*) Location of impact.

$$y_B = (3)(1.3295) - (4.905)(1.3295)^2 = -4.68 \text{ m}$$

$$y_E = -10 + (4)(1.3295) = -4.68 \text{ m} \qquad \text{(checks)}$$

4.68 m below the man ◀




## PROBLEM 11.45

Two rockets are launched at a fireworks display. Rocket $A$ is launched with an initial velocity $v_0 = 100$ m/s and rocket $B$ is launched $t_1$ seconds later with the same initial velocity. The two rockets are timed to explode simultaneously at a height of 300 m as $A$ is falling and $B$ is rising. Assuming a constant acceleration $g = 9.81\ \text{m/s}^2$, determine (*a*) the time $t_1$, (*b*) the velocity of $B$ relative to $A$ at the time of the explosion.

## SOLUTION

Place origin at ground level. The motion of rockets $A$ and $B$ is

Rocket $A$:
$$v_A = (v_A)_0 - gt = 100 - 9.81t \tag{1}$$

$$y_A = (y_A)_0 + (v_A)_0 t - \frac{1}{2}gt^2 = 100t - 4.905t^2 \tag{2}$$

Rocket $B$:
$$v_B = (v_B)_0 - g(t - t_1) = 100 - 9.81(t - t_1) \tag{3}$$

$$y_B = (y_B)_0 + (v_B)_0(t - t_1) - \frac{1}{2}g(t - t_1)^2$$

$$= 100(t - t_1) - 4.905(t - t_1)^2 \tag{4}$$

Time of explosion of rockets $A$ and $B$. $y_A = y_B = 300$ ft

From (2),
$$300 = 100t - 4.905t^2$$
$$4.905t^2 - 100t + 300 = 0$$
$$t = 16.732 \text{ s} \quad \text{and} \quad 3.655 \text{ s}$$

From (4),
$$300 = 100(t - t_1) - 4.905(t - t_1^2)$$
$$t - t_1 = 16.732 \text{ s} \quad \text{and} \quad 3.655 \text{ s}$$

Since rocket $A$ is falling, $t = 16.732$ s

Since rocket $B$ is rising, $t - t_1 = 3.655$ s

(*a*) Time $t_1$: $t_1 = t - (t - t_1)$ $\qquad t_1 = 13.08 \text{ s}$ ◀

(*b*) Relative velocity at explosion.

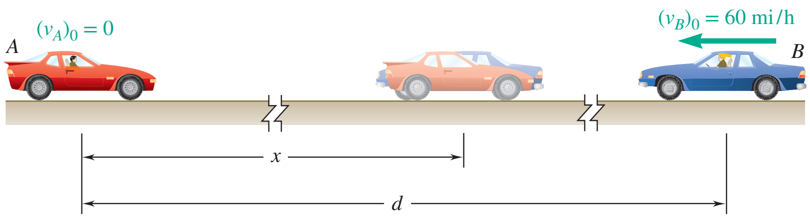
From (1), $v_A = 100 - (9.81)(16.732) = -64.15$ m/s

From (3), $v_B = 100 - (9.81)(16.732 - 13.08) = 64.15$ m/s

Relative velocity: $v_{B/A} = v_B - v_A$ $\qquad v_{B/A} = 128.3 \text{ m/s} \uparrow$ ◀

## PROBLEM 11.46

Car $A$ is parked along the northbound lane of a highway, and car $B$ is traveling in the southbound lane at a constant speed of 60 mi/h. At $t = 0$, $A$ starts and accelerates at a constant rate $a_A$, while at $t = 5$ s, $B$ begins to slow down with a constant deceleration of magnitude $a_A/6$. Knowing that when the cars pass each other $x = 294$ ft and $v_A = v_B$, determine ($a$) the acceleration $a_A$, ($b$) when the vehicles pass each other, ($c$) the distance $d$ between the vehicles at $t = 0$.




## SOLUTION


For $t \geq 0$:
$$v_A = 0 + a_A t$$
$$x_A = 0 + 0 + \frac{1}{2} a_A t^2$$

$0 \leq t < 5$ s:
$$x_B = 0 + (v_B)_0 t \quad (v_B)_0 = 60 \text{ mi/h} = 88 \text{ ft/s}$$

At $t = 5$ s:
$$x_B = (88 \text{ ft/s})(5 \text{ s}) = 440 \text{ ft}$$

For $t \geq 5$ s:
$$v_B = (v_B)_0 + a_B(t-5) \qquad a_B = -\frac{1}{6} a_A$$
$$x_B = (x_B)_5 + (v_B)_0(t-5) + \frac{1}{2} a_B (t-5)^2$$

Assume $t > 5$ s when the cars pass each other.

At that time $(t_{AB})$,

$v_A = v_B$:
$$a_A t_{AB} = (88 \text{ ft/s}) - \frac{a_A}{6}(t_{AB} - 5)$$

$x_A = 294$ ft:
$$294 \text{ ft} = \frac{1}{2} a_A t_{AB}^2$$

Then
$$\frac{a_A\left(\frac{7}{6} t_{AB} - \frac{5}{6}\right)}{\frac{1}{2} a_A t_{AB}^2} = \frac{88}{294}$$

or
$$44 t_{AB}^2 - 343 t_{AB} + 245 = 0$$

## PROBLEM 11.46 (CONTINUED)

Solving $t_{AB} = 0.795 \text{ s} \quad \text{and} \quad t_{AB} = 7.00 \text{ s}$

(*a*) With $t_{AB} > 5$ s, $294 \text{ ft} = \frac{1}{2}a_A(7.00 \text{ s})^2$

or $a_A = 12.00 \text{ ft/s}^2$ ◀

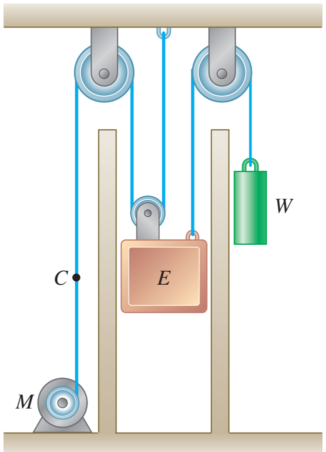
(*b*) From above $t_{AB} = 7.00$ s ◀

*Note:* An acceptable solution cannot be found if it is assumed that $t_{AB} \leq 5$ s.

(*c*) We have

$$d = x + (x_B)_{t_{AB}}$$
$$= 294 \text{ ft} + 440 \text{ ft} + (88 \text{ ft/s})(2.00 \text{ s})$$
$$+ \frac{1}{2}\left(-\frac{1}{6} \times 12.00 \text{ ft/s}^2\right)(2.00 \text{ s})^2$$

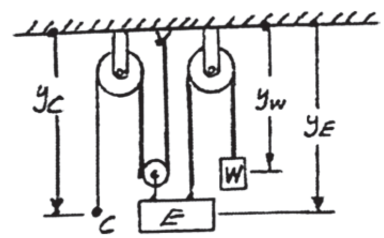
or $d = 906$ ft ◀




## PROBLEM 11.47

The elevator shown in the figure moves downward with a constant velocity of 4 m/s. Determine (*a*) the velocity of the cable $C$, (*b*) the velocity of the counterweight $W$, (*c*) the relative velocity of the cable $C$ with respect to the elevator, (*d*) the relative velocity of the counterweight $W$ with respect to the elevator.

## SOLUTION




Choose the positive direction downward.

(*a*) Velocity of cable $C$.

$$y_C + 2y_E = \text{constant}$$

$$v_C + 2v_E = 0$$

But, $$v_E = 4 \text{ m/s}$$

or $$v_C = -2v_E = -8 \text{ m/s}$$

$$\mathbf{v}_C = 8.00 \text{ m/s} \uparrow \blacktriangleleft$$

(*b*) Velocity of counterweight $W$.

$$y_W + y_E = \text{constant}$$

$$v_W + v_E = 0 \quad v_W = -v_E = -4 \text{ m/s}$$

$$\mathbf{v}_W = 4.00 \text{ m/s} \uparrow \blacktriangleleft$$

(*c*) Relative velocity of $C$ with respect to $E$.

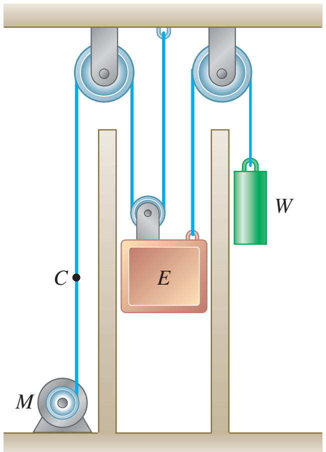
$$v_{C/E} = v_C - v_E = (-8 \text{ m/s}) - (+4 \text{ m/s}) = -12 \text{ m/s}$$

$$\mathbf{v}_{C/E} = 12.00 \text{ m/s} \uparrow \blacktriangleleft$$

(*d*) Relative velocity of $W$ with respect to $E$.

$$v_{W/E} = v_W - v_E = (-4 \text{ m/s}) - (4 \text{ m/s}) = -8 \text{ m/s}$$

$$\mathbf{v}_{W/E} = 8.00 \text{ m/s} \uparrow \blacktriangleleft$$

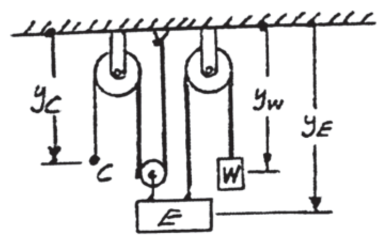



## PROBLEM 11.48

The elevator shown starts from rest and moves upward with a constant acceleration. If the counterweight $W$ moves through 30 ft in 5 s, determine ($a$) the acceleration of the elevator and the cable $C$, ($b$) the velocity of the elevator after 5 s.

## SOLUTION

We choose positive direction downward for motion of counterweight.




$$y_W = \frac{1}{2}a_W t^2$$

At $t = 5$ s,

$$y_W = 30 \text{ ft}$$

$$30 \text{ ft} = \frac{1}{2}a_W (5 \text{ s})^2$$

$$a_W = 2.4 \text{ ft/s}^2 \qquad \mathbf{a}_W = 2.4 \text{ ft/s}^2 \downarrow$$

($a$) Accelerations of $E$ and $C$.

Since $\quad y_W + y_E = \text{constant} \quad v_W + v_E = 0, \quad \text{and} \quad a_W + a_E = 0$

Thus: $\quad a_E = -a_W = -(2.4 \text{ ft/s}^2), \qquad \mathbf{a}_E = 2.40 \text{ ft/s}^2 \uparrow$ ◀

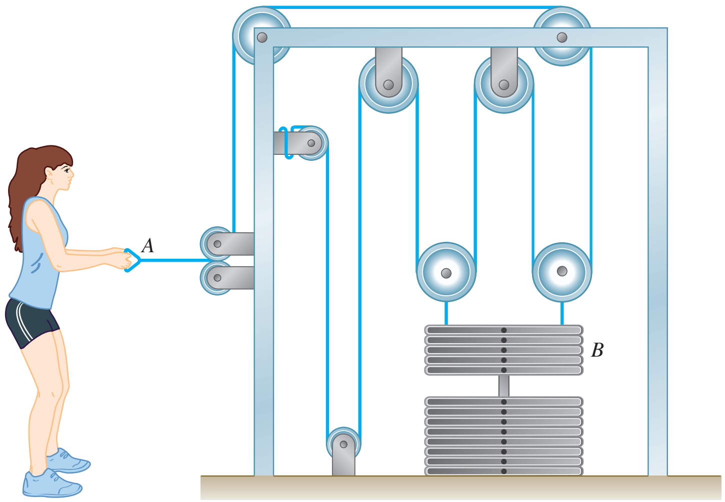
Also, $\quad y_C + 2y_E = \text{constant}, \quad v_C + 2v_E = 0, \quad \text{and} \quad a_C + 2a_E = 0$

Thus: $\quad a_C = -2a_E = -2(-2.4 \text{ ft/s}^2) = +4.8 \text{ ft/s}^2, \qquad \mathbf{a}_C = 4.80 \text{ ft/s}^2 \downarrow$ ◀

($b$) Velocity of elevator after 5 s.

$$v_E = (v_E)_0 + a_E t = 0 + (-2.4 \text{ ft/s}^2)(5 \text{ s}) = -12 \text{ ft/s} \qquad (\mathbf{v}_E)_5 = 12.00 \text{ ft/s} \uparrow$$ ◀

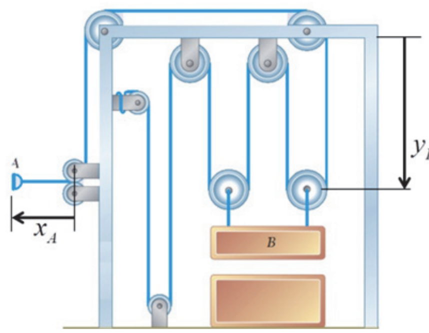
## Problem 11.49




An athlete pulls handle $A$ to the left with a constant velocity of 0.5 m/s. Determine ($a$) the velocity of the weight $B$, ($b$) the relative velocity of weight $B$ with respect to the handle $A$.

## SOLUTION

From the diagram: $x_A + 4y_B = \text{constant}$ Sketch:




*(a)* Differentiate $v_A + 4v_B = 0$ (1)

$\dot{x}_A = 0.5 \text{ m/s}$

Solve (1) for $v_B$ $v_B = -0.125 \text{ m/s}$ $v_B = 0.125 \text{ m/s} \uparrow$ ◀

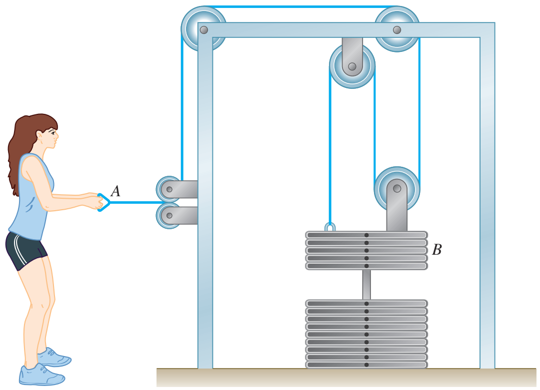
*(b)* Finding the relative velocity:

$$\vec{v}_B = 0.125 \uparrow \text{ m/s}$$

$$\vec{v}_A = 0.5 \leftarrow \text{ m/s}$$

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$$
$$= 0.125 \uparrow - (0.5 \leftarrow) \text{ m/s}$$

or $\mathbf{v}_{B/A} = 0.5 \rightarrow + 0.125 \uparrow \text{ m/s}$ $\mathbf{v}_{B/A} = 0.5154 \text{ m/s} \measuredangle 14°$ ◀



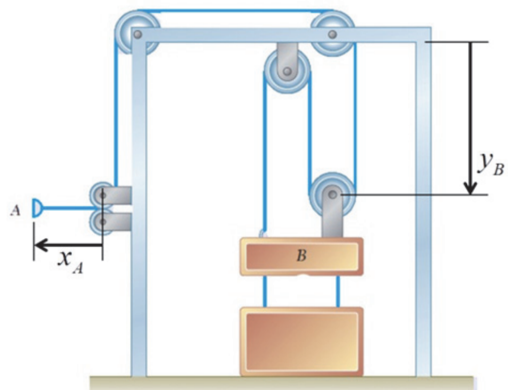

**Problem 11.50**

An athlete pulls handle $A$ to the left with a constant acceleration. Knowing that after the weight $B$ has been lifted 4 in. its velocity is 2 ft/s, determine (*a*) the accelerations of handle $A$ and weight $B$ (*b*) the velocity and change in position of handle $A$ after 0.5 sec.

**SOLUTION**

(*a*) From the diagram:

Sketch:




$$x_A + 3y_B = \text{constant}$$

$$v_A + 3v_B = 0 \qquad (1)$$

$$a_A + 3a_B = 0 \qquad (2)$$

Uniform Acceleration of weight B

$$v_B{}^2 = (v_B)_o^2 + 2a_B\left(y_B - (y_B)_o\right)$$

Substitute in known values

$$(-2\text{ ft/s})^2 = 0^2 + 2a_B\left(-\frac{4}{12}\text{ ft} - 0\right)$$

$$a_B = -6\text{ ft/s}^2$$

Using (2)

$$a_A + 3(-6\text{ ft/s}^2) = 0$$

$$a_A = 18\text{ ft/s}^2 \leftarrow \blacktriangleleft$$

$$a_B = 6\text{ ft/s}^2 \uparrow \blacktriangleleft$$

(*b*) Uniform Acceleration of Handle A

$$v_A = (v_A)_o + a_A t$$

$$v_A = 0 + \left(18\text{ ft/s}^2\right)(0.5\text{ s})$$

or

$$v_A = 9\text{ ft/s} \leftarrow \blacktriangleleft$$

Also

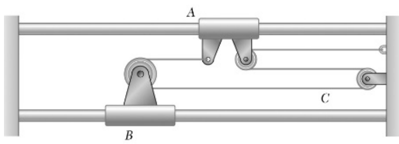
$$y_A = (y_A)_o + (v_A)_o t + \frac{1}{2}a_A t^2$$

$$y_A - (y_A)_o = 0 + \frac{1}{2}\left(18\text{ ft/s}^2\right)(0.5\text{ s})^2$$

or

$$\Delta y_A = 2.25\text{ ft} \leftarrow \blacktriangleleft$$

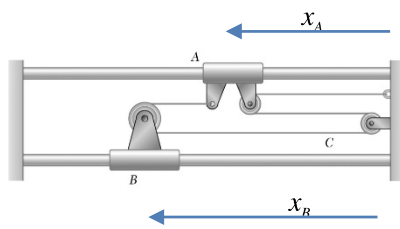
## PROBLEM 11.51




In the position shown, collar $B$ moves to the left with a constant velocity of 300 mm/s. Determine (*a*) the velocity of collar $A$, (*b*) the velocity of portion $C$ of the cable, (*c*) the relative velocity of portion $C$ of the cable with respect to collar $B$.

## SOLUTION

Let $x$ be position relative to the right supports, increasing to the left.




Constraint of entire cable: $$2x_A + x_B + (x_B - x_A) = \text{constant}$$

$$2v_B + v_A = 0 \qquad v_A = -2v_B$$

Constraint of point $C$ of cable: $$2x_A + x_C = \text{constant}$$

$$2v_A + v_C = 0 \qquad v_C = -2v_A$$

(*a*) Velocity of collar $A$.

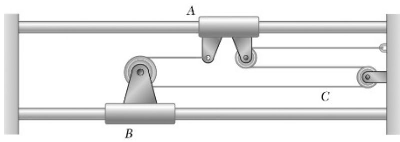
$$v_A = -2v_B = -(2)(300) = -600 \text{ mm/s} \qquad v_A = 600 \text{ mm/s} \longrightarrow \blacktriangleleft$$

(*b*) Velocity of point $C$ of cable.

$$v_C = -2v_A = -(2)(-600) = 1200 \text{ mm/s} \qquad v_C = 1200 \text{ mm/s} \longleftarrow \blacktriangleleft$$

(*c*) Velocity of point $C$ relative to collar $B$.

$$v_{C/B} = v_C - v_B = 1200 - 300 = 900 \text{ mm/s} \qquad v_{C/B} = 900 \text{ mm/s} \longleftarrow \blacktriangleleft$$

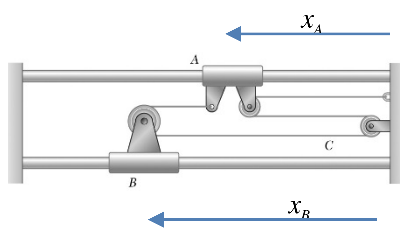



## PROBLEM 11.52

Collar $A$ starts from rest and moves to the right with a constant acceleration. Knowing that after 8 s the relative velocity of collar $B$ with respect to collar $A$ is 610 mm/s, determine ($a$) the accelerations of $A$ and $B$, ($b$) the velocity and the change in position of $B$ after 6 s.

## SOLUTION

Let $x$ be position relative to the right supports, increasing to the left.




Constraint of entire cable: $\quad 2x_A + x_B + (x_B - x_A) = \text{constant},$

$$2v_B + v_A = 0, \qquad \text{or} \qquad v_B = -\frac{1}{2}v_A, \qquad \text{and} \qquad a_B = -\frac{1}{2}a_A$$

($a$) Accelerations of $A$ and $B$.

$$v_{B/A} = v_B - v_A = -\frac{1}{2}v_A - v_A \qquad v_A = -\frac{2}{3}v_{B/A}$$

$$v_A = -\frac{2}{3}(610) = -406.67 \text{ mm/s}$$

$$v_A - (v_A)_0 = a_A t, \qquad \text{or} \qquad a_A = \frac{v_A - (v_A)_0}{t} = \frac{-406.67 - 0}{8} = -50.8 \text{ mm/s}^2$$

$$a_A = 50.8 \text{ mm/s}^2 \longrightarrow \blacktriangleleft$$

$$a_B = -\frac{1}{2}a_A = -\frac{1}{2}(-50.8) \qquad a_B = 25.4 \text{ mm/s}^2 \longleftarrow \blacktriangleleft$$

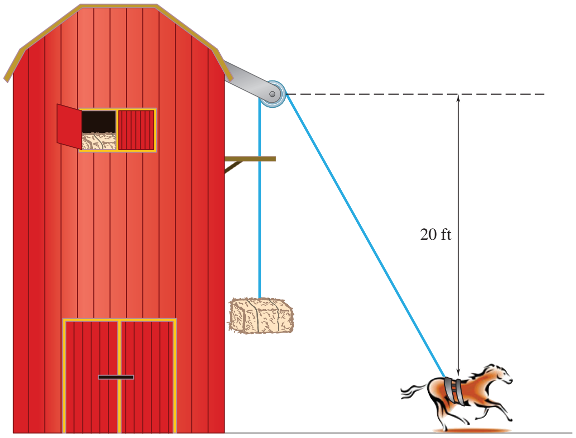
## PROBLEM 11.52 (CONTINUED)

(*b*) Velocity and change in position of *B* after 6 s.

$$v_B = (v_B)_0 + a_B t = 0 + (25.4)(6) \qquad v_B = 152.5 \text{ mm/s} \longleftarrow \blacktriangleleft$$

$$x_B - (x_B)_0 = (v_B)_0 t + \frac{1}{2} a_B t^2 = \frac{1}{2}(25.4)(6)^2 \qquad \Delta x_B = 458 \text{ mm} \longleftarrow \blacktriangleleft$$

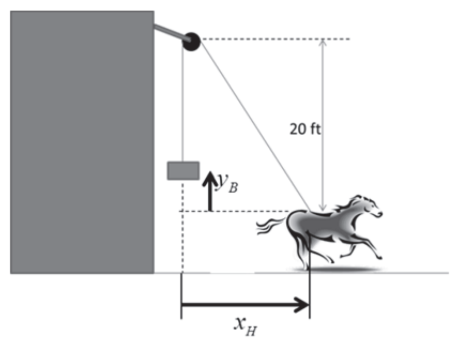
## PROBLEM 11.53




A farmer wants to get his hay bales into the top loft of his barn by walking his horse forward with a constant velocity of 1 ft/s. Determine the velocity and acceleration of the hay bale when the horse is 10 ft away from the barn.

## SOLUTION

From the diagram, we can write an expression for the total length of rope:




$$L = 20 - y_B + \sqrt{x_H^2 + (20)^2}$$

Differentiate

$$\frac{dL}{dt} = 0$$

$$0 = -\dot{y}_B + \frac{x_H \dot{x}_H}{\sqrt{x_H^2 + (20)^2}}$$

Velocities

$$v_B = \dot{y}_B \text{ and } v_H = \dot{x}_H$$

$$v_B = \frac{x_H v_H}{\sqrt{x_H^2 + (20)^2}}$$

Differentiate again:

$$a_B = \frac{dv_B}{dt}$$

$$a_B = \frac{\dot{x}_H v_H}{\sqrt{x_H^2 + (20)^2}} + \frac{x_H \dot{v}_H}{\sqrt{x_H^2 + (20)^2}} + \frac{x_H v_H (-\frac{1}{2})(2x_H)\dot{x}_H}{\left(x_H^2 + (20)^2\right)^{3/2}}$$

or

$$a_B = \frac{v_H^2}{\sqrt{x_H^2 + (20)^2}} + \frac{x_H a_H}{\sqrt{x_H^2 + (20)^2}} - \frac{x_H^2 v_H^2}{\left(x_H^2 + (20)^2\right)^{3/2}}$$

at $x_H = 10$

$$v_B = \frac{10(1)}{\sqrt{(10)^2 + (20)^2}}$$

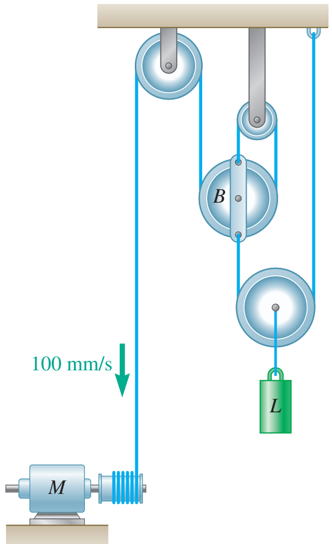
$v_B = 0.447$ ft/s ◀

and

$$a_B = \frac{(1)^2}{\sqrt{(10)^2 + (20)^2}} + \frac{(10)(0)}{\sqrt{(10)^2 + (20)^2}} - \frac{(10)^2 (1)^2}{\left((10)^2 + (20)^2\right)^{3/2}}$$

$$a_B = 0.0447 + 0 - 0.000894$$

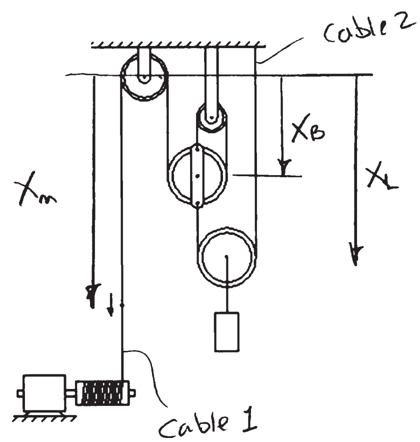
$a_B = 0.0358$ ft/s$^2$ ◀




## PROBLEM 11.54

The motor $M$ reels in the cable at a constant rate of 100 mm/s. Determine (*a*) the velocity of load $L$, (*b*) the velocity of pulley $B$ with respect to load $L$.

## SOLUTION




Let $x_B$ and $x_L$ be the positions, respectively, of pulley $B$ and load $L$ measured downward from a fixed elevation above both. Let $x_M$ be the position of a point on the cable about to enter the reel driven by the motor. Then, considering the lengths of the two cables,

$$x_M + 3x_B = \text{constant} \qquad v_M + 3v_B = 0$$

$$x_L + (x_L - x_B) = \text{constant} \qquad 2v_L + v_B = 0$$

with

$$v_M = 100 \text{ mm/s}$$

$$v_B = -\frac{v_M}{3} = -33.333 \text{ m/s}$$

$$v_L = \frac{v_B}{2} = -16.667 \text{ mm/s}$$

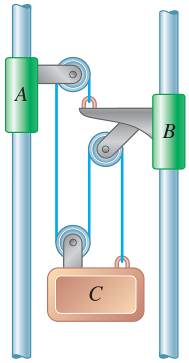
(*a*) Velocity of load $L$.

$$\mathbf{v}_L = 16.67 \text{ mm/s} \uparrow \blacktriangleleft$$

(*b*) Velocity of pulley $B$ with respect to load $L$. $v_{B/L} = v_B - v_L = -33.333 - (-16.667) = -16.667$

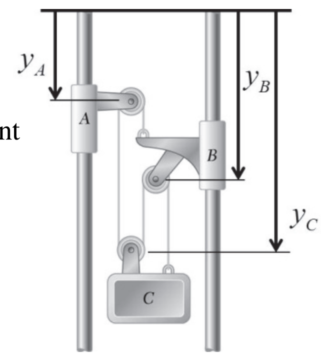
$$\mathbf{v}_{B/L} = 16.67 \text{ mm/s} \uparrow \blacktriangleleft$$

## PROBLEM 11.55




Collar $A$ starts from rest at $t = 0$ and moves upward with a constant acceleration of 3.6 in./s$^2$. Knowing that collar $B$ moves downward with a constant velocity of 18 in./s, determine ($a$) the time at which the velocity of block $C$ is zero, ($b$) the corresponding position of block $C$.

## SOLUTION




Define positions as positive downward from a fixed level.

Constraint of cable. $\quad (y_B - y_A) + (y_C - y_A) + 2(y_C - y_B) = \text{constant}$

$$3y_C - y_B - 2y_A = \text{constant}$$

Differentiate $\quad 3v_C - v_B - 2v_A = 0 \qquad (1)$

Differentiate again $\quad 3a_C - a_B - 2a_A = 0 \qquad (2)$

Motion of block C:

$$(v_A)_0 = 0, \ \ a_A = -3.6 \text{ in./s}^2, \ \ v_B = (v_B)_0 = 18 \text{ in./s}, \ \ a_B = 0$$

Using (1) $\quad (v_C)_0 = \frac{1}{3}\left[(v_B)_0 + 2(v_A)_0\right] = 6 \text{ in./s}$

Using (2) $\quad a_C = \frac{1}{3}(a_B + 2a_A) = \frac{1}{3}\left[0 + (2)(-3.6)\right] = -2.4 \text{ in./s}^2$

Constant acceleration $\quad v_C = (v_C)_0 + a_C t = 6 - 1.2t \qquad (3)$

Constant acceleration $\quad x_C - (x_C)_0 = (v_C)_0 t + \frac{1}{2}a_C t^2 = 6t - 0.6t^2 \qquad (4)$

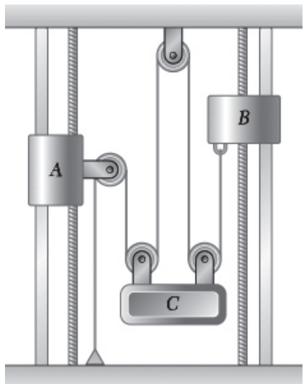
($a$) Time at $v_C = 0$ :

Using (3) $\quad 0 = 6 - 2.4t \qquad t = 2.5 \text{ s}$ ◀

($b$) Corresponding position of block C:

Using (4) $\quad x_C - (x_C)_0 = (6)(2.5) + \left(\frac{1}{2}\right)(-2.4)(2.5)^2 \qquad x_C - (x_C)_0 = 7.5 \text{ in.}\downarrow$ ◀

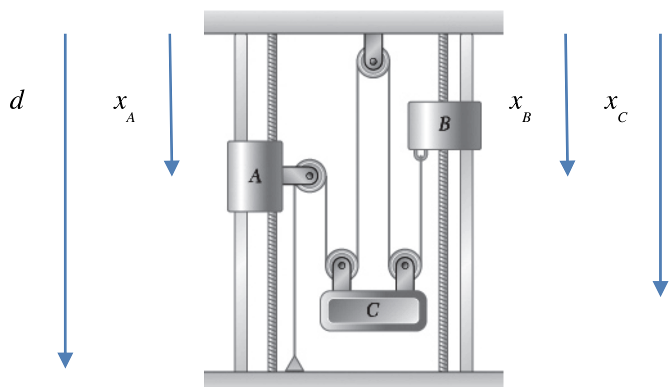
## PROBLEM 11.56




Collars $A$ and $B$ start from rest, and collar $A$ moves upward with an acceleration of $3t^2$ mm/s$^2$. Knowing that collar $B$ moves downward with a constant acceleration and that its velocity is 150 mm/s after moving 700 mm, determine (*a*) the acceleration of block $C$, (*b*) the distance through which block $C$ will have moved after 3 s.

## SOLUTION

Let $x$ be position relative to upper support, positive downward.




Let $d =$ value of $x$ at lower support.

Constraint of cable: $(d - x_A) + (x_C - x_A) + 2x_C + (x_C - x_B) = \text{constant}$

$$4v_C - v_B - 2v_A = 0 \qquad \text{and} \qquad 4a_C - a_B - 2a_A = 0$$

Accelerations: $$v_B^2 - (v_B)_0^2 = 2a_B\left[x_B - (x_B)_0\right]$$

$$a_B = \frac{v_B^2 - (v_B)_0^2}{2\left[x_B - (x_B)_0\right]} = \frac{150^2 - 0}{2(700 - 0)} = 16.07 \text{ mm/s}^2$$

$$a_A = -3t^2$$

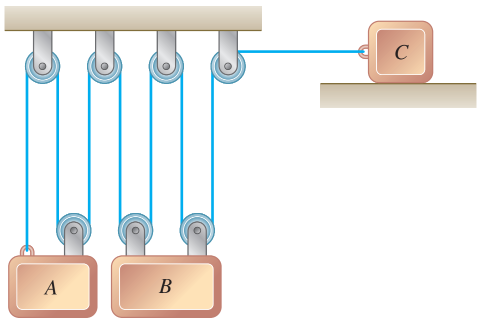
## PROBLEM 11.56 (CONTINUED)

(*a*) $$a_C = \frac{1}{4}(a_B + 2a_A) = \frac{1}{4}\left[16.07 + (2)(-3t^2)\right]$$ $a_C = 4.02 - 1.5t^2 \text{ mm/s}^2 \downarrow$ ◀

(*b*) Velocity and position: $$v_C = (v_C)_0 + \int_0^t a_C dt = 0 + 4.018t - 0.5t^3$$

$$x_C - (x_C)_0 = \int_0^t v_C dt = 2.009t^2 - 0.125t^4$$

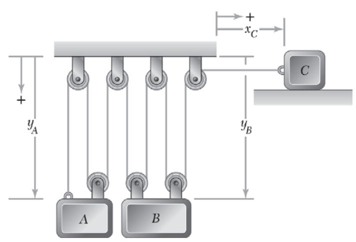
At $t = 3$ s, $$x_C - (x_C)_0 = (2.009)(3)^2 - (0.125)(3)^4$$ $\Delta x_C = 7.96 \text{ mm} \downarrow$ ◀




## PROBLEM 11.57

Block $B$ starts from rest, block $A$ moves with a constant acceleration, and slider block $C$ moves to the right with a constant acceleration of $75 \text{ mm/s}^2$. Knowing that at $t = 2$ s the velocities of $B$ and $C$ are 480 mm/s downward and 280 mm/s to the right, respectively, determine (*a*) the accelerations of $A$ and $B$, (*b*) the initial velocities of $A$ and $C$, (*c*) the change in position of slider block $C$ after 3 s.

## SOLUTION




From the diagram

$$3y_A + 4y_B + x_C = \text{constant}$$

Then $$3v_A + 4v_B + v_C = 0 \quad (1)$$

and $$3a_A + 4a_B + a_C = 0 \quad (2)$$

Given: $$(v_B) = 0,$$

$$a_A = \text{constent}$$

$$(\mathbf{a}_C) = 75 \text{ mm/s}^2 \longrightarrow$$

At $t = 2$ s, $$\mathbf{v}_B = 480 \text{ mm/s} \downarrow$$

$$\mathbf{v}_C = 280 \text{ mm/s} \longrightarrow$$

(*a*) Eq. (2) and $a_A = \text{constant}$ and $a_C = \text{constant} \Rightarrow a_B = \text{constant}$

Then $$v_B = 0 + a_B t$$

At $t = 2$ s: $$480 \text{ mm/s} = a_B (2 \text{ s})$$

$$a_B = 240 \text{ mm/s}^2 \quad \text{or} \quad \mathbf{a}_B = 240 \text{ mm/s}^2 \downarrow \blacktriangleleft$$

Substituting into Eq. (2)

$$3a_A + 4(240 \text{ mm/s}^2) + (75 \text{ mm/s}^2) = 0$$

$$a_A = -345 \text{ mm/s} \quad \text{or} \quad \mathbf{a}_A = 345 \text{ mm/s}^2 \uparrow \blacktriangleleft$$

## PROBLEM 11.57 (CONTINUED)

(*b*) We have $$v_C = (v_C)_0 + a_C t$$

At $t = 2$ s: $$280 \text{ mm/s} = (v_C)_0 + (75 \text{ mm/s})(2 \text{ s})$$

$$v_C = 130 \text{ mm/s} \qquad \text{or} \qquad (\mathbf{v}_C)_0 = 130.0 \text{ mm/s} \longrightarrow \blacktriangleleft$$

Then, substituting into Eq. (1) at $t = 0$

$$3(v_A)_0 + 4(0) + (130 \text{ mm/s}) = 0$$

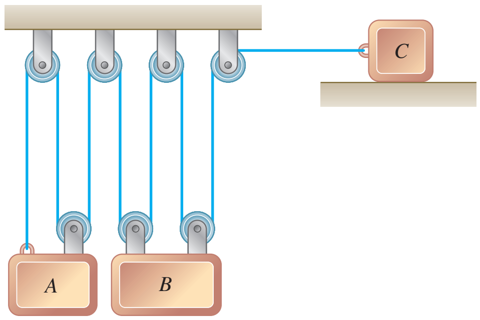
$$v_A = -43.3 \text{ mm/s} \qquad \text{or} \qquad (\mathbf{v}_A)_0 = 43.3 \text{ mm/s} \uparrow \blacktriangleleft$$

(*c*) We have $$x_C = (x_C)_0 + (v_C)_0 t + \frac{1}{2} a_C t^2$$

At $t = 3$ s: $$x_C - (x_C)_0 = (130 \text{ mm/s})(3 \text{ s}) + \frac{1}{2}(75 \text{ mm/s}^2)(3 \text{ s})^2$$

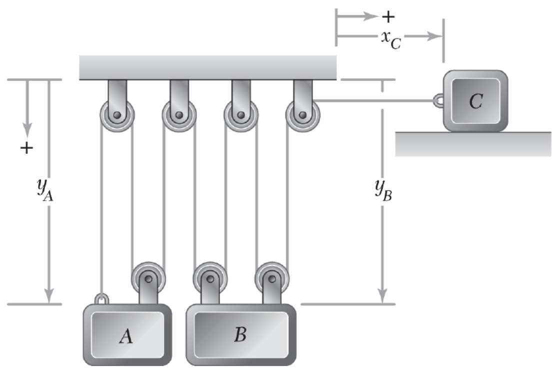
$$= 728 \text{ mm} \qquad \text{or} \quad \mathbf{x}_C - (\mathbf{x}_C)_0 = 728 \text{ mm} \longrightarrow \blacktriangleleft$$

## PROBLEM 11.58




Block $B$ moves downward with a constant velocity of 20 mm/s. At $t = 0$, block $A$ is moving upward with a constant acceleration, and its velocity is 30 mm/s. Knowing that at $t = 3$ s slider block $C$ has moved 57 mm to the right, determine (*a*) the velocity of slider block $C$ at $t = 0$, (*b*) the accelerations of $A$ and $C$, (*c*) the change in position of block $A$ after 5 s.

## SOLUTION




From the diagram

$$3y_A + 4y_B + x_C = \text{constant}$$

Then $$3v_A + 4v_B + v_C = 0 \quad (1)$$

and $$3a_A + 4a_B + a_C = 0 \quad (2)$$

<u>Given</u>: $$\mathbf{v}_B = 20 \text{ mm/s} \downarrow;$$

$$(\mathbf{v}_A)_0 = 30 \text{ mm/s} \uparrow$$

(*a*) Substituting into Eq. (1) at $t = 0$

$$3(-30 \text{ mm/s}) + 4(20 \text{ mm/s}) + (v_C)_0 = 0$$

$$(v_C)_0 = 10 \text{ mm/s} \quad \text{or} \quad (\mathbf{v}_C)_0 = 10.00 \text{ mm/s} \longrightarrow \blacktriangleleft$$

(*b*) We have $$x_C = (x_C)_0 + (v_C)_0 t + \frac{1}{2} a_C t^2$$

At $t = 3$ s: $$57 \text{ mm} = (10 \text{ mm/s})(3 \text{ s}) + \frac{1}{2} a_C (3 \text{ s})^2$$

$$a_C = 6 \text{ mm/s}^2 \quad \text{or} \quad \mathbf{a}_C = 6.00 \text{ mm/s}^2 \longrightarrow \blacktriangleleft$$

Now $$\mathbf{v}_B = \text{constant} \rightarrow a_B = 0$$

## PROBLEM 11.58 (CONTINUED)

Then, substituting into Eq. (2)

$$3a_A + 4(0) + (6 \text{ mm/s}^2) = 0$$

$$a_A = -2 \text{ mm/s}^2 \qquad \text{or} \qquad \mathbf{a}_A = 2.00 \text{ mm/s}^2 \uparrow \blacktriangleleft$$

(*c*) We have

$$y_A = (y_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2$$

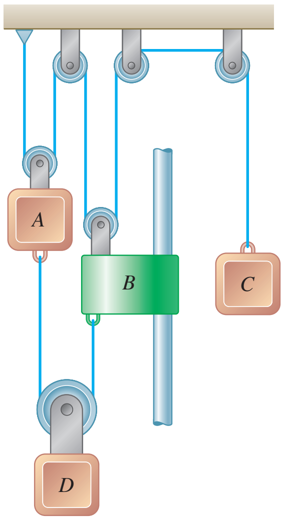
At $t = 5$ s:

$$y_A - (y_A)_0 = (-30 \text{ mm/s})(5 \text{ s}) + \frac{1}{2}(-2 \text{ mm/s}^2)(5 \text{ s})^2$$
$$= -175 \text{ mm}$$

or

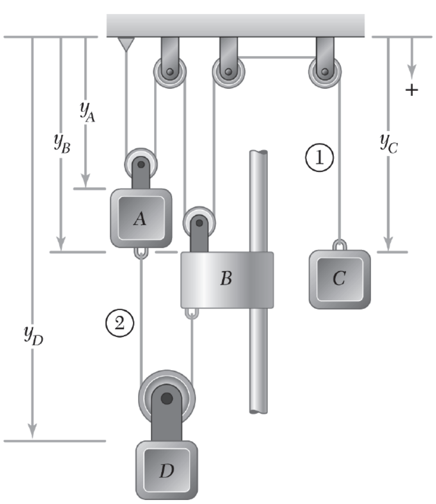
$$\mathbf{y}_A - (\mathbf{y}_A)_0 = 175.0 \text{ mm} \uparrow \blacktriangleleft$$

## PROBLEM 11.59




The system shown starts from rest, and each component moves with a constant acceleration. If the relative acceleration of block $C$ with respect to collar $B$ is $60\ \text{mm/s}^2$ upward and the relative acceleration of block $D$ with respect to block $A$ is $110\ \text{mm/s}^2$ downward, determine (*a*) the velocity of block $C$ after 3 s, (*b*) the change in position of block $D$ after 5 s.

## SOLUTION




From the diagram

Cable 1: $\quad 2y_A + 2y_B + y_C = \text{constant}$

Then $\quad 2v_A + 2v_B + v_C = 0 \qquad (1)$

and $\quad 2a_A + 2a_B + a_C = 0 \qquad (2)$

Cable 2: $\quad (y_D - y_A) + (y_D - y_B) = \text{constant}$

Then $\quad -v_A - v_B + 2v_D = 0 \qquad (3)$

and $\quad -a_A - a_B + 2a_D = 0 \qquad (4)$

Given: At $t = 0,\ v = 0;$ all accelerations constant;
$a_{C/B} = 60\ \text{mm/s}^2 \uparrow,\ a_{D/A} = 110\ \text{mm/s}^2 \downarrow$

(a) We have $\quad a_{C/B} = a_C - a_B = -60 \quad \text{or} \quad a_B = a_C + 60$

and $\quad a_{D/A} = a_D - a_A = 110 \quad \text{or} \quad a_A = a_D - 110$

Substituting into Eqs. (2) and (4)

Eq. (2): $\quad 2(a_D - 110) + 2(a_C + 60) + a_C = 0$

or $\quad 3a_C + 2a_D = 100 \qquad (5)$

Eq. (4): $\quad -(a_D - 110) - (a_C + 60) + 2a_D = 0$

or $\quad -a_C + a_D = -50 \qquad (6)$

## PROBLEM 11.59 (CONTINUED)

Solving Eqs. (5) and (6) for $a_C$ and $a_D$

$$a_C = 40 \text{ mm/s}^2$$

$$a_D = -10 \text{ mm/s}^2$$

Now $$v_C = 0 + a_C t$$

At $t = 3$ s: $$v_C = (40 \text{ mm/s}^2)(3 \text{ s})$$

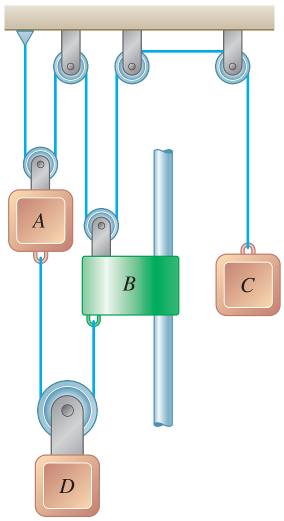
or $$\mathbf{v}_C = 120.0 \text{ mm/s}\downarrow \blacktriangleleft$$

(*b*) We have $$y_D = (y_D)_0 + (0)t + \frac{1}{2}a_D t^2$$

At $t = 5$ s: $$y_D - (y_D)_0 = \frac{1}{2}(-10 \text{ mm/s}^2)(5 \text{ s})^2$$

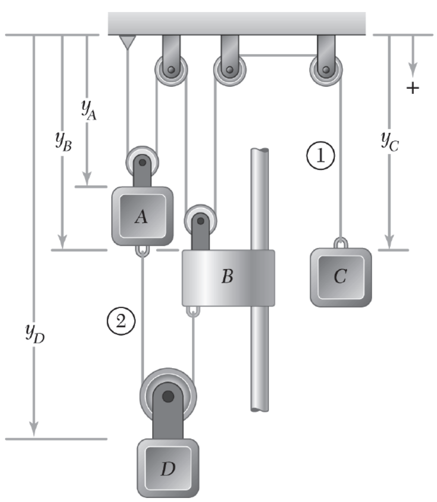
or $$\mathbf{y}_D - (\mathbf{y}_D)_0 = 125.0 \text{ mm}\uparrow \blacktriangleleft$$

## PROBLEM 11.60*




The system shown starts from rest, and the length of the upper cord is adjusted so that $A$, $B$, and $C$ are initially at the same level. Each component moves with a constant acceleration, and after 2 s the relative change in position of block $C$ with respect to block $A$ is 280 mm upward. Knowing that when the relative velocity of collar $B$ with respect to block $A$ is 80 mm/s downward, the displacements of $A$ and $B$ are 160 mm downward and 320 mm downward, respectively, determine ($a$) the accelerations of $A$ and $B$ if $a_B > 10\ \text{mm/s}^2$, ($b$) the change in position of block $D$ when the velocity of block $C$ is 600 mm/s upward.

## SOLUTION




From the diagram

Cable 1: $\quad 2y_A + 2y_B + y_C = \text{constant}$

Then $\quad 2v_A + 2v_B + v_C = 0 \qquad (1)$

and $\quad 2a_A + 2a_B + a_C = 0 \qquad (2)$

Cable 2: $\quad (y_D - y_A) + (y_D - y_B) = \text{constant}$

Then $\quad -v_A - v_B - 2v_D = 0 \qquad (3)$

and $\quad -a_A - a_B + 2a_D = 0 \qquad (4)$

Given: At

$$t = 0$$

$$v = 0$$

$$(y_A)_0 = (y_B)_0 = (y_C)_0$$

All accelerations constant.

At $t = 2$ s

$$y_{C/A} = 280\ \text{mm}\uparrow$$

When

$$v_{B/A} = 80\ \text{mm/s}\downarrow$$

$$y_A - (y_A)_0 = 160\ \text{mm}\uparrow$$

$$y_B - (y_B)_0 = 320\ \text{mm}\downarrow$$

$$a_B > 10\ \text{mm/s}^2$$

## PROBLEM 11.60* (CONTINUED)

(*a*) We have $$y_A = (y_A)_0 + (0)t + \frac{1}{2}a_A t^2$$

and $$y_C = (y_C)_0 + (0)t + \frac{1}{2}a_C t^2$$

Then $$y_{C/A} = y_C - y_A = \frac{1}{2}(a_C - a_A)t^2$$

At $t = 2$ s, $y_{C/A} = -280$ mm:

$$-280 \text{ mm} = \frac{1}{2}(a_C - a_A)(2 \text{ s})^2$$

or $$a_C = a_A - 140 \qquad (5)$$

Substituting into Eq. (2)

$$2a_A + 2a_B + (a_A - 140) = 0$$

or $$a_A = \frac{1}{3}(140 - 2a_B) \qquad (6)$$

Now $$v_B = 0 + a_B t$$

$$v_A = 0 + a_A t$$

$$v_{B/A} = v_B - v_A = (a_B - a_A)t$$

Also $$y_B = (y_B)_0 + (0)t + \frac{1}{2}a_B t^2$$

When $$\mathbf{v}_{B/A} = 80 \text{ mm/s}\downarrow: \quad 80 = (a_B - a_A)t \qquad (7)$$

$$\Delta y_A = 160 \text{ mm}\downarrow: \quad 160 = \frac{1}{2}a_A t^2$$

$$\Delta y_B = 320 \text{ mm}\downarrow: \quad 320 = \frac{1}{2}a_B t^2$$

Then $$160 = \frac{1}{2}(a_B - a_A)t^2$$

Using Eq. (7) $$320 = (80)t \quad \text{or} \quad t = 4 \text{ s}$$

Then $$160 = \frac{1}{2}a_A(4)^2 \quad \text{or} \quad \mathbf{a}_A = 20.0 \text{ mm/s}^2\downarrow \blacktriangleleft$$

and $$320 = \frac{1}{2}a_B(4)^2 \quad \text{or} \quad \mathbf{a}_B = 40.0 \text{ mm/s}^2\downarrow \blacktriangleleft$$

Note that Eq. (6) is not used; thus, the problem is over-determined.

## PROBLEM 11.60* (CONTINUED)

(*b*) Substituting into Eq. (5)

$$a_C = 20 - 140 = -120 \text{ mm/s}^2$$

and into Eq. (4)

$$-(20 \text{ mm/s}^2) - (40 \text{ mm/s}^2) + 2a_D = 0$$

or $$a_D = 30 \text{ mm/s}^2$$

Now $$v_C = 0 + a_C t$$

When $v_C = -600$ mm/s: $$-600 \text{ mm/s} = (-120 \text{ mm/s}^2)t$$

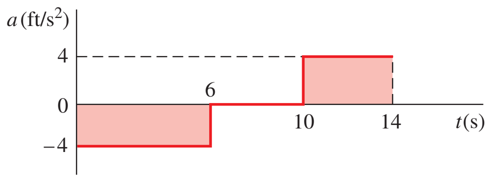
or $$t = 5 \text{ s}$$

Also $$y_D = (y_D)_0 + (0)t + \frac{1}{2}a_D t^2$$

At $t = 5$ s: $$y_D - (y_D)_0 = \frac{1}{2}(30 \text{ mm/s}^2)(5 \text{ s})^2$$

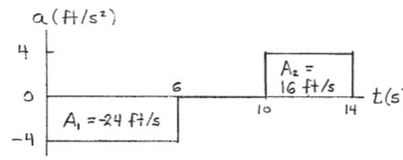
or $$\mathbf{y}_D - (\mathbf{y}_D)_0 = 375 \text{ mm} \downarrow \blacktriangleleft$$

## PROBLEM 11.61




A particle moves in a straight line with a constant acceleration of $-4\ \text{ft/s}^2$ for 6s, zero acceleration for the next 4s, and a constant acceleration of $+4\ \text{ft/s}^2$ for the next 4s. Knowing that the particle starts from the origin and that its velocity is $-8$ ft/s during the zero acceleration time interval, ($a$) construct the $v-t$ and $x-t$ curves for $0 \le t \le 14$s, ($b$) determine the position and the velocity of the particle and the total distance traveled when $t = 14$s.

## SOLUTION




From the $a-t$ curve

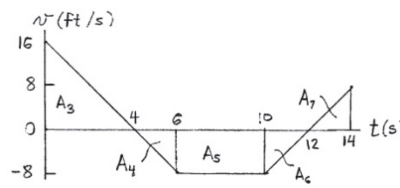
$$A_1 = -24\ \text{ft/s},\ A_2 = 16\ \text{ft/s}$$

($a$) Construct the $v-t$ curve : $v_6 = -8$ ft/s

$$v_0 = v_6 - A_1$$
$$= -8 - (-24)$$
$$= 16\ \text{ft/s}$$

$$v_{10} = -8\ \text{ft/s}$$

$$v_{14} = v_{10} + A_2$$
$$= -8 + 16$$
$$= 8\ \text{ft/s}$$




From the $v-t$ curve $A_3 = 32$ ft, $A_4 = -8$ ft

$$A_5 = -32\ \text{ft},\ A_6 = -8\ \text{ft},\ A_7 = 8\ \text{ft}$$

(b) Construct the $x-t$ curve $x_0 = 0$

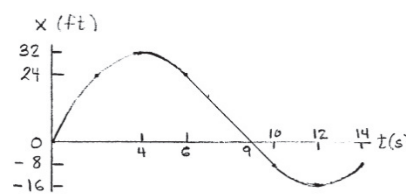
$$x_4 = x_0 + A_3 = 32\ \text{ft}$$

$$x_6 = x_4 + A_4 = 24\ \text{ft}$$

$$x_{10} = x_6 + A_5 = -8\ \text{ft}$$

$$x_{12} = x_{10} + A_6 = -16\ \text{ft}$$

$$x_{14} = x_{12} + A_7 = -8\ \text{ft}$$




Total Distance Traveled

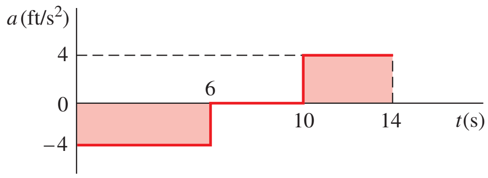
$0 \le t \le 4$ s, $\quad d_1 = |32 - 0| = 32$ ft

$4\ \text{s} \le t \le 12$ s, $\quad d_2 = |-16 - 32| = 48$ ft

$12\ \text{s} \le t \le 14$ s, $\quad d_3 = |-8 - (-16)| = 8$ ft

$d = 32 + 48 + 8$ $\qquad$ $d = 88$ ft

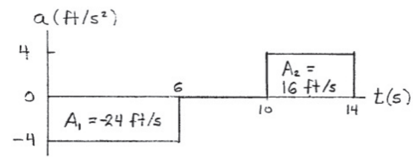
## Problem 11.62




A particle moves in a straight line with a constant acceleration of $-4$ ft/s$^2$ for 6s, zero acceleration for the next 4 s, and a constant acceleration of $+4$ ft/s$^2$ for the next 4s. Knowing that the particle starts from the origin with $v_0 = 16$ ft/s, ($a$) construct the $v-t$ and $x-t$ curves for $0 \le t \le 14$s, ($b$) determine the amount of time during which the particle is further than 16 ft from the origin.

## SOLUTION

From the $a-t$ curve

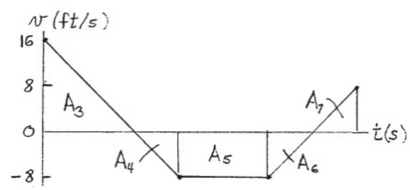



$$A_1 = -24 \text{ ft/s}, \; A_2 = 16 \text{ ft/s}$$

($a$) Construct the $v-t$ curve : $v_0 = 16$ ft/s

$$v_6 = v_0 + A_1$$
$$= 16 + (-24)$$
$$= -8 \text{ ft/s}$$

$$v_{10} = v_6 = -8 \text{ ft/s}$$




$$v_{14} = v_{10} + A_2$$
$$= -8 + 16$$
$$= 8 \text{ ft/s}$$

From the $v-t$ curve $A_3 = 32$ ft, $A_4 = -8$ ft

$A_5 = -32$ ft, $A_6 = -8$ ft , $A_7 = 8$ ft

Construct the $x-t$ curve $x_0 = 0$

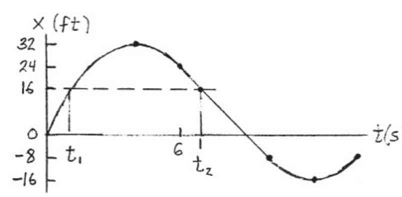
$$x_4 = x_0 + A_3 = 32 \text{ ft}$$

$$x_6 = x_4 + A_4 = 24 \text{ ft}$$

$$x_{10} = x_6 + A_5 = -8 \text{ ft}$$

$$x_{12} = x_{10} + A_6 = -16 \text{ ft}$$

$$x_{14} = x_{12} + A_7 = -8 \text{ ft}$$




($b$) Time for $|x| > 16$ ft.

From the $x-t$ diagram, this is time interval $t_1$ to $t_2$.

From $0 < t < 6$ s, $\dfrac{dx}{dt} = v = 16 - 4t$

Integrating, using limits $x = 0$ when $t = 0$ and $x = 16$ ft when $t = t_1$

$$[x]_0^{16} = \left[16t - 2t^2\right]_0^{t} \qquad \text{or} \qquad 16 = 16t_1 - 2t_1^2$$

## PROBLEM 11.62 (CONTINUED)

or $$t_1^2 - 8t_1 + 8 = 0$$

Using the quadratic formula: $$t_1 = \frac{8 \pm \sqrt{(8)^2 - (4)(1)(8)}}{(2)(1)} = 4 \pm 2.828 = 1.172 \text{ s} \quad \text{and} \quad 6.828 \text{ s}$$

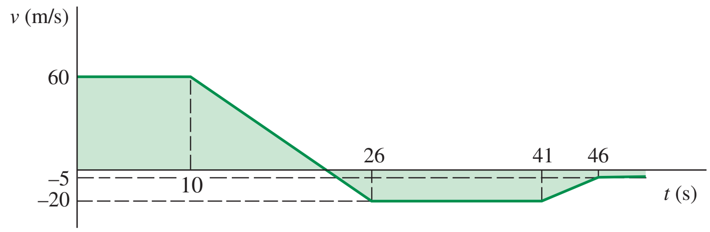
The larger root is out of range, thus $t_1 = 1.172$ s

From $6 < t < 10$, $$x = 24 - 8(t - 6) = 72 - 8t$$

Setting $x = 16$, $$16 = 72 - 8t_2 \quad \text{or} \quad t_2 = 7 \text{ s}$$

Time Interval: $$\Delta t = t_2 - t_1$$

$\Delta t = 5.83$ s ◀




## PROBLEM 11.63

A particle moves in a straight line with the velocity shown in the figure. Knowing that $x = -540$ m at $t = 0$, (*a*) construct the $a$–$t$ and $x$–$t$ curves for $0 < t < 50$ s, and determine (*b*) the total distance traveled by the particle when $t = 50$ s, (*c*) the two times at which $x = 0$.

## SOLUTION

(*a*) $a_t =$ slope of $v$–$t$ curve at time $t$

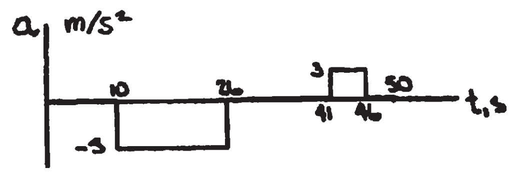
From $t = 0$ to $t = 10$ s: $v = \text{constant} \Rightarrow a = 0$

$t = 10$ s to $t = 26$ s: $$a = \frac{-20 - 60}{26 - 10} = -5 \text{ m/s}^2$$

$t = 26$ s to $t = 41$ s: $v = \text{constant} \Rightarrow a = 0$

$t = 41$ s to $t = 46$ s: $$a = \frac{-5 - (-20)}{46 - 41} = 3 \text{ m/s}^2$$

$t = 46$ s: $v = \text{constant} \Rightarrow a = 0$




$x_2 = x_1 +$ (area under $v$–$t$ curve from $t_1$ to $t_2$)

At $t = 10$ s: $x_{10} = -540 + 10(60) + 60$ m

Next, find time at which $v = 0$. Using similar triangles

$$\frac{t_{v=0} - 10}{60} = \frac{26 - 10}{80} \quad \text{or} \quad t_{v=0} = 22 \text{ s}$$

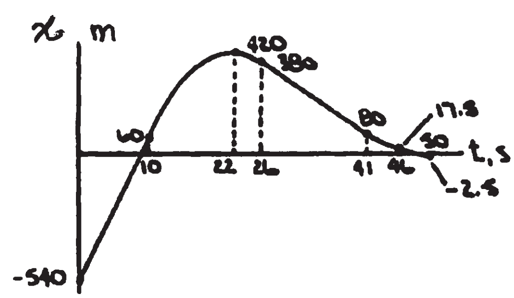
At $t = 22$ s: $x_{22} = 60 + \frac{1}{2}(12)(60) = 420$ m

$t = 26$ s: $x_{26} = 420 - \frac{1}{2}(4)(20) = 380$ m

$t = 41$ s: $x_{41} = 380 - 15(20) = 80$ m

$t = 46$ s: $x_{46} = 80 - 5\left(\frac{20 + 5}{2}\right) = 17.5$ m

$t = 50$ s: $x_{50} = 17.5 - 4(5) = -2.5$ m



## PROBLEM 11.63 (CONTINUED)

(*b*) From $t = 0$ to $t = 22$ s: Distance traveled $= 420 - (-540)$

$$= 960 \text{ m}$$

$t = 22$ s to $t = 50$ s: Distance traveled $= |-2.5 - 420|$

$$= 422.5 \text{ m}$$

Total distance traveled $= (960 + 422.5)$ ft $= 1382.5$ m

Total distance traveled $= 1383$ m ◀

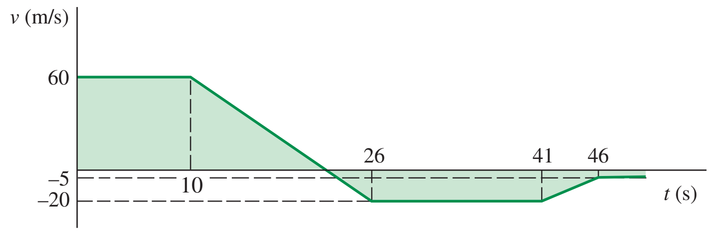
(*c*) Using similar triangles

Between 0 and 10 s: $\dfrac{(t_{x=0})_1 - 0}{540} = \dfrac{10}{600}$

$(t_{x=0})_1 = 9.00$ s ◀

Between 46 s and 50 s: $\dfrac{(t_{x=0})_2 - 46}{17.5} = \dfrac{4}{20}$

$(t_{x=0})_2 = 49.5$ s ◀




## PROBLEM 11.64

A particle moves in a straight line with the velocity shown in the figure. Knowing that $x = -540$ m at $t = 0$, (*a*) construct the $a-t$ and $x-t$ curves for $0 < t < 50$ s, and determine (*b*) the maximum value of the position coordinate of the particle, (*c*) the values of $t$ for which the particle is at $x = 100$ m.

## SOLUTION

(*a*) $a_t$ = slope of $v-t$ curve at time $t$

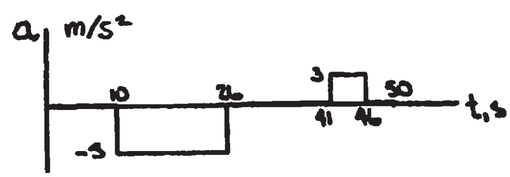
From $t = 0$ to $t = 10$ s: $v = \text{constant} \Rightarrow a = 0$

$t = 10$ s to $t = 26$ s: $a = \dfrac{-20-60}{26-10} = -5 \text{ m/s}^2$

$t = 26$ s to $t = 41$ s: $v = \text{constant} \Rightarrow a = 0$

$t = 41$ s to $t = 46$ s: $a = \dfrac{-5-(-20)}{46-41} = 3 \text{ m/s}^2$

$t = 46$ s: $v = \text{constant} \Rightarrow a = 0$




$x_2 = x_1 +$ (area under $v-t$ curve from $t_1$ to $t_2$)

At $t = 10$ s: $x_{10} = -540 + 10(60) = 60$ m

Next, find time at which $v = 0$. Using similar triangles

$$\frac{t_{v=0} - 10}{60} = \frac{26-10}{80} \quad \text{or} \quad t_{v=0} = 22 \text{ s}$$

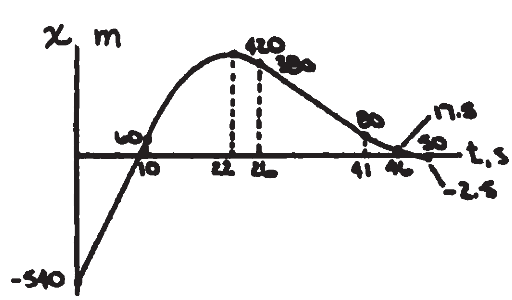
At $t = 22$ s: $x_{22} = 60 + \dfrac{1}{2}(12)(60) = 420$ m

$t = 26$ s: $x_{26} = 420 - \dfrac{1}{2}(4)(20) = 380$ m

$t = 41$ s: $x_{41} = 380 - 15(20) = 80$ m

$t = 46$ s: $x_{46} = 80 - 5\left(\dfrac{20+5}{2}\right) = 17.5$ m

$t = 50$ s: $x_{50} = 17.5 - 4(5) = -2.5$ m

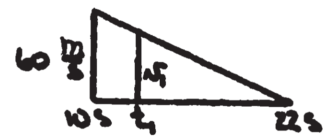


## PROBLEM 11.64 (CONTINUED)

(*b*) Reading from the $x{-}t$ curve $\qquad x_{\max} = 420\text{ m}$ ◀

(*c*) Between 10 s and 22 s

$100\text{ m} = 420\text{ m} - (\text{area under } v{-}t \text{ curve from } t_1 \text{ to } 22\text{ s})\text{ m}$




$$100 = 420 - \frac{1}{2}(22 - t_1)(v_1)$$

$$(22 - t_1)(v_1) = 640$$

Using similar triangles

$$\frac{v_1}{22 - t_1} = \frac{60}{12} \qquad \text{or} \qquad v_1 = 5(22 - t_1)$$

Then

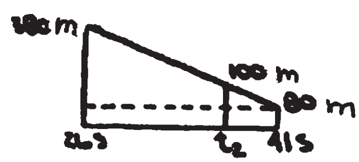
$$(22 - t_1)[5(22 - t_1)] = 640$$

$$t_1 = 10.69\text{ s} \quad \text{and} \quad t_1 = 33.3\text{ s}$$

We have $\qquad 10\text{ s} < t_1 < 22\text{ s} \Rightarrow \qquad t_1 = 10.69\text{ s}$ ◀

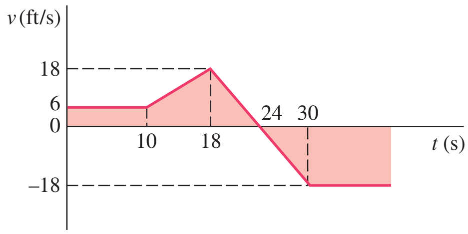
Between 26 s and 41 s:

Using similar triangles




$$\frac{41 - t_2}{20} = \frac{15}{300}$$

$t_2 = 40.0\text{ s}$ ◀




## Problem 11.65

A particle moves in a straight line with the velocity shown in the figure. Knowing that $x = -48$ ft at $t = 0$, draw the $a - t$ and $x - t$ curves for $0 < t < 40$ s and determine (*a*) the maximum value of the position coordinate of the particle, (*b*) the values of $t$ for which the particle is at a distance of 108 ft from the origin.

## SOLUTION

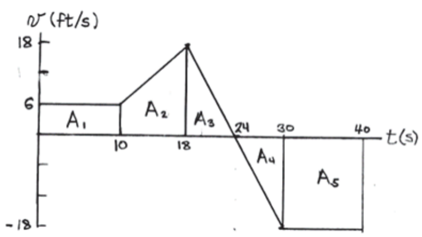
The $a - t$ curve is the slope of the $v - t$ curve

$0 < t < 10$ s, $\qquad a = 0$

$10$ s $< t < 18$ s, $\qquad a = \dfrac{18 - 6}{18 - 10} = 1.5 \text{ ft/s}^2$

$18$ s $< t < 30$ s, $\qquad a = \dfrac{-18 - 18}{30 - 18} = -3 \text{ ft/s}^2$

$30$ s $< t < 40$ s $\qquad a = 0$




◀

Points on the $x$–$t$ curve may be calculated using areas of the $v$–$t$ curve.

$$A_1 = (10)(6) = 60 \text{ ft}$$

$$A_2 = \frac{1}{2}(6 + 18)(18 - 10) = 96 \text{ ft}$$

$$A_3 = \frac{1}{2}(18)(24 - 18) = 54 \text{ ft}$$

$$A_4 = \frac{1}{2}(-18)(30 - 24) = -54 \text{ ft}$$

$$A_5 = (-18)(40 - 30) = -180 \text{ ft}$$

Value of x at specific times:

$$x_0 = -48 \text{ ft}$$

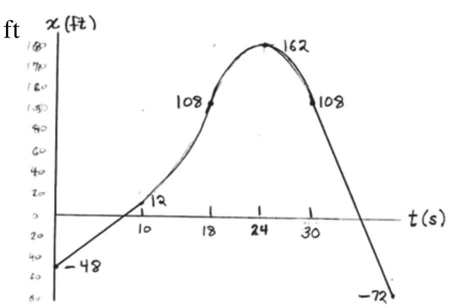
$$x_{10} = x_0 + A_1 = 12 \text{ ft}$$

$$x_{18} = x_{10} + A_2 = 108 \text{ ft}$$

$$x_{24} = x_{18} + A_3 = 162 \text{ ft}$$

$$x_{30} = x_{24} + A_4 = 108 \text{ ft}$$

$$x_{40} = x_{30} + A_5 = -72 \text{ ft}$$




◀

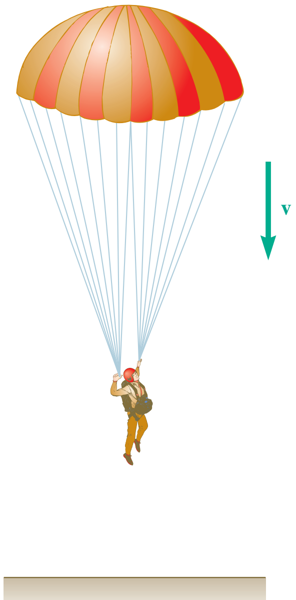
## PROBLEM 11.65 (CONTINUED)

(*a*) Maximum value of x occurs when: $v = 0$, i.e. $t = 24$ s. $x_{max} = 162$ ft ◀

(*b*) Times when $x = 108$ ft.

From the $x - t$ curve $t = 18$ s and $t = 30$ s ◀

## PROBLEM 11.66



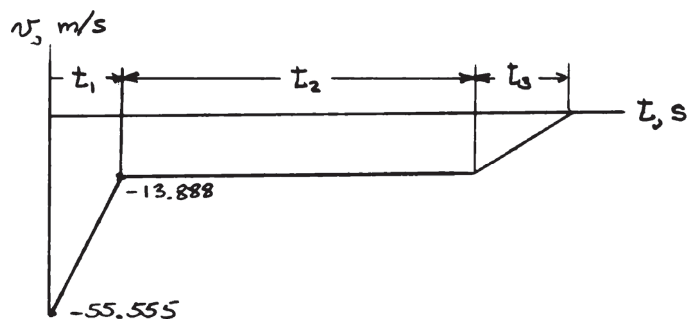

A parachutist is in free fall at a rate of 200 km/h when he opens his parachute at an altitude of 600 m. Following a rapid and constant deceleration, he then descends at a constant rate of 50 km/h from 586 m to 30 m, where he maneuvers the parachute into the wind to further slow his descent. Knowing that the parachutist lands with a negligible downward velocity, determine (*a*) the time required for the parachutist to land after opening his parachute, (*b*) the initial deceleration.

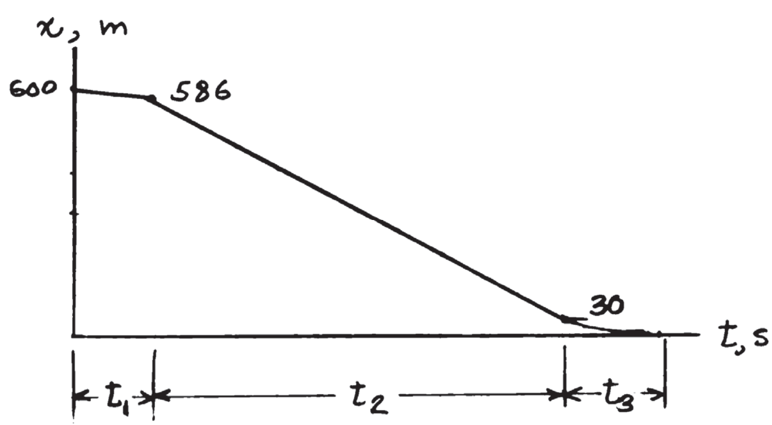
## SOLUTION

Assume second deceleration is constant. Also, note that

$200 \text{ km/h} = 55.555 \text{ m/s},$

$50 \text{ km/h} = 13.888 \text{ m/s}$



## PROBLEM 11.66 (CONTINUED)

(*a*) Now $\Delta x =$ area under $v-t$ curve for given time interval

Then

$$(586-600)\text{ m} = -t_1\left(\frac{55.555+13.888}{2}\right)\text{ m/s}$$

$$t_1 = 0.4032\text{ s}$$

$$(30-586)\text{ m} = -t_2(13.888\text{ m/s})$$

$$t_2 = 40.0346\text{ s}$$

$$(0-30)\text{ m} = -\frac{1}{2}(t_3)(13.888\text{ m/s})$$

$$t_3 = 4.3203\text{ s}$$

$$t_{\text{total}} = (0.4032 + 40.0346 + 4.3203)\text{ s}$$

$t_{\text{total}} = 44.8\text{ s}$ ◀

(*b*) We have

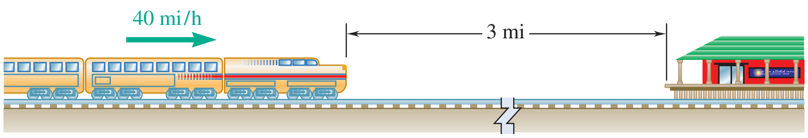
$$a_{\text{initial}} = \frac{\Delta v_{\text{initial}}}{t_1}$$

$$= \frac{[-13.888-(-55.555)]\text{ m/s}}{0.4032\text{ s}}$$

$$= 103.3\text{ m/s}^2$$

$\mathbf{a}_{\text{initial}} = 103.3\text{ m/s}^2 \uparrow$ ◀

## PROBLEM 11.67



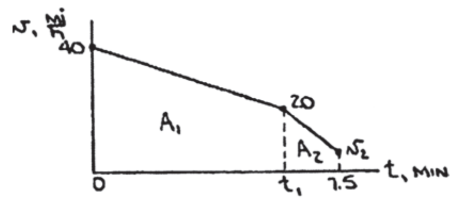

A commuter train traveling at 40 mi/h is 3 mi from a station. The train then decelerates so that its speed is 20 mi/h when it is 0.5 mi from the station. Knowing that the train arrives at the station 7.5 min after beginning to decelerate and assuming constant decelerations, determine (*a*) the time required for the train to travel the first 2.5 mi, (*b*) the speed of the train as it arrives at the station, (*c*) the final constant deceleration of the train.

## SOLUTION

Given: At $t = 0$, $v = 40$ mi/h, $x = 0$; when $x = 2.5$ mi, $v = 20$ mi/h;

at $t = 7.5$ min, $x = 3$ mi; constant decelerations.

The $v-t$ curve is first drawn as shown.




(*a*) We have $$A_1 = 2.5 \text{ mi}$$

$$(t_1 \text{ min})\left(\frac{40+20}{2}\right) \text{mi/h} \times \frac{1 \text{ h}}{60 \text{ min}} = 2.5 \text{ mi}$$

$$t_1 = 5.00 \text{ min} \blacktriangleleft$$

(*b*) We have $$A_2 = 0.5 \text{ mi}$$

$$(7.5-5)\text{min} \times \left(\frac{20+v_2}{2}\right)\text{mi/h} \times \frac{1 \text{ h}}{60 \text{ min}} = 0.5 \text{ mi}$$

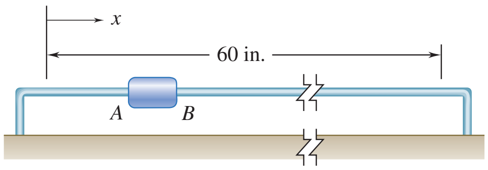
$$v_2 = 4.00 \text{ mi/h} \blacktriangleleft$$

(*c*) We have $$a_{\text{final}} = a_{12}$$

$$= \frac{(4-20)\text{ mi/h}}{(7.5-5)\text{ min}} \times \frac{5280 \text{ ft}}{\text{mi}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ h}}{3600 \text{ s}}$$

$$a_{\text{final}} = -0.1564 \text{ ft/s}^2 \blacktriangleleft$$

## PROBLEM 11.68




A temperature sensor is attached to slider $AB$ which moves back and forth through 60 in. The maximum velocities of the slider are 12 in./s to the right and 30 in./s to the left. When the slider is moving to the right, it accelerates and decelerates at a constant rate of 6 in./s$^2$; when moving to the left, the slider accelerates and decelerates at a constant rate of 20 in./s$^2$. Determine the time required for the slider to complete a full cycle, and construct the $v-t$ and $x-t$ curves of its motion.

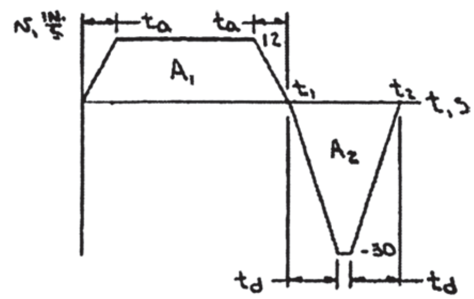
## SOLUTION

The $v-t$ curve is first drawn as shown. Then

$$t_a = \frac{v_{\text{right}}}{a_{\text{right}}} = \frac{12 \text{ in./s}}{6 \text{ in./s}^2} = 2 \text{ s}$$

$$t_d = \frac{v_{\text{left}}}{a_{\text{left}}} = \frac{30 \text{ in./s}}{20 \text{ in./s}}$$

$$= 1.5 \text{ s}$$




Now $\quad A_1 = 60$ in.

or $\quad [(t_1 - 2) \text{ s}](12 \text{ in./s}) = 60$ in.

or $\quad t_1 = 7$ s

and $\quad A_2 = 60$ in.

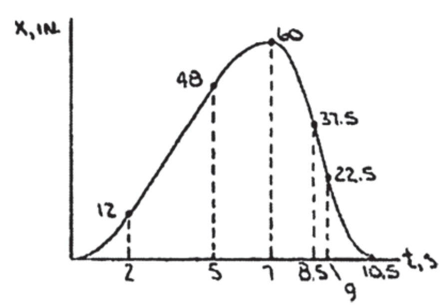
or $\quad \{[(t_2 - 7) - 1.5] \text{ s}\}(30 \text{ in./s}) = 60$ in.

or $\quad t_2 = 10.5$ s

Now $\quad t_{\text{cycle}} = t_2$

$t_{\text{cycle}} = 10.5$ s ◀

We have $x_{ii} = x_i +$ (area under $v-t$ curve from $t_i$ to $t_{ii}$)




At $\quad t = 2$ s: $\quad x_2 = \frac{1}{2}(2)(12) = 12$ in.

$t = 5$ s: $\quad x_5 = 12 + (5-2)(12)$

$= 48$ in.

$t = 7$ s: $\quad x_7 = 60$ in.

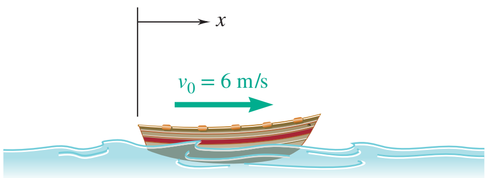
$t = 8.5$ s: $\quad x_{8.5} = 60 - \frac{1}{2}(1.5)(30)$

$= 37.5$ in.

$t = 9$ s: $\quad x_9 = 37.5 - (0.5)(30)$

$= 22.5$ in.

$t = 10.5$ s: $\quad x_{10.5} = 0$




## PROBLEM 11.69

In a water-tank test involving the launching of a small model boat, the model's initial horizontal velocity is 6 m/s, and its horizontal acceleration varies linearly from $-12$ m/s$^2$ at $t = 0$ to $-2$ m/s$^2$ at $t = t_1$ and then remains equal to $-2$ m/s$^2$ until $t = 1.4$ s. Knowing that $v = 1.8$ m/s when $t = t_1$, determine (*a*) the value of $t_1$, (*b*) the velocity and the position of the model at $t = 1.4$ s.

## SOLUTION

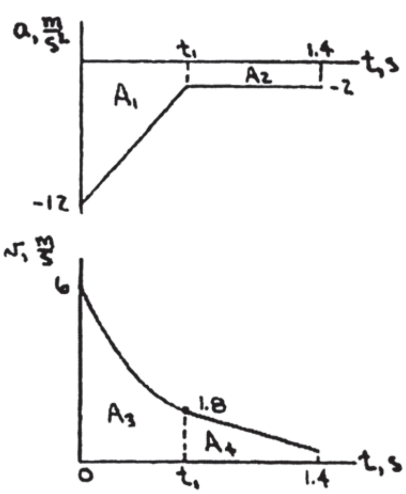
<u>Given</u>: $v_0 = 6$ m/s; for $0 < t < t_1$,

for $t_1 < t < 1.4$ s $\quad a = -2$ m/s$^2$;

at $t = 0 \quad a = -12$ m/s$^2$;

at $t = t_1$ $\quad a = -2$ m/s$^2$, $v = 1.8$ m/s$^2$

The $a$–$t$ and $v$–$t$ curves are first drawn as shown. The time axis is not drawn to scale.




(*a*) We have $\quad v_{t_1} = v_0 + A_1$

$$1.8 \text{ m/s} = 6 \text{ m/s} - (t_1 \text{ s})\left(\frac{12+2}{2}\right) \text{m/s}^2$$

$$t_1 = 0.6 \text{ s} \blacktriangleleft$$

(*b*) We have $\quad v_{1.4} = v_{t_1} + A_2$

$$v_{1.4} = 1.8 \text{ m/s} - (1.4 - 0.6) \text{ s} \times 2 \text{ m/s}^2$$

$$v_{1.4} = 0.20 \text{ m/s} \blacktriangleleft$$

Now $x_{1.4} = A_3 + A_4$, where $A_3$ is most easily determined using integration. Thus,

for $0 < t < t_1$: $\quad a = \dfrac{-2-(-12)}{0.6}t - 12 = \dfrac{50}{3}t - 12$

Now $\quad \dfrac{dv}{dt} = a = \dfrac{50}{3}t - 12$

## PROBLEM 11.69 (CONTINUED)

At $t = 0,\ v = 6$ m/s: $\displaystyle\int_6^v dv = \int_0^t \left(\frac{50}{3}t - 12\right)dt$

or $$v = 6 + \frac{25}{3}t^2 - 12t$$

We have $$\frac{dx}{dt} = v = 6 - 12t + \frac{25}{3}t^2$$

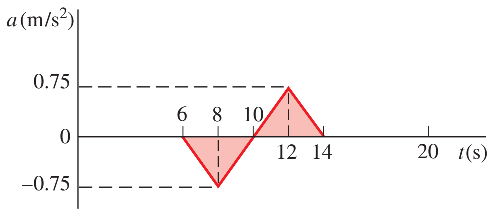
Then $$A_3 = \int_0^{x_{t_1}} dx = \int_0^{0.6} (6 - 12t + \frac{25}{3}t^2)dt$$

$$= \left[6t - 6t^2 + \frac{25}{9}t^3\right]_0^{0.6} = 2.04 \text{ m}$$

Also $$A_4 = (1.4 - 0.6)\left(\frac{1.8 + 0.2}{2}\right) = 0.8 \text{ m}$$

Then $$x_{1.4} = (2.04 + 0.8) \text{ m}$$

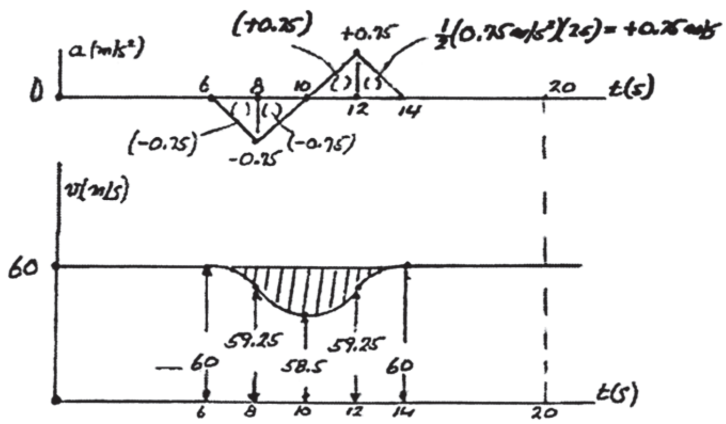
or $$x_{1.4} = 2.84 \text{ m} \blacktriangleleft$$



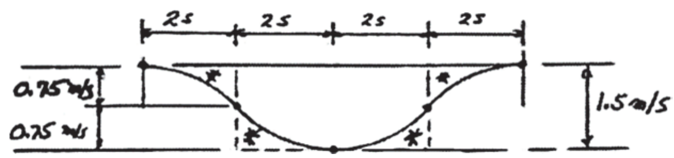

## PROBLEM 11.70

The acceleration record shown was obtained for a small airplane traveling along a straight course. Knowing that $x = 0$ and $v = 60$ m/s when $t = 0$, determine (*a*) the velocity and position of the plane at $t = 20$ s, (*b*) its average velocity during the interval $6 \text{ s} < t < 14 \text{ s}$.

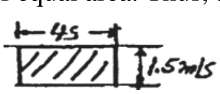
## SOLUTION




Geometry of "bell-shaped" portion of $v{-}t$ curve




The parabolic spandrels marked by * are of equal area. Thus, total area of shaded portion of $v{-}t$ diagram is:




$= \Delta x = 6 \text{ m}$

(*a*) When $t = 20$ s: $\qquad v_{20} = 60 \text{ m/s}$ ◀

$$x_{20} = (60 \text{ m/s})(20 \text{ s}) - (\text{shaded area})$$

$$= 1200 \text{ m} - 6 \text{ m}$$

$x_{20} = 1194 \text{ m}$ ◀

(*b*) From $t = 6$ s to $t = 14$ s: $\qquad \Delta t = 8 \text{ s}$

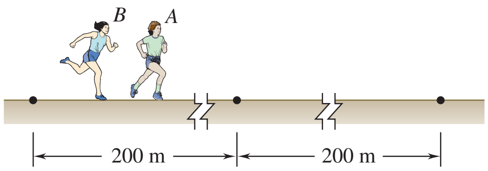
$$\Delta x = (60 \text{ m/s})(14 \text{ s} - 6 \text{ s}) - (\text{shaded area})$$

$$= (60 \text{ m/s})(8 \text{ s}) - 6 \text{ m} = 480 \text{ m} - 6 \text{ m} = 474 \text{ m}$$

$$v_{\text{average}} = \frac{\Delta x}{\Delta t} = \frac{474 \text{ m}}{8 \text{ s}}$$

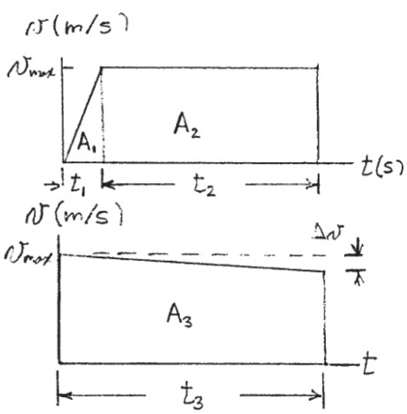
$v_{\text{average}} = 59.25 \text{ m/s}$ ◀

## PROBLEM 11.71




In a 400-m race, runner $A$ reaches her maximum velocity $v_A$ in 4 s with constant acceleration and maintains that velocity until she reaches the half-way point with a split time of 25 s. Runner $B$ reaches her maximum velocity $v_B$ in 5 s with constant acceleration and maintains that velocity until she reaches the half-way point with a split time of 25.2 s. Both runners then run the second half of the race with the same constant deceleration of $0.1\ \text{m/s}^2$. Determine ($a$) the race times for both runners, ($b$) the position of the winner relative to the loser when the winner reaches the finish line.

## SOLUTION




Sketch $v-t$ curves for first 200 m.

Runner $A$: $t_1 = 4$ s, $t_2 = 25 - 4 = 21$ s

$$A_1 = \frac{1}{2}(4)(v_A)_{\max} = 2(v_A)_{\max}$$

$$A_2 = 21(v_A)_{\max}$$

$$A_1 + A_2 = \Delta x = 200 \text{ m}$$

$$23(v_A)_{\max} = 200 \qquad \text{or} \qquad (v_A)_{\max} = 8.6957 \text{ m/s}$$

Runner $B$: $t_1 = 5$ s, $t_2 = 25.2 - 5 = 20.2$ s

$$A_1 = \frac{1}{2}(5)(v_B)_{\max} = 2.5(v_B)_{\max}$$

$$A_2 = 20.2(v_B)_{\max}$$

$$A_1 + A_2 = \Delta x = 200 \text{ m}$$

$$22.7(v_B)_{\max} = 200 \qquad \text{or} \qquad (v_B)_{\max} = 8.8106 \text{ m/s}$$

Sketch $v-t$ curve for second 200 m. $\Delta v = |a| t_3 = 0.1 t_3$

$$A_3 = v_{\max} t_3 - \frac{1}{2}\Delta v t_3 = 200 \qquad \text{or} \qquad 0.05 t_3^2 - v_{\max} t_3 + 200 = 0$$

$$t_3 = \frac{v_{\max} \pm \sqrt{(v_{\max})^2 - (4)(0.05)(200)}}{(2)(0.05)} = 10\left(v_{\max} \pm \sqrt{(v_{\max})^2 - 40}\right)$$

Runner $A$: $(v_{\max})_A = 8.6957,$ $(t_3)_A = 146.64$ s and 27.279 s

Reject the larger root. Then total time $t_A = 25 + 27.279 = 52.279$ s

$t_A = 52.2$ s ◀

## PROBLEM 11.71 (CONTINUED)

Runner $B$: $(v_{\max})_B = 8.8106$, $(t_3)_B = 149.45$ s and 26.765 s

Reject the larger root. Then total time $t_B = 25.2 + 26.765 = 51.965$ s

$t_B = 52.0$ s ◀

Velocity of $A$ at $t = 51.965$ s:

$$v_1 = 8.6957 - (0.1)(51.965 - 25) = 5.999 \text{ m/s}$$

Velocity of $A$ at $t = 51.279$ s:

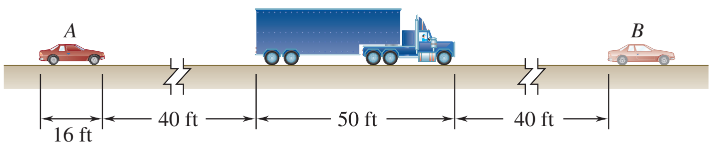
$$v_2 = 8.6957 - (0.1)(52.279 - 25) = 5.968 \text{ m/s}$$

Over $51.965 \text{ s} \le t \le 52.965 \text{s}$, runner $A$ covers a distance $\Delta x$

$$\Delta x = v_{ave}(\Delta t) = \frac{1}{2}(5.999 + 5.968)(52.279 - 51.965)$$

$\Delta x = 1.879$ m ◀

## PROBLEM 11.72

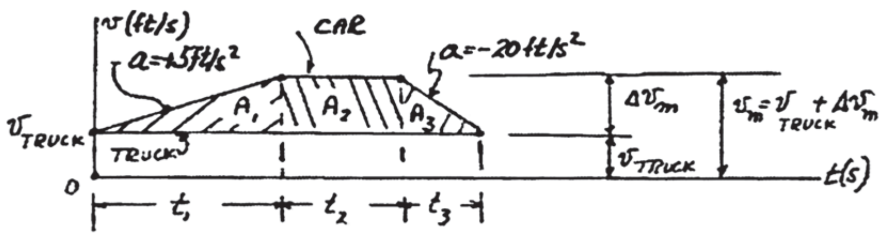



A car and a truck are both traveling at the constant speed of $35\,\text{mi/h}$; the car is 40 ft behind the truck. The driver of the car wants to pass the truck, i.e., he wishes to place his car at *B*, 40 ft in front of the truck, and then resume the speed of $35\,\text{mi/h}$; The maximum acceleration of the car is $5\,\text{ft/s}^2$ and the maximum deceleration obtained by applying the brakes is $20\,\text{ft/s}^2$ What is the shortest time in which the driver of the car can complete the passing operation if he does not at any time exceed a speed of $50\,\text{mi/h}$? Draw the $v-t$ curve.

## SOLUTION

Relative to truck, car must move a distance: $\Delta x = 16 + 40 + 50 + 40 = 146$ ft

Allowable increase in speed: $\Delta v_m = 50 - 35 = 15$ mi/h $= 22$ ft/s




Acceleration Phase: $t_1 = 22/5 = 4.4$ s $\quad A_1 = \dfrac{1}{2}(22)(4.4) = 48.4\,\text{ft}$

Deceleration Phase: $t_3 = 22/20 = 1.1$ s $\quad A_3 = \dfrac{1}{2}(22)(1.1) = 12.1\,\text{ft}$

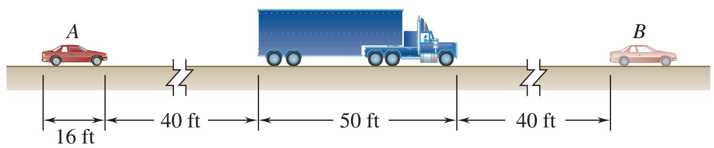
But: $\Delta x = A_1 + A_2 + A_3$: $\quad 146\text{ ft} = 48.4 + (22)t_2 + 12.1 \quad t_2 = 3.89$ s

$t_{\text{total}} = t_1 + t_2 + t_3 = 4.4\,\text{s} + 3.89\,\text{s} + 1.1\,\text{s} = 9.39\,\text{s} \quad t_B = 9.39$ s ◀

## PROBLEM 11.73

Solve Problem 11.72, assuming that the driver of the car does not pay any attention to the speed limit while passing and concentrates on reaching position *B* and resuming a speed of 35 mi/h in the shortest possible time. What is the maximum speed reached? Draw the $v-t$ curve.

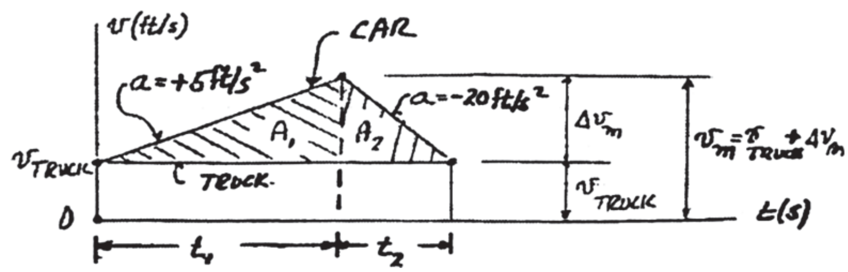
**PROBLEM 11.72** A car and a truck are both traveling at the constant speed of 35 mi/h; the car is 40 ft behind the truck. The driver of the car wants to pass the truck, i.e., he wishes to place his car at *B*, 40 ft in front of the truck, and then resume the speed of 35 mi/h; The maximum acceleration of the car is 5 ft/s$^2$ and the maximum deceleration obtained by applying the brakes is 20 ft/s$^2$ What is the shortest time in which the driver of the car can complete the passing operation if he does not at any time exceed a speed of 50 mi/h? Draw the $v-t$ curve.




## SOLUTION

Relative to truck, car must move a distance:

$$\Delta x = 16 + 40 + 50 + 40 = 146 \text{ ft}$$




$$\Delta v_m = 5t_1 = 20t_2; \; t_2 = \frac{1}{4}t_1$$

$\Delta x = A_1 + A_2$:

$$146 \text{ ft} = \frac{1}{2}(\Delta v_m)(t_1 + t_2)$$

$$146 \text{ ft} = \frac{1}{2}(5t_1)\left(t_1 + \frac{1}{4}t_1\right)$$

$$t_1^2 = 46.72 \qquad t_1 = 6.835 \text{ s} \qquad t_2 = \frac{1}{4}t_1 = 1.709$$

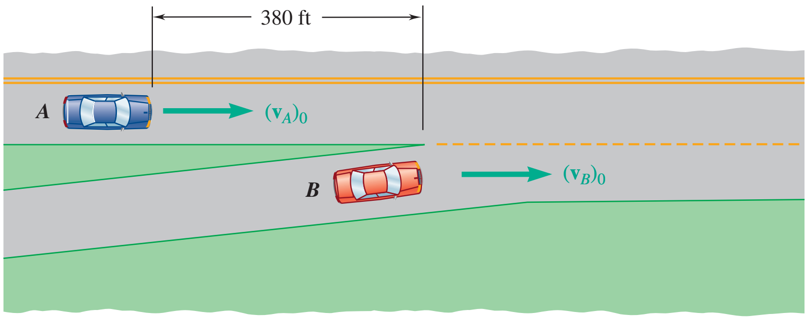
$$t_{\text{total}} = t_1 + t_2 = 6.835 + 1.709$$

$t_B = 8.54$ s ◀

$$\Delta v_m = 5t_1 = 5(6.835) = 34.18 \text{ ft/s} = 23.3 \text{ mi/h}$$

Speed $v_{\text{total}} = 35$ mi/h, $v_m = 35$ mi/h $+ 23.3$ mi/h

$v_m = 58.3$ mi/h ◀



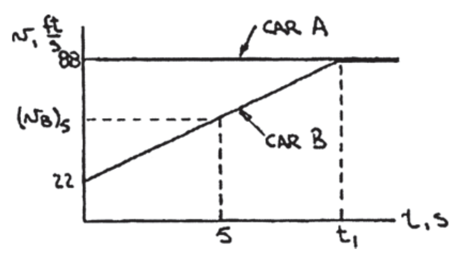

## PROBLEM 11.74

Car $A$ is traveling on a highway at a constant speed $(v_A)_0 = 60\,\text{mi/h}$, and is 380 ft from the entrance of an access ramp when car $B$ enters the acceleration lane at that point at a speed $(v_B)_0 = 15\,\text{mi/h}$, Car $B$ accelerates uniformly and enters the main traffic lane after traveling 200 ft in 5 s. It then continues to accelerate at the same rate until it reaches a speed of $60\,\text{mi/h}$, which it then maintains. Determine the final distance between the two cars.

## SOLUTION

<u>Given</u>:
$$(v_A)_0 = 60 \text{ mi/h}, \ (v_B)_0 = 1.5 \text{ mi/h}, \ \text{at } t = 0,$$
$$(x_A)_0 = -380 \text{ ft}, \ (x_B)_0 = 0; \ \text{at } t = 5 \text{ s},$$
$$x_B = 200 \text{ ft}; \ \text{for } 15 \text{ mi/h} < v_B \le 60 \text{ mi/h},$$
$$a_B = \text{constant; for } v_B = 60 \text{ mi/h},$$
$$a_B = 0$$

First note
$$60 \text{ mi/h} = 88 \text{ ft/s}$$
$$15 \text{ mi/h} = 22 \text{ ft/s}$$




The $v-t$ curves of the two cars are then drawn as shown.

Using the coordinate system shown, we have


at $t = 5$ s, $x_B = 200$ ft:
$$(5 \text{ s})\left[\frac{22 + (v_B)_5}{2}\right] \text{ft/s} = 200 \text{ ft}$$

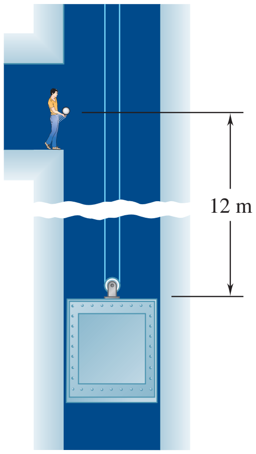
or
$$(v_B)_5 = 58 \text{ ft/s}$$

Then, using similar triangles, we have
$$\frac{(88-22) \text{ ft/s}}{t_1} = \frac{(58-22) \text{ ft/s}}{5 \text{ s}} (= a_B)$$

or
$$t_1 = 9.1667 \text{ s}$$

Finally, at $t = t_1$
$$x_{B/A} = x_B - x_A = \left[(9.1667 \text{ s})\left(\frac{22+88}{2}\right) \text{ft/s}\right] - [-380 \text{ ft} + (9.1667 \text{ s})(88 \text{ ft/s})]$$

or
$$x_{B/A} = 77.5 \text{ ft} \blacktriangleleft$$




## PROBLEM 11.75

An elevator starts from rest and moves upward, accelerating at a rate of 1.2 m/s$^2$ until it reaches a speed of 7.8 m/s, which it then maintains. Two seconds after the elevator begins to move, a man standing 12 m above the initial position of the top of the elevator throws a ball upward with an initial velocity of 20 m/s. Determine when the ball will hit the elevator.

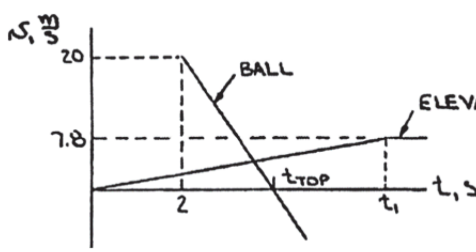
## SOLUTION

<u>Given</u>: At $t = 0$ $v_E = 0$; For $0 < v_E \le 7.8$ m/s, $a_E = 1.2$ m/s$^2$ $\uparrow$;

For $v_E = 7.8$ m/s, $a_E = 0$;

At $t = 2$ s, $v_B = 20$m/s $\uparrow$

The $v - t$ curves of the ball and the elevator are first drawn as shown. Note that the initial slope of the curve for the elevator is 1.2 m/s$^2$, while the slope of the curve for the ball is $-g(-9.81$ m/s$^2)$.




The time $t_1$ is the time when $v_E$ reaches 7.8 m/s

Thus, $$v_E = (0) + a_E t$$

or $$7.8 \text{ m/s} = (1.2 \text{ m/s}^2)t_1$$

or $$t_1 = 6.5 \text{ s}$$

The time $t_{top}$ is the time at which the ball reaches the top of its trajectory.

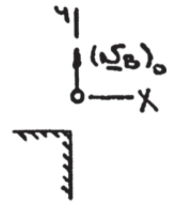
Thus, $$v_B = (v_B)_0 - g(t-2)$$

or $$0 = 20 \text{m/s} - (9.81 \text{ m/s}^2)(t_{top} - 2) \text{ s}$$

or $$t_{top} = 4.0387 \text{ s}$$

## PROBLEM 11.75 (CONTINUED)

Using the coordinate system shown, we have




$0 < t < t_1:$ $$y_E = -12\,\text{m} + \left(\frac{1}{2}a_E t^2\right)\text{m}$$

At $t = t_{\text{top}}$: $$y_B = \frac{1}{2}(4.0387 - 2)\ \text{s} \times (20\ \text{m/s})$$
$$= 20.387\ \text{m}$$

and $$y_E = -12\ \text{m} + \frac{1}{2}(1.2\ \text{m/s}^2)(4.0387\,\text{s})^2$$
$$= -2.213\ \text{m}$$

At $$t = [2{+}2(4.0387 - 2)]\text{s} = 6.0774\ \text{s},\ y_B = 0$$

and at $t = t_1$, $$y_E = -12\ \text{m} + \frac{1}{2}(6.5\ \text{s})(7.8\ \text{m/s}) = 13.35\ \text{m}$$

The ball hits the elevator $(y_B = y_E)$ when $t_{\text{top}} \le t \le t_1$.

For $t \ge t_{\text{top}}$: $$y_B = 20.387\,\text{m} - \left[\frac{1}{2}g(t - t_{\text{top}})^2\right]\text{m}$$

Then,

when $$y_B = y_B$$

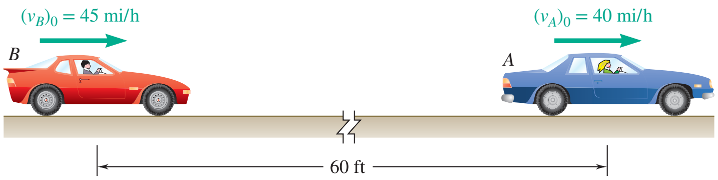
$$20.387\,\text{m} - \frac{1}{2}(9.81\,\text{m/s}^2)(t - 4.0387)^2$$
$$= -12\,\text{m} + \frac{1}{2}(1.2\,\text{m/s}^2)(t\,\text{s})^2$$

or $$5.505t^2 - 39.6196t + 47.619 = 0$$

Solving $$t = 1.525\,\text{s} \quad \text{and} \quad t = 5.67\,\text{s}$$

Since 1.525 s is less than 2 s, $t = 5.67\,\text{s}$ ◀

## PROBLEM 11.76




Car $A$ is traveling at 40 mi/h when it enters a 30 mi/h speed zone. The driver of car $A$ decelerates at a rate of 16 ft/s$^2$ until reaching a speed of 30 mi/h, which she then maintains. When car $B$, which was initially 60 ft behind car $A$ and traveling at a constant speed of 45 mi/h, enters the speed zone, its driver decelerates at a rate of 20 ft/s$^2$ until reaching a speed of 28 mi/h. Knowing that the driver of car $B$ maintains a speed of 28 mi/h, determine ($a$) the closest that car $B$ comes to car $A$, ($b$) the time at which car $A$ is 70 ft in front of car $B$.

## SOLUTION

Given: $(v_A)_0 = 40$ mi/h; For 30 mi/h $< v_A \le 40$ mi/h, $a_A = -16\text{ft/s}^2$; For $v_A = 30$ mi/h, $a_A = 0$;

$(x_{A/B})_0 = 60$ ft; $(v_B)_0 = 45$ mi/h;

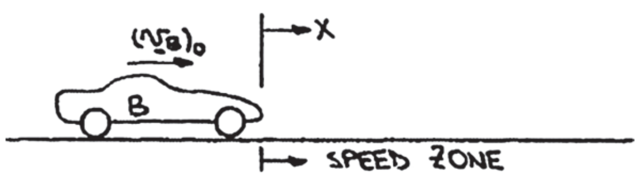
When $x_B = 0$, $a_B = -20$ ft/s$^2$;

For $v_B = 28$ mi/h, $a_B = 0$

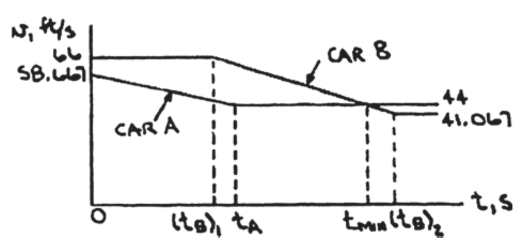
First note 40 mi/h = 58.667 ft/s 30 mi/h = 44 ft/s

45 mi/h = 66 ft/s 28 mi/h = 41.067 ft/s

At $t = 0$




The $v-t$ curves of the two cars are as shown.




At $t = 0$: Car $A$ enters the speed zone.

$t = (t_B)_1$: Car $B$ enters the speed zone.

$t = t_A$: Car $A$ reaches its final speed.

$t = t_{\min}$: $v_A = v_B$

$t = (t_B)_2$: Car $B$ reaches its final speed.

## PROBLEM 11.76 (CONTINUED)

(*a*) We have
$$a_A = \frac{(v_A)_{\text{final}} - (v_A)_0}{t_A}$$

or
$$-16 \text{ ft/s}^2 = \frac{(44 - 58.667) \text{ ft/s}}{t_A}$$

or
$$t_A = 0.91669 \text{ s}$$

Also
$$60 \text{ ft} = (t_B)_1 (v_B)_0$$

or
$$60 \text{ ft} = (t_B)_1 (66 \text{ ft/s}) \quad \text{or} \quad (t_B)_1 = 0.90909 \text{ s}$$

and
$$a_B = \frac{(v_B)_{\text{final}} - (v_B)_0}{(t_B)_2 - (t_B)_1}$$

or
$$-20 \text{ ft/s}^2 = \frac{(41.067 - 66) \text{ ft/s}}{[(t_B)_2 - 0.90909]\text{s}}$$

Car *B* will continue to overtake car *A* while $v_B > v_A$. Therefore, $(x_{A/B})_{\min}$ will occur when $v_A = v_B$, which occurs for
$$(t_B)_1 < t_{\min} < (t_B)_2$$

For this time interval
$$v_A = 44 \text{ ft/s}$$
$$v_B = (v_B)_0 + a_B[t - (t_B)_1]$$

Then at $t = t_{\min}$:
$$44 \text{ ft/s} = 66 \text{ ft/s} + (-20 \text{ ft/s}^2)(t_{\min} - 0.90909)\text{s}$$

or
$$t_{\min} = 2.00909 \text{ s}$$

Finally
$$\begin{aligned}
(x_{A/B})_{\min} &= (x_A)_{t_{\min}} - (x_B)_{t_{\min}} \\
&= \left\{ t_A \left[ \frac{(v_A)_0 + (v_A)_{\text{final}}}{2} \right] + (t_{\min} - t_A)(v_A)_{\text{final}} \right\} \\
&\quad - \left\{ (x_B)_0 + (t_B)_1 (v_B)_0 + [t_{\min} - (t_B)_1] \left[ \frac{(v_B)_0 + (v_A)_{\text{final}}}{2} \right] \right\} \\
&= \left[ (0.91669 \text{ s}) \left( \frac{58.667 + 44}{2} \right) \text{ft/s} + (2.00909 - 0.91669)\text{s} \times (44 \text{ ft/s}) \right] \\
&\quad - \left[ -60 \text{ ft} + (0.90909 \text{ s})(66 \text{ ft/s}) + (2.00909 - 0.90909) \text{ s} \times \left( \frac{66 + 44}{2} \right) \text{ft/s} \right] \\
&= (47.057 + 48.066) \text{ ft} - (-60 + 60.000 + 60.500) \text{ ft} \\
&= 34.623 \text{ ft}
\end{aligned}$$

or $(x_{A/B})_{\min} = 34.6 \text{ ft}$ ◀

## PROBLEM 11.76 (CONTINUED)

(*b*) Since $(x_{A/B}) \leq$ 60 ft for $t \leq t_{\min}$, it follows that $x_{A/B} = 70$ ft for $t > (t_B)_2$

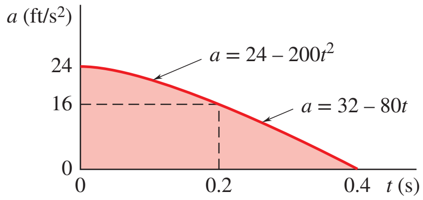
[Note $(t_B)_2 \simeq t_{\min}$]. Then, for $t > (t_B)_2$

$$x_{A/B} = (x_{A/B})_{\min} + [(t - t_{\min})(v_A)_{\text{final}}]$$
$$- \left\{[(t_B)_2 - (t_{\min})]\left[\frac{(v_A)_{\text{final}} + (v_B)_{\text{final}}}{2}\right] + [t - (t_B)_2](v_B)_{\text{final}}\right\}$$

or

$$70\text{ft} = 34.623\text{ft} + [(t - 2.00909)\text{s} \times (44\text{ft/s})]$$
$$- \left[(2.15574 - 2.00909)\text{s} \times \left(\frac{44 + 41.06}{2}\right)\text{ft/s} + (t - 2.15574)\text{s} \times (41.067)\text{ft/s}\right]$$

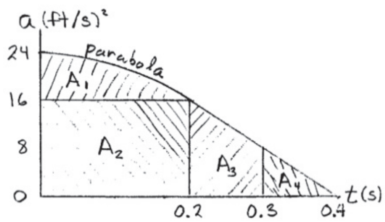
or $t = 14.14$ s ◀

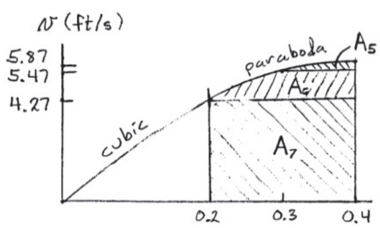



## PROBLEM 11.77

An accelerometer record for the motion of a given part of a mechanism is approximated by an arc of a parabola for 0.2 s and a straight line for the next 0.2 s as shown in the figure. Knowing that $v = 0$ when $t = 0$ and $x = 0.8$ ft when $t = 0.4$ s, (*a*) construct the $v - t$ curve for $0 \leq t \leq 0.4$ s, (*b*) determine the position of the part at $t = 0.3$ s and $t = 0.2$ s.

## SOLUTION




Divide the area of the $a - t$ curve into the four areas $A_1$, $A_2$, $A_3$ and $A_4$.

$$A_1 = \frac{2}{3}(8)(0.2) = 1.0667 \text{ ft/s}$$

$$A_2 = (16)(0.2) = 3.2 \text{ ft/s}$$

$$A_3 = \frac{1}{2}(16 + 8)(0.1) = 1.2 \text{ ft/s}$$

$$A_4 = \frac{1}{2}(8)(0.1) = 0.4 \text{ ft/s}$$

Velocities: $v_0 = 0$

$v_{0.2} = v_0 + A_1 + A_2$ $\qquad v_{0.2} = 4.27$ ft/s ◀

$v_{0.3} = v_{0.2} + A_3$ $\qquad v_{0.3} = 5.47$ ft/s ◀

$v_{0.4} = v_{0.3} + A_4$ $\qquad v_{0.4} = 5.87$ ft/s ◀

Sketch the $v - t$ curve and divide its area into $A_5$, $A_6$, and $A_7$ as shown.

$$\int_x^{0.8} dx = 0.8 - x = \int_t^{0.4} v dt \quad \text{or} \quad x = 0.8 - \int_t^{0.4} v dt$$

At $t = 0.3$ s, $\qquad x_{0.3} = 0.8 - A_5 - (5.47)(0.1)$

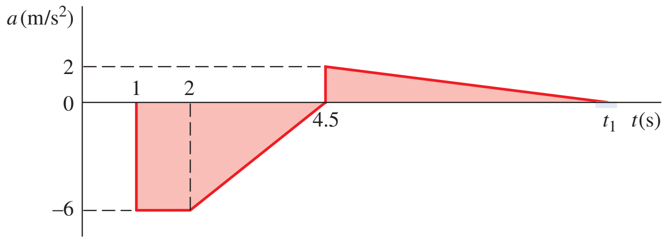
With $A_5 = \frac{2}{3}(0.4)(0.1) = 0.0267$ ft, $\qquad x_{0.3} = 0.227$ ft ◀

At $t = 0.2$ s, $\qquad x_{0.2} = 0.8 - (A_5 + A_6) - A_7$

With $A_5 + A_6 = \frac{2}{3}(1.6)(0.2) = 0.2133$ ft,

and $A7 = (4.27)(0.2) = 0.8533$ ft

$x_{0.2} = 0.8 - 0.2133 - 0.8533$ $\qquad x_{0.2} = -0.267$ ft ◀




## PROBLEM 11.78

A car is traveling at a constant speed of 54 km/h when its driver sees a child run into the road. The driver applies her brakes until the child returns to the sidewalk and then accelerates to resume her original speed of 54 km/h; the acceleration record of the car is shown in the figure. Assuming $x = 0$ when $t = 0$, determine (*a*) the time $t_1$ at which the velocity is again 54 km/h, (*b*) the position of the car at that time, (*c*) the average velocity of the car during the interval $1\text{ s} \le t \le t_1$.

## SOLUTION

<u>Given</u>: At $t = 0,\quad x = 0,\quad v = 54\ \text{km/h};$

For $t = t_1,\quad v = 54\ \text{km/h}$

First note $54\ \text{km/h} = 15\ \text{m/s}$

(*a*) We have $v_b = v_a + (\text{area under } a - t \text{ curve from } t_a \text{ to } t_b)$

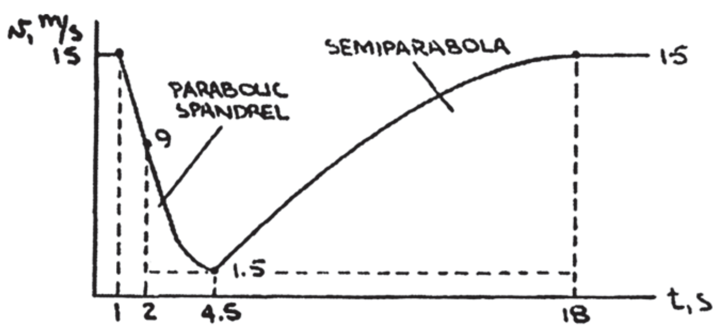
Then at $t = 2\text{ s}:\quad v = 15 - (1)(6) = 9\ \text{m/s}$

$$t = 4.5\text{ s}:\quad v = 9 - \frac{1}{2}(2.5)(6) = 1.5\ \text{m/s}$$

$$t = t_1:\quad 15 = 1.5 + \frac{1}{2}(t_1 - 4.5)(2)$$

or $t_1 = 18.00\text{ s}$ ◀

(*b*) Using the above values of the velocities, the $v - t$ curve is drawn as shown.



## PROBLEM 11.78 (CONTINUED)

Now $x$ at $t = 18\,\text{s}$

$$x_{18} = 0 + \sum \text{(area under the } v - t \text{ curve from } t = 0 \text{ to } t = 18 \text{ s)}$$

$$= (1\,\text{s})(15\,\text{m/s}) + (1\,\text{s})\left(\frac{15+9}{2}\right)\text{m/s}$$

$$+ \left[(2.5\text{ s})(1.5\text{ m/s}) + \frac{1}{3}(2.5\,\text{s})(7.5\,\text{m/s})\right]$$

$$+ \left[(13.5\text{ s})(1.5\text{ m/s}) + \frac{2}{3}(13.5\,\text{s})(13.5\,\text{m/s})\right]$$

$$= [15 + 12 + (3.75 + 6.25) + (20.25 + 121.50)]\text{m}$$

$$= 178.75\,\text{m}$$

or $x_{18} = 178.8\,\text{m}$ ◀

(*c*) First note

$$x_1 = 15\,\text{m}$$

$$x_{18} = 178.75\,\text{m}$$

Now

$$v_{\text{ave}} = \frac{\Delta x}{\Delta t} = \frac{(178.75 - 15)\,\text{m}}{(18 - 1)\,\text{s}} = 9.6324\,\text{m/s}$$

or $v_{\text{ave}} = 34.7\,\text{km/h}$ ◀

## PROBLEM 11.79

An airport shuttle train travels between two terminals that are 1.6 mi apart. To maintain passenger comfort, the acceleration of the train is limited to $\pm 4$ ft/s$^2$, and the jerk, or rate of change of acceleration, is limited to $\pm 0.8$ ft/s$^2$ per second. If the shuttle has a maximum speed of 20 mi/h, determine (*a*) the shortest time for the shuttle to travel between the two terminals, (*b*) the corresponding average velocity of the shuttle.

## SOLUTION

Given:
$$x_{\max} = 1.6\,\text{mi}; \; |a_{\max}| = 4 \text{ ft/s}^2$$
$$\left|\left(\frac{da}{dt}\right)_{\max}\right| = 0.8\,\text{ft/s}^2\text{/s}; \; v_{\max} = 20 \text{ mi/h}$$

First note
$$20 \text{ mi/h} = 29.333 \text{ ft/s}$$
$$1.6 \text{ mi} = 8448 \text{ ft}$$

(*a*) To obtain $t_{\min}$, the train must accelerate and decelerate at the maximum rate to maximize the time for which $v = v_{\max}$. The time $\Delta t$ required for the train to have an acceleration of 4 ft/s$^2$ is found from
$$\left(\frac{da}{dt}\right)_{\max} = \frac{a_{\max}}{\Delta t}$$

or
$$\Delta t = \frac{4 \text{ ft/s}^2}{0.8 \text{ ft/s}^2\text{/s}}$$

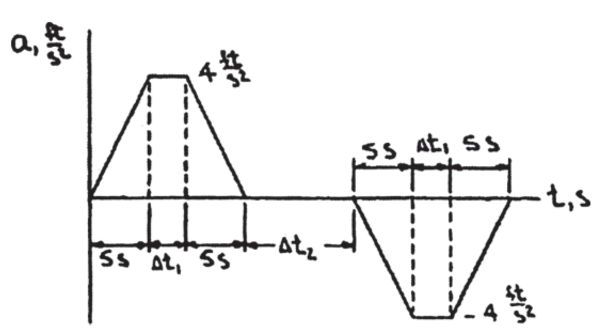
or
$$\Delta t = 5 \text{ s}$$

Now,

after 5 s, the speed of the train is
$$v_5 = \frac{1}{2}(\Delta t)(a_{\max}) \qquad \left(\text{since}\,\frac{da}{dt} = \text{constant}\right)$$

or
$$v_5 = \frac{1}{2}(5 \text{ s})(4 \text{ ft/s}^2) = 10 \text{ ft/s}$$

Then, since $v_5 < v_{\max}$, the train will continue to accelerate at 4 ft/s$^2$ until $v = v_{\max}$. The $a - t$ curve must then have the shape shown. Note that the magnitude of the slope of each inclined portion of the curve is 0.8 ft/s$^2$/s.



## PROBLEM 11.79 (CONTINUED)

Now at $t = (10 + \Delta t_1)\text{s}, \quad v = v_{\max}:$

$$2\left[\frac{1}{2}(5\text{ s})(4\text{ ft/s}^2)\right] + (\Delta t_1)(4\text{ ft/s}^2) = 29.333\text{ ft/s}$$

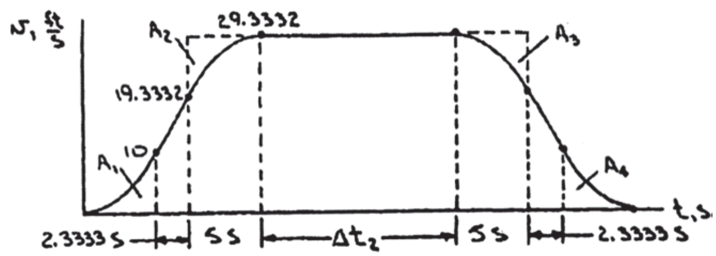
or $\Delta t_1 = 2.3333\text{ s}$

Then at $t = 5\text{ s}:\quad v = 0 + \frac{1}{2}(5)(4) = 10\text{ ft/s}$

$t = 7.3333\text{ s}:\quad v = 10 + (2.3333)(4) = 19.3332\text{ ft/s}$

$t = 12.3333\text{ s}:\quad v = 19.3332 + \frac{1}{2}(5)(4) = 29.3332\text{ ft/s}$

Using symmetry, the $v - t$ curve is then drawn as shown.




Noting that $A_1 = A_2 = A_3 = A_4$ and that the area under the $v - t$ curve is equal to $x_{\max}$, we have

$$2\left[(2.3333\text{ s})\left(\frac{10 + 19.3332}{2}\right)\text{ft/s}\right] + (10 + \Delta t_2)\text{s} \times (29.3332\text{ ft/s}) = 8448\text{ ft}$$

or $\Delta t_2 = 275.67\text{ s}$

Then $t_{\min} = 4(5\text{ s}) + 2(2.3333\text{ s}) + 275.67\text{ s}$

$= 300.34\text{ s}$

or $t_{\min} = 5.01\text{min}$ ◀

(*b*) We have $v_{\text{ave}} = \dfrac{\Delta x}{\Delta t} = \dfrac{1.6\text{ mi}}{300.34\text{ s}} \times \dfrac{3600\text{ s}}{1\text{ h}}$

or $v_{\text{ave}} = 19.18\text{ mi/h}$ ◀

## PROBLEM 11.80

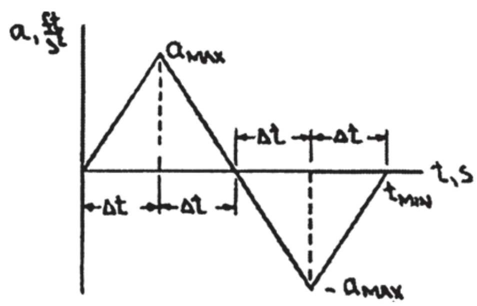
During a manufacturing process, a conveyor belt starts from rest and travels a total of 1.2 ft before temporarily coming to rest. Knowing that the jerk, or rate of change of acceleration, is limited to $\pm 4.8$ ft/s$^2$ per second, determine (*a*) the shortest time required for the belt to move 1.2 ft, (*b*) the maximum and average values of the velocity of the belt during that time.

## SOLUTION

Given: At $t = 0, \quad x = 0, \quad v = 0; \quad x_{\max} = 1.2$ ft;

when $x = x_{\max}, \quad v = 0; \quad \left|\left(\dfrac{da}{dt}\right)_{\max}\right| = 4.8$ ft/s$^2$

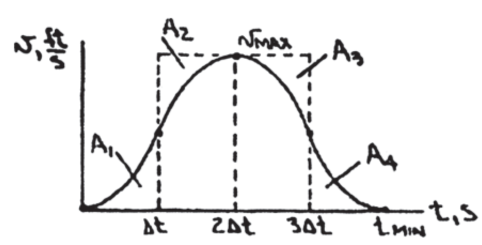
(*a*) Observing that $v_{\max}$ must occur at $t = \frac{1}{2}t_{\min}$, the $a-t$ curve must have the shape shown. Note that the magnitude of the slope of each portion of the curve is 4.8 ft/s$^2$/s.




We have at $t = \Delta t$: $\quad v = 0 + \frac{1}{2}(\Delta t)(a_{\max}) = \frac{1}{2}a_{\max}\Delta t$

$t = 2\Delta t$: $\quad v_{\max} = \frac{1}{2}a_{\max}\Delta t + \frac{1}{2}(\Delta t)(a_{\max}) = a_{\max}\Delta t$

Using symmetry, the $v-t$ is then drawn as shown.




Noting that $A_1 = A_2 = A_3 = A_4$ and that the area under the $v-t$ curve is equal to $x_{\max}$, we have

$$(2\Delta t)(v_{\max}) = x_{\max}$$

$$v_{\max} = a_{\max}\Delta t \Rightarrow 2a_{\max}\Delta t^2 = x_{\max}$$

## PROBLEM 11.80 (CONTINUED)

Now $\dfrac{a_{max}}{\Delta t} = 4.8\text{ ft/s}^2/\text{s}$ so that

or
$$2(4.8\Delta t\text{ ft/s}^3)\Delta t^2 = 1.2\text{ ft}$$
$$\Delta t = 0.5\text{ s}$$

Then $t_{min} = 4\Delta t$

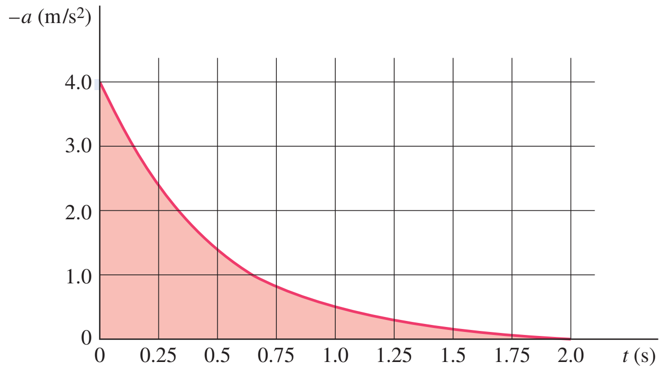
or $t_{min} = 2.00\text{ s}$ ◀

(*b*) We have
$$v_{max} = a_{max}\Delta t$$
$$= (4.8\text{ft/s}^2/\text{s} \times \Delta\text{t})\Delta t$$
$$= 4.8\text{ft/s}^2/\text{s} \times (0.5\text{ s})^2$$

or $v_{max} = 1.2\text{ ft/s}$ ◀

Also $v_{ave} = \dfrac{\Delta x}{\Delta t_{total}} = \dfrac{1.2\text{ ft}}{2.00\text{ s}}$

or $v_{ave} = 0.6\text{ ft/s}$ ◀




## PROBLEM 11.81

Two seconds are required to bring the piston rod of an air cylinder to rest; the acceleration record of the piston rod during the 2 s is as shown. Determine by approximate means (*a*) the initial velocity of the piston rod, (*b*) the distance traveled by the piston rod as it is brought to rest.

## SOLUTION

Given: $a-t$ curve; at $t = 2$ s, $v = 0$

1. The $a-t$ curve is first approximated with a series of rectangles, each of width $\Delta t = 0.25$ s. The area $(\Delta t)\,(a_{\text{ave}})$ of each rectangle is approximately equal to the change in velocity $\Delta v$ for the specified interval of time. Thus,

$$\Delta v \cong a_{\text{ave}}\ \Delta t$$

where the values of $a_{\text{ave}}$ and $\Delta v$ are given in columns 1 and 2, respectively, of the following table.

2. Now

$$v(2) = v_0 + \int_0^2 a\ dt = 0$$

and approximating the area $\int_0^2 a\,dt$ under the $a-t$ curve by $\sum a_{\text{ave}}\Delta t \approx \sum \Delta v$, the initial velocity is then equal to

$$v_0 = -\sum \Delta v$$

Finally, using

$$v_2 = v_1 + \Delta v_{12}$$

where $\Delta v_{12}$ is the change in velocity between times $t_1$ and $t_2$, the velocity at the end of each 0.25 interval can be computed; see column 3 of the table and the $v-t$ curve.

3. The $v-t$ curve is then approximated with a series of rectangles, each of width 0.25 s. The area $(\Delta t)(v_{\text{ave}})$ of each rectangle is approximately equal to the change in position $\Delta x$ for the specified interval of time. Thus

$$\Delta x \approx v_{\text{ave}}\Delta t$$

where $v_{\text{ave}}$ and $\Delta x$ are given in columns 4 and 5, respectively, of the table.

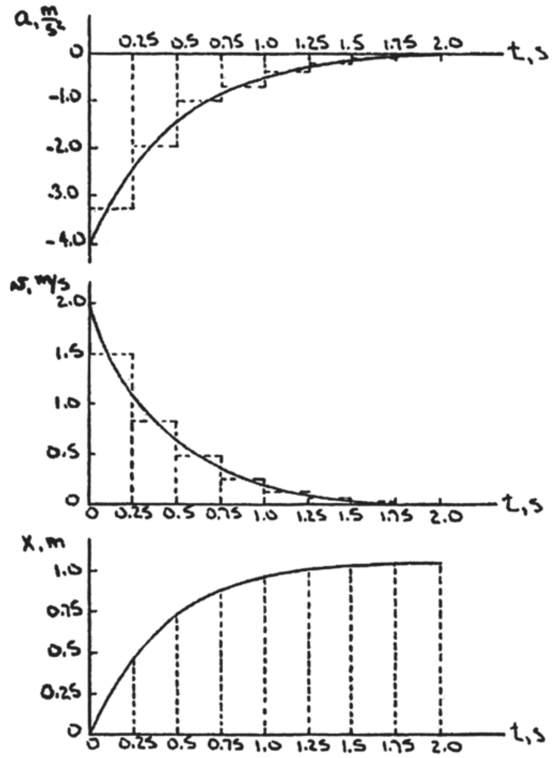
**PROBLEM 11.81 (CONTINUED)**

4. With $x_0 = 0$ and noting that

$$x_2 = x_1 + \Delta x_{12}$$

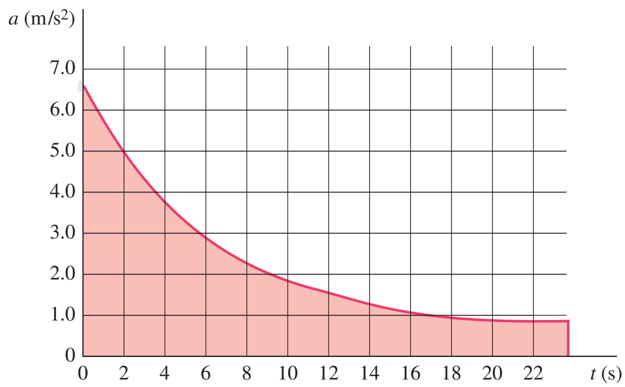
where $\Delta x_{12}$ is the change in position between times $t_1$ and $t_2$, the position at the end of each 0.25 s interval can be computed; see column 6 of the table and the $x - t$ curve.

| | | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $t$, s | $a$, m/s² | $a_{ave}$, m/s² | $\Delta v$, m/s | $v$, m/s | $v_{ave}$, m/s | $\Delta x$, m | $x$, m |
| 0 | -4.00 | | | 1.914 | | | 0 |
| | | -3.215 | -0.804 | | 1.512 | 0.378 | |
| 0.25 | -2.43 | | | 1.110 | | | 0.378 |
| | | -1.915 | -0.479 | | 0.871 | 0.218 | |
| 0.50 | -1.40 | | | 0.631 | | | 0.596 |
| | | -1.125 | -0.281 | | 0.491 | 0.123 | |
| 0.75 | -0.85 | | | 0.350 | | | 0.719 |
| | | -0.675 | -0.169 | | 0.266 | 0.067 | |
| 1.00 | -0.50 | | | 0.181 | | | 0.786 |
| | | -0.390 | -0.098 | | 0.132 | 0.033 | |
| 1.25 | -0.28 | | | 0.083 | | | 0.819 |
| | | -0.205 | -0.051 | | 0.058 | 0.015 | |
| 1.50 | -0.13 | | | 0.032 | | | 0.834 |
| | | -0.095 | -0.024 | | 0.020 | 0.005 | |
| 1.75 | -0.06 | | | 0.008 | | | 0.839 |
| | | -0.030 | -0.008 | | 0.004 | 0.001 | |
| 2.00 | 0 | | | 0 | | | 0.840 |




(*a*) We had found $v_0 = 1.914$ m/s ◀

(*b*) At $t = 2$ s $x = 0.840$ m ◀




## PROBLEM 11.82

The acceleration record shown was obtained during the speed trials of a sports car. Knowing that the car starts from rest, determine by approximate means (*a*) the velocity of the car at $t = 8$ s, (*b*) the distance the car has traveled at $t = 20$ s.

## SOLUTION

Given: $a-t$ curve; at $t = 0,\ x = 0,\ v = 0$

1. The $a-t$ curve is first approximated with a series of rectangles, each of width $\Delta t = 2\text{s}.$ The area $(\Delta t)(a_{ave})$ of each rectangle is approximately equal to the change in velocity $\Delta v$ for the specified interval of time. Thus,

$$\Delta v \cong a_{ave}\Delta t$$

where the values of $a_{ave}$ and $\Delta v$ are given in columns 1 and 2, respectively, of the following table.

2. Noting that $v_0 = 0$ and that

$$v_2 = v_1 + \Delta v_{12}$$

where $\Delta v_{12}$ is the change in velocity between times $t_1$ and $t_2$, the velocity at the end of each 2 s interval can be computed; see column 3 of the table and the $v-t$ curve.

3. The $v-t$ curve is next approximated with a series of rectangles, each of width $\Delta t = 2$ s. The area $(\Delta t)(v_{ave})$ of each rectangle is approximately equal to the change in position $\Delta x$ for the specified interval of time.

Thus, $$\Delta x \cong v_{ave}\Delta t$$

where $v_{ave}$ and $\Delta x$ are given in columns 4 and 5, respectively, of the table.

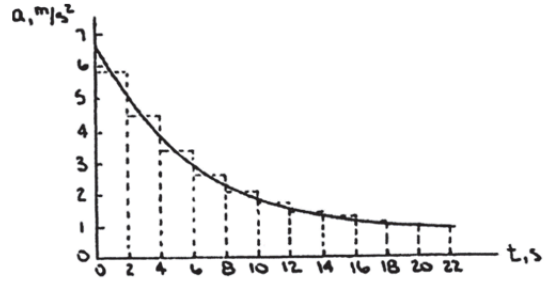
4. With $x_0 = 0$ and noting that

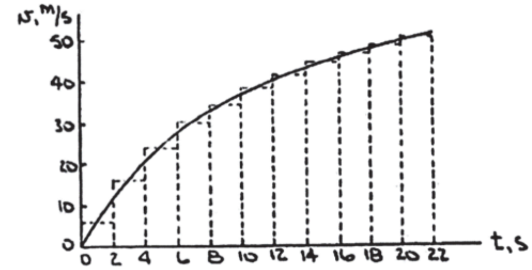
$$x_2 = x_1 + \Delta x_{12}$$

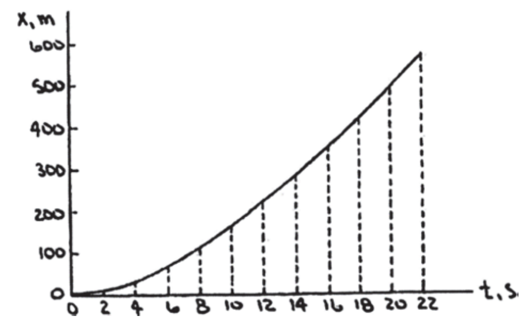
where $\Delta x_{12}$ is the change in position between times $t_1$ and $t_2$, the position at the end of each 2 s interval can be computed; see column 6 of the table and the $x-t$ curve.

## PROBLEM 11.82 (CONTINUED)

| | | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| t, s | a, m/s² | $a_{AVE}$, m/s² | Δv, m/s | v, m/s | $v_{AVE}$, m/s | Δx, m | x, m |
| 0 | 6.63 | | | 0 | | | 0 |
| | | 5.86 | 11.72 | | 5.86 | 11.72 | |
| 2 | 5.08 | | | 11.72 | | | 11.72 |
| | | 4.47 | 8.94 | | 16.19 | 32.38 | |
| 4 | 3.86 | | | 20.66 | | | 44.10 |
| | | 3.38 | 6.76 | | 24.04 | 48.08 | |
| 6 | 2.90 | | | 27.42 | | | 92.18 |
| | | 2.58 | 5.16 | | 30.00 | 60.00 | |
| 8 | 2.25 | | | 32.58 | | | 152.18 |
| | | 2.06 | 4.12 | | 34.64 | 69.28 | |
| 10 | 1.87 | | | 36.70 | | | 221.46 |
| | | 1.71 | 3.42 | | 38.41 | 76.82 | |
| 12 | 1.54 | | | 40.12 | | | 298.28 |
| | | 1.42 | 2.84 | | 41.54 | 83.08 | |
| 14 | 1.29 | | | 42.96 | | | 381.36 |
| | | 1.23 | 2.46 | | 44.19 | 88.38 | |
| 16 | 1.16 | | | 45.42 | | | 469.74 |
| | | 1.10 | 2.20 | | 46.52 | 93.04 | |
| 18 | 1.03 | | | 47.62 | | | 562.78 |
| | | 1.00 | 2.00 | | 48.62 | 97.24 | |
| 20 | 0.97 | | | 49.62 | | | 660.02 |
| | | 0.94 | 1.88 | | 50.56 | 101.12 | |
| 22 | 0.90 | | | 51.50 | | | 761.14 |

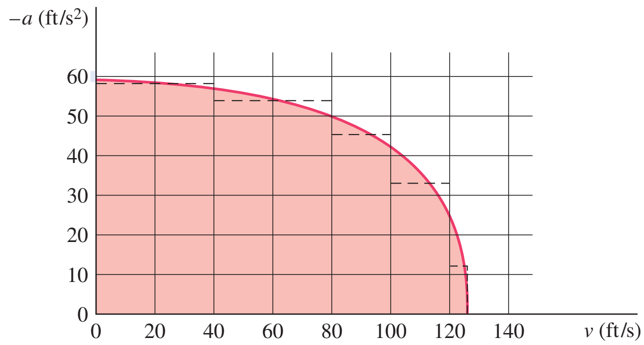



(*a*) At $t = 8$ s, $v = 32.58$ m/s or $v = 117.3$ km/h ◀

(*b*) At $t = 20$ s $x = 660$ m ◀




## PROBLEM 11.83

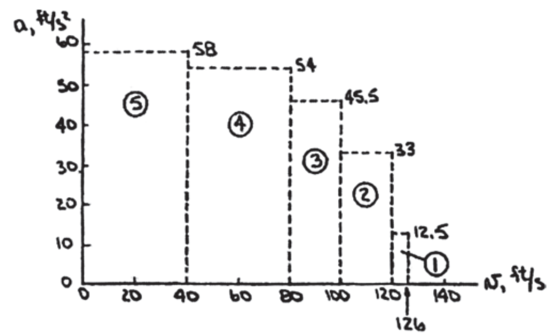
A training airplane has a velocity of 126 ft/s when it lands on an aircraft carrier. As the arresting gear of the carrier brings the airplane to rest, the velocity and the acceleration of the airplane are recorded; the results are shown (solid curve) in the figure. Determine by approximate means (*a*) the time required for the airplane to come to rest, (*b*) the distance traveled in that time.

## SOLUTION

<u>Given</u>: $a - v$ curve:

$$v_0 = 126 \text{ ft/s}$$

The given curve is approximated by a series of uniformly accelerated motions (the horizontal dashed lines on the figure).




For uniformly accelerated motion

$$v_2^2 = v_1^2 + 2a(x_2 - x_1)$$

$$v_2 = v_1 + a(t_2 - t_1)$$

or

$$\Delta x = \frac{v_2^2 - v_1^2}{2a}$$

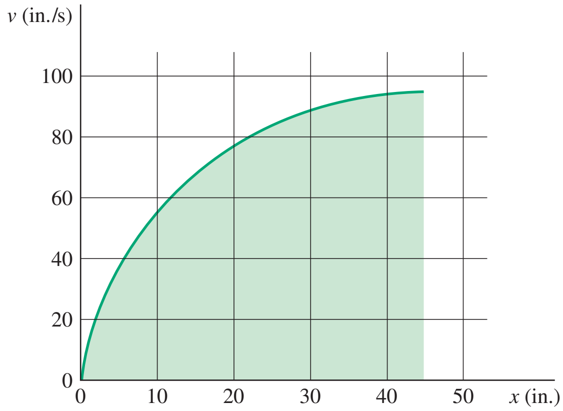
$$\Delta t = \frac{v_2 - v_1}{a}$$

For the five regions shown above, we have

| Region | $v_1$, ft/s | $v_2$, ft/s | $a$, ft/s$^2$ | $\Delta x$, ft | $\Delta t$, s |
|---|---|---|---|---|---|
| 1 | 126 | 120 | −12.5 | 59.0 | 0.480 |
| 2 | 120 | 100 | −33 | 66.7 | 0.606 |
| 3 | 100 | 80 | −45.5 | 39.6 | 0.440 |
| 4 | 80 | 40 | −54 | 44.4 | 0.741 |
| 5 | 40 | 0 | −58 | 13.8 | 0.690 |
| $\sum$ | | | | 223.5 | 2.957 |

(*a*) From the table, when $v = 0$ $\quad t = 2.96$ s ◀

(*b*) From the table and assuming $x_0 = 0$, when $v = 0$ $\quad x = 224$ ft ◀

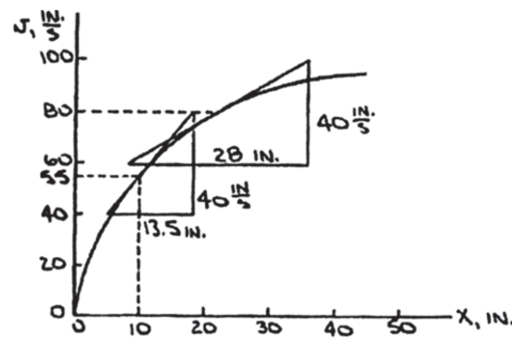



## PROBLEM 11.84

Shown in the figure is a portion of the experimentally determined $v-x$ curve for a shuttle cart. Determine by approximate means the acceleration of the cart (*a*) when $x = 10$ in., (*b*) when $v = 80$ in./s.

## SOLUTION

Given: $v-x$ curve




First note that the slope of the above curve is $\frac{dv}{dx}$. Now

$$a = v\frac{dv}{dx}$$

(*a*) When $x = 10$ in., $v = 55$ in./s

Then

$$a = 55 \text{ in./s}\left(\frac{40 \text{ in./s}}{13.5 \text{ in.}}\right)$$

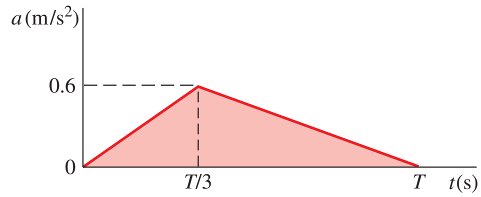
or $a = 163.0 \text{ in./s}^2$ ◀

(*b*) When $v = 80$ in./s, we have

$$a = 80 \text{ in./s}\left(\frac{40 \text{ in./s}}{28 \text{ in.}}\right)$$

or $a = 114.3 \text{ in./s}^2$ ◀

*Note:* To use the method of measuring the subnormal outlined at the end of Section 11.8, it is necessary that the same scale be used for the *x* and *v* axes (e.g., 1 in. = 50 in., 1 in. = 50 in./s). In the above solution, $\Delta v$ and $\Delta x$ were measured directly, so different scales could be used.



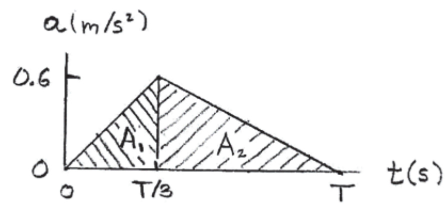

## PROBLEM 11.85

An elevator starts from rest and rises 40 m to its maximum velocity in $T$ s with the acceleration record shown in the figure. Determine (*a*) the required time $T$, (*b*) the maximum velocity, (*c*) the velocity and position of the elevator at $t = T/2$.

## SOLUTION

Find the position of the elevator from the $a-t$ curve using the moment-area method.

First, find the areas $A_1$ and $A_2$ on the $a-t$ curve




$$A_1 = \frac{1}{2}(0.6)\frac{T}{3} = 0.1T \text{ m/s}$$

$$A_2 = \frac{1}{2}(0.6)\frac{2T}{3} = 0.2T \text{ m/s}$$

(*a*) Apply the moment-area formula: $x - x_0 = v_0 t + \int_0^T (T-t)\,a\,dt$

$$x = v_0 t + (A_1)\left(T - \frac{2}{3}\left(\frac{T}{3}\right)\right) + A_2\left[T - \left(\frac{T}{3} + \frac{1}{3}\left(\frac{2T}{3}\right)\right)\right]$$

$$40 = 0 + \frac{7}{90}T^2 + \frac{8}{90}T^2$$

$$= \frac{15}{90}T^2 \quad = \frac{1}{6}T^2$$

$$T^2 = (40)(6)$$

$$= 240 \text{ s}^2$$

$T = 15.49$ s ◀

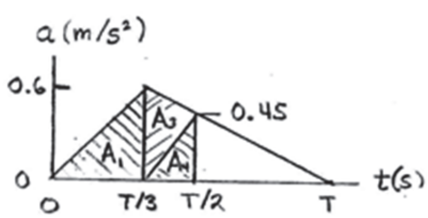
(*b*)
$$v_{max} = v_0 + A_1 + A_2 = 0 + 0.1T + 0.2T = 0.3T$$

$v_{max} = 4.65$ m/s ◀

(*c*) To find the position at t = T/2, first find the areas $A_1$ and $A_2$ on the $a-t$ curve

$$A_1 = 0.1T \qquad A_3 = \frac{1}{2}(0.6)\frac{T}{6} = 0.05T$$

$$A_4 = \frac{1}{2}(0.45)\frac{T}{6} = 0.0375T$$




Apply the moment-area formula: $x - x_0 = v_0 t + \int_0^{T/2}\left(\frac{T}{2} - t\right)a\,dt$

## PROBLEM 11.85 (CONTINUED)

$$x = v_0 \frac{T}{2} + A_1\left(\frac{T}{2} - \frac{2T}{9}\right) + A_3\left(\frac{2}{3}\cdot\frac{T}{6}\right) + A_4\left(\frac{1}{3}\cdot\frac{T}{6}\right)$$

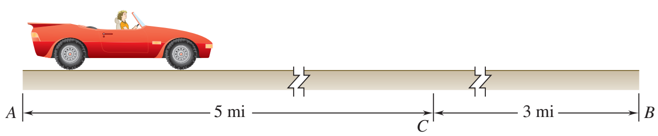
$$= 0 + (0.1T)\left(\frac{5T}{18}\right) + (0.05T)\left(\frac{T}{9}\right) + (0.0375T)\frac{T}{18} = 0.035417T^2$$

$$= (0.035417)(15.49)^2$$

$x = 8.50$ m ◀

$v = v_0 + A_1 + A_3 + A_4 = 0.1875T$ $\qquad$ $v = 2.90$ m/s ◀

## PROBLEM 11.86

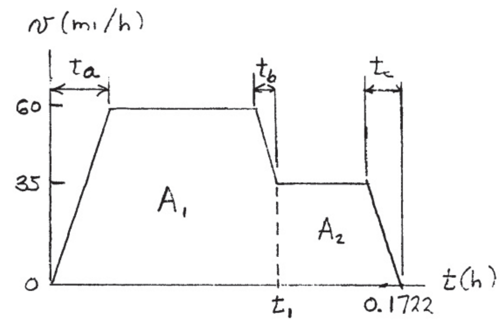



Two road rally checkpoints $A$ and $B$ are located on the same highway and are 8 mi apart. The speed limits for the first 5 mi and the last 3 mi are 60 mi/h and 35 mi/h, respectively. Drivers must stop at each checkpoint, and the specified time between points $A$ and $B$ is 10 min 20 s. Knowing that a driver accelerates and decelerates at the same constant rate, determine the magnitude of her acceleration if she travels at the speed limit as much as possible.

## SOLUTION

Given

$$10\,\text{min } 20\text{ s} = \frac{10}{60} + \frac{20}{3600}$$
$$= 0.1722\text{ h}$$




Sketch the $v - t$ curve and note $t_a = \dfrac{60}{a},\ t_b = \dfrac{25}{a},\ t_c = \dfrac{35}{a}$

Find Areas $A_1$ and $A_2$

$$A_1 = 60t_1 - \frac{1}{2}(60)(t_a) - \frac{1}{2}(25)t_b$$
$$= 60t_1 - 1800\frac{1}{a} - 312.5\frac{1}{a} \qquad (1)$$

$$A_2 = 35(0.1722 - t_1) - \frac{1}{2}(35)t_c$$
$$= 6.0278 - 35t_1 - 612.5\frac{1}{a} \qquad (2)$$

Also given $\qquad A_1 = 5\text{ mi and } A_2 = 3\text{ mi}$

Setting (1) equal to 5 mi: $\qquad 60t_1 - 2112.5\dfrac{1}{a} = 5 \qquad (3)$

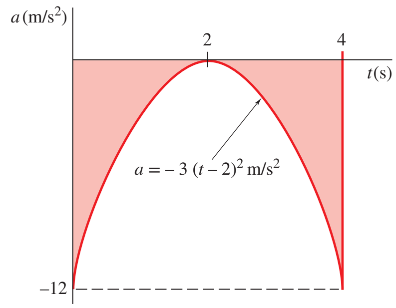
Setting (2) equal to 3 mi: $\qquad 35t_1 + 612.5\dfrac{1}{a} = 3\ 0278 \qquad (4)$

Solving equations (3) and (4) for $t_1$ and $\dfrac{1}{a}$: $\quad t_1 = 85.45\times 10^{-3}\ \text{h} = 5.13\,\text{min}$

$$\frac{1}{a} = 60.23\times 10^{-6}\ \text{h}^2/\text{mi}$$

$$a = 16.616\times 10^3\ \text{mi/h}^2 = \frac{(16.616\times 10^3)(5280)}{(3600)^2}$$

$a = 6.77\ \text{ft/s}^2$ ◀




## PROBLEM 11.87

As shown in the figure, from $t = 0$ to $t = 4$ s, the acceleration of a given particle is represented by a parabola. Knowing that $x = 0$ and $v = 8$ m/s when $t = 0$, (*a*) construct the $v - t$ and $x - t$ curves for $0 < t < 4$ s, (*b*) determine the position of the particle at $t$ =3s. (*Hint:* Use table inside the front cover.)

## SOLUTION

Given At $t = 0,\ x = 0,\ v = 8$ m/s

(*a*) We have $v_2 = v_1$ (area under $a - t$ curve from $t_1$ to $t_2$)

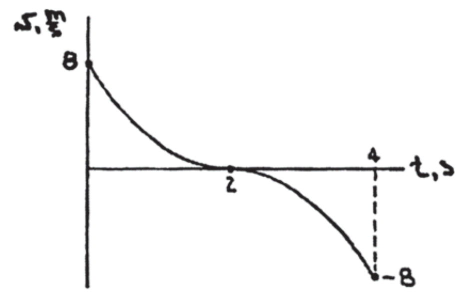
and $x_2 = x_1$ (area under $v - t$ curve from $t_1$ to $t_2$)

Then, using the formula for the area of a parabolic spandrel, we have

$$\text{at } t = 2 \text{ s:}\quad v = 8 - \frac{1}{3}(2)(12) = 0$$

$$t = 4 \text{ s:}\quad v = 0 - \frac{1}{3}(2)(12) = -8 \text{ m/s}$$

The $v - t$ curve is then drawn as shown.

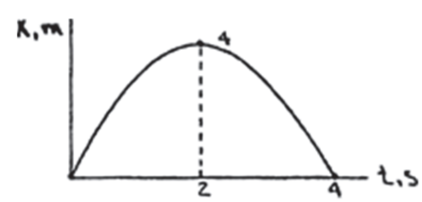



*Note:* The area under each portion of the curve is a spandrel of Order no. 3.

$$\text{Now at } t = 2 \text{ s:}\quad x = 0 + \frac{(2)(8)}{3+1} = 4 \text{ m}$$

$$t = 4 \text{ s:}\quad x = 4 - \frac{(2)(8)}{3+1} = 0$$

The $x - t$ curve is then drawn as shown. ◀

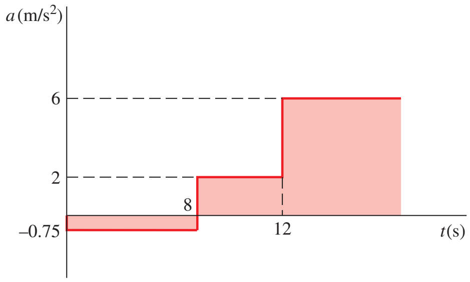



(*b*) We have At t = 3s: $a = -3(3-2)^2 = -3 \text{ m/s}^2$

$$v = 0 - \frac{1}{3}(1)(3) = -1 \text{ m/s}$$

$$x = 4 - \frac{(1)(1)}{3+1}$$

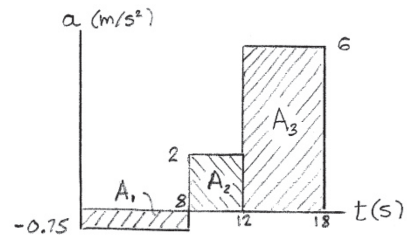
or $x_3 = 3.75$ m ◀




## PROBLEM 11.88

A particle moves in a straight line with the acceleration shown in the figure. Knowing that the particle starts from the origin with $v_0 = -2$ m/s, (*a*) construct the $v-t$ and $x-t$ curves for $0 < t < 18$ s, (*b*) determine the position and the velocity of the particle and the total distance traveled when $t = 18$ s.

## SOLUTION




Find the indicated areas under the $a-t$ curve:

$$A_1 = (-0.75)(8) = -6 \text{ m/s}$$

$$A_2 = (2)(4) = 8 \text{ m/s}$$

$$A_3 = (6)(6) = 36 \text{ m/s}$$

Find the values of velocity at times indicated:

$$v_0 = -2 \text{ m/s}$$

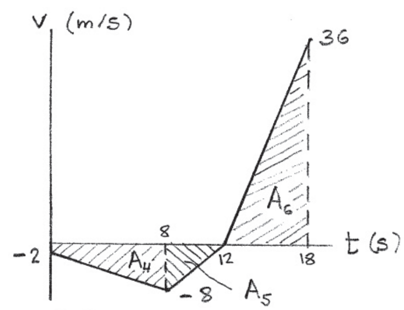
$$v_8 = v_0 + A_1 = -8 \text{ m/s}$$

$$v_{12} = v_8 + A_2 = 0$$

$$v_{18} = v_{12} + A_3 = 36 \text{ m/s}$$

(*a*) Sketch $v-t$ curve using straight line portions over the constant acceleration periods.

Find the indicated areas under the $v-t$ curve:




$$A_4 = \frac{1}{2}(-2-8)(8) = -40 \text{ m}$$

$$A_5 = \frac{1}{2}(-8)(4) = -16 \text{ m}$$

$$A_6 = \frac{1}{2}(36)(6) = 108 \text{ m}$$

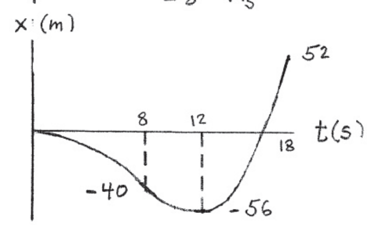
Find the values of position at times indicated:

$$x_0 = 0$$

$$x_8 = x_0 + A_4 = -40 \text{ m}$$

$$x_{12} = x_8 + A_5 = -56 \text{ m}$$

$$x_{18} = x_{12} + A_6 = 52 \text{ m}$$

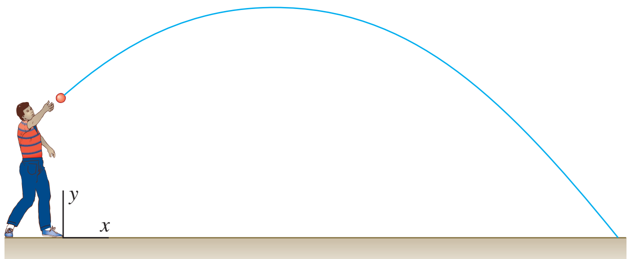



Sketch the $x-t$ curve using the positions and times calculated:

(*b*) Position at t = 18s $\qquad x_{18} = 52$ m ◀

Velocity at t – 18s $\qquad v_{18} = 36$ m/s ◀

Total Distance Traveled: $= 56 + 108$ $\qquad d = 164$ m ◀




## PROBLEM 11.89

A ball is thrown so that the motion is defined by the equations $x = 5t$ and $y = 2 + 6t - 4.9t^2$, where $x$ and $y$ are expressed in meters and $t$ is expressed in seconds. Determine (*a*) the velocity at $t = 1\,\text{s}$, (*b*) the horizontal distance the ball travels before hitting the ground.

## SOLUTION

Units are meters and seconds.

Horizontal motion: $$v_x = \frac{dx}{dt} = 5$$

Vertical motion: $$v_y = \frac{dy}{dt} = 6 - 9.8t$$

(*a*) Velocity at $t = 1$ s.

$$v_x = 5$$
$$v_y = 6 - 9.8 = -3.8$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{5^2 + 3.8^2} = 6.28 \text{ m/s}$$

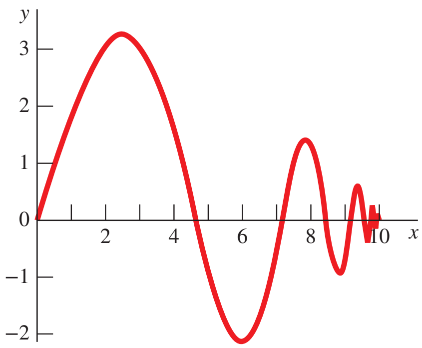
$$\tan\theta = \frac{v_y}{v_x} = \frac{-3.8}{5} \qquad \theta = -37.2°$$

$v = 6.28$ m/s $\measuredangle$ 37.2° ◀

(*b*) Horizontal distance: $(y = 0)$

$$y = 2 + 6t - 4.9t^2$$
$$t = 1.4971 \text{ s}$$
$$x = (5)(1.4971) = 7.4856 \text{ m}$$

$x = 7.49$ m ◀




## PROBLEM 11.90

The motion of a vibrating particle is defined by the position vector $\mathbf{r} = 10(1 - e^{-3t})\mathbf{i} + (4e^{-2t}\sin 15t)\mathbf{j}$, where $\mathbf{r}$ and $t$ are expressed in millimeters and seconds, respectively. Determine the velocity and acceleration when (*a*) $t = 0$, (*b*) $t = 0.5$ s.

## SOLUTION

$$\mathbf{r} = 10(1 - e^{-3t})\mathbf{i} + (4e^{-2t}\sin 15t)\mathbf{j}$$

Then $$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 30e^{-3t}\,\mathbf{i} + [60e^{-2t}\cos 15t - 8e^{-2t}\sin 15t]\mathbf{j}$$

and $$\mathbf{a} = \frac{d\mathbf{v}}{dt} = -90e^{-3t}\,\mathbf{i} + [-120e^{-2t}\cos 15t - 900e^{-2t}\sin 15t - 120e^{-2t}\cos 15t + 16e^{-2t}\sin 15t]\mathbf{j}$$

$$= -90e^{-3t}\mathbf{i} + [-240e^{-2t}\cos 15t - 884e^{-2t}\sin 15t]\mathbf{j}$$

(*a*) When $t = 0$:

$$\mathbf{v} = 30\mathbf{i} + 60\mathbf{j}\text{ mm/s}$$

$\mathbf{v} = 67.1$ mm/s $63.4°$ ◀

$$\mathbf{a} = -90\mathbf{i} - 240\mathbf{j}\text{ mm/s}^2$$

$\mathbf{a} = 256$ mm/s$^2$ $69.4°$ ◀

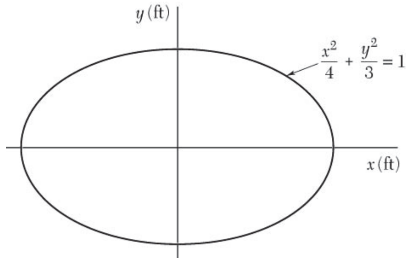
When $t = 0.5$ s:

$$\mathbf{v} = 30e^{-1.5}\mathbf{i} + [60e^{-1}\cos 7.5 - 8e^{-1}\sin 7.5]$$
$$= 6.694\mathbf{i} + 4.8906\mathbf{j}\text{ mm/s}$$

$\mathbf{v} = 8.29$ mm/s $36.2°$ ◀

$$\mathbf{a} = 90e^{-1.5}\mathbf{i} + [-240e^{-1}\cos 7.5 - 884e^{-1}\sin 7.5\mathbf{j}]$$
$$= -20.08\mathbf{i} - 335.65\mathbf{j}\text{ mm/s}^2$$

$\mathbf{a} = 336$ mm/s$^2$ $86.6°$ ◀




## PROBLEM 11.91

The motion of a particle is defined by the equations $x=(4\cos\pi t-2)/(2-\cos\pi t)$ and $y=3\sin\pi t/(2-\cos\pi t)$, where $x$ and $y$ are expressed in feet and $t$ is expressed in seconds. Show that the path of the particle is part of the ellipse shown, and determine the velocity when (*a*) $t=0$, (*b*) $t=1/3$ s, (*c*) $t=1$ s.

## SOLUTION

Substitute the given expressions for $x$ and $y$ into the given equation of the ellipse, and note that the equation is satisfied.

$$\frac{x^2}{4}+\frac{y^2}{3}=\frac{\left(16\cos^2\pi t-16\cos\pi t+4\right)}{4\left(2-\cos\pi t\right)^2}+\frac{9\sin^2\pi t}{3\left(2-\cos\pi t\right)^2}$$

$$=\frac{4\cos^2\pi t-4\cos\pi t+1+3\sin^2\pi t}{\left(2-\cos\pi t\right)^2}=\frac{4-4\cos\pi t+\cos^2\pi t}{\left(2-\cos\pi t\right)^2}=1$$

Calculate $\dot{x}$ and $\dot{y}$ by differentiation.

$$\dot{x}=\frac{-4\pi\sin\pi t}{\left(2-\cos\pi t\right)}-\frac{\left(4\cos\pi t-2\right)\left(\pi\sin\pi t\right)}{\left(2-\cos\pi t\right)^2}=\frac{-6\pi\sin\pi t}{\left(2-\cos\pi t\right)^2}$$

$$\dot{y}=\frac{3\pi\cos\pi t}{\left(2-\cos\pi t\right)}-\frac{3\sin\pi t\left(\pi\sin\pi t\right)}{\left(2-\cos\pi t\right)^2}=\frac{3\pi\left(2\cos\pi t-1\right)}{\left(2-\cos\pi t\right)^2}$$

(*a*) When $t=0$ s, $\dot{x}=0$ and $\dot{y}=3\pi$, $\boldsymbol{v}=9.42$ ft/s $\uparrow$ ◀

(*b*) When $t=\frac{1}{3}$ s, $\dot{x}=\frac{-6\pi\left(\frac{\sqrt{3}}{2}\right)}{\left(2-\frac{1}{2}\right)^2}=-\frac{4}{3}\pi-\sqrt{3},\ \dot{y}=0$ $\boldsymbol{v}=7.26$ ft/s $\longleftarrow$ ◀

(*c*) When $t=1$ s, $\dot{x}=0$ and $\dot{y}=\frac{3\pi\left(-3\right)}{\left(3\right)^2}=-\pi,$ $\boldsymbol{v}=3.14$ ft/s $\downarrow$ ◀

## PROBLEM 11.92

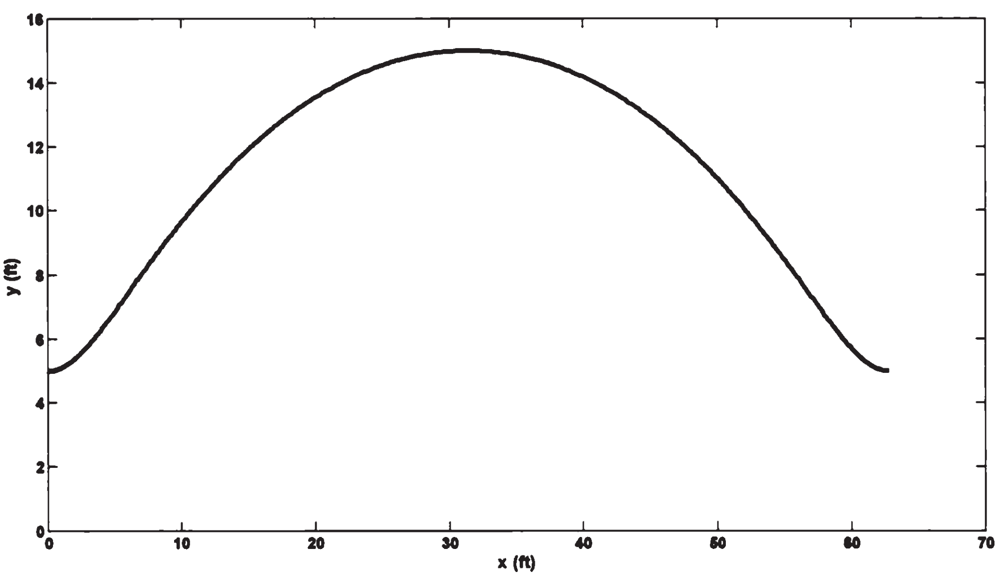
The motion of a particle is defined by the equations $x = 10t - 5\sin t$ and $y = 10 - 5\cos t$, where $x$ and $y$ are expressed in feet and $t$ is expressed in seconds. Sketch the path of the particle for the time interval $0 \le t \le 2\pi$, and determine (*a*) the magnitudes of the smallest and largest velocities reached by the particle, (*b*) the corresponding times, positions, and directions of the velocities.

## SOLUTION

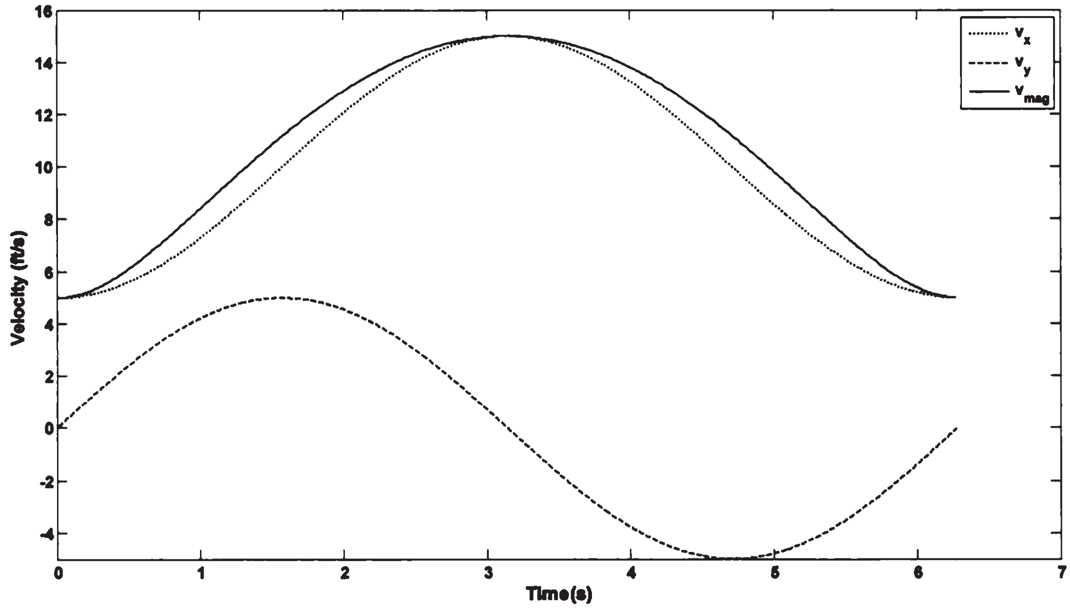
Sketch the path of the particle, i.e., plot of $y$ versus $x$.

Using $x = 10t - 5\sin t$, and $y = 10 - 5\cos t$ obtain the values in the table below. Plot as shown.

| $t(s)$ | $x$(ft) | $y$(ft) |
|---|---|---|
| 0 | 0.00 | 5 |
| $\frac{\pi}{2}$ | 10.71 | 10 |
| $\pi$ | 31.41 | 15 |
| $3\frac{\pi}{2}$ | 52.12 | 10 |
| $2\pi$ | 62.83 | 5 |



## PROBLEM 11.92 (Continued)




(*a*) Differentiate with respect to $t$ to obtain velocity components.

$$v_x = \frac{dx}{dt} = 10 - 5\cos t \quad \text{and} \quad v_y = 5\sin t$$

$$v^2 = v_x^2 + v_y^2 = (10 - 5\ \cos t)^2 + 25\sin^2 t = 125 - 100\ \cos t$$

$$\frac{d(v)^2}{dt} = 100\sin t = 0 \quad t = 0.\ \pm\pi.\ \pm 2\pi \ldots \pm N\pi$$

When $t = 2N\pi.$ $\cos t = 1.$ and $v^2$ is minimum.

When $t = (2N+1)\pi.$ $\cos t = 1.$ and $v^2$ is maximum.

$$(v^2)_{\min} = 125 - 100 = 25(\text{ft/s})^2 \qquad v_{\min} = 5 \text{ ft/s} \blacktriangleleft$$

$$(v^2)_{\max} = 125 + 100 = 225(\text{ft/s})^2 \qquad v_{\max} = 15 \text{ ft/s} \blacktriangleleft$$

(*b*) When $v = v_{\max}.$

When $N = 0,1,2,\ldots$ $\quad x = 10(2\pi N) - 5\sin(2\pi N)$ $\qquad x = 20\pi N$ ft $\blacktriangleleft$

$$y = 10 - 5\cos(2\pi N) \qquad y = 5 \text{ ft} \blacktriangleleft$$

$$v_x = 10 - 5\cos(2\pi N) \qquad v_x = 5 \text{ ft/s} \blacktriangleleft$$

$$v_y = 5\sin(2\pi N) \qquad v_y = 0 \blacktriangleleft$$

$$\tan\theta = \frac{v_y}{v_x} = 0, \qquad \theta = 0 \blacktriangleleft$$

## PROBLEM 11.92 (CONTINUED)

When $v = v_{\max}$. $\qquad t = (2N+1)\pi$ s ◀

$$x = 10[2\pi(N-1)] - 5\sin[2\pi(N+1)] \qquad x = 20\pi(N+1)\,\text{ft}$$ ◀

$$y = 10 - 5\cos[2\pi(N+1)] \qquad y = 15 \text{ ft}$$ ◀

$$v_x = 10 - 5\cos[2\pi(N+1)] \qquad v_x = 15 \text{ ft/s}$$ ◀

$$v_y = 5\sin[2\pi(N+1)] \qquad v_y = 0$$ ◀

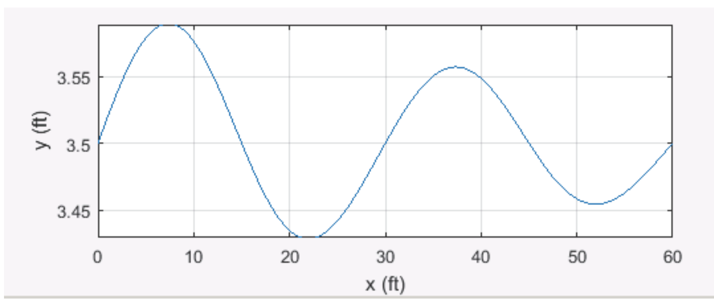
$$\tan\theta = \frac{v_y}{v_x} = 0, \qquad \theta = 0$$ ◀

## PROBLEM 11.93

Test engineers are examining how shock absorber designs affect the displacement of a mountain biker's hip after she lands from a jump. Immediately after the jump, the $x$ and $y$ locations of her hip can be described by $x(t) = 20t + 0.2\sin(\frac{4\pi}{3}t)$ and $y(t) = 3.5 + 0.1e^{-0.3t}\sin(\frac{4\pi}{3}t)$ where $x$ and $y$ are expressed in feet and $t$ is expressed in seconds. Plot the path of the hip for the time interval $0 \le t \le 3$ s and determine the hip's vertical position and the components of the hip velocity at $t = 2$ s.

## SOLUTION

Graph y(t) vs x(t)

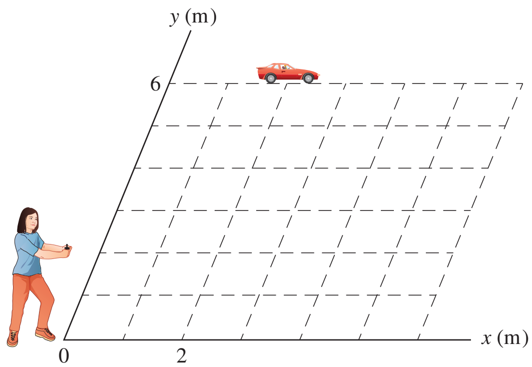



Evaluate y(t) at t = 2 seconds $\quad y(2) = 3.55\text{ ft}$ ◀

Next, differentiate v and y with respect to t to get the velocities and evaluate at t=2 sec,

$$v_x = 20 + \frac{4\pi}{15}\cos(4\pi t/3) \qquad v_x(2) = 19.58\text{ ft/s} ◀$$

$$v_y = \frac{1}{15}\left[2\pi e^{(-0.3t)}\cos(4\pi t/3)\right] \qquad v_y(2) = -0.129\text{ ft/s} ◀$$




## Problem 11.94

A girl operates a radio-controlled model car in a vacant parking lot. The girl's position is at the origin of the $xy$ coordinate axes, and the surface of the parking lot lies in the $x$-$y$ plane. The motion of the car is defined by the position vector $\vec{r} = (2+2t^2)\hat{i} + (6+t^3)\hat{j}$ where $r$ and $t$ are expressed in meters and seconds, respectively. Determine (a) the distance between the car and the girl when $t = 2$ s, ($b$) the distance the car traveled in the interval from $t = 0$ to $t = 2$ s, ($c$) the speed and direction of the car's velocity at $t = 2$ s, (d) the magnitude of the car's acceleration at $t = 2$ s.

## SOLUTION

Given: $\mathbf{r} = (2+2t^2)\mathbf{i} + (6+t^3)\mathbf{j}$

($a$) At t = 2s $\quad \mathbf{r}(2) = 10\mathbf{i} + 14\mathbf{j}$ m

$$|\mathbf{r}(2)| = \sqrt{10^2 + 14^2} \text{ m} \qquad |\mathbf{r}(2)| = 17.20 \text{ m} \blacktriangleleft$$

($b$) The car is traveling on a curved path. The distance that the car travels during any infinitesimal interval is given by: $\quad ds = \sqrt{dx^2 + dy^2}$

where: $\quad dx = 4t\,dt$ and $dy = 3t^2\,dt$

Substituting $\quad ds = \sqrt{16t^2 + 9t^4}\,dt$

Integrating: $\quad \int_0^s ds = \int_0^2 t\sqrt{16+9t^2}\,dt$

$$s = \frac{1}{27}(16+9t^2)^{3/2}\Big|_0^2 \qquad s = 11.52 \text{ m} \blacktriangleleft$$

($c$) Velocity can be found by $\quad \mathbf{v} = \frac{d\mathbf{r}}{dt}$

$$\mathbf{v} = 4t\mathbf{i} + 3t^2\mathbf{j} \text{ m/s}$$

At t = 2 s $\quad \mathbf{v} = 8\mathbf{i} + 12\mathbf{j}$ m/s $\qquad |\mathbf{v}| = 14.42 \text{ m/s} \measuredangle 56.31° \blacktriangleleft$

($d$) Acceleration can be found by $\mathbf{a} = \frac{d\mathbf{v}}{dt}$

$$\mathbf{a} = 4\mathbf{i} + 6t\mathbf{j} \text{ m/s}^2$$

At $t = 2$ s $\quad \mathbf{a} = 4\mathbf{i} + 12\mathbf{j}$ m/s$^2$ $\qquad |\mathbf{a}| = 12.65 \text{ m/s}^2 \blacktriangleleft$

## PROBLEM 11.95

The three-dimensional motion of a particle is defined by the position vector $\mathbf{r} = (Rt \cos \omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt \sin \omega_n t)\mathbf{k}$. Determine the magnitudes of the velocity and acceleration of the particle. (The space curve described by the particle is a conic helix.)

## SOLUTION

We have $\quad \mathbf{r} = (Rt \cos \omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt \sin \omega_n t)\mathbf{k}$

Then $\quad \mathbf{v} = \dfrac{d\mathbf{r}}{dt} = R(\cos\omega_n t - \omega_n t \sin\omega_n t)\mathbf{i} + c\mathbf{j} + R(\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}$

and

$$\begin{aligned}\mathbf{a} &= \frac{d\mathbf{v}}{dt}\\ &= R(-\omega_n \sin\omega_n t - \omega_n \sin\omega_n t - \omega_n^2 t\cos\omega_n t)\mathbf{i}\\ &\quad + R(\omega_n \cos\omega_n t + \omega_n \cos\omega_n t - \omega_n^2 t \sin\omega_n t)\mathbf{k}\\ &= R(-2\omega_n \sin\omega_n t - \omega_n^2 t\cos\omega_n t)\mathbf{i} + R(2\omega_n \cos\omega_n t - \omega_n^2 t\sin\omega_n t)\mathbf{k}\end{aligned}$$

Now

$$\begin{aligned}v^2 &= v_x^2 + v_y^2 + v_z^2\\ &= [R(\cos\omega_n t - \omega_n t\sin\omega_n t)]^2 + (c)^2 + [R(\sin\omega_n t + \omega_n t\cos\omega_n t)]^2\\ &= R^2[(\cos^2\omega_n t - 2\omega_n t\sin\omega_n t\cos\omega_n t + \omega_n^2 t^2 \sin^2\omega_n t)\\ &\quad + (\sin^2\omega_n t + 2\omega_n t \sin\omega_n t\cos\omega_n t + \omega_n^2 t^2\cos^2\omega_n t)] + c^2\\ &= R^2(1+\omega_n^2 t^2) + c^2\end{aligned}$$

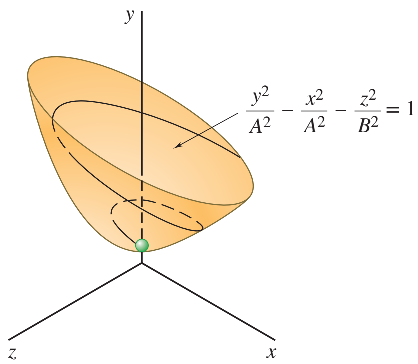
or $\quad v = \sqrt{R^2(1+\omega_n^2 t^2) + c^2}$ ◀

Also,

$$\begin{aligned}a^2 &= a_x^2 + a_y^2 + a_z^2\\ &= [R(-2\omega_n \sin\omega_n t - \omega_n^2 t\cos\omega_n t)]^2 + (0)^2\\ &\quad + [R(2\omega_n\cos\omega_n t - \omega_n^2 t\sin\omega_n t)]^2\\ &= R^2[(4\omega_n^2\sin^2\omega_n t + 4\omega_n^3 t\sin\omega_n t\cos\omega_n t + \omega_n^4 t^2\cos^2\omega_n t)\\ &\quad + (4\omega_n^2\cos^2\omega_n t - 4\omega_n^3 t\sin\omega_n t\cos\omega_n t + \omega_n^4 t^2 \sin^2\omega_n t)]\\ &= R^2\left(4\omega_n^2 + \omega_n^4 t^2\right)\end{aligned}$$

or $\quad a = R\omega_n\sqrt{4+\omega_n^2 t^2}$ ◀

## PROBLEM 11.96




The three-dimensional motion of a particle is defined by the position vector $\mathbf{r} = (At\cos t)\mathbf{i} + (A\sqrt{t^2+1})\mathbf{j} + (Bt\sin t)\mathbf{k}$, where $r$ and $t$ are expressed in feet and seconds, respectively. Show that the curve described by the particle lies on the hyperboloid $(y/A)^2 - (x/A)^2 - (z/B)^2 = 1$. For $A = 3$ and $B = 1$, determine (*a*) the magnitudes of the velocity and acceleration when $t = 0$, (*b*) the smallest nonzero value of $t$ for which the position vector and the velocity are perpendicular to each other.

## SOLUTION

We have $$\mathbf{r} = (At\cos t)\mathbf{i} + (A\sqrt{t^2+1})\mathbf{j} + (Bt\sin t)\mathbf{k}$$

or $$x = At\cos t \quad y = A\sqrt{t^2+1}) \quad z = Bt\sin t$$

Then $$\cos t = \frac{x}{At} \quad \sin t = \frac{z}{Bt} \quad t^2 = \left(\frac{y}{A}\right)^2 - 1$$

Now $$\cos^2 t + \sin^2 t = 1 \Rightarrow \left(\frac{x}{At}\right)^2 + \left(\frac{z}{Bt}\right)^2 = 1$$

or $$t^2 = \left(\frac{x}{A}\right)^2 + \left(\frac{z}{B}\right)^2$$

Then $$\left(\frac{y}{A}\right)^2 - 1 = \left(\frac{x}{A}\right)^2 + \left(\frac{z}{B}\right)^2$$

or $$\left(\frac{y}{A}\right)^2 - \left(\frac{x}{A}\right)^2 - \left(\frac{z}{B}\right)^2 = 1$$ Q.E.D. ◀

(*a*) With $A = 3$ and $B = 1$, we have

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 3(\cos t - t\sin t)\mathbf{i} + 3\frac{t}{\sqrt{t^2+1}}\mathbf{j} + (\sin t + t\cos t)\mathbf{k}$$

and $$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 3(-\sin t - \sin t - t\cos t)\mathbf{i} + 3\frac{\sqrt{t^2+1} - t\left(\frac{t}{\sqrt{t^2+1}}\right)}{(t^2+1)}\mathbf{j}$$
$$+ (\cos t + \cos t - t\sin t)\mathbf{k}$$
$$= -3(2\sin t + t\cos t)\mathbf{i} + 3\frac{1}{(t^2+1)^{3/2}}\mathbf{j} + (2\cos t - t\sin t)\mathbf{k}$$

## PROBLEM 11.96 (CONTINUED)

At $t = 0$:
$$\mathbf{v} = 3(1-0)\mathbf{i} + (0)\mathbf{j} + (0)\mathbf{k}$$
$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

or $$v = 3 \text{ ft/s} \blacktriangleleft$$

and $$\mathbf{a} = -3(0)\mathbf{i} + 3(1)\mathbf{j} + (2-0)\mathbf{k}$$

Then $$a^2 = (0)^2 + (3)^2 + (2)^2 = 13$$

or $$a = 3.61 \text{ ft/s}^2 \blacktriangleleft$$

(*b*) If **r** and **v** are perpendicular, $\mathbf{r}\cdot\mathbf{v} = 0$

$$[(3t\cos t)\mathbf{i} + (3\sqrt{t^2+1})\mathbf{j} + (t\sin t)\mathbf{k}]\cdot[3(\cos t - t\sin t)\mathbf{i} + \left(3\frac{t}{\sqrt{t^2+1}}\right)\mathbf{j} + (\sin t + t\cos t)\mathbf{k}] = 0$$

or $$(3t\cos t)[3(\cos t - t\sin t)] + (3\sqrt{t^2+1})\left(3\frac{t}{\sqrt{t^2+1)}}\right) + (t\sin t)(\sin t + t\cos t) = 0$$

Expanding $$(9t\cos^2 t - 9t^2\sin t\cos t) + (9t) + (t\sin^2 t + t^2\sin t\cos t) = 0$$

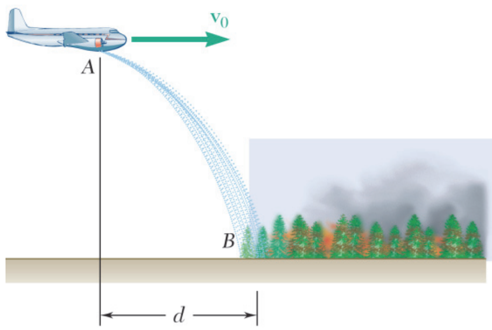
or (with $t \neq 0$) $$10 + 8\cos^2 t - 8t\,\sin t\cos t = 0$$

or $$7 + 2\cos 2t - 2t\,\sin 2t = 0$$

Using "trial and error" or numerical methods, the smallest root is $t = 3.82$ s $\blacktriangleleft$

*Note:* The next root is $t = 4.38$ s.

## PROBLEM 11.97




An airplane used to drop water on brushfires is flying horizontally in a straight line at 130 knots (1 knot = 1.15 mph) at an altitude of 175 ft. Determine the distance $d$ at which the pilot should release the water so that it will hit the fire at $B$.

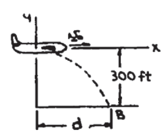
## SOLUTION

First note $$v_o = 130\text{knots}\left(\frac{1.15\text{mph}}{\text{knot}}\right)\left(\frac{1\text{hr}}{3600\text{s}}\right)\left(\frac{5280\text{ft}}{\text{mi}}\right) = 219.27\text{ft/s}$$

Place origin of coordinates at Point $A$.

<u>Vertical motion</u>. (Uniformly accelerated motion)

$$y = 0 + (0)t - \frac{1}{2}gt^2$$




At $B$: $$-175\text{ ft} = -\frac{1}{2}(32.2\text{ ft/s}^2)t^2$$

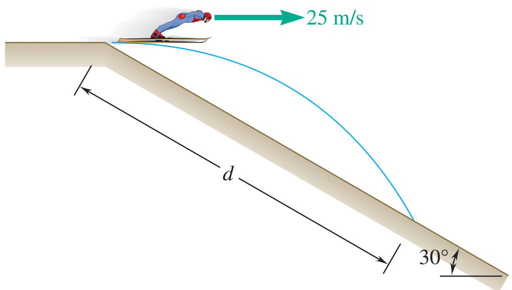
or $$t_B = 3.297\text{s}$$

Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At $B$: $$d = (219.27\text{ ft/s})(3.297\text{ s}) = 722.9\text{ ft}$$

or $$d = 723\text{ ft} \blacktriangleleft$$




## PROBLEM 11.98

A ski jumper starts with a horizontal take-off velocity of 25 m/s and lands on a straight landing hill inclined at 30° Determine (*a*) the time between take-off and landing, (*b*) the length $d$ of the jump, (*c*) the maximum vertical distance between the jumper and the landing hill.

## SOLUTION

(*a*) At the landing point, $$y = -x\tan 30^\circ$$

Horizontal motion: $$x = x_0 + (v_x)_0 t = v_0 t$$

Vertical motion: $$y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2 = -\frac{1}{2}gt^2$$

from which $$t^2 = -\frac{2y}{g} = \frac{2x\tan 30^\circ}{g} = \frac{2v_0 t\tan 30^\circ}{g}$$

Rejecting the $t = 0$ solution gives $$t = \frac{2v_0\tan 30^\circ}{g} = \frac{(2)(25)\tan 30^\circ}{9.81}$$ $t = 2.94\text{ s}$ ◀

(*b*) Landing distance: $$d = \frac{x}{\cos 30^\circ} = \frac{v_0 t}{\cos 30^\circ} = \frac{(25)(2.94)}{\cos 30^\circ}$$ $d = 84.9\text{ m}$ ◀

(*c*) Vertical distance: $$h = x\tan 30^\circ + y$$

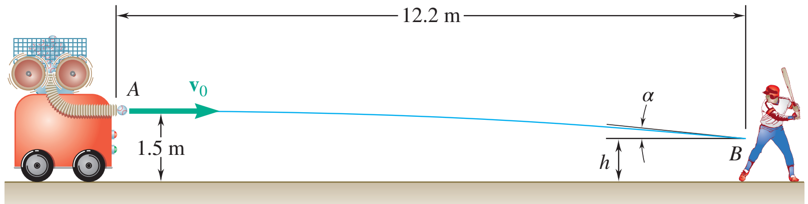
or $$h = v_0 t\tan 30^\circ - \frac{1}{2}gt^2$$

Differentiating and setting equal to zero,

$$\frac{dh}{dt} = v_0\tan 30^\circ - gt = 0 \qquad \text{or} \qquad t = \frac{v_0\tan 30^\circ}{g}$$

Then, $$h_{max} = \frac{(v_0)(v_0\tan 30^\circ)\tan 30^\circ}{g} - \frac{1}{2}g\left(\frac{v_0\tan 30^\circ}{g}\right)^2$$

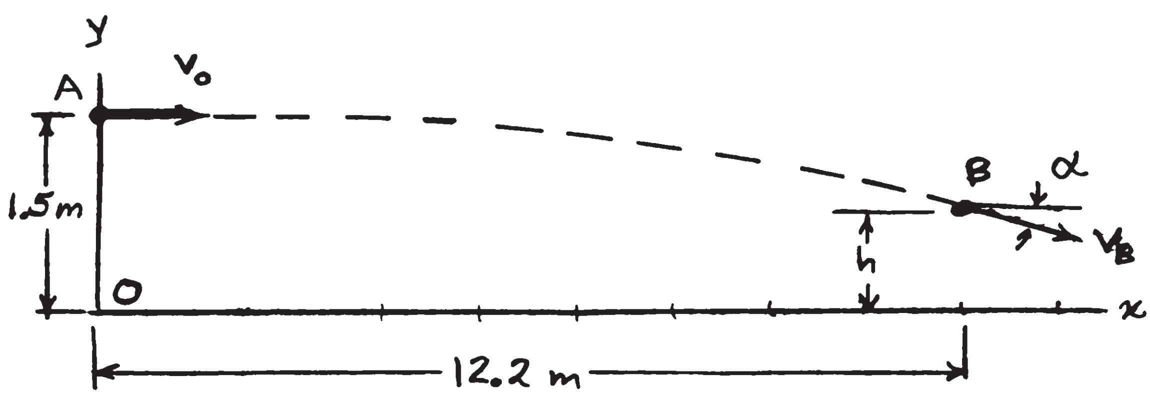
$$= \frac{v_0^2\tan^2 30^\circ}{2g} = \frac{(25)^2(\tan 30^\circ)^2}{(2)(9.81)}$$ $h_{max} = 10.62\text{ m}$ ◀




## PROBLEM 11.99

A baseball pitching machine "throws" baseballs with a horizontal velocity $\mathbf{v}_0$. Knowing that height $h$ varies between 788 mm and 1068 mm, determine (*a*) the range of values of $v_0$, (*b*) the values of $\alpha$ corresponding to $h$ = 788 mm and $h$ = 1068 mm.

## SOLUTION




(*a*) Vertical motion: $y_0 = 1.5\,\text{m}, \quad (v_y)_0 = 0$

$$y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2 \qquad \text{or} \qquad t = \sqrt{\frac{2(y_0 - y)}{g}}$$

At Point $B$, $\quad y = h \quad$ or $\quad t_B = \sqrt{\dfrac{2(y_0 - h)}{g}}$

When $h = 788\text{ mm} = 0.788\,\text{m}$, $\quad t_B = \sqrt{\dfrac{(2)(1.5 - 0.788)}{9.81}} = 0.3810\,\text{s}$

When $h = 1068\text{ mm} = 1.068\,\text{m}$, $\quad t_B = \sqrt{\dfrac{(2)(1.5 - 1.068)}{9.81}} = 0.2968\,\text{s}$

Horizontal motion: $x_0 = 0,\ (v_x)_0 = v_0,$

$$x = v_0 t \qquad \text{or} \qquad v_0 = \frac{x}{t} = \frac{x_B}{t_B}$$

## PROBLEM 11.99 (CONTINUED)

With $x_B = 12.2\,\text{m}$, we get $v_0 = \dfrac{12.2}{0.3810} = 32.02 \text{ m/s}$

and $v_0 = \dfrac{12.2}{0.2968} = 41.11 \text{ m/s}$

$32.02 \text{ m/s} \le v_0 \le 41.11 \text{ m/s}$ or $115.3 \text{ km/h} \le v_0 \le 148.0 \text{ km/h}$ ◀

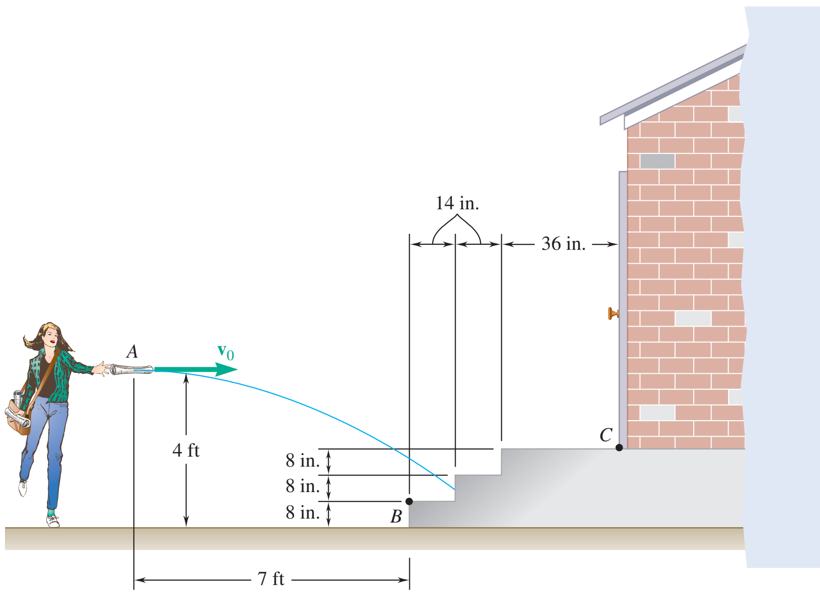
(*b*) Vertical motion: $v_y = (v_y)_0 - gt = -gt$

Horizontal motion: $v_x = v_0$

$$\tan\alpha = -\frac{dy}{dx} = -\frac{(v_y)B}{(v_x)B} = \frac{gt_B}{v_0}$$

For $h = 0.788 \text{ m}$, $\tan\alpha = \dfrac{(9.81)(0.3810)}{32.02} = 0.11673,$ $\alpha = 6.66°$ ◀

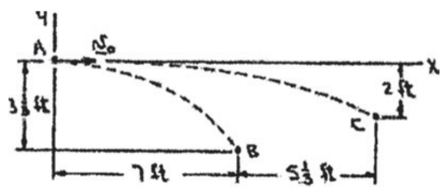
For $h = 1.068 \text{ m}$, $\tan\alpha = \dfrac{(9.81)(0.2968)}{41.11} = 0.07082,$ $\alpha = 4.05°$ ◀




## PROBLEM 11.100

While delivering newspapers, a girl throws a newspaper with a horizontal velocity $\mathbf{v}_0$. Determine the range of values of $v_0$ if the newspaper is to land between Points $B$ and $C$.

## SOLUTION




Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (0)t - \frac{1}{2}gt^2$$

Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = v_0 t$$

At $B$: $\quad y$: $\quad -3\frac{1}{3}\,\text{ft} = -\frac{1}{2}(32.2\,\text{ft/s}^2)t^2$

or $\quad t_B = 0.455016\,\text{s}$

Then $\quad x$: $\quad 7\ \text{ft} = (v_0)_B(0.455016\,\text{s})$

or $\quad (v_0)_B = 15.38\ \text{ft/s}$

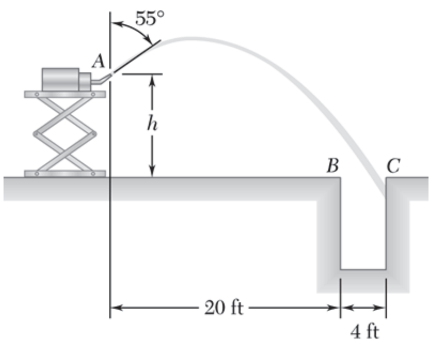
At $C$: $\quad y$: $\quad -2\,\text{ft} = -\frac{1}{2}(32.2\ \text{ft/s}^2)t^2$

or $\quad t_c = 0.352454\,\text{s}$

Then $\quad x$: $\quad 12\frac{1}{3}\,\text{ft} = (v_0)_c(0.352454\,\text{s})$

or $\quad (v_0)_c = 35.0\,\text{ft/s}$

$$15.38\,\text{ft/s} < v_0 < 35.0\,\text{ft/s} \blacktriangleleft$$




## PROBLEM 11.101

A pump is located near the edge of the horizontal platform shown. The nozzle at $A$ discharges water with an initial velocity of 25 ft/s at an angle of 55° with the vertical. Determine the range of values of the height $h$ for which the water enters the opening $BC$.

## SOLUTION

Data: $v_0 = 25$ ft/s, $\alpha = 90° - 55° = 35°$, $g = 32.2$ ft/s$^2$

Horizontal motion. $x = (v_0 \cos\alpha)t$

Vertical motion. $y = h + (v_0 \sin\alpha)t - \frac{1}{2}gt^2$

Eliminate $t$. $t = \dfrac{x}{v_0 \cos\alpha}$

$$y = h + x\tan\alpha - \frac{gx^2}{2v_0^2\cos^2\alpha}$$

Solve for $h$. $h = y - x\tan\alpha + \dfrac{gx^2}{2v_0^2\cos^2\alpha}$

To hit point $B$. $x = 20$ ft, $y = 0$

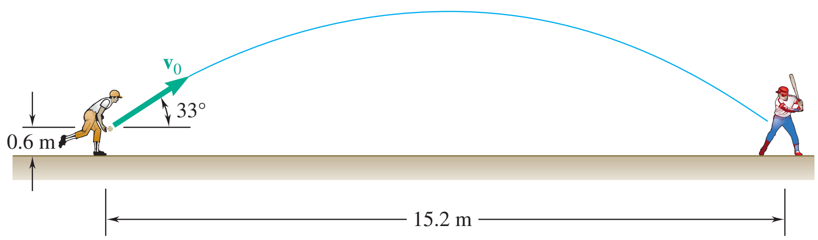
$$h = 0 - 20\tan 35° + \frac{(32.2)(20)^2}{(2)(25\cos 35°)^2} = 1.352 \text{ ft}$$

To hit point $C$. $x = 24$ ft, $y = 0$

$$h = 0 - 24\tan 35° + \frac{(32.2)(24)^2}{(2)(25\cos 35°)^2} = 5.31 \text{ ft}$$

Range of values of $h$. $1.352 \text{ ft} < h < 5.31 \text{ ft}$ ◀

## PROBLEM 11.102




In slow pitch softball the underhand pitch must reach a maximum height of between 1.8 m and 3.7 m above the ground. A pitch is made with an initial velocity $v_0$ of magnitude 13 m/s at an angle of 33° with the horizontal. Determine (*a*) if the pitch meets the maximum height requirement, (*b*) the height of the ball as it reaches the batter.

## SOLUTION

$$v_0 = 13\,\text{m/s},\ \alpha = 33°, x_0 = 0,\ y_0 = 0.6\,\text{m}$$

Vertical motion:
$$v_y = v_0 \sin\alpha - gt$$

$$y = y_0 + (v_0 \sin\alpha)t - \frac{1}{2}gt^2$$

At maximum height,
$$v_y = 0 \quad or \quad t = \frac{v_0 \sin\alpha}{g}$$

(*a*)
$$t = \frac{13\sin 33°}{9.81} = 0.7217\,\text{s}$$

$$y_{max} = 0.6 + (13\sin 33°)(0.7217) - \frac{1}{2}(9.81)(0.7217)^2$$

$y_{\text{max}} = 3.16\,\text{m}$ ◀

$$1.8\,\text{m} < 3.16\,\text{m} < 3.7\,\text{m}$$

yes ◀

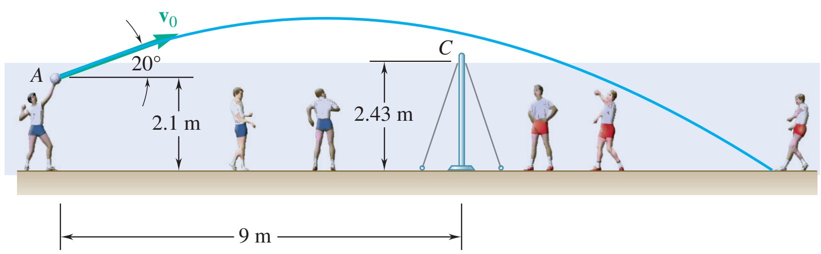
Horizontal motion:
$$x = x_0 + (v_0\cos\alpha)t \quad \text{or} \quad t = \frac{x - x0}{v_0\cos\alpha}$$

At $x = 15.2$ m,
$$t = \frac{15.2 - 0}{13\cos 33°} = 1.3941\,\text{s}$$

(*b*) Corresponding value of $y$:
$$y = 0.6 + (13\sin 33°)(1.3941) - \frac{1}{2}(9.81)(1.3941)^2$$

$y = 0.937\,\text{m}$ ◀

## PROBLEM 11.103



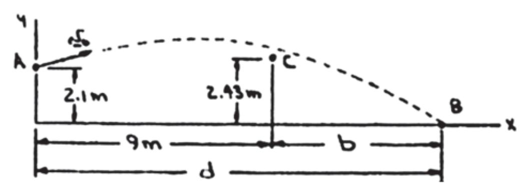

A volleyball player serves the ball with an initial velocity $\mathbf{v}_0$ of magnitude 13.40 m/s at an angle of 20° with the horizontal. Determine (*a*) if the ball will clear the top of the net, (*b*) how far from the net the ball will land.

## SOLUTION

First note

$$(v_x)_0 = (13.40\,\text{m/s})\cos 20° = 12.5919\,\text{m/s}$$

$$(v_y)_0 = (13.40\,\text{m/s})\sin 20° = 4.5831\,\text{m/s}$$




(*a*) Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At $C$

$$9\,\text{m} = (12.5919\,\text{m/s})t \quad \text{or} \quad t_c = 0.71475\,\text{s}$$

Vertical motion. (Uniformly accelerated motion)

$$y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2$$

At $C$:

$$y_c = 2.1\,\text{m} + (4.5831\,\text{m/s})(0.71475\,\text{s})$$
$$-\frac{1}{2}(9.81\,\text{m/s}^2)(0.71475\,\text{s})^2$$
$$= 2.87\,\text{m}$$

$y_c > 2.43\,\text{m}$ (height of net) $\Rightarrow$ ball clears net ◀

(*b*) At $B$, $y = 0$

$$0 = 2.1\,\text{m} + (4.5831\,\text{m/s})t - \frac{1}{2}(9.81\,\text{m/s}^2)t^2$$

Solving

$$t_B = 1.2471175\,\text{s} \quad \text{(the other root is negative)}$$

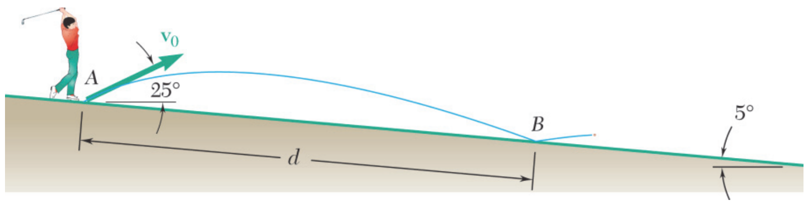
Then

$$d = (v_x)_0 t_B = (12.5919\,\text{m/s})(1.271175\,\text{s})$$
$$= 16.01\,\text{m}$$

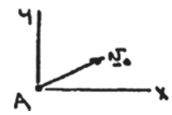
The ball lands $b = (16.01 - 9.00)\,\text{m} = 7.01\,\text{m}$ from the net ◀

**PROBLEM 11.104**

A golfer hits a golf ball with an initial velocity of 140 ft/s at an angle of 25° with the horizontal. Knowing that the fairway slopes downward at an average angle of 5°, determine the distance $d$ between the golfer and Point $B$ where the ball first lands.




**SOLUTION**




First note

$$v_{x0} = 140\cos 25^\circ$$
$$v_{y0} = 140\sin 25^\circ$$

and at $B$

$$x_B = d\cos 5^\circ \quad y_B = -d\sin 5^\circ$$

Now Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At $B$

$$d\cos(5^\circ) = (140\cos(25^\circ))t \quad or \quad t_B = \frac{\cos(5^\circ)}{140\cos(25^\circ)}d$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2}gt^2 \quad (g = 32.2 \text{ ft/s}^2)$$

At $B$:

$$-d\sin 5^\circ = (140\sin 25^\circ)t_B - \frac{1}{2}gt_B{}^2$$

Substituting for $t_B$

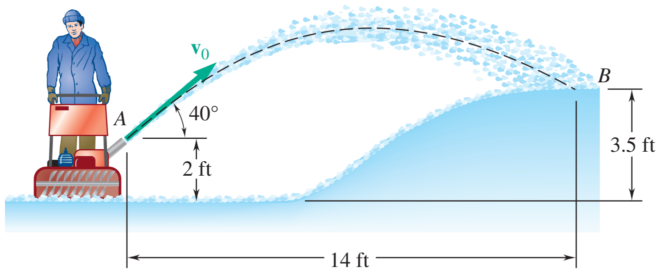
$$-d\sin 5^\circ = (140\sin 25^\circ)\frac{\cos(5^\circ)}{140\cos(25^\circ)}d - \frac{1}{2}g\left(\frac{\cos(5^\circ)}{140\cos(25^\circ)}\right)d^2$$

or

$$d = \frac{2}{32.2\cos 5^\circ}(140\cos 25^\circ)^2(\tan 5^\circ + \tan 25^\circ)$$
$$d = 555.89 \text{ ft}$$

or $d = 185$ yd ◀

## PROBLEM 11.105

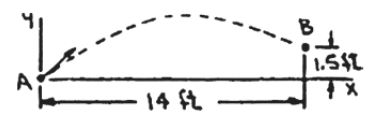



A homeowner uses a snowblower to clear his driveway. Knowing that the snow is discharged at an average angle of 40° with the horizontal, determine the initial velocity $v_0$ of the snow.

## SOLUTION

First note $\quad (v_x)_0 = v_0 \cos 40°$

$$(v_y)_0 = v_0 \sin 40°$$




Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At $B$: $\quad 14 = (v_0 \cos 40°)t \quad \text{or} \quad t_B = \dfrac{14}{v_0 \cos 40°}$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2}gt^2 \qquad (g = 32.2 \text{ ft/s}^2)$$

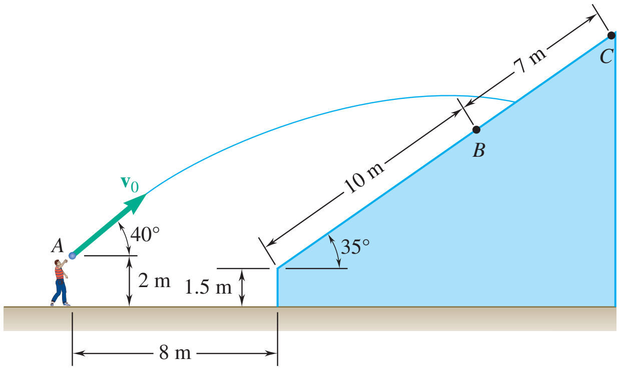
At $B$: $\quad 1.5 = (v_0 \sin 40°)t_B - \dfrac{1}{2}gt_B^2$

Substituting for $t_B$

$$1.5 = (v_0 \sin 40°)\left(\frac{14}{v_0 \cos 40°}\right) - \frac{1}{2}g\left(\frac{14}{v_0 \cos 40°}\right)^2$$

or $\quad v_0^2 = \dfrac{\frac{1}{2}(32.2)(196)/\cos^2 40°}{-1.5 + 14 \tan 40}$

or $\quad v_0 = 22.9 \text{ ft/s}$ ◀




## PROBLEM 11.106

At halftime of a football game souvenir balls are thrown to the spectators with a velocity $\mathbf{v}_0$. Determine the range of values of $v_0$ if the balls are to land between Points $B$ and $C$.

## SOLUTION

The motion is projectile motion. Place the origin of the $xy$-coordinate system at ground level just below Point $A$. The coordinates of Point $A$ are $x_0 = 0,\ y_0 = 2\,\text{m}$. The components of initial velocity are $(v_x)_0 = v_0 \cos 40°$ m/s and $(v_y)_0 = v_0 \sin 40°$.

Horizontal motion: $$x = x_0 + (v_x)_0 t = (v_0 \cos 40°)t \tag{1}$$

Vertical motion: $$y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2$$

$$= 2 + (v_0 \sin 40°) - \frac{1}{2}gt^2 \tag{2}$$

From (1), $$v_0 t = \frac{x}{\cos 40°} \tag{3}$$

Then $$y = 2 + x \tan 40° - 4.905t^2$$

$$t^2 = \frac{2 + x \tan 40° - y}{4.905} \tag{4}$$

Point $B$: $$x = 8 + 10\cos 35° = 16.1915 \text{ m}$$

$$y = 1.5 + 10\sin 35° = 7.2358 \text{ m}$$

$$v_0 t = \frac{16.1915}{\cos 40°} = 21.1365 \text{ m}$$

$$t^2 = \frac{2 + 16.1915\tan 40° - 7.2358}{4.905} \qquad t = 1.3048 \text{ s}$$

$$v_0 = \frac{21.1365}{1.3048} \qquad v_0 = 16.199 \text{ m/s}$$

## PROBLEM 11.106 (CONTINUED)

Point $C$:

$$x = 8 + (10 + 7)\cos 35° = 21.9256 \text{ m}$$

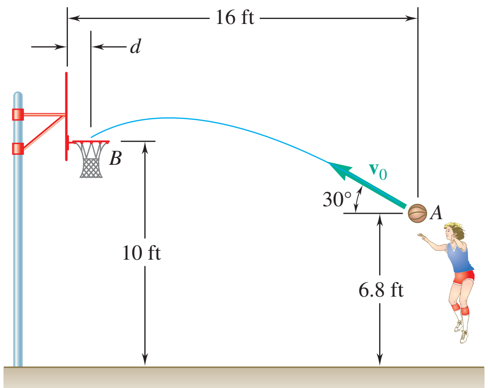
$$y = 1.5 + (10 + 7)\sin 35° = 11.2508 \text{ m}$$

$$v_0 t = \frac{21.9256}{\cos 40°} = 28.622 \text{ m}$$

$$t^2 = \frac{2 + 21.9256\tan 40° - 11.2508}{4.905} \qquad t = 1.3656 \text{ s}$$

$$v_0 = \frac{28.622}{1.3656} \qquad v_0 = 20.96 \text{ m/s}$$

Range of values of $v_0$. $\quad 16.20 \text{ m/s} < v_0 < 21.0 \text{ m/s}$ ◀

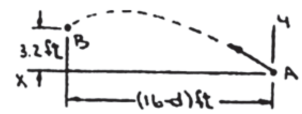



## PROBLEM 11.107

A basketball player shoots when she is 16 ft from the backboard. Knowing that the ball has an initial velocity $\mathbf{v}_0$ at an angle of 30° with the horizontal, determine the value of $v_0$ when $d$ is equal to (*a*) 9 in., (*b*) 17 in.

## SOLUTION

First note $\quad (v_x)_0 = v_0 \cos 30° \qquad (v_y)_0 = v_0 \sin 30°$




Horizontal motion. (Uniform) $\quad x = 0 + (v_x)_0 t$

At $B$: $\quad (16 - d) = (v_0 \cos 30°)t \quad \text{or} \quad t_B = \dfrac{16 - d}{v_0 \cos 30°}$

Vertical motion. (Uniformly accelerated motion) $\quad y = 0 + (v_y)_0 t - \dfrac{1}{2} g t^2 \quad (g = 32.2 \text{ ft/s}^2)$

At $B$: $\quad 3.2 = (v_0 \sin 30°) t_B - \dfrac{1}{2} g t_B^2$

Substituting for $t_B$

$$3.2 = (v_0 \sin 30°)\left(\frac{16 - d}{v_0 \cos 30°}\right) - \frac{1}{2} g \left(\frac{16 - d}{v_0 \cos 30°}\right)^2$$

or

$$v_0^2 = \frac{2g(16 - d)^2}{3\left[\frac{1}{\sqrt{3}}(16 - d) - 3.2\right]}$$

(*a*) $\quad d = 9 \text{ in.}:$

$$v_0^2 = \frac{2(32.2)\left(16 - \frac{9}{12}\right)^2}{3\left[\frac{1}{\sqrt{3}}\left(16 - \frac{9}{12}\right) - 3.2\right]}$$

$v_0 = 29.8$ ft/s ◀

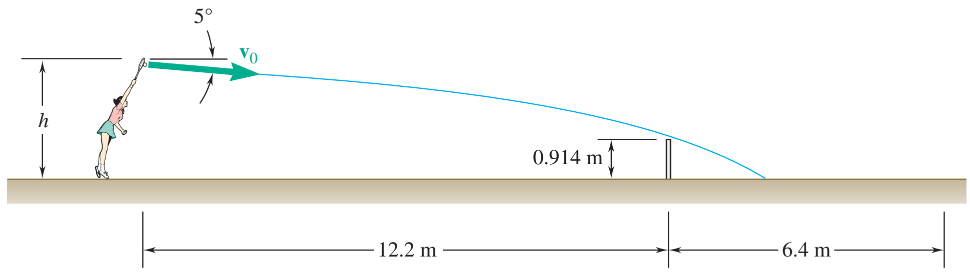
(*b*) $\quad d = 17 \text{ in.}:$

$$v_0^2 = \frac{2(32.2)\left(16 - \frac{17}{12}\right)^2}{3\left[\frac{1}{\sqrt{3}}\left(16 - \frac{17}{12}\right) - 3.2\right]}$$

$v_0 = 29.6$ ft/s ◀

## PROBLEM 11.108

A tennis player serves the ball at a height $h = 2.5$ m with an initial velocity of $\mathbf{v}_0$ at an angle of 5° with the horizontal. Determine the range for which of $v_0$ for which the ball will land in the service area which extends to 6.4 m beyond the net.




## SOLUTION

The motion is projectile motion. Place the origin of the $xy$-coordinate system at ground level just below the point where the racket impacts the ball. The coordinates of this impact point are $x_0 = 0,\ y_0 = h = 2.5\text{m}$. The components of initial velocity are $(v_x)_0 = v_0 \cos 5°$ and $(v_y)_0 = v_0 \sin 5°$.

Horizontal motion:
$$x = x_0 + (v_x)_0 t = (v_0 \cos 5°)t \tag{1}$$

Vertical motion:
$$y = y_0 + (v_y)_0 t = \frac{1}{2} g t^2$$
$$= 2.5 - (v_0 \sin 5°)t = -\frac{1}{2}(9.81)t^2 \tag{2}$$

From (1),
$$v_0 t = \frac{x}{\cos 5°} \tag{3}$$

Then
$$y = 2.5 - x \tan 5° - 4.905 t^2$$
$$t^2 = \frac{2.5 - x \tan 5° - y}{4.905} \tag{4}$$

At the minimum speed the ball just clears the net.

$$x = 12.2 \text{ m}, \quad y = 0.914 \text{ m}$$
$$v_0 t = \frac{12.2}{\cos 5°} = 12.2466 \text{ m}$$
$$t^2 = \frac{2.5 - 12.2 \tan 5° - 0.914}{4.905} \qquad t = 0.32517 \text{ s}$$
$$v_0 = \frac{12.2466}{0.32517} \qquad v_0 = 37.66 \text{ m/s}$$

## PROBLEM 11.108 (CONTINUED)

At the maximum speed the ball lands 6.4 m beyond the net.

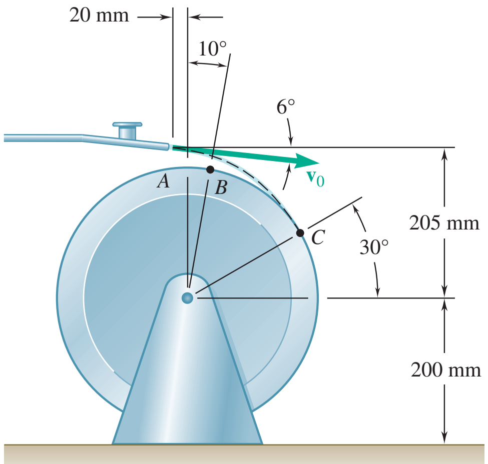
$$x = 12.2 + 6.4 = 18.6 \text{ m} \qquad y = 0$$

$$v_0 t = \frac{18.6}{\cos 5°} = 18.6710 \text{ m}$$

$$t^2 = \frac{2.5 - 18.6\tan 5° - 0}{4.905} \qquad t = 0.42181 \text{ s}$$

$$v_0 = \frac{18.6710}{0.42181} \qquad v_0 = 44.26 \text{ m/s}$$

Range for $v_0$. $37.7 \text{ m/s} < v_0 < 44.3 \text{ m/s}$ ◀



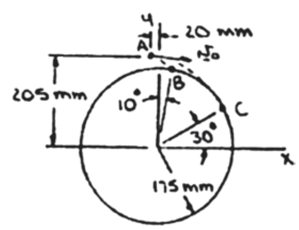

## PROBLEM 11.109

The nozzle at $A$ discharges cooling water with an initial velocity $\mathbf{v}_0$ at an angle of 6° with the horizontal onto a grinding wheel 350 mm in diameter. Determine the range of values of the initial velocity for which the water will land on the grinding wheel between Points $B$ and $C$.

## SOLUTION

First note

$$(v_x)_0 = v_0 \cos 6^\circ$$
$$(v_y)_0 = -v_0 \sin 6^\circ$$




Horizontal motion. (Uniform)

$$x = x_0 + (v_x)_0 t$$

Vertical motion. (Uniformly accelerated motion)

$$y = t_0 + (v_y)_0 t - \frac{1}{2} g t^2 \, (g = 9.81 \text{ m/s}^2)$$

At Point $B$:

$$x = (0.175 \text{ m}) \sin 10^\circ$$
$$y = (0.175 \text{ m}) \cos 10^\circ$$

$$x: \;\; 0.175 \sin 10^\circ = -0.020 + (v_0 \cos 6^\circ) t$$

or

$$t_B = \frac{0.050388}{v_0 \cos 6^\circ}$$

$$y: \;\; 0.175 \cos 10^\circ = 0.205 + (-v_0 \sin 6^\circ) t_B - \frac{1}{2} g t_B^2$$

Substituting for $t_B$

$$-0.032659 = (-v_0 \sin 6^\circ)\left(\frac{0.050388}{v_0 \cos 6^\circ}\right) - \frac{1}{2}(9.81)\left(\frac{0.050388}{v_0 \cos 6^\circ}\right)^2$$

## PROBLEM 11.109 (CONTINUED)

or
$$v_0^2 = \frac{\frac{1}{2}(9.81)(0.050388)^2}{\cos^2 6°(0.032659 - 0.050388\tan 6°}$$

or
$$(v_0)_B = 0.678 \text{ m/s}$$

At Point $C$:
$$x = (0.175 \text{ m})\cos 30°$$
$$y = (0.175 \text{ m})\sin 30°$$

$$x: \quad 0.175\cos 30° = -0.020 + (v_0 \cos 6°)t$$

or
$$t_c = \frac{0.171554}{v_0 \cos 6°}$$

$$y: \quad 0.175\sin 30° = 0.205 + (-v_0 \sin 6°)t_c - \frac{1}{2}gt_c^2$$

Substituting for $t_c$

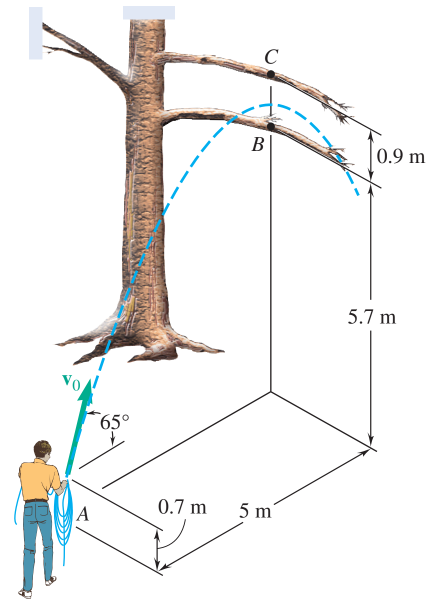
$$-0.117500 = (-v_0 \sin 6°)\left(\frac{0.171554}{v_0 \cos 6°}\right) - \frac{1}{2}(9.81)\left(\frac{0.171554}{v_0 \cos 6°}\right)^2$$

or
$$v_0^2 = \frac{\frac{1}{2}(9.81)(0.171554)^2}{\cos^2 6°(0.117500 - 0.171554\tan 6°)}$$

or
$$(v_0)_c = 1.211 \text{ m/s}$$

$$0.678 \text{ m/s} < v_0 < 1.211 \text{ m/s} \blacktriangleleft$$

## PROBLEM 11.110



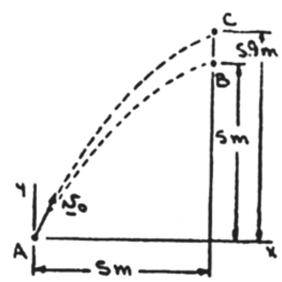

While holding one of its ends, a worker lobs a coil of rope over the lowest limb of a tree. If he throws the rope with an initial velocity $\mathbf{v}_0$ at an angle of 65° with the horizontal, determine the range of values of $v_0$ for which the rope will go over only the lowest limb.

## SOLUTION

First note

$$(v_x)_0 = v_0 \cos 65°$$

$$(v_y)_0 = -v_0 \sin 65°$$




<u>Horizontal motion</u>. (Uniform)

$$x = 0 + (v_x)_0 t$$

At either $B$ or $C$, $x = 5\,\text{m}$

$$s = (v_0 \cos 65°) t_{B,C}$$

or

$$t_{B,C} = \frac{5}{(v_0 \cos 65°)}$$

<u>Vertical motion</u>. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2} g t^2 \qquad (g = 9.81\ \text{m/s}^2)$$

At the tree limbs, $t = t_{B,C}$

$$y_{B,C} = (v_0 \sin 65°)\left(\frac{5}{v_0 \cos 65°}\right) - \frac{1}{2} g \left(\frac{5}{v_0 \cos 65°}\right)^2$$

## PROBLEM 11.110 (CONTINUED)

or $$v_0^2 = \frac{\frac{1}{2}(9.81)(25)}{\cos^2 65°(5\tan 65° - y_{B,C})}$$

$$= \frac{686.566}{5\tan 65° - y_{B,C}}$$

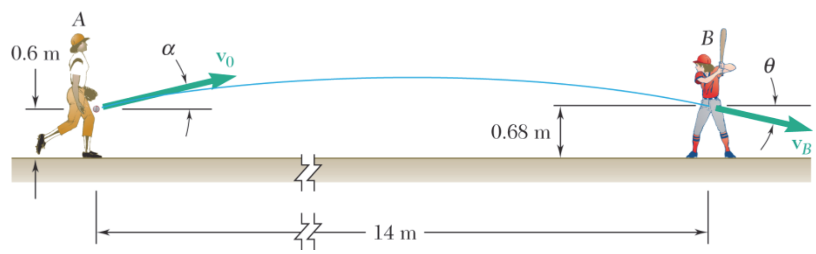
At Point $B$: $$v_0^2 = \frac{686.566}{5\tan 65° - 5} \qquad \text{or} \qquad (v_0)_B = 10.95 \text{ m/s}$$

At Point $C$: $$v_0^2 = \frac{686.566}{5\tan 65° - 5.9} \qquad \text{or} \qquad (v_0)_C = 11.93 \text{ m/s}$$

$$10.95 \text{ m/s} > v_0 < 11.93 \text{ m/s} \blacktriangleleft$$

## PROBLEM 11.111

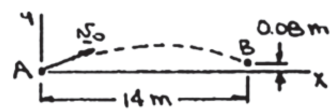
The pitcher in a softball game throws a ball with an initial velocity $\mathbf{v}_0$ of 108 km/h at an angle $\alpha$ with the horizontal. If the height of the ball at Point $B$ is 0.68 m, determine ($a$) the angle $\alpha$, ($b$) the angle $\theta$ that the velocity of the ball at Point $B$ forms with the horizontal.




## SOLUTION

First note

$$v_0 = 108 \text{ km/h} = 30 \text{ m/s}$$




and

$$(v_x)_0 = v_0 \cos\alpha = (30 \text{ m/s})\cos\alpha$$
$$(v_y)_0 = v_0 \sin\alpha = (30 \text{ m/s})\sin\alpha$$

*(a)* Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = (30\cos\alpha)t$$

At Point $B$: $$14 = (30\cos\alpha)t \quad \text{or} \quad t_B = \frac{7}{15\cos\alpha}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2}gt^2 = (30\sin\alpha)t - \frac{1}{2}gt^2 \qquad (g = 9.81 \text{ m/s}^2)$$

At Point $B$: $$0.08 = (30\sin\alpha)t_B - \frac{1}{2}gt_B^2$$

Substituting for $t_B$

$$0.08 = (30\sin\alpha)\left(\frac{7}{15\cos\alpha}\right) - \frac{1}{2}g\left(\frac{7}{15\cos\alpha}\right)^2$$

or

$$0.08 = 14\tan\alpha - \frac{1}{2}g\frac{49}{225\cos^2\alpha}$$

Solving $\alpha = 4.718^\circ$ and $\alpha = 85.61^\circ$ $\alpha = 4.72^\circ$ ◀

Rejecting the second root because it is not physically reasonable.

*(b)* We have
$$v_x = (v_x)_0 = 30\cos\alpha$$

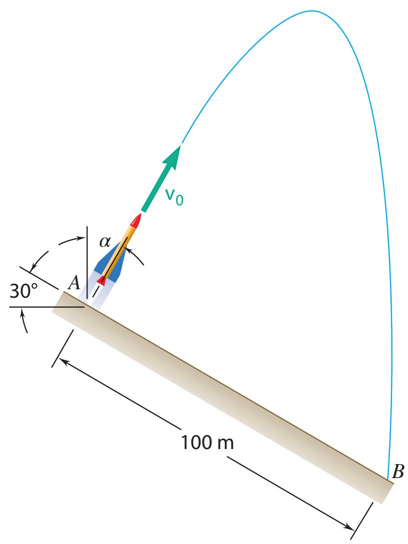
and
$$v_y = (v_y)_0 - gt = 30\sin\alpha - gt$$

At Point $B$:
$$\begin{aligned}(v_y)_B &= 30\sin\alpha - gt_B \\ &= 30\sin\alpha - \frac{7g}{15\cos\alpha}\end{aligned}$$

Noting that at Point $B$, $v_y < 0$, we have

$$\begin{aligned}\tan\theta &= \frac{\left|(v_y)_B\right|}{v_x} \\ &= \frac{\frac{7g}{15\cos\alpha} - 30\sin\alpha}{30\cos\alpha} \\ &= \frac{\frac{7}{15}\frac{9.81}{\cos 4.718^\circ} - 30\sin 4.718^\circ}{30\cos 4.718^\circ}\end{aligned}$$

Or $\theta = 4.07^\circ$ ◀




## PROBLEM 11.112

A model rocket is launched from Point $A$ with an initial velocity $\mathbf{v}_0$ of 75 m/s. If the rocket's descent parachute does not deploy and the rocket lands a distance $d = 100$ m from $A$, determine (*a*) the angle $\alpha$ that $\mathbf{v}_0$ forms with the vertical, (*b*) the maximum height above Point $A$ reached by the rocket, and (*c*) the duration of the flight.

## SOLUTION

Set the origin at Point $A$. $\quad x_0 = 0, \quad y_0 = 0$

Horizontal motion: $\quad x = v_0 t \sin\alpha \qquad \sin\alpha = \dfrac{x}{v_0 t} \qquad (1)$

Vertical motion: $\quad y = v_0 t \cos\alpha - \dfrac{1}{2} g t^2$

$$\cos\alpha = \frac{1}{v_0 t}\left(y + \frac{1}{2} g t^2\right) \qquad (2)$$

$$\sin^2\alpha + \cos^2\alpha = \frac{1}{(v_0 t)^2}\left[x2 + \left(y + \frac{1}{2} g t^2\right)^2\right] = 1$$

$$x^2 + y^2 + gyt^2 + \frac{1}{4} g^2 t^4 = v_0^2 t^2$$

$$\frac{1}{4} g^2 t^4 - (v_0^2 - gy)t^2 + (x^2 + y^2) = 0 \qquad (3)$$

At Point $B$, $\quad \sqrt{x^2 + y^2} = 100\,\text{m}, \quad x = 100\cos 30°\,\text{m}$

$$y = -100\sin 30° = -50\,\text{m}$$

$$\frac{1}{4}(9.81)^2 t^4 - [75^2 - (9.81)(-50)]t^2 + 100^2 = 0$$

$$24.0590\,t^4 - 6115.5\,t^2 + 10000 = 0$$

$$t^2 = 252.54\,\text{s}^2 \quad \text{and} \quad 1.6458\,\text{s}^2$$

$$t = 15.8916\,\text{s} \quad \text{and} \quad 1.2829\,\text{s}$$

## PROBLEM 11.112 (CONTINUED)

Restrictions on $\alpha$: $0 < \alpha < 120°$

$$\tan\alpha = \frac{x}{y + \frac{1}{2}gt^2} = \frac{100\cos 30°}{-50 + (4.905)(15.8916)^2} = 0.0729$$

$$\alpha = 4.1669°$$

and $$\frac{100\cos 30°}{-50 + (4.905)(1.2829)^2} = 2.0655$$

$$\alpha = 115.8331°$$

Use $\alpha = 4.1669°$ corresponding to the steeper possible trajectory.

(*a*) Angle $\alpha$. $\alpha = 4.17°$ ◀

(*b*) Maximum height. $v_y = 0$ at $y = y_{\text{max}}$

$$v_y = v_0\cos\alpha - gt = 0$$

$$t = \frac{v_0\cos\alpha}{g}$$

$$y_{\text{max}} = v_0 t\cos\alpha - \frac{1}{2}gt = \frac{v_0^2\cos^2\alpha}{2g}$$

$$= \frac{(75)^2\cos^2 4.1669°}{(2)(9.81)}$$

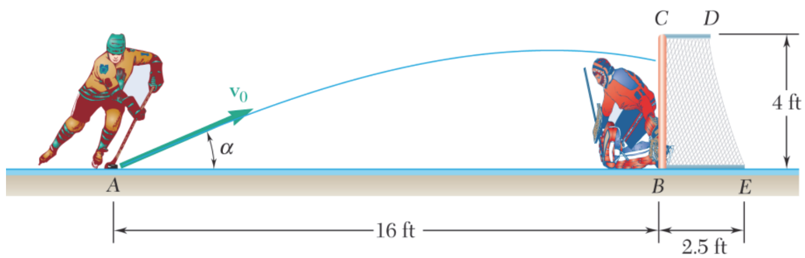
$y_{\text{max}} = 285\text{ m}$ ◀

(*c*) Duration of the flight. (time to reach $B$)

$t = 15.89\text{ s}$ ◀

**PROBLEM 11.113**

The initial velocity $\mathbf{v}_0$ of a hockey puck is 90 mi/h. Determine (*a*) the largest value (less than 45°) of the angle $\alpha$ for which the puck will enter the net, (*b*) the corresponding time required for the puck to reach the net.




**SOLUTION**


First note $\qquad v_0 = 90 \text{ mi/h} = 132 \text{ ft/s}$

and

$$(v_x)_0 = v_0 \cos\alpha = (132 \text{ ft/s})\cos\alpha$$
$$(v_y)_0 = v_0 \sin\alpha = (132 \text{ ft/s})\sin\alpha$$

*(a)* Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = (132\cos\alpha)t$$

At the front of the net, $\qquad x = 16 \text{ ft}$

Then $\qquad 16 = (132\cos\alpha)t$

or $\qquad t_{\text{enter}} = \dfrac{8}{66\cos\alpha}$

Vertical motion. (Uniformly accelerated motion)

$$\begin{aligned} y &= 0 + (v_y)_0 t - \frac{1}{2}gt^2 \\ &= (132\sin\alpha)t - \frac{1}{2}gt^2 \qquad (g = 32.2 \text{ ft/s}^2) \end{aligned}$$

## PROBLEM 11.113 (CONTINUED)

At the front of the net,

$$y_{\text{front}} = (132\sin\alpha)t_{\text{enter}} - \frac{1}{2}gt_{\text{enter}}^2$$

$$= (132\sin\alpha)\left(\frac{8}{66\cos\alpha}\right) - \frac{1}{2}g\left(\frac{8}{66\cos\alpha}\right)^2 = 16\tan\alpha - \frac{32g}{4356\cos^2\alpha}$$

Now $$\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$$

Then $$y_{\text{front}} = 16\tan\alpha - \frac{32g}{4356}(1 + \tan^2\alpha)$$

or $$\tan^2\alpha - \frac{4356}{2g}\tan\alpha + \left(1 + \frac{4356}{32g}y_{\text{front}}\right) = 0$$

Then $$\tan\alpha = \frac{\frac{4356}{2g} \pm \left[\left(-\frac{4356}{2g}\right)^2 - 4\left(1 + \frac{4356}{2g}y_{\text{front}}\right)\right]^{1/2}}{2}$$

or $$\tan\alpha = \frac{4356}{4\times 32.2} \pm \left[\left(-\frac{4356}{4\times 32.2}\right)^2 - \left(1 + \frac{4356}{32\times 32.2}y_{\text{front}}\right)\right]^{1/2}$$

or $$\tan\alpha = 33.82 \pm [(33.82)^2 - (1 + 4.2275y_{\text{front}})]^{1/2}$$

Now $0 < y_{\text{front}} < 4\,\text{ft}$ so that the positive root will yield values of $\alpha > 45°$ for all values of $y_{\text{front}}$.

When the negative root is selected, $\alpha$ increases as $y_{\text{front}}$ is increased. Therefore, for $\alpha_{\max}$, set

$$y_{\text{front}} = y_c = 4\text{ ft}$$

Then $$\tan\alpha = 33.82 - [(33.82)^2 - (1 + 16.910y_{\text{front}})]^{1/2}$$

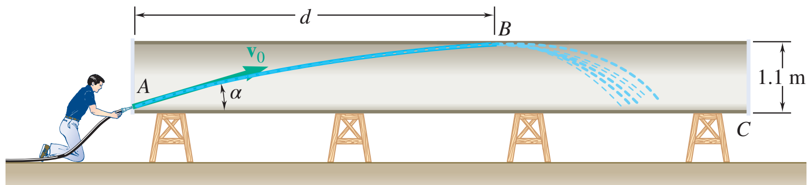
or $$\alpha_{\max} = 14.887°$$ $\alpha_{\max} = 14.89°$ ◀

*(b)* We had found $$t_{\text{enter}} = \frac{8}{66\cos\alpha} = \frac{8}{66\cos 14.887°}$$

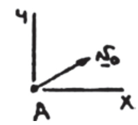
or $t_{\text{enter}} = 0.1254\,\text{s}$ ◀

## PROBLEM 11.114

A worker uses high-pressure water to clean the inside of a long drainpipe. If the water is discharged with an initial velocity $\mathbf{v}_0$ of 11.5 m/s, determine (*a*) the distance $d$ to the farthest Point $B$ on the top of the pipe that the worker can wash from his position at $A$, (*b*) the corresponding angle $\alpha$.




## SOLUTION




First note
$$(v_x)_0 = v_0 \cos\alpha = (11.5 \text{ m/s})\cos\alpha$$
$$(v_y)_0 = v_0 \sin\alpha = (11.5 \text{ m/s})\sin\alpha$$

By observation, $d_{\max}$ occurs when $y_{\max} = 1.1\,\text{m}$.

Vertical motion. (Uniformly accelerated motion)

$$v_y = (v_y)_0 - gt \qquad y = 0 + (v_y)_0 t - \frac{1}{2}gt^2$$
$$= (11.5\sin\alpha) - gt \qquad = (11.5\sin\alpha)t - \frac{1}{2}gt^2$$

When $y = y_{\max}$ at $B$, $(v_y)_B = 0$

Then
$$(v_y)_B = 0 = (11.5\sin\alpha) - gt$$

or
$$t_B = \frac{11.5\sin\alpha}{g} \quad (g = 9.81\,\text{m/s}^2)$$

and
$$y_B = (11.5\sin\alpha)t_B - \frac{1}{2}gt_B^2$$

Substituting for $t_B$ and noting $y_B = 1.1\,\text{m}$

$$1.1 = (11.5\sin\alpha)\left(\frac{11.5\sin\alpha}{g}\right) - \frac{1}{2}g\left(\frac{11.5\sin\alpha}{g}\right)^2$$
$$= \frac{1}{2g}(11.5)^2 \sin^2\alpha$$

or
$$\sin^2\alpha = \frac{2.2\times 9.81}{11.5^2} \qquad \alpha = 23.8265°$$

## PROBLEM 11.114 (CONTINUED)

(*a*) Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = (11.5\cos\alpha)t$$

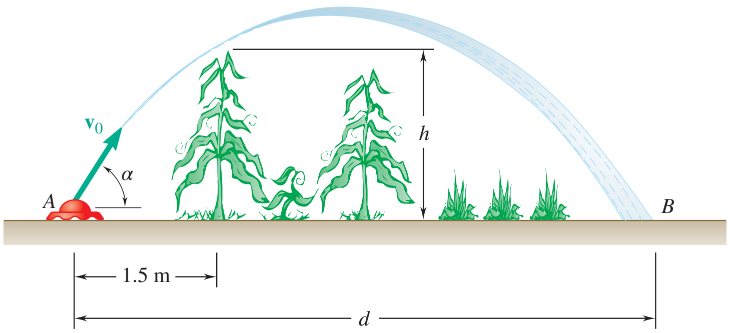
At Point *B*: $x = d_{\max}$ and $t = t_B$

where $$t_B = \frac{11.5}{9.81}\sin 23.8265° = 0.47356\,\text{s}$$

Then $$d_{\max} = (11.5)(\cos 23.8265°)(0.47356)$$

or $$d_{\max} = 4.98\,\text{m} \blacktriangleleft$$

(*b*) From above $$\alpha = 23.8° \blacktriangleleft$$




## PROBLEM 11.115

An oscillating garden sprinkler which discharges water with an initial velocity $\mathbf{v}_0$ of 8 m/s is used to water a vegetable garden. Determine the distance $d$ to the farthest Point $B$ that will be watered and the corresponding angle $\alpha$ when ($a$) the vegetables are just beginning to grow, ($b$) the height $h$ of the corn is 1.8 m.

## SOLUTION

First note

$$(v_x)_0 = v_0 \cos\alpha = (8 \text{ m/s})\cos\alpha$$
$$(v_y)_0 = v_0 \sin\alpha = (8 \text{ m/s})\sin\alpha$$


Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = (8\cos\alpha)t$$

At Point $B$:

$$x = d: \quad d = (8\cos\alpha)t$$

or

$$t_B = \frac{d}{8\cos\alpha}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2}gt^2$$
$$= (8\sin\alpha)t - \frac{1}{2}gt^2 \qquad (g = 9.81 \text{m/s}^2)$$

At Point $B$:

$$0 = (8\sin\alpha)t_B - \frac{1}{2}gt_B^2$$

Simplifying and substituting for $t_B$

$$0 = 8\sin\alpha - \frac{1}{2}g\left(\frac{d}{8\cos\alpha}\right)$$

or

$$d = \frac{64}{g}\sin 2\alpha \qquad (1)$$

($a$) When $h = 0,$ the water can follow any physically possible trajectory. It then follows from Eq. (1) that $d$ is maximum when $2\alpha = 90°$

or $\alpha = 45°$ ◀

Then

$$d = \frac{64}{9.81}\sin(2\times 45°)$$

or $d_{\max} = 6.52 \text{ m}$ ◀

## PROBLEM 11.115 (CONTINUED)

(*b*) Based on Eq. (1) and the results of Part *a*, it can be concluded that *d* increases in value as $\alpha$ increases in value from 0 to 45° and then *d* decreases as $\alpha$ is further increased. Thus, $d_{max}$ occurs for the value of $\alpha$ closest to 45° and for which the water just passes over the first row of corn plants. At this row, $x_{corn} = 1.5\text{ m}$

so that
$$t_{corn} = \frac{1.5}{8\cos\alpha}$$

Also, with $y_{corn} = h$, we have

$$h = (8\sin\alpha)t_{corn} - \frac{1}{2}gt_{corn}^2$$

Substituting for $t_{corn}$ and noting $h = 1.8\text{ m}$,

$$1.8 = (8\sin\alpha)\left(\frac{1.5}{8\cos\alpha}\right) - \frac{1}{2}g\left(\frac{1.5}{8\cos\alpha}\right)^2$$

or
$$1.8 = 1.5\tan\alpha - \frac{2.25g}{128\cos^2\alpha}$$

Now
$$\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$$

Then
$$1.8 = 1.5\tan\alpha - \frac{2.25(9.81)}{128}(1 + \tan^2\alpha)$$

or
$$0.172441\tan^2\alpha - 1.5\tan\alpha + 1.972441 = 0$$

Solving
$$\alpha = 58.229° \quad \text{and} \quad \alpha = 81.965°$$

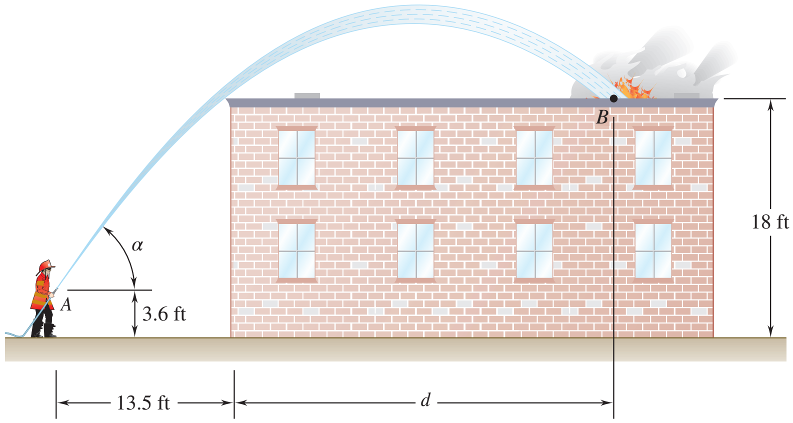
From the above discussion, it follows that $d = d_{max}$ when

$$\alpha = 58.2° \blacktriangleleft$$

Finally, using Eq. (1)

$$d = \frac{64}{9.81}\sin(2\times 58.229°)$$

or
$$d_{max} = 5.84\text{ m} \blacktriangleleft$$




## PROBLEM 11.116*

A nozzle at $A$ discharges water with an initial velocity of 36 ft/s at an angle $\alpha$ with the horizontal. Determine ($a$) the distance $d$ to the farthest point $B$ on the roof that the water can reach, ($b$) the corresponding angle $\alpha$ Check that the stream will clear the edge of the roof.

## SOLUTION

Horizontal motion: $x = (v_0 \cos\alpha)t$ or $t = \dfrac{x}{v_0 \cos\alpha}$

Vertical motion: $y = y_0 + (v_0 \sin\alpha)t - \dfrac{1}{2}gt^2 = y_0 + x\tan\alpha - \dfrac{1}{2}\dfrac{gx^2}{(v_0\cos\alpha)^2}$

$$y = y_0 + x\tan\alpha - \frac{gx^2}{2v_0^2}(1+\tan^2\alpha)$$

Let $u = x\tan\alpha$ so that $y = y_0 + u - \dfrac{g}{2v_0^2}(x^2+u^2)$

Solving for $x^2$: $x^2 = \dfrac{2v_0^2}{g}(u + y_0 - y) - u^2$

The maximum value of $x^2$ is required: $\dfrac{d(x^2)}{du} = 0.$

$$\frac{d(x^2)}{du} = \frac{2v_0^2}{g} - 2u = 0 \qquad \text{or} \qquad u = \frac{v_0^2}{g}$$

Data: $v_0 = 36\ \text{ft/s},\quad y_0 = 3.6\ \text{ft},\quad y_B = 18\ \text{ft} \qquad u = \dfrac{(36)^2}{32.2} = 40.2484\ \text{ft}$

$$(x_{\max})^2 = \frac{(2)(36)^2}{32.2}(40.2484 + 3.6 - 18) - (40.2484)^2 = 460.78\,\text{ft}^2 \qquad x_{\max} = 21.466\ \text{ft}$$

($a$) Maximum distance: $d = x_{\max} - 13.5$ $\quad d = 7.97\ \text{ft}$ ◀

## PROBLEM 11.116* (CONTINUED)

(*b*) Angle $\alpha$.

$$\tan\alpha = \frac{x_{\max}\tan\alpha}{x\max} = \frac{u}{x_{\max}} = \frac{40.2484}{21.466} = 1.875$$

$\alpha = 61.93°$ $\qquad\qquad$ $\alpha = 61.9°$ ◀

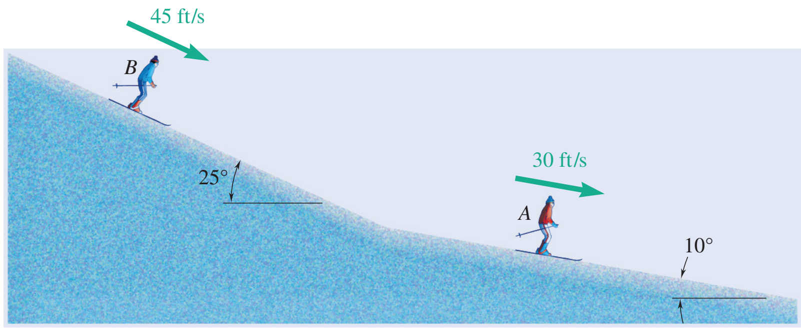
Check the edge.

$$y = y_0 + x\tan\alpha + \frac{gx^2}{2(v_0\cos\alpha)^2}$$

$$y = 3.6 + (13.5)(1.875) - \frac{(32.2)(13.5)^2}{2[36\cos 61.93°]^2}$$

$y = 18.69$ ft ◀

Since $y > 18$ ft, the stream clears the edge.

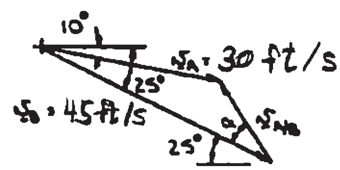



## PROBLEM 11.117

The velocities of skiers $A$ and $B$ are as shown. Determine the velocity of $A$ with respect to $B$.

## SOLUTION

We have $$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$




The graphical representation of this equation is then as shown.

Then $$v_{A/B}^2 = 30^2 + 45^2 - 2(30)(45)\cos 15°$$

or $$v_{A/B} = 17.80450 \text{ ft/s}$$

and $$\frac{30}{\sin\alpha} = \frac{17.80450}{\sin 15°}$$

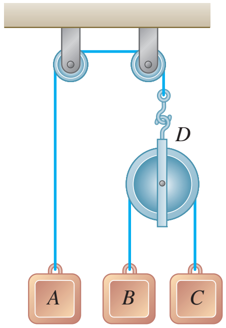
or $$\alpha = 25.8554°$$

$$\alpha + 25° = 50.8554°$$

$$\mathbf{v}_{A/B} = 17.8 \text{ ft/s} \searrow 50.9° \blacktriangleleft$$

<u>Alternative solution</u>.

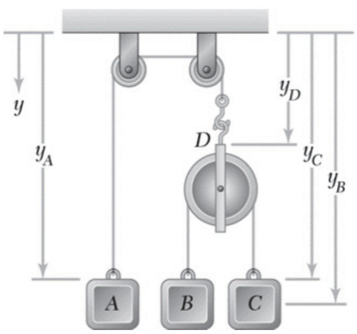
$$\mathbf{v}_{A/B} = \mathbf{v}_A + \mathbf{v}_B$$
$$= 30 \cos 10° \,\mathbf{i} - 30 \sin 10° \,\mathbf{j} - (45 \cos 25° \mathbf{i} - 45° \sin 25° \mathbf{j})$$
$$= 11.2396\mathbf{i} + 13.8084\mathbf{j}$$
$$= 5.05 \text{ m/s} = 17.8 \text{ ft/s} \searrow 50.9°$$




## PROBLEM 11.118

The three blocks shown move with constant velocities. Find the velocity of each block, knowing that the relative velocity of $A$ with respect to $C$ is 300 mm/s upward and that the relative velocity of $B$ with respect to $A$ is 200 mm/s downward.

## SOLUTION




From the diagram

Cable 1: $$y_A + y_D = \text{constant}$$

Then $$v_A + v_D = 0 \qquad (1)$$

Cable 2: $$(y_B + y_D) + (y_C + y_D) = \text{constant}$$

Then $$v_B + v_C - 2v_D = 0 \qquad (2)$$

Combining Eqs. (1) and (2) to eliminate $v_D$,

$$2v_A + v_B + v_C = 0 \qquad (3)$$

Now $$v_{A/C} = v_A - v_C = -300 \text{ mm/s} \qquad (4)$$

and $$v_{B/A} = v_B - v_A = 200 \text{ mm/s} \qquad (5)$$

Then $$(3) + (4) - (5) \Rightarrow$$

$$(2v_A + v_B + v_C) + (v_A - v_C) - (v_B - v_A) = (-300) - (200)$$

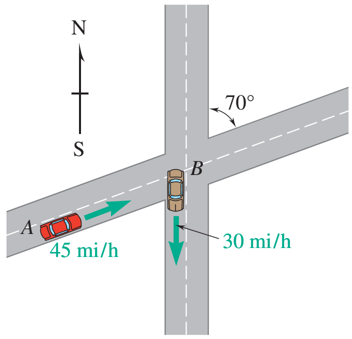
or $$\mathbf{v}_A = 125 \text{ mm/s}\uparrow \blacktriangleleft$$

and using Eq. (5) $$v_B - (-125) = 200$$

or $$\mathbf{v}_B = 75 \text{ mm/s}\downarrow \blacktriangleleft$$

Eq. (4) $$-125 - v_C = -300$$

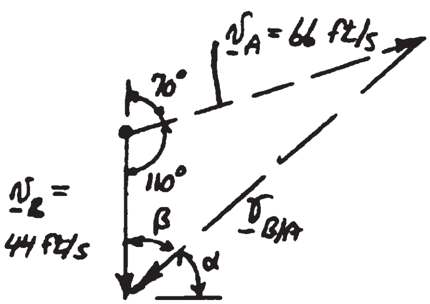
or $$\mathbf{v}_C = 175 \text{ mm/s}\downarrow \blacktriangleleft$$




## PROBLEM 11.119

Three seconds after automobile $B$ passes through the intersection shown, automobile $A$ passes through the same intersection. Knowing that the speed of each automobile is constant, determine ($a$) the relative velocity of $B$ with respect to $A$, ($b$) the change in position of $B$ with respect to $A$ during a 4-s interval, ($c$) the distance between the two automobiles 2 s after $A$ has passed through the intersection.

## SOLUTION




$$v_A = 45 \text{ mi/h} = 66 \text{ ft/s}$$

$$v_B = 30 \text{ mi/h} = 44 \text{ ft/s}$$

Law of cosines

$$v_{B/A}^2 = 66^2 + 44^2 - 2(66)(44)\cos 110°$$

$$v_{B/A} = 90.99 \text{ ft/s}$$

$$v_B = v_A + v_{B/A}$$

Law of sines

$$\frac{\sin\beta}{66} = \frac{\sin 110°}{90.99} \qquad \beta = 42.97°$$

$$\alpha = 90° - \beta = 90° - 42.97° = 47.03°$$

($a$) Relative velocity:

$$\mathbf{v}_{B/A} = 91.0 \text{ ft/s} \; 47.0° \blacktriangleleft$$

($b$) Change in position for $\Delta t = 4$ s.

$$\Delta r_{B/A} = r_{B/A}\Delta t = (91.0 \text{ ft/s})(4 \text{ s})$$

$$\mathbf{r}_{B/A} = 364 \text{ ft} \; 47.0° \blacktriangleleft$$

($c$) Distance between autos 2 seconds after auto $A$ has passed intersection.

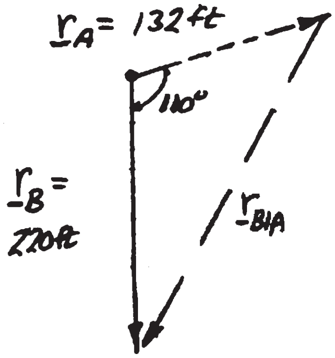
Auto $A$ travels for 2 s.

$$v_A = 66 \text{ ft/s} \; 20°$$

$$r_A = v_A t = (66 \text{ ft/s})(2 \text{ s}) = 132 \text{ ft/s}$$

$$\mathbf{r}_A = 132 \text{ ft} \; 20°$$

## PROBLEM 11.119 (CONTINUED)




Auto *B*

$$\mathbf{v}_B = 44 \text{ ft/s}\downarrow$$

$$\mathbf{r}_B = \mathbf{v}_B t = (44 \text{ ft/s})(5 \text{ s}) = 200 \text{ ft}\downarrow$$

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$

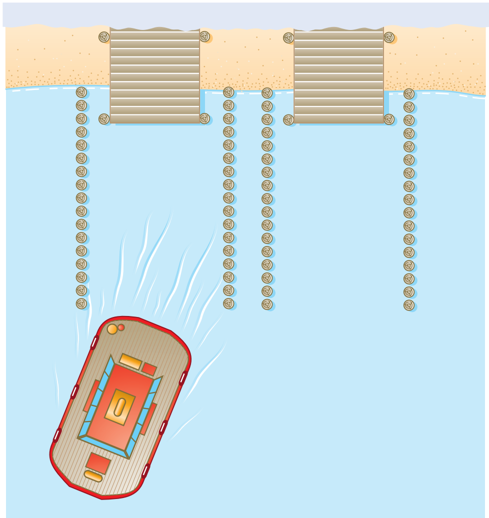
Law of cosines

$$r_{B/A}^2 = (132)^2 + (220)^2 - 2(132)(220)\cos 110°$$

$$\mathbf{r}_{B/A} = 292.7 \text{ ft}$$

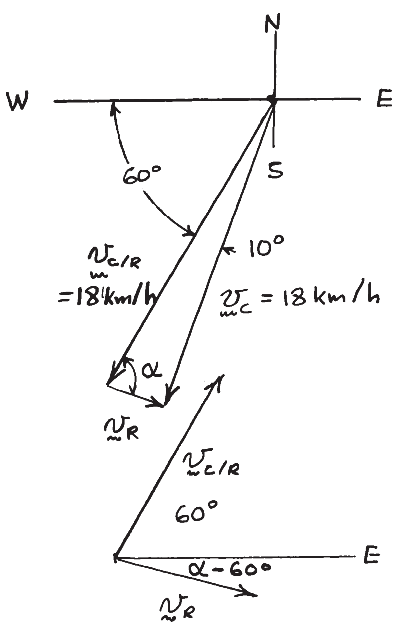
Distance between autos = 293 ft ◀

## PROBLEM 11.120



Shore-based radar indicates that a ferry leaves its slip with a velocity $\mathbf{v} = 18\ \text{km/h} \measuredangle\ 70°$, while instruments aboard the ferry indicate a speed of 18.4 km/h and a heading of 30° west of south relative to the river. Determine the velocity of the river.

## SOLUTION




We have $\mathbf{v}_F = \mathbf{v}_R + \mathbf{v}_{F/R}$ or $\mathbf{v}_F = \mathbf{v}_{F/R} + \mathbf{v}_R$

The graphical representation of the second equation is then as shown.

We have $v_R^2 = 18^2 + 18.4^2 - 2(18)(18.4)\cos 10°$

or $v_R = 3.1974\ \text{km/h}$

and $\dfrac{18}{\sin\alpha} = \dfrac{3.1974}{\sin 10°}$

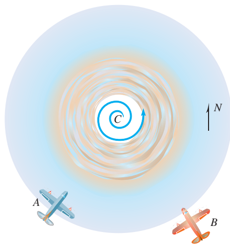

or $\alpha = 77.84°$

Noting that

$\mathbf{v}_R = 3.20\ \text{km/h} \measuredangle\ 17.8°$ ◀

Alternatively one could use vector algebra.

## PROBLEM 11.121




Airplanes $A$ and $B$ are flying at the same altitude and are tracking the eye of hurricane $C$. The relative velocity of $C$ with respect to $A$ is $\mathbf{v}_{C/A} = 350$ km/h ∠ 75°, and the relative velocity of $C$ with respect to $B$ is $\mathbf{v}_{C/B} = 400$ km/h ∠ 40°. Determine (*a*) the relative velocity of $B$ with respect to $A$, (*b*) the velocity of $A$ if ground-based radar indicates that the hurricane is moving at a speed of 30 km/h due north, (*c*) the change in position of $C$ with respect to $B$ during a 15-min interval.

## SOLUTION

(*a*) We have $\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$

and $\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

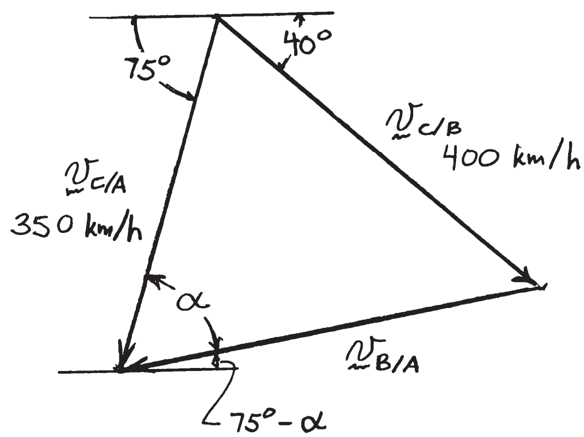
Then $\mathbf{v}_A + \mathbf{v}_{C/A} = \mathbf{v}_B + \mathbf{v}_{C/B}$

or $\mathbf{v}_B - \mathbf{v}_A = \mathbf{v}_{C/A} - \mathbf{v}_{C/B}$

Now $\mathbf{v}_B - \mathbf{v}_A = \mathbf{v}_{B/A}$

so that $\mathbf{v}_{B/A} = \mathbf{v}_{C/A} - \mathbf{v}_{C/B}$

or $\mathbf{v}_{C/A} = \mathbf{v}_{C/B} + \mathbf{v}_{B/A}$




The graphical representation of the last equation is then as shown.

We have $v_{B/A}^2 = 350^2 + 400^2 - 2(350)(400)\cos 65°$

or $v_{B/A} = 405.175$ km/h

and $\dfrac{400}{\sin\alpha} = \dfrac{405.175}{\sin 65°}$

or $\alpha = 63.474°$

$75° - \alpha = 11.526°$

$\mathbf{v}_{B/A} = 405$ km/h ∠ 11.53° ◀

## PROBLEM 11.121 (CONTINUED)

(*b*) We have $\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$

or $\mathbf{v}_A = (30 \text{ km/h})\mathbf{j} - (350 \text{ km/h})(-\cos 75°\mathbf{i} - \sin 75°\mathbf{j})$

$\mathbf{v}_A = (90.587 \text{ km/h})\mathbf{i} + (368.07 \text{ km/h})\mathbf{j}$

or $\mathbf{v}_A = 379 \text{ km/h} \measuredangle 76.17°$ ◀

(*c*) Noting that the velocities of *B* and *C* are constant, we have

$$\mathbf{r}_B = (\mathbf{r}_B)_0 + \mathbf{v}_B t \qquad \mathbf{r}_C = (\mathbf{r}_C)_0 + \mathbf{v}_C t$$

Now
$$\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = [(\mathbf{r}_C)_0 - (\mathbf{r}_B)_0] + (\mathbf{v}_C - \mathbf{v}_B)t$$
$$= [(\mathbf{r}_C)_0 - (\mathbf{r}_B)_0] + \mathbf{v}_{C/B}t$$

Then
$$\Delta\mathbf{r}_{C/B} = (\mathbf{r}_{C/B})_{t_2} - (\mathbf{r}_{C/B})_{t_1} = \mathbf{v}_{C/B}(t_2 - t_1) = v_{C/B}\Delta t$$

For $\Delta t = 15 \text{ min}$:
$$\Delta r_{C/B} = (400 \text{ km/h})\left(\frac{1}{4}\text{h}\right) = 100 \text{ km}$$

$\Delta\mathbf{r}_{C/B} = 100 \text{ km} \searrow 40°$ ◀

## PROBLEM 11.122

Instruments in an airplane indicate that with respect to the air, the plane is moving north at a speed of 500 km/h. At the same time ground-based radar indicates that the plane is moving at a speed of 530 km/h in a direction 5° east of north. Determine the magnitude and direction of the velocity of the air.

## SOLUTION

Starting with

$$\mathbf{v}_{\text{Plane}} = \mathbf{v}_{\text{Air}} + \mathbf{v}_{\text{Plane/Air}}$$

or

$$\mathbf{v}_{\text{Air}} = \mathbf{v}_{\text{Plane}} - \mathbf{v}_{\text{Plane/Air}}$$

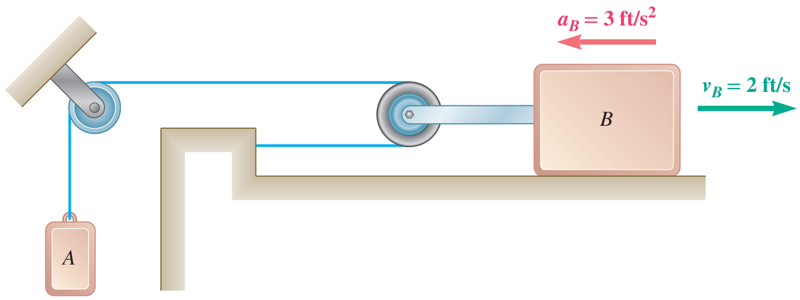
$$\mathbf{v}_{\text{Air}} = 530(\sin(5)\mathbf{i} + \cos(5)\mathbf{j}) - 500\mathbf{j}$$

$$\mathbf{v}_{\text{Air}} = 46.19\mathbf{i} + 527.9\mathbf{j} - 500\mathbf{j}$$

$$\mathbf{v}_{\text{Air}} = 46.19\mathbf{i} + 27.9\mathbf{j}$$

$$\mathbf{v}_{\text{Air}} = 54\angle 31.2^\circ\ \text{km/h}$$

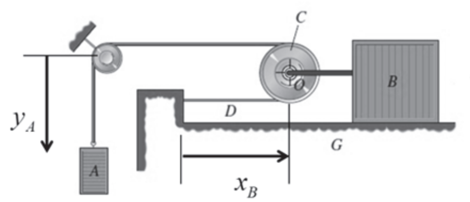
$$\mathbf{v}_{\text{Air}} = 54 \measuredangle 31.2^\circ\ \text{km/h} \blacktriangleleft$$




## Problem 11.123

Knowing that at the instant shown block $B$ has a velocity of 2 ft/s to the right and an acceleration of 3 ft/s$^2$ to the left, determine (*a*) the velocity of block $A$, (*b*) the acceleration of block A.

## SOLUTION




Given: $v_B = 2 \text{ ft/s}, a_B = -3 \text{ ft/s}$

Length of Cable $L = y_A + 2x_B + \text{constants}$

Differentiate Length Equation

$$0 = v_A + 2v_B \qquad (1)$$

Differentiate again $$0 = a_A + 2a_B \qquad (2)$$

(*e*) Rearrange (1)

$$v_A = -2v_B$$
$$= -2(2) \text{ ft/s}$$

$$v_A = -4 \text{ ft/s}$$

$v_A = 4 \text{ ft/s} \uparrow$ ◀

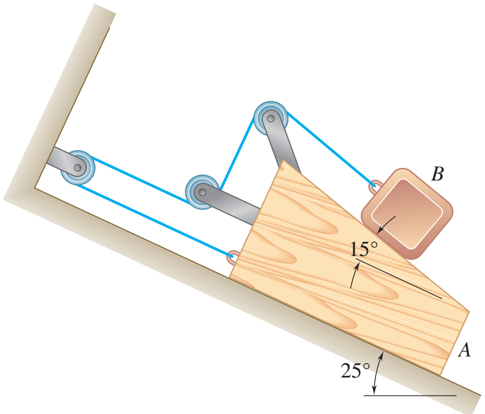
(*f*) Rearrange (2)

$$a_A = -2a_B$$
$$= -2(-3) \text{ ft/s}^2$$

$$a_A = 6 \text{ ft/s}^2$$

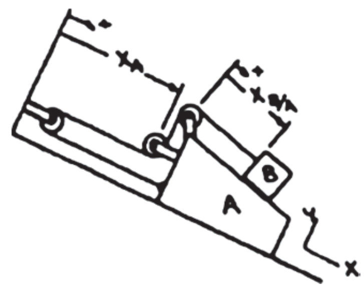
$a_A = 6 \text{ ft/s}^2 \downarrow$ ◀

## PROBLEM 11.124




Knowing that at the instant shown block $A$ has a velocity of 8 in./s and an acceleration of 6 in./s$^2$ both directed down the incline, determine ($a$) the velocity of block $B$, ($b$) the acceleration of block $B$.

## SOLUTION




From the diagram $\qquad 2x_A + x_{B/A} = \text{constant}$

Then $\qquad 2v_A + v_{B/A} = 0$

or $\qquad |v_{B/A}| = 16 \text{ in./s}$

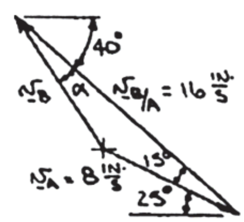
and $\qquad 2a_A + a_{B/A} = 0$

or $\qquad |a_{B/A}| = 12 \text{ in./s}^2$

Note that $\mathbf{v}_{B/A}$ and $\mathbf{a}_{B/A}$ must be parallel to the top surface of block $A$.

($a$) We have $\qquad \mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

The graphical representation of this equation is then as shown. Note that because $A$ is moving downward, $B$ must be moving upward relative to $A$.




We have $\qquad v_B^2 = 8^2 + 16^2 - 2(8)(16)\cos 15°$

or $\qquad v_B = 8.5278 \text{ in./s}$

and $\qquad \dfrac{8}{\sin\alpha} = \dfrac{8.5278}{\sin 15°}$

or $\qquad \alpha = 14.05°$

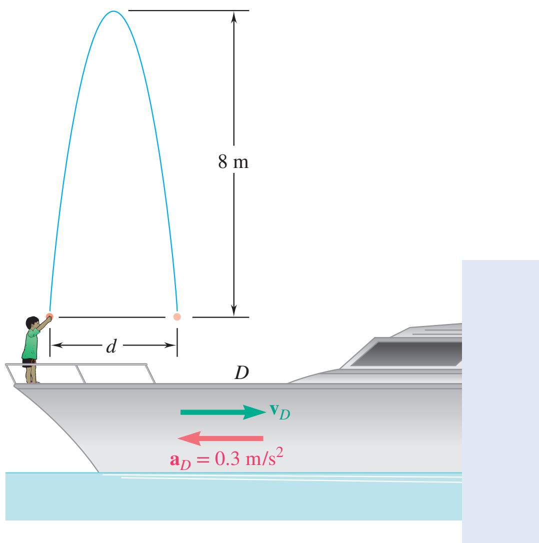
$$\mathbf{v}_B = 8.53 \text{ in./s} \measuredangle 54.1° \blacktriangleleft$$

($b$) The same technique that was used to determine $\mathbf{v}_B$ can be used to determine $\mathbf{a}_B$. An alternative method is as follows.

We have
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$
$$= (6\mathbf{i}) + 12(-\cos 15°\mathbf{i} + \sin 15°\mathbf{j})\,*$$
$$= -(5.5911 \text{ in./s}^2)\mathbf{i} + (3.1058 \text{ in./s}^2)\mathbf{j}$$

or
$$\mathbf{a}_B = 6.40 \text{ in./s}^2 \measuredangle 54.1° \blacktriangleleft$$

* Note the orientation of the coordinate axes on the sketch of the system.




## PROBLEM 11.125

A boat is moving to the right with a constant deceleration of 0.3 m/s$^2$ when a boy standing on the deck $D$ throws a ball with an initial velocity relative to the deck which is vertical. The ball rises to a maximum height of 8 m above the release point and the boy must step forward a distance $d$ to catch it at the same height as the release point. Determine ($a$) the distance $d$, ($b$) the relative velocity of the ball with respect to the deck when the ball is caught.

## SOLUTION

Horizontal motion of the ball: $v_x = (v_x)_0, \quad x_{\text{ball}} = (v_x)_0 t$

Vertical motion of the ball: $v_y = (v_y)_0 - gt$

$$y_B = (v_y)_0 t - \frac{1}{2} g t^2, \quad (v_y)^2 - (v_y)_0^2 = -2gy$$

At maximum height, $(v_y) = 0$ and $y = y_{\max}$

$$(v_y)^2 = 2gy_{\max} = (2)(9.81)(8) = 156.96\ \text{m}^2/\text{s}^2$$

$$(v_y)_0 = 12.528\ \text{m/s}$$

At time of catch, $y = 0 = 12.528 - \frac{1}{2}(9.81)t^2$

or $t_{\text{catch}} = 2.554$ s and $v_y = 12.528$ m/s$\downarrow$

Motion of the deck: $v_x = (v_x)_0 + a_D t, \quad x_{\text{deck}} = (v_x)_0 t + \frac{1}{2} a_D t^2$

Motion of the ball relative to the deck:

$$(v_{B/D})_x = (v_x)_0 - [(v_x)_0 + a_D t] = -a_D t$$

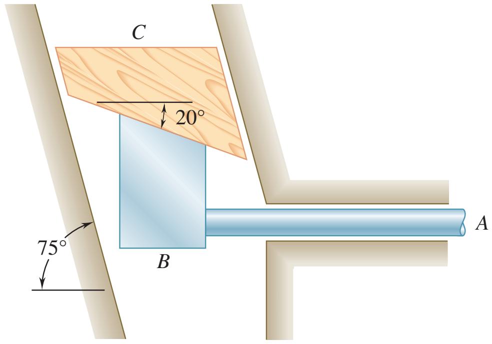
$$x_{B/D} = (v_x)_0 t - \left[(v_x)_0 t + \frac{1}{2} a_D t^2\right] = -\frac{1}{2} a_D t^2$$

$$(v_{B/D})_y = (v_y)_0 - gt, \quad y_{B/D} = y_B$$

($a$) At time of catch, $d = x_{D/B} = -\frac{1}{2}(-0.3)(2.554)^2$ $\qquad d = 0.979\ \text{m}$ ◀

($b$) $(v_{B/D})_x = -(-0.3)(2.554) = +0.766$ m/s $\qquad$ or 0.766 m/s $\longrightarrow$

$(v_{B/D})_y = 12.528$ m/s $\downarrow$ $\qquad v_{B/D} = 12.55$ m/s $\searrow$ $86.5^\circ$ ◀



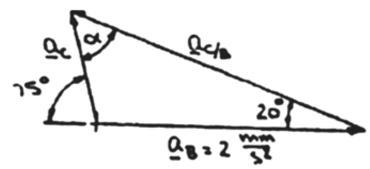

## PROBLEM 11.126

The assembly of rod $A$ and wedge $B$ starts from rest and moves to the right with a constant acceleration of 2 mm/s$^2$. Determine (*a*) the acceleration of wedge $C$, (*b*) the velocity of wedge $C$ when $t = 10$ s.

## SOLUTION

(*a*) We have $\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C/B}$

The graphical representation of this equation is then as shown.




First note
$$\alpha = 180° - (20° + 105°)$$
$$= 55°$$

Then
$$\frac{a_C}{\sin 20°} = \frac{2}{\sin 55°}$$
$$a_C = 0.83506 \text{ mm/s}^2$$

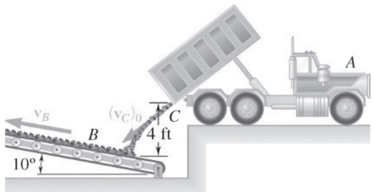
$$\mathbf{a}_C = 0.835 \text{ mm/s}^2 \ \searrow 75° \ ◀$$

(*b*) For uniformly accelerated motion

$$v_C = 0 + a_C t$$

At $t = 10$ s:
$$v_C = (0.83506 \text{ mm/s}^2)(10 \text{ s})$$
$$= 8.3506 \text{ mm/s}$$

or
$$\mathbf{v}_C = 8.35 \text{ mm/s} \ \searrow 75° \ ◀$$




## PROBLEM 11.127

Coal discharged from a dump truck with an initial velocity $(v_C)_0 =$ 5 ft/s $\measuredangle$ 50° falls onto conveyor belt $B$. Determine the required velocity $\mathbf{v}_B$ of the belt if the relative velocity with which the coal hits the belt is to be (*a*) vertical, (*b*) as small as possible.

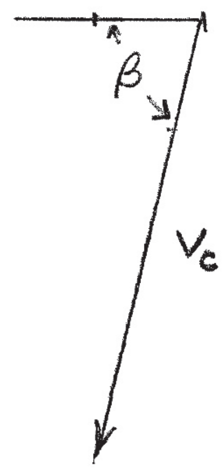
## SOLUTION

First determine the velocity $v_C$ of the coal at the point where the coal impacts on the belt.

Horizontal motion:

$$(v_C)_x = \left[(v_C)_x\right]_0 = -5\cos 50°$$

$$= -3.2139 \text{ m/s}$$




Vertical motion:

$$(v_C)_y^2 = \left[(v_C)_y\right]_0^2 - 2g(y - y_0)$$

$$= (5\sin 50°)^2 - (2)(32.2)(-4)$$
$$= 272.27 \text{ ft}^2/\text{s}^2$$
$$(v_C)_y = -16.501 \text{ ft/s}$$

$$\tan\beta = \frac{v_{cy}}{v_{cx}} = \frac{-16.5006}{-3.2139} = 5.134, \quad \beta = 79.98°$$
$$v_C = \sqrt{(v_C)_x^2 + (v_C)_y^2} = 16.81 \text{ ft/s}$$

$$v_C = 16.81 \text{ ft/s} \measuredangle\ 79.98°$$

or

$$\mathbf{v_C} = (-3.214 \text{ ft/s})\mathbf{i} + (-16.501 \text{ ft/s})\mathbf{j}$$

Velocity of the belt:

$$\mathbf{v_B} = v_B(-\cos 10°\mathbf{i} + \sin 10°\mathbf{j})$$

Relative velocity:

$$\mathbf{v_{C/B}} = \mathbf{v_C} - \mathbf{v_B} = \mathbf{v_C} + (-\mathbf{v_B})$$

(*a*) $\boldsymbol{v}_{C/B}$ is vertical.

$$\left(v_{C/B}\right)_x = 0$$

$$\left(v_{C/B}\right)_y \mathbf{j} = (-3.214\mathbf{i} - 16.501\mathbf{j}) - (-.9848v_B\mathbf{i} + .1736v_B\mathbf{j})$$
$$\mathbf{i}: \quad 0 = -3.214 + 0.9848v_B$$
$$v_B = 3.264$$

## PROBLEM 11.127 (CONTINUED)

$\boldsymbol{v}_B = 3.26 \text{ ft/s} \measuredangle 10° ◀$

(*b*) for minimum $v_{C/B}$

$$\mathbf{v}_{C/B} = \mathbf{v}_C - \mathbf{v}_B$$

$$\left(v_{C/B}\right)_x \mathbf{i} + \left(v_{C/B}\right)_y \mathbf{j} = (-3.214\mathbf{i} - 16.501\mathbf{j}) - (-0.9848 v_B \mathbf{i} + 0.1736 v_B \mathbf{j})$$

$\mathbf{i}: \quad v_{C/Bx} = -3.214 + 0.9848 v_B$

$\mathbf{j}: \quad v_{C/By} = -16.501 - 0.1736 v_B$

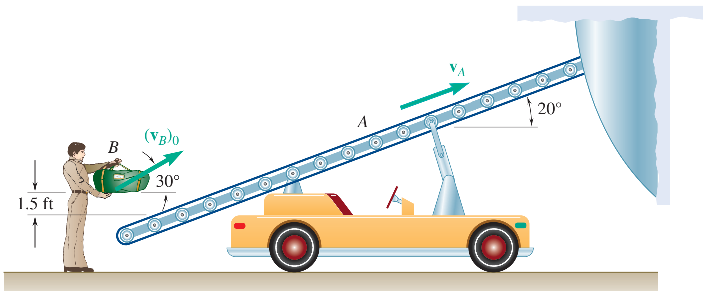
$(v_{C/B})^2 = (v_{C/B})_x{}^2 + (v_{C/B})_y{}^2 = 282.6 - 0.6 v_B + v_B{}^2$

To find the minimum

$$\frac{d(v_{C/B})^2}{dv_B} = -0.6 + 2v_B = 0$$

$$v_B = 0.300$$

$\boldsymbol{v}_B = 0.300 \text{ ft/s} \measuredangle 10° ◀$

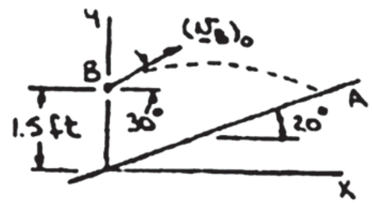



## PROBLEM 11.128

Conveyor belt $A$, which forms a 20° angle with the horizontal, moves at a constant speed of 4 ft/s and is used to load an airplane. Knowing that a worker tosses duffel bag $B$ with an initial velocity of 2.5 ft/s at an angle of 30° with the horizontal, determine the velocity of the bag relative to the belt as it lands on the belt.

## SOLUTION

First determine the velocity of the bag as it lands on the belt. Now




$$[(v_B)_x]_0 = (v_B)_0 \cos 30°$$
$$= (2.5 \text{ ft/s}) \cos 30°$$
$$[(v_B)_y]_0 = (v_B)_0 \sin 30°$$
$$= (2.5 \text{ ft/s}) \sin 30°$$

Horizontal motion. (Uniform)

$$x = 0 + [(v_B)_x]_0 t \qquad (v_B)_x = [(v_B)_x]_0$$
$$= (2.5 \cos 30°)t \qquad = 2.5 \cos 30°$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + [(v_B)_y]_0 t - \frac{1}{2}gt^2 \qquad (v_B)_y = [(v_B)_y]_0 - gt$$
$$= 1.5 + (2.5 \sin 30°)t - \frac{1}{2}gt^2 \qquad = 2.5 \sin 30° - gt$$

The equation of the line collinear with the top surface of the belt is

$$y = x \tan 20°$$

Thus, when the bag reaches the belt

$$= 1.5 + (2.5 \sin 30°)t - \frac{1}{2}gt^2 = [(2.5 \cos 30°)t] \tan 20°$$

or
$$\frac{1}{2}(32.2)t^2 + 2.5(\cos 30° \tan 20° - \sin 30°)t - 1.5 = 0$$

or
$$16.1t^2 - 0.46198t - 1.5 = 0$$

Solving
$$t = 0.31992 \text{ s} \quad \text{and} \quad t = -0.29122 \text{ s} \quad \text{(Reject)}$$

## PROBLEM 11.128 (CONTINUED)

The velocity $\mathbf{v}_B$ of the bag as it lands on the belt is then

$$\mathbf{v}_B = (2.5\cos 30°)\mathbf{i} + [2.5\sin 30° - 32.2(0.31992)]\mathbf{j}$$
$$= (2.1651 \text{ ft/s})\mathbf{i} - (9.0514 \text{ ft/s})\mathbf{j}$$

Finally $$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

or $$\mathbf{v}_{B/A} = (2.1651\mathbf{i} - 9.0514\mathbf{j}) - 4(\cos 20°\mathbf{i} + \sin 20°\mathbf{j})$$
$$= (1.59367 \text{ ft/s})\mathbf{i} - (10.4195 \text{ ft/s})\mathbf{j}$$

or $$\mathbf{v}_{B/A} = 10.54 \text{ ft/s} \measuredangle 81.3° \blacktriangleleft$$

## PROBLEM 11.129

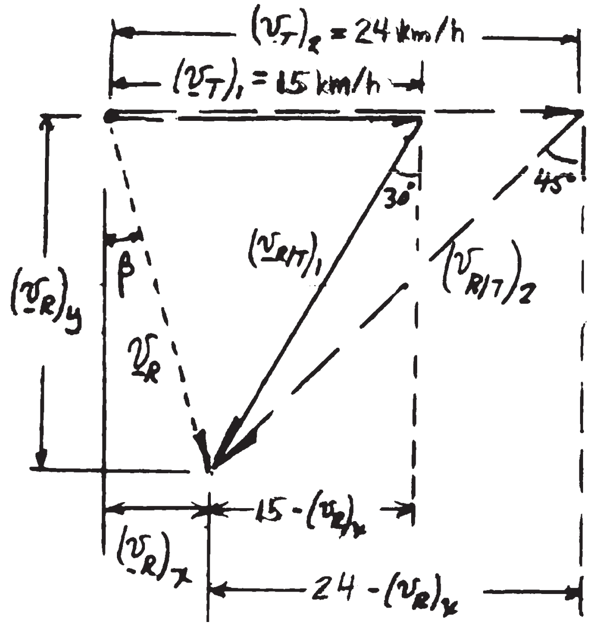
During a rainstorm the paths of the raindrops appear to form an angle of 30° with the vertical and to be directed to the left when observed from a side window of a train moving at a speed of 15 km/h. A short time later, after the speed of the train has increased to 24 km/h, the angle between the vertical and the paths of the drops appears to be 45°. If the train were stopped, at what angle and with what velocity would the drops be observed to fall?

## SOLUTION

$$v_{\text{rain}} = v_{\text{train}} + v_{\text{rain/train}}$$

<u>Case</u> ①: $v_T = 15 \text{ km/h} \longrightarrow; \qquad v_{R/T} \swarrow 30°$

<u>Case</u> ②: $v_T = 24 \text{ km/h} \longrightarrow; \qquad v_{R/T} \swarrow 45°$




<u>Case</u> ①: $$(v_R)_y \tan 30° = 15 - (v_R)_x \qquad (1)$$

<u>Case</u> ②: $$(v_R)_y \tan 45° = 24 - (v_R)_x \qquad (2)$$

Substract (1) from (2)
$$(v_R)_y (\tan 45° - \tan 30°) = 9$$
$$(v_R)_y = 21.294 \text{ km/h}$$

Eq. (2): $$21.294 \tan 45° = 25 - (v_R)_x$$
$$(v_R)_x = 3.706 \text{ km/h}$$

## PROBLEM 11.129 (CONTINUED)

$$\tan\beta = \frac{3.706}{21.294}$$
$$\beta = 7.24°$$
$$v_R = \frac{21.294}{\cos 7.24°} = 21.47 \text{ km/h} = 5.96 \text{ m/s}$$

$v_R = 5.96$ m/s $\searrow$ 82.8° ◀

**Alternate solution**

Alternate, vector equation $\quad \mathbf{v}_R = \mathbf{v}_T + \mathbf{v}_{R/T}$

For first case, $\quad \mathbf{v}_R = 15\mathbf{i} + v_{R/T-1}(-\sin 30°\mathbf{i} - \cos 30°\mathbf{j})$

For second case, $\quad \mathbf{v}_R = 24\mathbf{i} + v_{R/T-2}(-\sin 45°\mathbf{i} - \cos 45°\mathbf{j})$

Set equal

$$15\mathbf{i} + v_{R/T-1}(-\sin 30°\mathbf{i} - \cos 30°\mathbf{j}) = 24\mathbf{i} + v_{R/T-2}(-\sin 45°\mathbf{i} - \cos 45°\mathbf{j})$$

Separate into components:

**i**: $\quad 15 - v_{R/T-1}\sin 30° = 24 - v_{R/T-2}\sin 45°$

$$-v_{R/T-1}\sin 30° + v_{R/T-2}\sin 45° = 9 \qquad (3)$$

**j**: $\quad -v_{R/T-1}\cos 30° = -v_{R/T-2}\cos 45°$

$$-v_{R/T-1}\cos 30° + v_{R/T-2}\cos 45° = 0 \qquad (4)$$

Solving Eqs. (3) and (4) simultaneously,

$v_{R/T-1} = 24.5885$ km/h $\qquad v_{R/T-2} = 30.1146$ km/h

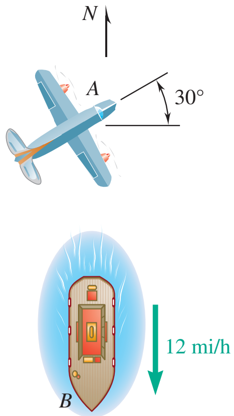
Substitute $v_{R/T-2}$ back into equation for $\mathbf{v}_R$.

$$\mathbf{v}_R = 24\mathbf{i} + 30.1146(-\sin 45°\mathbf{i} - \cos 45°\mathbf{j})$$
$$\mathbf{v}_R = 2.71\mathbf{i} - 21.29\mathbf{j}$$

$\mathbf{v}_R = 21.4654$ km/hr $= 5.96$ m/s

$$\theta = \tan^{-1}\left(\frac{-21.29}{2.71}\right) = -82.7585°$$

$\mathbf{v}_R = 5.96$ m/s $\searrow$ 82.8° ◀




## PROBLEM 11.130

Instruments in airplane $A$ indicate that with respect to the air the plane is headed 30° north of east with an airspeed of 300 mi/h. At the same time radar on ship $B$ indicates that the relative velocity of the plane with respect to the ship is 280 mi/h in the direction 33° north of east. Knowing that the ship is steaming due south at 12 mi/h, determine ($a$) the velocity of the airplane, ($b$) the wind speed and direction.

## SOLUTION

Let the $x$-axis be directed east, and the $y$-axis be directed north.

Airspeed: $$\mathbf{v}_{A/W} = 300 \text{ mi/h} \measuredangle 30^\circ = 300(\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{j}) \text{ mi/h}$$

Plane relative to ship: $$\mathbf{v}_{A/B} = 280(\cos 33^\circ \mathbf{i} + \sin 33^\circ \mathbf{j}) \text{ mi/h}$$

Ship: $$\mathbf{v}_B = 12 \text{ mi/h} \downarrow = -12\mathbf{j}$$

($a$) Velocity of airplane.

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} = -12\mathbf{j} + 280(\cos 33^\circ \mathbf{i} + \sin 33^\circ \mathbf{j})$$

$$= 234.83\mathbf{i} + 140.50\mathbf{j}$$

$$\mathbf{v}_A = 274 \text{ mi/h} \measuredangle 30.9^\circ \blacktriangleleft$$

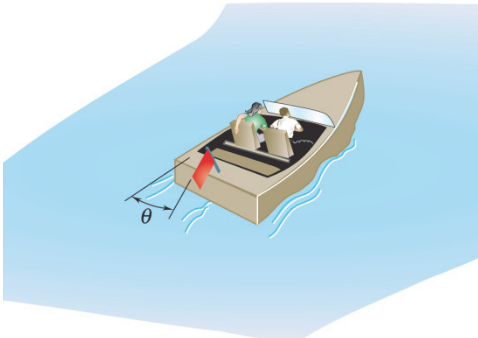
($b$) Wind velocity.

$$\mathbf{v}_W = \mathbf{v}_A + \mathbf{v}_{W/A} = \mathbf{v}_A - \mathbf{v}_{A/W}$$

$$= 234.83\mathbf{i} + 140.50\mathbf{j} - 300(\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{j})$$

$$= -24.98\mathbf{i} - 9.50\mathbf{j}$$

$$\mathbf{v}_W = 26.7 \text{ mi/h} \swarrow 20.8^\circ \blacktriangleleft$$



## PROBLEM 11.131

When a small boat travels north at 15 km/h, a flag mounted on its stern forms an angle $\theta = 50°$ with the centerline of the boat as shown. A short time later, when the boat travels east at 25 km/h, angle $\theta$ is again 50°. Determine the speed and the direction of the wind.

## SOLUTION

For both cases we have $\qquad \mathbf{v_w} = \mathbf{v_B} + \mathbf{v_{B/W}}$

Case 1: $\qquad v_{wx}\mathbf{i} + v_{wy}\mathbf{j} = 15\mathbf{j} + \left(v_{w/B}\right)_1 \sin(50)\mathbf{i} - \left(v_{w/B}\right)_1 \cos(50)\mathbf{j}$

Separate i and j components to get

$$\mathbf{i}: \; v_{wx} = 0 + \left(v_{w/B}\right)_1 \sin(50)$$
$$\mathbf{j}: \; v_{wy} = 15 - \left(v_{w/B}\right)_1 \cos(50)$$

Case 2: $\qquad v_{wx}\mathbf{i} + v_{wy}\mathbf{j} = 25\mathbf{i} - \left(v_{w/B}\right)_2 \cos(50)\mathbf{i} - \left(v_{w/B}\right)_2 \sin(50)\mathbf{j}$

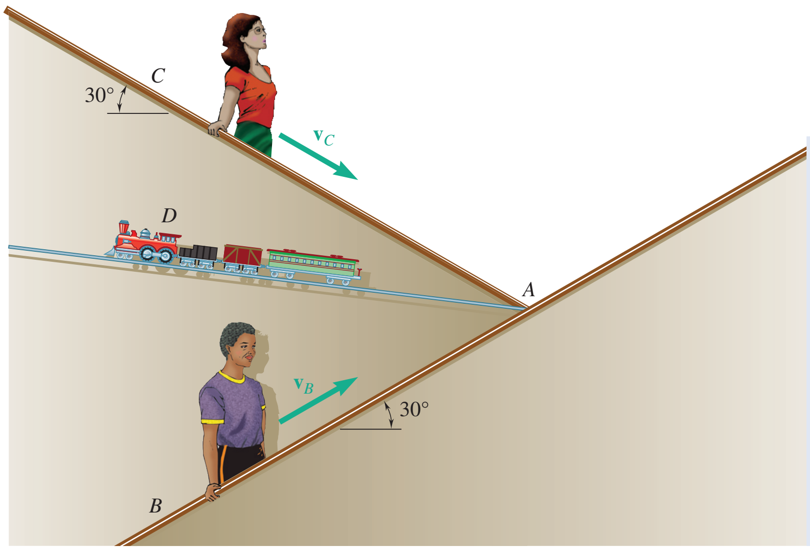
Separate i and j components to get

$$\mathbf{i}: \; v_{wx} = 25 - \left(v_{w/B}\right)_2 \cos(50)$$
$$\mathbf{j}: \; v_{wy} = 0 - \left(v_{w/B}\right)_2 \sin(50)$$

Solving the 4 scalar equations to get,

$$\left(v_{w/B}\right)_1 = 28.79 \text{ km/h}$$
$$\left(v_{w/B}\right)_2 = 4.579 \text{ km/h}$$
$$v_{wx} = 22.057 \text{ km/h}$$
$$v_{wy} = -3.5078 \text{ km/h}$$

$$v_w = 22.33 \text{ km/h} \;\searrow 9.04° \;◀$$




## PROBLEM 11.132

As part of a department store display, a model train $D$ runs on a slight incline between the store's up and down escalators. When the train and shoppers pass Point $A$, the train appears to a shopper on the up escalator $B$ to move downward at an angle of 22° with the horizontal, and to a shopper on the down escalator $C$ to move upward at an angle of 23° with the horizontal and to travel to the left. Knowing that the speed of the escalators is 3 ft/s, determine the speed and the direction of the train.

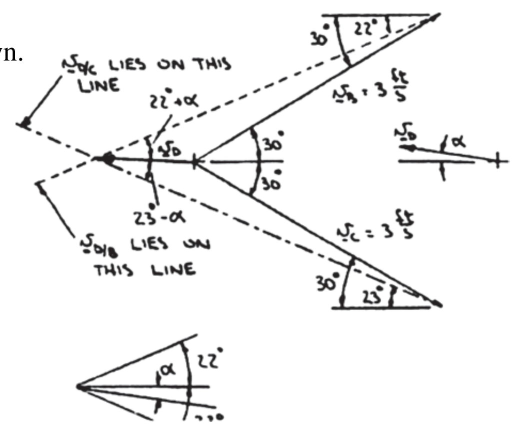
## SOLUTION

We have

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$

$$\mathbf{v}_D = \mathbf{v}_C + \mathbf{v}_{D/C}$$

The graphical representations of these equations are then as shown.




Then $\dfrac{v_D}{\sin 8°} = \dfrac{3}{\sin(22° + \alpha)}$ $\quad \dfrac{v_D}{\sin 7°} = \dfrac{3}{\sin(23° - \alpha)}$

Equating the expressions for $\dfrac{v_D}{3}$

$$\frac{\sin 8°}{\sin(22° + \alpha)} = \frac{\sin 7°}{\sin(23° - \alpha)}$$

or

$$\sin 8°(\sin 23° \cos\alpha - \cos 23° \sin\alpha) = \sin 7°(\sin 22° \cos\alpha + \cos 22° \sin\alpha)$$

or

$$\tan\alpha = \frac{\sin 8° \sin 23° - \sin 7° \sin 22°}{\sin 8° \cos 23° + \sin 7° \cos 22°}$$

or

$$\alpha = 2.0728°$$

Then

$$v_D = \frac{3 \sin 8°}{\sin(22° + 2.0728°)} = 1.024 \text{ ft/s}$$

$$\mathbf{v}_D = 1.024 \text{ ft/s} \searrow 2.07° \blacktriangleleft$$

## PROBLEM 11.132 (CONTINUED)

<u>Alternate solution using components</u>.

$$\mathbf{v}_B = (3 \text{ ft/s}) \ 30° = (2.5981 \text{ ft/s})\mathbf{i} + (1.5 \text{ ft/s})\mathbf{j}$$

$$\mathbf{v}_C = (3 \text{ ft/s}) \ 30° = (2.5981 \text{ ft/s})\mathbf{i} - (1.5 \text{ ft/s})\mathbf{j}$$

$$\mathbf{v}_{D/B} = u_1 \ 22° = -(u_1 \cos 22°)\mathbf{i} - (u_1 \sin 22°)\mathbf{j}$$

$$\mathbf{v}_{D/C} = u_2 \ 23° = -(u_2 \cos 23°)\mathbf{i} + (u_2 \sin 23°)\mathbf{j}$$

$$\mathbf{v}_D = v_D \ \alpha = -(v_D \cos\alpha)\mathbf{i} + (v_D \sin\alpha)\mathbf{j}$$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B} = \mathbf{v}_C + \mathbf{v}_{D/C}$$

$$2.5981\mathbf{i} + 1.5\mathbf{j} - (u_1 \cos 22°)\mathbf{i} - (u_1 \sin 22°)\mathbf{j} = 2.5981\mathbf{i} - 1.5\mathbf{j} - (u_2 \cos 23°)\mathbf{i} + (u_2 \sin 23°)\mathbf{j}$$

Separate into components, transpose, and change signs.

$$u_1 \cos 22° - u_2 \cos 23° = 0$$

$$u_1 \sin 22° + u_1 \sin 23° = 3$$

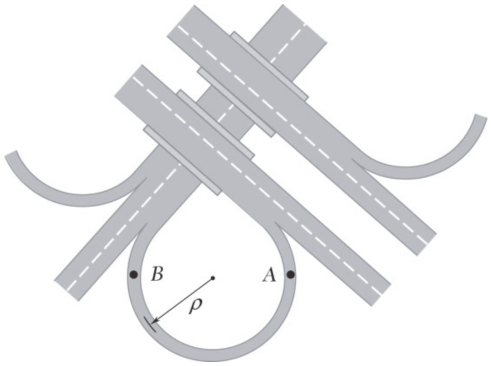
Solving for $u_1$ and $u_2$, $\quad u_1 = 3.9054$ ft/s $\quad u_2 = 3.9337$ ft/s

$$\mathbf{v}_D = 2.5981\mathbf{i} + 1.5\mathbf{j} - (3.9054 \cos 22°)\mathbf{i} - (3.9054 \sin 22°)\mathbf{j}$$
$$= -(1.0029 \text{ ft/s})\mathbf{i} + (0.0370 \text{ ft/s})\mathbf{j}$$

or

$$\mathbf{v}_D = 2.5981\mathbf{i} + 1.5\mathbf{j} - (3.9337 \cos 23°)\mathbf{i} - (3.9337 \sin 23°)\mathbf{j}$$
$$= -(1.0029 \text{ ft/s})\mathbf{i} + (0.0370 \text{ ft/s})\mathbf{j}$$

$$\mathbf{v}_D = 1.024 \text{ ft/s} \ 2.07° \ ◀$$




## PROBLEM 11.133

Determine the normal component of acceleration of a car traveling at 72 km/h on an exit ramp of radius ρ = 150 m.

## SOLUTION

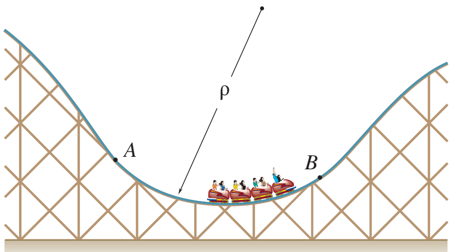
$$a_n = \frac{v^2}{\rho}$$

$$a_n = \frac{\left(75\text{ km/h}\left(\frac{1\text{ hr}}{3600\text{ s}}\right)\frac{1000\text{ m}}{\text{km}}\right)^2}{150\text{ m}} = 2.89\text{ m/s}^2$$

$$a_n = 2.89\text{ m/s}^2 \blacktriangleleft$$

◀

## PROBLEM 11.134




Determine the maximum speed that the cars of the roller-coaster can reach along the circular portion $AB$ of the track if $\rho$ is 25 m and the normal component of their acceleration cannot exceed 3 g.

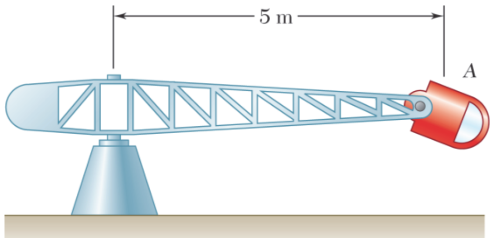
## SOLUTION

We have $$a_n = \frac{v^2}{\rho}$$

Then $$(v_{\text{max}})_{AB}^2 = (3 \times 9.81\ \text{m/s}^2)(25\ \text{m})$$

or $$(v_{\text{max}})_{AB} = 27.124\ \text{m/s}$$

or $$(v_{\text{max}})_{AB} = 97.6\ \text{km/h} \blacktriangleleft$$




## PROBLEM 11.135

Human centrifuges are often used to simulate different acceleration levels for pilots and astronauts. Space shuttle pilots typically face inwards towards the center of the gondola in order to experience a simulated 5$g$ forward acceleration. Knowing that the astronaut sits 5 m from the axis of rotation and experiences 5 g's inward, determine her velocity.

## SOLUTION

Given: $a_n = 5g,\ \rho = 5\text{ m}$

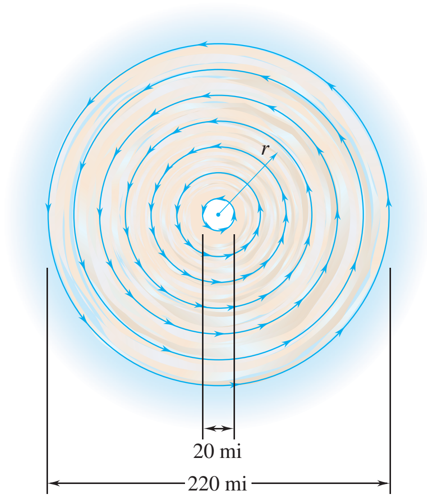
$$a_n = \frac{v^2}{\rho}$$

Rearrange $v = \sqrt{\rho a_n}$

$$v = \sqrt{\rho 5g}$$

Substitute in known values $v = \sqrt{5 * 5 * 9.81}$

$$v_A = 15.66\text{ m/s} \blacktriangleleft$$




## PROBLEM 11.136

The diameter of the eye of a stationary hurricane is 20 mi and the maximum wind speed is 100 mi/h at the eye wall, $r = 10$ mi. Assuming that the wind speed is constant for constant $r$ and decreases uniformly with increasing $r$ to 40 mi/h at $r = 110$ mi, determine the magnitude of the acceleration of the air at (*a*) $r = 10$ mi, (*b*) $r = 60$ mi, (*c*) $r = 110$ mi.

## SOLUTION

(*a*)

$$r = 10 \text{ mi} = 52800 \text{ ft}, \qquad v = 100 \text{ mi/h} = 146.67 \text{ ft/s}$$

$$a = \frac{v^2}{r} = \frac{(146.67)^2}{52800} \qquad a = 0.407 \text{ ft/s}^2 \blacktriangleleft$$

(*b*)

$$r = 60 \text{ mi} = 316800 \text{ ft}, \qquad v = 100 - \frac{40-100}{110-10}(60-10) = 70 \text{ mi/h} = 102.67 \text{ ft/s}$$

$$a = \frac{v^2}{r} = \frac{(102.67)^2}{316800} \qquad a = 0.0333 \text{ ft/s}^2 \blacktriangleleft$$

(*c*)

$$r = 110 \text{ mi} = 580800 \text{ ft}, \qquad v = 40 \text{ mi/h} = 58.67 \text{ ft/s}$$

$$a = \frac{v^2}{r} = \frac{(58.67)^2}{580800} \qquad a = 0.00593 \text{ ft/s}^2 \blacktriangleleft$$

## PROBLEM 11.137

The peripheral speed of the tooth of a 10-in.-diameter circular saw blade is $150 \text{ ft/s}$ when the power to the saw is turned off. The speed of the tooth decreases at a constant rate, and the blade comes to rest in 9 s. Determine the time at which the total acceleration of the tooth is $130 \text{ ft/s}^2$.

## SOLUTION

For uniformly decelerated motion: $\quad v = v_0 + a_t t$

At $t = 9 \text{ s}$, $\quad 0 = 150 - a_t(9), \quad$ or $\quad a_t = -16.667 \text{ ft/s}^2$

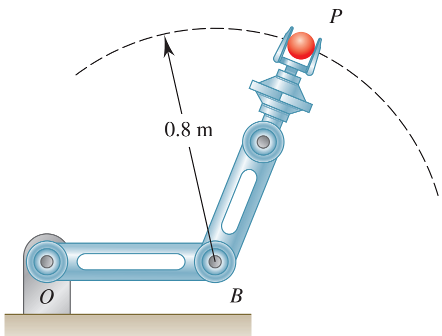
Total acceleration: $\quad a^2 = a_t^2 + a_n^2$

$$a_n = \left[a^2 - a_t^2\right]^{1/2} = \left[(130)^2 - (-16.667)^2\right]^{1/2} = 128.93 \text{ ft/s}^2$$

Normal acceleration: $\quad a_n = \dfrac{v^2}{\rho}, \quad$ where $\quad \rho = \dfrac{1}{2}\text{diameter} = \dfrac{5}{12} \text{ ft}$

$$v^2 = \rho a_n = \left(\frac{5}{12}\right)(128.93) = 53.72 \text{ ft}^2/\text{s}^2, \qquad v = 7.329 \text{ ft/s}$$

Time: $\quad t = \dfrac{v - v_0}{a_t} = \dfrac{7.329 - 150}{-16.667}$ $\qquad t = 8.56 \text{ s}$ ◀




## PROBLEM 11.138

A robot arm moves so that $P$ travels in a circle about Point $B$, which is not moving. Knowing that $P$ starts from rest, and its speed increases at a constant rate of 10 mm/s$^2$, determine ($a$) the magnitude of the acceleration when $t$ = 4 s, ($b$) the time for the magnitude of the acceleration to be 80 mm/s$^2$.

## SOLUTION

Tangential acceleration: $a_t = 10 \text{ mm/s}^2$

Speed: $v = a_t t$

Normal acceleration: $a_n = \dfrac{v^2}{\rho} = \dfrac{a_t^2 t^2}{\rho}$

where $\rho = 0.8 \text{ m} = 800 \text{ mm}$

($a$) When $t = 4\text{ s}$ $\quad v = (10)(4) = 40 \text{ mm/s}$

$$a_n = \frac{(40)^2}{800} = 2 \text{ mm/s}^2$$

Acceleration: $a = \sqrt{a_t^2 + a_n^2} = \sqrt{(10)^2 + (2)^2}$

$$a = 10.20 \text{ mm/s}^2 \blacktriangleleft$$

($b$) Time when $a = 80 \text{ mm/s}^2$

$$a^2 = a_n^2 + a_t^2$$

$$(80)^2 = \left[\frac{(10)^2 t^2}{800}\right]^2 + 10^2 \qquad t^4 = 403200 \text{ s}^4$$

$$t = 25.2 \text{ s} \blacktriangleleft$$

## PROBLEM 11.139

A monorail train starts from rest on a curve of radius 400 m and accelerates at the constant rate $a_t$. If the maximum total acceleration of the train must not exceed 1.5 m/s$^2$, determine (*a*) the shortest distance in which the train can reach a speed of 72 km/h, (*b*) the corresponding constant rate of acceleration $a_t$.

## SOLUTION

When $v = 72 \text{ km/h} = 20 \text{ m/s}$ and $\rho = 400 \text{ m}$,

$$a_n = \frac{v^2}{\rho} = \frac{(20)^2}{400} = 1.000 \text{ m/s}^2$$

But

$$a = \sqrt{a_n^2 + a_t^2}$$

$$a_t = \sqrt{a^2 - a_n^2} = \sqrt{(1.5)^2 - (1.000)^2} = \pm 1.11803 \text{ m/s}^2$$

Since the train is accelerating, reject the negative value.

(*a*) Distance to reach the speed.

$$v_0 = 0$$

Let

$$x_0 = 0$$

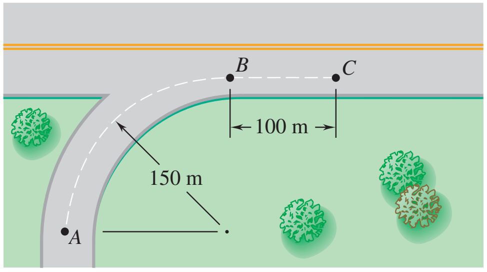
$$v_1^2 = v_0^2 + 2a_t(x_1 - x_0) = 2a_t x_1$$

$$x_1 = \frac{v_1^2}{2a_t} = \frac{(20)^2}{(2)(1.11803)}$$

$x_1 = 178.9 \text{ m}$ ◀

(*b*) Corresponding tangential acceleration.

$a_t = 1.118 \text{ m/s}^2$ ◀




## PROBLEM 11.140

A motorist starts from rest at Point $A$ on a circular entrance ramp when $t = 0$, increases the speed of her automobile at a constant rate and enters the highway at Point $B$. Knowing that her speed continues to increase at the same rate until it reaches 100 km/h at Point $C$, determine ($a$) the speed at Point $B$, ($b$) the magnitude of the total acceleration when $t = 20$ s.

## SOLUTION

Speeds: $v_0 = 0 \qquad v_1 = 100 \text{ km/h} = 27.78 \text{ m/s}$

Distance: $s = \frac{\pi}{2}(150) + 100 = 335.6 \text{ m}$

Tangential component of acceleration: $v_1^2 = v_0^2 + 2a_t s$

$$a_t = \frac{v_1^2 - v_0^2}{2s} = \frac{(27.78)^2 - 0}{(2)(335.6)} = 1.1495 \text{ m/s}^2$$

At Point $B$, $v_B^2 = v_0^2 + 2a_t s_B$ where $s_B = \frac{\pi}{2}(150) = 235.6 \text{ m}$

$$v_B^2 = 0 + (2)(1.1495)(235.6) = 541.69 \text{ m}^2/\text{s}^2$$

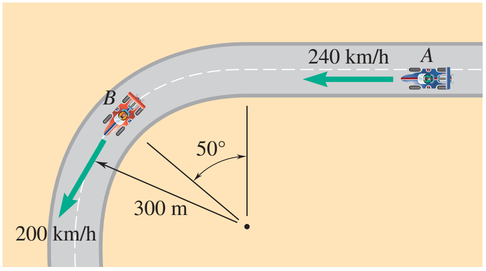
$v_B = 23.27 \text{ m/s}$ $\qquad v_B = 83.8 \text{ km/h}$ ◀

($a$) At $t = 20$ s, $v = v_0 + a_t t = 0 + (1.1495)(20) = 22.99 \text{ m/s}$

Since $v < v_B$, the car is still on the curve. $\rho = 150 \text{ m}$

Normal component of acceleration: $a_n = \frac{v^2}{\rho} = \frac{(22.99)^2}{150} = 3.524 \text{ m/s}^2$

($b$) Magnitude of total acceleration: $|a| = \sqrt{a_t^2 - a_n^2} = \sqrt{(1.1495)^2 + (3.524)^2}$ $\qquad |a| = 3.71 \text{ m/s}^2$ ◀




## PROBLEM 11.141

Racecar $A$ is traveling on a straight portion of the track while racecar $B$ is traveling on a circular portion of the track. At the instant shown, the speed of $A$ is increasing at the rate of 10 m/s², and the speed of $B$ is decreasing at the rate of 6 m/s². For the position shown, determine (*a*) the velocity of $B$ relative to $A$, (*b*) the acceleration of $B$ relative to $A$.

## SOLUTION

Speeds: $v_A = 240 \text{ km/h} = 66.67 \text{ m/s}$

$v_B = 200 \text{ km/h} = 55.67 \text{ m/s}$

Velocities: $\mathbf{v}_A = 66.67 \text{ m/s} \longleftarrow$ and $\mathbf{v}_B = 55.56 \text{ m/s} \measuredangle 50^\circ$

(*a*) Relative velocity: $\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A$

$$\mathbf{v}_{B/A} = (55.56\cos 50^\circ) \leftarrow + 55.56\sin 50^\circ \downarrow + 66.67 \longrightarrow$$

$$= 30.96 \rightarrow +42.56 \downarrow$$

$$= 52.63 \text{ m/s} \searrow 53.96^\circ$$

$$\mathbf{v}_{B/A} = 189.5 \text{ km/h} \searrow 54.0^\circ \blacktriangleleft$$

Tangential accelerations: $(\mathbf{a}_A)_t = 10 \text{ m/s}^2 \longleftarrow$

$(\mathbf{a}_B)_t = 6 \text{ m/s}^2 \measuredangle 50^\circ$

Normal accelerations: $a_n = \dfrac{v^2}{\rho}$

Car A: $(\rho = \infty)$ $(\mathbf{a}_A)_n = 0$

Car B: $(\rho = 300 \text{ m})$

$$(\mathbf{a}_B)_n = \frac{(55.56)^2}{300} = 10.288 \qquad (\mathbf{a}_B)_n = 10.288 \text{ m/s}^2 \searrow 40^\circ$$

(*b*) Acceleration of $B$ relative to $A$: $\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A$

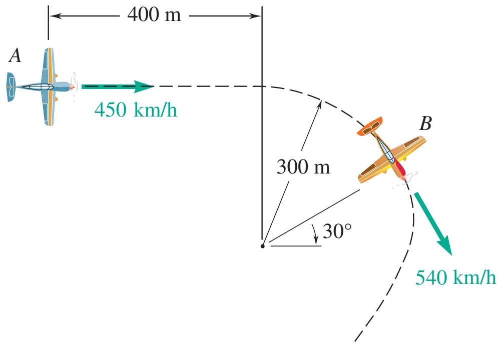
$$\mathbf{a}_{B/A} = (\mathbf{a}_B)_t + (\mathbf{a}_B)_n - (\mathbf{a}_A)_t - (\mathbf{a}_A)_n$$

$$= 6 \measuredangle 50^\circ + 10.228 \searrow 40^\circ + 10 \rightarrow +0$$

$$= (6\cos 50^\circ + 10.288\cos 40^\circ + 10) \longrightarrow$$

$$+ (6\sin 50^\circ - 10.288\sin 40^\circ) \uparrow$$

$$= 21.738 \rightarrow +2.017 \downarrow \qquad \mathbf{a}_{B/A} = 21.8 \text{ m/s}^2 \searrow 5.3^\circ \blacktriangleleft$$




## PROBLEM 11.142

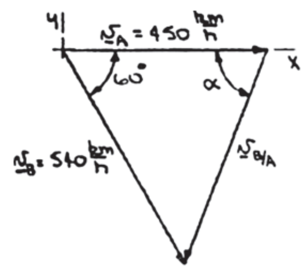
At a given instant in an airplane race, airplane $A$ is flying horizontally in a straight line, and its speed is being increased at the rate of $8\ \text{m/s}^2$. Airplane $B$ is flying at the same altitude as airplane $A$ and, as it rounds a pylon, is following a circular path of 300-m radius. Knowing that at the given instant the speed of $B$ is being decreased at the rate of $3\ \text{m/s}^2$, determine, for the positions shown, ($a$) the velocity of $B$ relative to $A$, ($b$) the acceleration of $B$ relative to $A$.

## SOLUTION

First note $\quad v_A = 450\ \text{km/h} \qquad v_B = 540\ \text{km/h} = 150\ \text{m/s}$

($a$) We have $\quad \mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

The graphical representation of this equation is then as shown.




We have
$$v_{B/A}^2 = 450^2 + 540^2 - 2(450)(540)\cos 60°$$
$$v_{B/A} = 501.10\ \text{km/h}$$

and
$$\frac{540}{\sin\alpha} = \frac{501.10}{\sin 60°}$$
$$\alpha = 68.9°$$

$$\mathbf{v}_{B/A} = 501\ \text{km/h} \swarrow 68.9° \blacktriangleleft$$

($b$) First note $\quad \mathbf{a}_A = 8\ \text{m/s}^2 \longrightarrow \qquad (\mathbf{a}_B)_t = 3\ \text{m/s}^2 \nwarrow 60°$

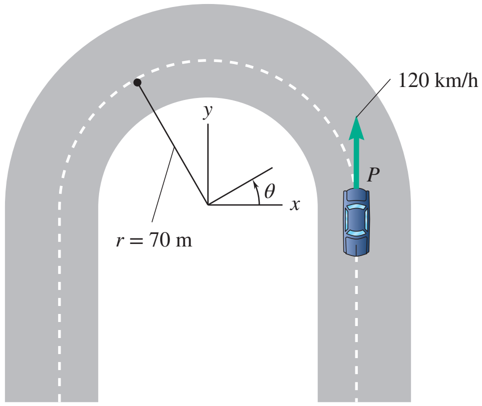
Now
$$(a_B)_n = \frac{v_B^2}{\rho_B} = \frac{(150\ \text{m/s})^2}{300\ \text{m}}$$
$$(a_B)_n = 75\ \text{m/s}^2 \swarrow 30°$$

Then
$$\mathbf{a}_B = (\mathbf{a}_B)_t + (\mathbf{a}_B)_n$$
$$= 3(-\cos 60°\mathbf{i} + \sin 60°\mathbf{j}) + 75(-\cos 30°\mathbf{i} - \sin 30°\mathbf{j})$$
$$= -(66.452\ \text{m/s}^2)\mathbf{i} - (34.902\ \text{m/s}^2)\mathbf{j}$$

Finally
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$
$$\mathbf{a}_{B/A} = -(66.452\mathbf{i} - 34.902\mathbf{j}) - (8\mathbf{i})$$
$$= -(74.452\ \text{m/s}^2)\mathbf{i} - (34.902\ \text{m/s}^2)\mathbf{j}$$

$$\mathbf{a}_{B/A} = 82.2\ \text{m/s}^2 \swarrow 25.1° \blacktriangleleft$$

## PROBLEM 11.143




A race car enters the circular portion of a track that has a radius of 70 m. When the car enters the curve at point P, it is travelling with a speed of 120 km/h that is increasing at 5 m/s². Three seconds later, determine (a) the total acceleration of the car in xy components, (b) the linear velocity of the car in xy components.

## SOLUTION

<u>Speeds</u>:

$$v_{t_0} = 120 \text{ km/h} = 33.33 \text{ m/s}$$

$$v_t = v_{t_0} + a_t t$$

$$v_t = 33.33 + 5(3) = 48.33 \text{ m/s}$$

<u>Distance</u>:

$$s = s_0 + v_{t_0} t + \frac{1}{2} a_t t^2$$

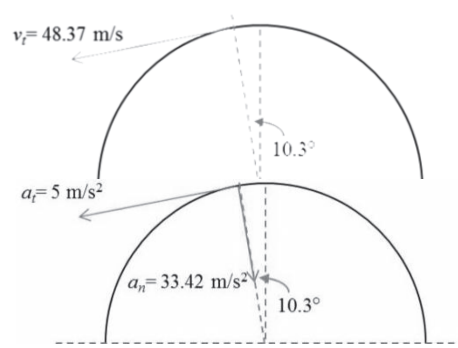
$$s = 33.33(3) + \frac{1}{2}(5)(3)^2 = 122.5 \text{ m}$$

<u>Location</u>:

$$s = R\theta$$

$$\theta = \frac{s}{R} = \frac{122.5}{70} = 1.75 \text{ rad}$$

$$\theta = 100.3°$$




<u>Tangential Acceleration</u>:

$$a_t = 5 \text{ m/s}^2$$

<u>Normal Acceleration</u>:

$$a_n = \frac{v^2}{\rho}$$

$$a_n = \frac{48.33^2}{70} = 33.37 \text{ m/s}^2$$

(*a*) <u>Accelerations</u>:

$$a_x = -5\cos(10.3°) + 33.37\sin(10.3°)$$

$$a_x = 1.047 \text{ m/s}^2 \blacktriangleleft$$

$$a_y = -5\sin(10.3°) + 33.37\cos(10.3°)$$

$$a_y = -33.726 \text{ m/s}^2 \blacktriangleleft$$

(*b*) <u>Velocities</u>:

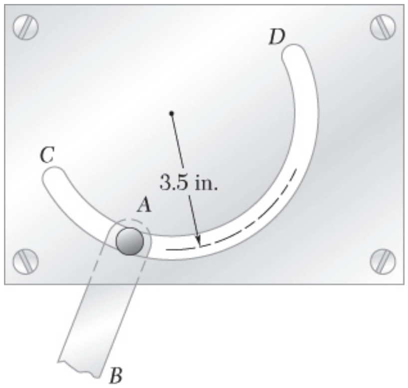
$$v_x = -48.33\cos(10.3°)$$

$$v_x = -47.55 \text{ m/s} \blacktriangleleft$$

$$v_y = -48.33\sin(10.3°)$$

$$v_y = -8.64 \text{ m/s} \blacktriangleleft$$

## PROBLEM 11.144




Pin $A$, which is attached to link $AB$, is constrained to move in the circular slot $CD$. Knowing that at $t=0$ the pin starts from rest and moves so that its speed increases at a constant rate of $0.8\text{ in./s}^2$, determine the magnitude of its total acceleration when (*a*) $t=0$, (*b*) $t=2$ s.

## SOLUTION

Given: $$(v_A)_0 = 0, \quad (a_A)_t = \frac{dv_A}{dt} = 0.8 \text{ s}$$

Then, $$v_A = (v_A)_0 + (a_A)_t\, t = 0.8\, t$$

(*a*) $t=0$, $$v_A = 0, \quad (a_A)_n = \frac{v_A^2}{\rho} = 0$$

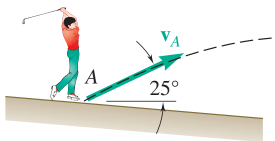
$$a_A = (a_A)_t$$ $a_A = 0.800 \text{ in./s}^2$ ◀

(*b*) $t=2$ s, $$v_A = 0 + (0.8)(2) = 1.6 \text{ in./s}$$

$$(a_A)_n = \frac{v_A^2}{\rho} = \frac{(1.6)^2}{3.5} = 0.731 \text{ in./s}^2$$

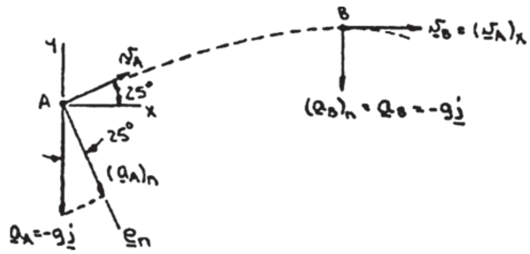
$$a_A = \left[(a_A)_t^2 + (a_A)_n^2\right]^{1/2} = \left[(0.8)^2 + (0.731)^2\right]^{1/2}$$ $a_A = 1.084 \text{ in./s}^2$ ◀

## PROBLEM 11.145




A golfer hits a golf ball from Point $A$ with an initial velocity of 50 m/s at an angle of 25° with the horizontal. Determine the radius of curvature of the trajectory described by the ball (*a*) at Point $A$, (*b*) at the highest point of the trajectory.

## SOLUTION




(*a*) We have $$(a_A)_n = \frac{v_A^2}{\rho_A}$$

or $$\rho_A = \frac{(50\text{ m/s})^2}{(9.81\text{ m/s}^2)\cos 25°}$$

or $$\rho_A = 281\text{ m} \blacktriangleleft$$

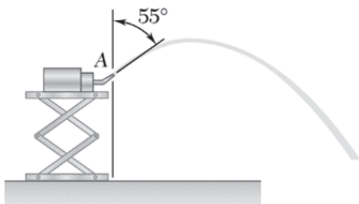
(*b*) We have $$(a_B)_n = \frac{v_B^2}{\rho_B}$$

where Point $B$ is the highest point of the trajectory, so that

$$v_B = (v_A)_x = v_A \cos 25°$$

Then $$\rho_B = \frac{[(50\text{ m/s})\cos 25°]^2}{9.81\text{ m/s}^2}$$

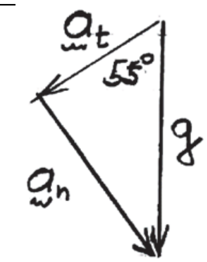
or $$\rho_B = 209\text{ m} \blacktriangleleft$$




## PROBLEM 11.146

A nozzle discharges a stream of water in the direction shown with an initial velocity of 24 m/s. Determine the radius of curvature of the stream (*a*) as it leaves the nozzle, (*b*) at the maximum height of the stream.

## SOLUTION




*(a)* As water leaves nozzle.

$$v = 24 \text{ ft/s}$$

$$a_n = g \sin 55° = 32.2 \sin 55° = 26.377 \text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho}$$

$$\rho = \frac{v^2}{a_n} = \frac{(24)^2}{26.377}$$

$\rho = 21.8$ ft ◀

*(b)* At maximum height of stream.

$$v = (v_x)_0 = 24 \sin 55° = 19.660 \text{ ft/s}$$

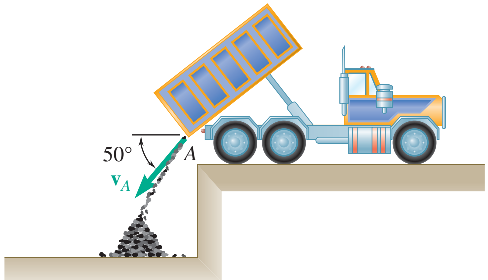
$$a_n = g = 32.2 \text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho}$$

$$\rho = \frac{v^2}{a_n} = \frac{(19.660)^2}{32.2}$$

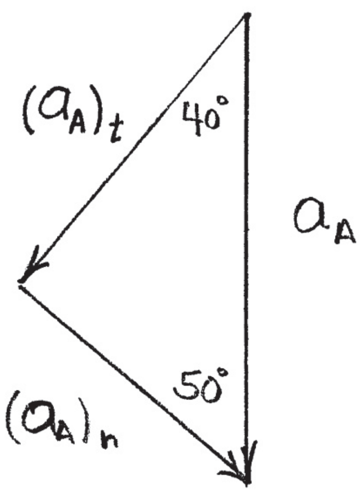
$\rho = 12.0$ ft ◀

## PROBLEM 11.147




Coal is discharged from the tailgate $A$ of a dump truck with an initial velocity $\mathbf{v}_A = 2\ \text{m/s} \measuredangle 50°$. Determine the radius of curvature of the trajectory described by the coal (*a*) at Point $A$, (*b*) at the point of the trajectory 1 m below Point $A$.

## SOLUTION




(*a*) At Point $A$. $a_A = g \downarrow = 9.81\ \text{m/s}^2 \downarrow$

Sketch tangential and normal components of acceleration at $A$.

$$(a_A)_n = g\cos 50°$$

$$\rho_A = \frac{v_A^2}{(a_A)_n} = \frac{(2)^2}{9.81\cos 50°} \qquad \rho_A = 0.634\ \text{m} \blacktriangleleft$$

(*b*) At Point $B$, 1 meter below Point $A$.

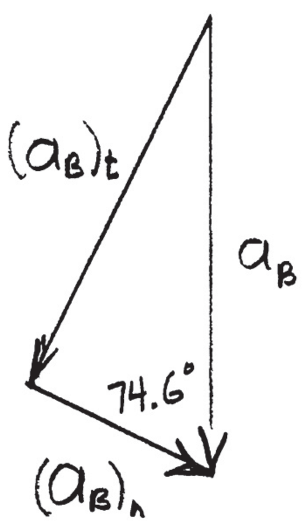
Horizontal motion: $(v_B)_x = (v_A)_x = 2\cos 50° = 1.286\ \text{m/s} \longleftarrow$

Vertical motion:

$$(v_B)_y^2 = (v_A)_y^2 + 2a_y(y_B - y_A)$$
$$= (2\cos 40°)^2 + (2)(-9.81)(-1)$$
$$= 21.97\ \text{m}^2/\text{s}^2$$

$$(v_B)_y = 4.687\ \text{m/s}\downarrow$$

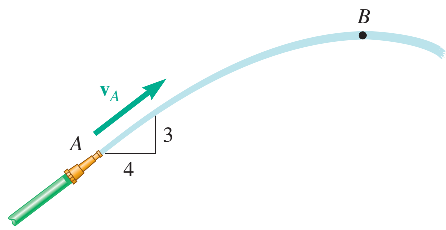
$$\tan\theta = \frac{(v_B)_y}{(v_B)_x} = \frac{4.687}{1.286}, \quad \text{or} \quad \theta = 74.6°$$




$$a_B = g\cos 74.6°$$

$$\rho_B = \frac{v_B{}^2}{(a_B)_n} = \frac{(v_B)_x^2 + (v_B)_y^2}{g\cos 74.6°}$$

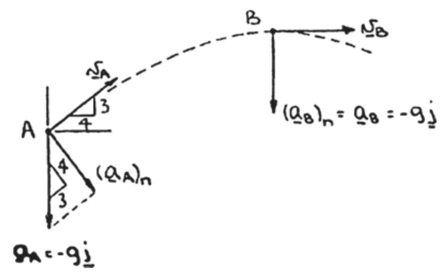
$$= \frac{(1.286)^2 + 21.97}{9.81\cos 74.6°} \qquad \rho_B = 9.07\ \text{m} \blacktriangleleft$$




## PROBLEM 11.148

From measurements of a photograph, it has been found that as the stream of water shown left the nozzle at $A$, it had a radius of curvature of 25 m. Determine ($a$) the initial velocity $\mathbf{v}_A$ of the stream, ($b$) the radius of curvature of the stream as it reaches its maximum height at $B$.

## SOLUTION




($a$) We have $$(a_A)_n = \frac{v_A^2}{\rho_A}$$

or $$v_A^2 = \left[\frac{4}{5}(9.81\text{ m/s}^2)\right](25\text{ m})$$

or $$v_A = 14.0071\text{ m/s}$$

$$\mathbf{v}_A = 14.01\text{ m/s} \measuredangle 36.9° \blacktriangleleft$$

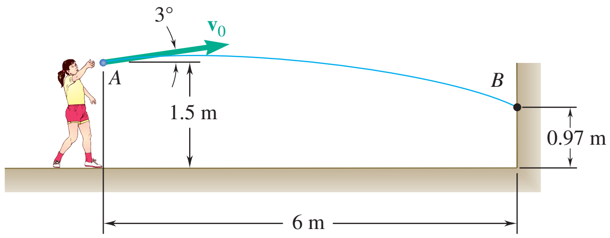
($b$) We have $$(a_B)_n = \frac{v_B^2}{\rho_B}$$

Where $$v_B = (v_A)_x = \frac{4}{5}v_A$$

Then $$\rho_B = \frac{\left(\frac{4}{5}\times 14.0071\text{ m/s}\right)^2}{9.81\text{ m/s}^2}$$

or $$\rho_B = 12.80\text{ m} \blacktriangleleft$$

## PROBLEM 11.149




A child throws a ball from point $A$ with an initial velocity $\mathbf{v}_0$ at an angle of 3° with the horizontal. Knowing that the ball hits a wall at point $B$, determine (*a*) the magnitude of the initial velocity, (*b*) the minimum radius of curvature of the trajectory.

## SOLUTION

Horizontal motion. $$v_x = v_0 \cos\alpha \qquad x = v_0 t \cos\alpha$$

Vertical motion. $$v_y = v_0 \sin\alpha - gt$$

$$y = y_0 + v_0 t \sin\alpha - \frac{1}{2} g t^2$$

Eliminate $t$. $$y = y_0 + x\tan\alpha - \frac{gx^2}{2v_0^2 \cos^2\alpha} \qquad (1)$$

Solving (1) for $v_0$ and applying result at point $B$

$$v_0 = \sqrt{\frac{gx^2}{2(y_0 + x\tan\alpha - y)\cos^2\alpha}} = \sqrt{\frac{(9.81)(6)^2}{(2)(1.5 + 6\tan 3° - 0.97)(\cos^2 3°)}}$$

(*a*) Magnitude of initial velocity. $v_0 = 14.48 \text{ m/s}$ ◀

(*b*) Minimum radius of curvature of trajectory.

$$a_n = g = \frac{v^2}{\rho} \qquad \rho = \frac{v^2}{a_n} = \frac{v^2}{g\cos\theta} \qquad (2)$$

where $\theta$ is the slope angle of the trajectory.

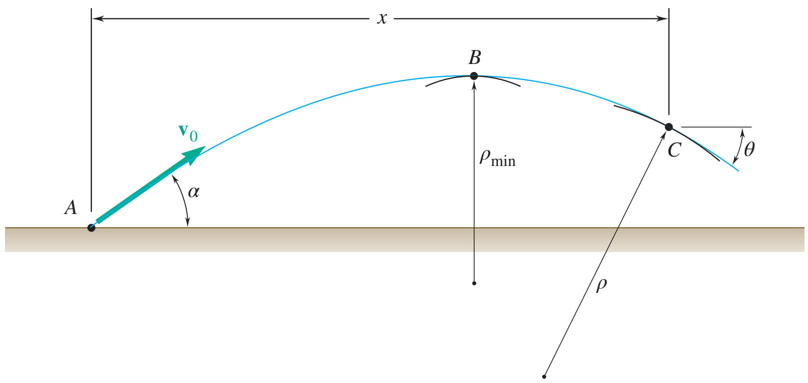
The minimum value of $\rho$ occurs at the highest point of the trajectory where $\cos\theta = 1$

and $v = v_x = v_0 \cos\alpha$

Then

$$\rho_{\min} = \frac{v_0^2 \cos^2\alpha}{g} = \frac{(14.48)^2 \cos^2 3°}{9.81}$$

$\rho_{\min} = 21.3 \text{ m}$ ◀

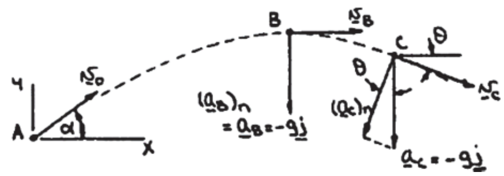



## PROBLEM 11.150

A projectile is fired from Point $A$ with an initial velocity $v_0$. ($a$) Show that the radius of curvature of the trajectory of the projectile reaches its minimum value at the highest Point $B$ of the trajectory.($b$) Denoting by $\theta$ the angle formed by the trajectory and the horizontal at a given Point $C$, show that the radius of curvature of the trajectory at $C$ is $\rho = \rho_{min}/\cos^3\theta$.

## SOLUTION

For the arbitrary Point $C$, we have



$$(a_C)_n = \frac{v_C^2}{\rho_C}$$

or

$$\rho_C = \frac{v_C^2}{g\cos\theta}$$

Noting that the horizontal motion is uniform, we have

$$(v_A)_x = (v_C)_x$$

where

$$(v_A)_x = v_0\cos\alpha \qquad (v_C)_x = v_C\cos\theta$$

Then

$$v_0\cos\alpha = v_C\cos\theta$$

or

$$v_C = \frac{\cos\alpha}{\cos\theta}v_0$$

so that

$$\rho_C = \frac{1}{g\cos\theta}\left(\frac{\cos\alpha}{\cos\theta}v_0\right)^2 = \frac{v_0^2\cos^2\alpha}{g\cos^3\theta}$$

($a$) In the expression for $\rho_C, v_0, \alpha,$ and $g$ are constants, so that $\rho_C$ is minimum where $\cos\theta$ is maximum. By observation, this occurs at Point $B$ where $\theta = 0$.

$$\rho_{min} = \rho_B = \frac{v_0^2\cos^2\alpha}{g}$$ Q.E.D. ◀

($b$)

$$\rho_C = \frac{1}{\cos^3\theta}\left(\frac{v_0^2\cos^2\alpha}{g}\right)$$

$$\rho_C = \frac{\rho_{min}}{\cos^3\theta}$$ Q.E.D. ◀

## PROBLEM 11.151*

Determine the radius of curvature of the path described by the particle of Problem 11.95 when $t = 0$.

**PROBLEM 11.95** The three-dimensional motion of a particle is defined by the position vector $\mathbf{r} = (Rt\cos\omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt\sin\omega_n t)\mathbf{k}$. Determine the magnitudes of the velocity and acceleration of the particle. (The space curve described by the particle is a conic helix.)

## SOLUTION

We have $$\mathbf{v} = \frac{d\mathbf{r}}{dt} = R(\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{i} + c\mathbf{j} + R(\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}$$

and $$\mathbf{a} = \frac{d\mathbf{v}}{dt} = R(-\omega_n\sin\omega_n t - \omega_n\sin\omega_n t - \omega_n^2 t\cos\omega_n t)\mathbf{i}$$
$$+ R(\omega_n\cos\omega_n t + \omega_n\cos\omega_n t - \omega_n^2 t\sin\omega_n t)\mathbf{k}$$

or $$\mathbf{a} = \omega_n R[-(2\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{i} + (2\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{k}]$$

Now $$v^2 = R^2(\cos\omega_n t - \omega_n t\sin\omega_n t)^2 + c^2 + R^2(\sin\omega_n t + \omega_n t\cos\omega_n t)^2$$
$$= R^2(1 + \omega_n^2 t^2) + c^2$$

Then $$v = \left[R^2\left(1 + \omega_n^2 t^2\right) + c^2\right]^{1/2}$$

and $$\frac{dv}{dt} = \frac{R^2\omega_n^2 t}{\left[R^2\left(1 + \omega_n^2 t^2\right) + c^2\right]^{1/2}}$$

Now $$a^2 = a_t^2 + a_n^2 = \left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{\rho}\right)^2$$

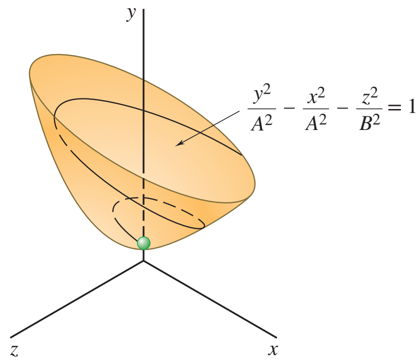
At $t = 0$: $$\frac{dv}{dt} = 0$$
$$\mathbf{a} = \omega_n R(2\mathbf{k}) \quad \text{or} \quad a = 2\omega_n R$$
$$v^2 = \mathrm{R}^2 + c^2$$

Then, with $$\frac{dv}{dt} = 0,$$

we have $$a = \frac{v^2}{\rho}$$

or $$2\omega_n R = \frac{R^2 + c^2}{\rho}$$ $$\rho = \frac{R^2 + c^2}{2\omega_n R} \blacktriangleleft$$

## PROBLEM 11.152*




Determine the radius of curvature of the path described by the particle of Problem 11.96 when $t = 0$, $A = 3$, and $B = 1$.

## PROBLEM 11.96

The three-dimensional motion of a particle is defined by the position vector $\mathbf{r} = (At\cos t)\mathbf{i} + (A\sqrt{t^2+1})\mathbf{j} + (Bt\sin t)\mathbf{k}$, where $r$ and $t$ are expressed in feet and seconds, respectively. Show that the curve described by the particle lies on the hyperboloid $(y/A)^2 - (x/A)^2 - (z/B)^2 = 1$. For $A = 3$ and $B = 1$, determine (*a*) the magnitudes of the velocity and acceleration when $t = 0$, (*b*) the smallest nonzero value of $t$ for which the position vector and the velocity are perpendicular to each other.

## SOLUTION

With $\quad A = 3, \quad B = 1$

we have
$$\mathbf{r} = (3t\cos t)\mathbf{i} + \left(3\sqrt{t^2+1}\right)\mathbf{j} + (t\sin t)\mathbf{k}$$

Now
$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 3(\cos t - t\sin t)\mathbf{i} + \left(\frac{3t}{\sqrt{t^2+1}}\right)\mathbf{j} + (\sin t + t\cos t)\mathbf{k}$$

and
$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 3(-\sin t - \sin t - t\cos t)\mathbf{i} + 3\left(\frac{\sqrt{t^2+1} - t\left(\frac{t}{\sqrt{t^2+1}}\right)}{t^2+1}\right)\mathbf{j}$$
$$+ \ (\cos t \ + \cos t = t\sin t)\mathbf{k}$$
$$= -3(2\sin t + t\cos t)\mathbf{i} + 3\frac{1}{(t^2+1)^{1/2}}\mathbf{j}$$
$$+ (2\cos t - t\sin t)\mathbf{k}$$

Then
$$v^2 = 9(\cos t - t\sin t)^2 + 9\frac{t^2}{t^2+1} + (\sin t + t\cos t)^2$$

Expanding and simplifying yields
$$v^2 = t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4\sin^2 t) - 8(t^3 + t)\sin 2t$$

Then
$$v = [t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4\sin^2 t) - 8(t^3 + t)\sin 2t]^{1/2}$$

and
$$\frac{dv}{dt} = \frac{4t^3 + 38t + 8(-2\cos t\sin t + 4t^3\sin^2 t + 2t^4\sin t\cos t - 8[(3t^2+1)\sin 2t + 2(t^3+t)\cos 2t]}{2[t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4\sin^2 t) - 8(t^3+t)\sin 2t]^{1/2}}$$

## PROBLEM 11.152* (CONTINUED)

Now $$a^2 = a_t^2 + a_n^2 = \left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{\rho}\right)^2$$

At $t = 0$: $$\mathbf{a} = 3\mathbf{j} + 2\mathbf{k}$$

or $$a = \sqrt{13}\ \text{ft/s}^2$$

$$\frac{dv}{dt} = 0$$
$$v^2 = 9(\text{ft/s})^2$$

Then, with $$\frac{dv}{dt} = 0,$$

we have $$a = \frac{v^2}{\rho}$$

or $$\rho = \frac{9\ \text{ft}^2/\text{s}^2}{\sqrt{13}\ \text{ft/s}^2}$$ $\rho = 2.50$ ft ◀

## PROBLEM 11.153

A satellite will travel indefinitely in a circular orbit around a planet if the normal component of the acceleration of the satellite is equal to $g(R/r)^2$, where $g$ is the acceleration of gravity at the surface of the planet, $R$ is the radius of the planet, and $r$ is the distance from the center of the planet to the satellite. Knowing that the diameter of the sun is 1.39 Gm and that the acceleration of gravity at its surface is $274 \text{ m/s}^2$, determine the radius of the orbit of the indicated planet around the sun assuming that the orbit is circular.

Earth: $(\upsilon_{\text{mean}})_{\text{orbit}} = 107 \text{ Mm/h}$.

## SOLUTION

For the sun, $$g = 274 \text{ m/s}^2,$$

and $$R = \frac{1}{2}D = \left(\frac{1}{2}\right)(1.39 \times 10^9) = 0.695 \times 10^9 \text{ m}$$

Given that $a_n = \dfrac{gR^2}{r^2}$ and that for a circular orbit $a_n = \dfrac{v^2}{r}$

Eliminating $a_n$ and solving for $r$, $$r = \frac{gR^2}{v^2}$$

For the planet Earth, $$v = 107 \times 10^6 \text{ m/h} = 29.72 \times 10^3 \text{ m/s}$$

Then $$r = \frac{(274)(0.695 \times 10^9)^2}{(29.72 \times 10^3)^2} = 149.8 \times 10^9 \text{ m}$$

$r = 149.8 \text{ Gm}$ ◀

## PROBLEM 11.154

A satellite will travel indefinitely in a circular orbit around a planet if the normal component of the acceleration of the satellite is equal to $g(R/r)^2$, where $g$ is the acceleration of gravity at the surface of the planet, $R$ is the radius of the planet, and $r$ is the distance from the center of the planet to the satellite. Knowing that the diameter of the sun is 1.39 Gm and that the acceleration of gravity at its surface is $274 \text{ m/s}^2$, determine the radius of the orbit of the indicated planet around the sun assuming that the orbit is circular.

Saturn: $(v_{\text{mean}})_{\text{orbit}} = 34.7 \text{ Mm/h}$.

## SOLUTION

For the sun, $$g = 274 \text{ m/s}^2$$

and $$R = \frac{1}{2}D = \left(\frac{1}{2}\right)(1.39\times 10^9) = 0.695\times 10^9 \text{ m}$$

Given that $a_n = \dfrac{gR^2}{r^2}$ and that for a circular orbit: $a_n = \dfrac{v^2}{r}$

Eliminating $a_n$ and solving for $r$, $$r = \frac{gR^2}{v^2}$$

For the planet Saturn, $$v = 34.7\times 10^6 \text{ m/h} = 9.639\times 10^3 \text{ m/s}$$

Then, $$r = \frac{(274)(0.695\times 10^9)^2}{(9.639\times 10^3)^2} = 1.425\times 10^{12} \text{ m}$$

$r = 1425 \text{ Gm}$ ◀

## PROBLEM 11.155

Determine the speed of a satellite relative to the indicated planet if the satellite is to travel indefinitely in a circular orbit 100 mi above the surface of the planet. (See information given in Problems 11.153–11.154).

Venus: $g = 29.20 \text{ ft/s}^2$, $R = 3761$ mi.

## SOLUTION

From Problems 11.153 and 11.154, $$a_n = \frac{gR^2}{r^2}$$

For a circular orbit, $$a_n = \frac{v^2}{r}$$

Eliminating $a_n$ and solving for $v$, $$v = R\sqrt{\frac{g}{r}}$$

For Venus, $g = 29.20 \text{ ft/s}^2$

$$R = 3761 \text{ mi} = 19.858 \times 10^6 \text{ ft.}$$

$$r = 3761 + 100 = 3861 \text{ mi} = 20.386 \times 10^6 \text{ ft}$$

Then, $$v = 19.858 \times 10^6 \sqrt{\frac{29.20}{20.386 \times 10^6}} = 23.766 \times 10^3 \text{ ft/s}$$

$v = 16200$ mi/h ◀

## PROBLEM 11.156

Determine the speed of a satellite relative to the indicated planet if the satellite is to travel indefinitely in a circular orbit 100 mi above the surface of the planet. (See information given in Problems 11.153–11.154).

Mars: $g = 12.17 \text{ ft/s}^2$, $R = 2102$ mi.

## SOLUTION

From Problems 11.153 and 11.154, $$a_n = \frac{gR^2}{r^2}$$

For a circular orbit, $$a_n = \frac{v^2}{r}$$

Eliminating $a_n$ and solving for $v$, $$v = R\sqrt{\frac{g}{r}}$$

For Mars, $g = 12.17 \text{ ft/s}^2$

$$R = 2102 \text{ mi} = 11.0986 \times 10^6 \text{ ft}$$

$$r = 2102 + 100 = 2202 \text{ mi} = 11.6266 \times 10^6 \text{ ft}$$

Then, $$v = 11.0986 \times 10^6 \sqrt{\frac{12.17}{11.6266 \times 10^6}} = 11.35 \times 10^3 \text{ ft/s}$$

$v = 7740$ mi/h ◀

## PROBLEM 11.157

Determine the speed of a satellite relative to the indicated planet if the satellite is to travel indefinitely in a circular orbit 100 mi above the surface of the planet. (See information given in Problems 11.153–11.154).

Jupiter: $g = 75.35 \text{ ft/s}^2, \;\; R = 44,432 \text{ mi}.$

## SOLUTION

From Problems 11.153 and 11.154, $$a_n = \frac{gR^2}{r^2}$$

For a circular orbit, $$a_n = \frac{v^2}{r}$$

Eliminating $a_n$ and solving for $v$, $$v = R\sqrt{\frac{g}{r}}$$

For Jupiter, $g = 75.35 \text{ ft/s}^2$

$$R = 44432 \text{ mi} = 234.60 \times 10^6 \text{ ft.}$$

$$r = 44432 + 100 = 44532 \text{ mi} = 235.13 \times 10^6 \text{ ft}$$

Then, $$v = (234.60 \times 10^6)\sqrt{\frac{75.35}{235.13 \times 10^6}} = 132.8 \times 10^3 \text{ ft/s}$$

$v = 90600$ mi/h ◀

## PROBLEM 11.158

A satellite will travel indefinitely in a circular orbit around the earth if the normal component of its acceleration is equal to $g(R/r)^2$, where $g = 9.81 \text{ m/s}^2$, $R$ = radius of the earth = 6370 km, and $r$ = distance from the center of the earth to the satellite. Assuming that the orbit of the moon is a circle of radius $384 \times 10^3$ km, determine the speed of the moon relative to the earth.

## SOLUTION

Normal acceleration:

$$a_n = \frac{gR^2}{r^2} \qquad \text{and} \qquad a_n = \frac{v^2}{\rho} = \frac{v^2}{r}$$

Solve for $v^2$:

$$v^2 = ra_n = \frac{gR^2}{r}$$

Data:

$$g = 9.81 \text{ m/s}^2, \qquad R = 6370 \text{ km} = 6.370 \times 10^6 \text{ m}$$

$$r = 384 \times 10^3 \text{ km} = 384 \times 10^6 \text{ m}$$

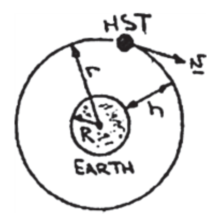
$$v^2 = \frac{(9.81)\left(6.370 \times 10^6\right)^2}{384 \times 10^6} = 1.0366 \times 10^6 \text{ m}^2/\text{s}^2$$

$v = 1.018$ m/s $\qquad v = 3670$ km/h ◀

## PROBLEM 11.159

Knowing that the radius of the earth is 6370 km, determine the time of one orbit of the Hubble Space Telescope, knowing that the telescope travels in a circular orbit 590 km above the surface of the earth. (See information given in Problems 11.153–11.155.)

## SOLUTION




We have $$a_n = g\frac{R^2}{r^2} \quad \text{and} \quad a_n = \frac{v^2}{r}$$

Then $$g\frac{R^2}{r^2} = \frac{v^2}{r}$$

or $$v = R\sqrt{\frac{g}{r}} \quad \textit{where} \quad r = R + h$$

The circumference $s$ of the circular orbit is equal to

$$s = 2\pi r$$

Assuming that the speed of the telescope is constant, we have

$$s = vt_{\text{orbit}}$$

Substituting for $s$ and $v$

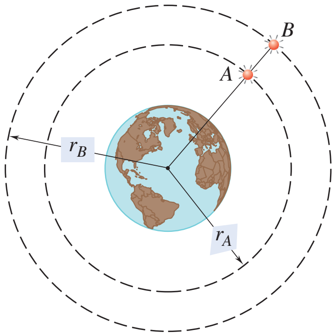
$$2\pi r = R\sqrt{\frac{g}{r}}t_{\text{orbit}}$$

or
$$t_{\text{orbit}} = \frac{2\pi}{R}\frac{r^{3/2}}{\sqrt{g}}$$

$$= \frac{2\pi}{6370\text{ km}}\frac{[(6370+590)\text{km}]^{3/2}}{[9.81\times 10^{-3}\text{ km/s}^2]^{1/2}} \times \frac{1\text{h}}{3600\text{s}}$$

or $$t_{\text{orbit}} = 1.606\text{ h} \blacktriangleleft$$




## PROBLEM 11.160

Satellites $A$ and $B$ are traveling in the same plane in circular orbits around the earth at altitudes of 120 and 200 mi, respectively. If at $t = 0$ the satellites are aligned as shown and knowing that the radius of the earth is $R = 3960$ mi, determine when the satellites will next be radially aligned. (See information given in Problems 11.153–11.155.)

## SOLUTION

We have $$a_n = g\frac{R^2}{r^2} \quad \text{and} \quad a_n = \frac{v^2}{r}$$

Then $$g\frac{R^2}{r^2} = \frac{v^2}{r} \quad \text{or} \quad v = R\sqrt{\frac{g}{r}}$$

where $$r = R + h$$

The circumference $s$ of a circular orbit is equal to

$$s = 2\pi r$$

Assuming that the speeds of the satellites are constant, we have

$$s = vT$$

Substituting for $s$ and $v$

$$2\pi r = R\sqrt{\frac{g}{r}}T$$

or $$T = \frac{2\pi}{R}\frac{r^{3/2}}{\sqrt{g}} = \frac{2\pi}{R}\frac{(R+h)^{3/2}}{\sqrt{g}}$$

Now $$h_B > h_A \Rightarrow (T)_B > (T)_A$$

Next let time $T_C$ be the time at which the satellites are next radially aligned. Then, if in time $T_C$ satellite $B$ completes $N$ orbits, satellite $A$ must complete $(N+1)$ orbits.

Thus,

$$T_C = N(T)_B = (N+1)(T)_A$$

or $$N\left[\frac{2\pi}{R}\frac{(R+h_B)^{3/2}}{\sqrt{g}}\right] = (N+1)\left[\frac{2\pi}{R}\frac{(R+h_A)^{3/2}}{\sqrt{g}}\right]$$

## PROBLEM 11.160 (CONTINUED)

or
$$N = \frac{(R+h_A)^{3/2}}{(R+h_B)^{3/2} - (R+h_A)^{3/2}} = \frac{1}{\left(\frac{R+h_B}{R+h_A}\right)^{3/2} - 1}$$
$$= \frac{1}{\left(\frac{3960+200}{3960+120}\right)^{3/2} - 1} = 33.835 \text{ orbits}$$

Then
$$T_C = N(T)_B = N\frac{2\pi}{R}\frac{(R+h_B)^{3/2}}{\sqrt{g}}$$
$$= 33.835\frac{2\pi}{3960 \text{ mi}}\frac{\left[(3960+200)\text{ mi}\right]^{3/2}}{\left(32.2 \text{ ft/s}^2 \times \frac{1 \text{mi}}{5280 \text{ ft}}\right)^{1/2}} \times \frac{1 \text{h}}{3600 \text{s}}$$

or $T_C = 51.2 \text{ h}$ ◀

<u>Alternative solution</u>

From above, we have $(T)_B > (T)_A$. Thus, when the satellites are next radially aligned, the angles $\theta_A$ and $\theta_B$ swept out by radial lines drawn to the satellites must differ by $2\pi$. That is,

$$\theta_A = \theta_B + 2\pi$$

For a circular orbit $s = r\theta$

From above $s = vt$ and $v = R\sqrt{\frac{g}{r}}$

Then
$$\theta = \frac{s}{r} = \frac{vt}{r} = \frac{1}{r}\left(R\sqrt{\frac{g}{r}}\right)t = \frac{R\sqrt{g}}{r^{3/2}}t = \frac{R\sqrt{g}}{(R+h)^{3/2}}t$$

At time $T_C$ :
$$\frac{R\sqrt{g}}{(R+h)^{3/2}}T_C = \frac{R\sqrt{g}}{(R+h_B)^{3/2}}T_C + 2\pi$$

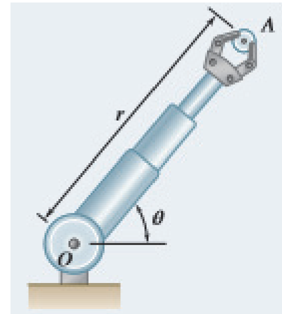
$$T_C = \frac{2\pi}{R\sqrt{g}\frac{1}{(R+h_A)^{3/2}} - \frac{1}{(R+h_B)^{3/2}}}$$

or
$$= \frac{2\pi}{(3960 \text{ mi})\left(32.2 \text{ ft/s}^2 \times \frac{1 \text{mi}}{5280 \text{ft}}\right)^{1/2}}$$
$$\times \frac{1}{\frac{1}{[(3960+120)\text{mi}]^{3/2}} - \frac{1}{[(3960+200)\text{mi}]^{3/2}}}$$
$$\times \frac{1 \text{h}}{3600 \text{s}}$$

or $T_C = 51.2 \text{ h}$ ◀

## PROBLEM 11.161

Robots with telescoping arms are sometimes used to perform tasks (e.g., welding or placing screws) where access may be difficult for other robotic types. During a test run, a robot arm is programmed to extend according to the relationship $r = 3 + 0.5\cos(4\theta)$ and the arm rotates according to the relationship $\theta = -\frac{\pi}{4}t^2 + \pi t$, where $r$ is in feet, $q$ is in radians, and $t$ is in seconds. Determine ($a$) the velocity and acceleration of the robot tip $A$ at t = 3 s and ($b$) use a computer program to plot the path of tip $A$ in $x$ and y coordinates for $0 \le t \le 4$ s.




## SOLUTION

Differentiate both functions with respect to time

$$r = 3 + 0.5\cos(4\theta) \qquad \theta = -\frac{\pi}{4}t^2 + \pi t$$

$$\dot{r} = -\sin(4\theta)2\dot{\theta} \qquad \dot{\theta} = -\frac{\pi}{2}t + \pi$$

$$\ddot{r} = -8\dot{\theta}^2(4\theta) - 2\ddot{\theta}\sin(4\theta) \qquad \ddot{\theta} = -\frac{\pi}{2}$$

Next, evaluate the results at t=2 to find

$$\theta = \pi, \dot{\theta} = 0, \ddot{\theta} = -\frac{\pi}{2}$$

$$r = 3.5, \dot{r} = 0, \ddot{r} = 0$$

To find the velocity and acceleration,

$$\mathbf{v} = \dot{r}\mathbf{e_r} + r\dot{\theta}\mathbf{e_\theta}$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e_\theta}$$

Using the values above

$$\mathbf{v} = 0$$

$$\mathbf{a} = 3.5(-\frac{\pi}{2})\mathbf{e_\theta} = -5.5 \text{ ft/s } \mathbf{e_\theta}$$

$\mathbf{v} = 0$ ◀

$\mathbf{a} = -5.5 \text{ ft/s } \mathbf{e_\theta} \text{ ft/s}^2$ ◀

Similarly, at t=3

$$\theta = \frac{3}{4}\pi, \dot{\theta} = -\frac{\pi}{2}, \ddot{\theta} = -\frac{\pi}{2}$$
$$r = 2.5, \dot{r} = 0, \ddot{r} = 19.739$$

And

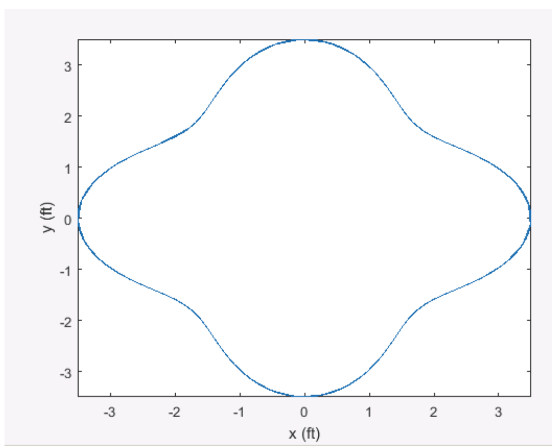
$$\mathbf{v} = 2.5(-\frac{\pi}{2})\mathbf{e_\theta} = -3.93\mathbf{e_\theta}$$
$$\mathbf{a} = -3.927\mathbf{e_\theta} + 13.571\mathbf{e_r}$$

$\mathbf{v} = -3.93\mathbf{e_\theta}$ ft/s ◀

$\mathbf{a} = -3.927\mathbf{e_\theta} + 13.571\mathbf{e_r}$ ft/s$^2$ ◀

A graph of the path of the robot grip is found by graphing a parametric plot of x vs y for values of t between 0 and 4 seconds, using

$$x = \cos(\theta)$$
$$y = \sin(\theta)$$



## PROBLEM 11.162

The angular displacement of the robotic arm is programmed according to the relationship $\theta = (1/\pi)(\sin \pi t)$, where $\theta$ and $t$ are expressed in radians and seconds, respectively. Simultaneously, the arm is programmed to extend so that the distance to $A$ follows the relationship $r = 4\left(1 + e^{-2t}\right)$, where $r$ and $t$ are expressed in feet and seconds, respectively. When $t = 1.5$ s, determine ($a$) the velocity of point $A$, ($b$) the acceleration of point $A$.

## SOLUTION

Evaluate the angle at t=1.5 s to get,

$$\theta = (1/\pi)(\sin \pi 1.5) = -1/\pi$$

Differentiate to get angular velocity and acceleration and evaluate at t = 1.5 seconds,

$$\dot{\theta} = \cos(\pi t) = \cos(\pi 1.5) = 0$$
$$\ddot{\theta} = -\pi \sin(\pi t) = -\pi \sin(\pi 1.5) = \pi$$

Next, differentiate r(t) with respect to time and evaluate at t = 1.5 seconds

$$r = 4\left(1 + e^{-2t}\right) = 4\left(1 + e^{-2*1.5}\right) = 4.200$$
$$\dot{r} = 8\left(e^{-2t}\right) = 8\left(e^{-2*1.5}\right) = -0.398$$
$$\ddot{r} = 16\left(e^{-2t}\right) = 16\left(e^{-2*1.5}\right) = 0.800$$

Next compute and evaluate the velocity in radial and traverse directions,

$$\mathbf{v} = \dot{r}\hat{\mathbf{e}}_{\mathbf{r}} + r\dot{\theta}\hat{\mathbf{e}}_{\theta}$$
$$\mathbf{v} = -.398\hat{\mathbf{e}}_{\mathbf{r}} + 4.200(0)\hat{\mathbf{e}}_{\theta}$$
$$\mathbf{v} = -.398\hat{\mathbf{e}}_{\mathbf{r}}$$

$\mathbf{v} = -.398\hat{\mathbf{e}}_{\mathbf{r}}$ ft/s ◀

Next compute and evaluate the acceleration in radial and traverse directions

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{e}}_{\mathbf{r}} + (r\ddot{\theta} + 2\dot{r}\ddot{\theta})\hat{\mathbf{e}}_{\theta}$$
$$\mathbf{a} = (0.797 - 0)\hat{\mathbf{e}}_{\mathbf{r}} + (4.200(\pi) + 2(-.398)\pi)\hat{\mathbf{e}}_{\theta}$$
$$\mathbf{a} = (0.797)\hat{\mathbf{e}}_{\mathbf{r}} + (13.192)\hat{\mathbf{e}}_{\theta}$$

$\mathbf{a} = (0.797)\hat{\mathbf{e}}_{\mathbf{r}} + (13.19)\hat{\mathbf{e}}_{\theta}$ ft/s$^2$ ◀

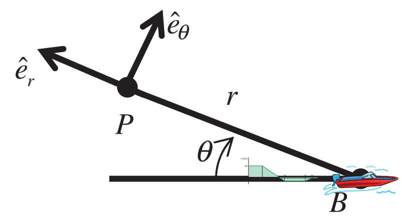

## PROBLEM 11.163

During a parasailing ride, the boat is traveling at a constant 30 km/hr with a 200 m long tow line. At the instant shown, the angle between the line and the water is 30° and is increasing at a constant rate of 2°/s. Determine the velocity and acceleration of the parasailer at this instant.

## SOLUTION

<u>Given</u>:

$$\mathbf{v}_B = 30\mathbf{i} \text{ km/hr} = 8.333\mathbf{i} \text{ m/s}$$
$$\mathbf{a}_B = 0$$
$$r = 200 \text{ m}, \dot{r} = 0, \ddot{r} = 0$$
$$\theta = 30°$$
$$\dot{\theta} = 2°/s = 0.0349 \text{ rad/s}$$
$$\ddot{\theta} = 0$$




<u>Relative Motion relations</u>:

$$\mathbf{v}_P = \mathbf{v}_B + \mathbf{v}_{P/B}$$
$$\mathbf{a}_P = \mathbf{a}_B + \mathbf{a}_{P/B}$$

<u>Using Radial and Transverse components</u>:

$$\mathbf{v}_{P/B} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$
$$\mathbf{a}_{P/B} = (\ddot{r} + r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

<u>Substitute in known values</u>:

$$\mathbf{v}_{P/B} = 6.981\mathbf{e}_\theta \text{ m/s}$$
$$\mathbf{a}_{P/B} = -0.2437\mathbf{e}_r \text{ m/s}^2$$

<u>Change to rectangular coordinates:</u>

$$\mathbf{v}_{P/B} = 6.981 * \sin 30°\mathbf{i} + 6.981 * \cos 30°\mathbf{j} \text{ m/s}$$
$$= 3.491\mathbf{i} = 6.046\mathbf{j} \text{ m/s}$$
$$\mathbf{a}_{P/B} = -0.2437 * \left(-\cos 30°\right)\mathbf{i} - 0.2437 * \sin 30°\mathbf{j} \text{ m/s}^2$$
$$= 0.2111\mathbf{i} - 0.1219\mathbf{j} \text{ m/s}^2$$

<u>Substitute into Relative Motion relations:</u>

$$\mathbf{v}_P = 8.33\mathbf{i} + 3.491\mathbf{i} + 6.046\mathbf{j} \text{ m/s}$$
$$= 11.824\mathbf{i} = 6.046\mathbf{j} \text{ m/s}$$
$$\mathbf{a}_P = 0 + 0.2111\mathbf{i} - 0.1219\mathbf{j} \text{ m/s}^2$$
$$= 0.2111\mathbf{i} - 0.1219\mathbf{j} \text{ m/s}^2$$

<u>Velocity:</u> $v_P = 13.280 \text{ m/s} \measuredangle 27.08°$ ◀

<u>Acceleration:</u> $a_P = 0.2437 \text{ m/s}^2 \ 30.00°$ ◀

## PROBLEM 11.164

Some parasailing systems use a winch to pull the rider back to the boat. During the interval when $\theta$ is between $20^\circ$ and $40^\circ$, (where t = 0 at $\theta = 20^\circ$) the angle increases at the constant rate of $2^\circ$/s. During this time, the length of the rope is defined by the relationship $r = 600 - \frac{1}{8}t^{5/2}$, where r and t are expressed in ft and s, respectively. Knowing that the boat is travelling at a constant rate of 15 knots (where 1 knot = 1.15 mi/h), (*a*) plot the magnitude of the velocity of the parasailer as a function of time (b) determine the magnitude of the acceleration of the parasailer when t = 5 s.

## SOLUTION

<u>Given</u>:

$$\mathbf{v}_B = 15\mathbf{i}\ \text{knots} * \frac{1.15\text{mph}}{\text{knot}} * \frac{5280\ ft\ /\ mi}{3600s\ /\ hr} = 25.30\mathbf{i}\ \text{ft/s}$$

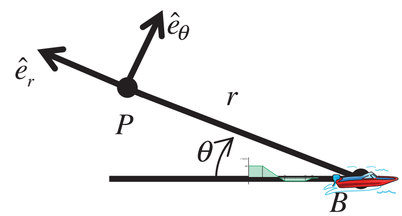
$$\mathbf{a}_B = 0$$

$$r = 600 - \frac{1}{8}t^{5/2}\ \text{m}$$

$$20° \le \theta \le 40°$$

$$\dot{\theta} = 2°/s = 0.0349\ \text{rad/s}$$

$$\ddot{\theta} = 0$$




<u>Take Derivatives of r(t)</u>:

$$\dot{r} = -\frac{5}{16}t^{3/2}\ \text{m/s}$$

$$\ddot{r} = -\frac{15}{32}t^{1/2}\ \text{m/s}^2$$

<u>Relative Motion relations</u>:

$$\mathbf{v}_P = \mathbf{v}_B + \mathbf{v}_{P/B}$$

$$\mathbf{a}_P = \mathbf{a}_B + \mathbf{a}_{P/B}$$

<u>Using Radial and Transverse components</u>:

$$\mathbf{v}_{P/B} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$

$$\mathbf{a}_{P/B} = \left(\ddot{r} + r\dot{\theta}^2\right)\mathbf{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta$$

<u>Change velocity to rectangular coordinates:</u>

$$\mathbf{v}_{P/B} = \dot{r}\left(-\cos\theta\mathbf{i} + \sin\theta\mathbf{j}\right) + r\dot{\theta}\left(\sin\theta\mathbf{i} + \cos\theta\mathbf{j}\right)$$

<u>Substitute into Relative Motion relations:</u>

$$\mathbf{v}_P = \left(v_B - \dot{r}\cos\theta + r\dot{\theta}\sin\theta\right)\mathbf{i} + \left(\dot{r}\sin\theta + r\dot{\theta}\cos\theta\right)\mathbf{j}$$

$$\mathbf{a}_P = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta$$

<u>Note that:</u>

$$\mathbf{v}_P = v_{Px}\mathbf{i} + v_{Py}\mathbf{j}$$

$$\mathbf{a}_P = a_r\mathbf{e}_r + a_\theta\mathbf{e}_\theta$$

## PROBLEM 11.164 (CONTINUED)

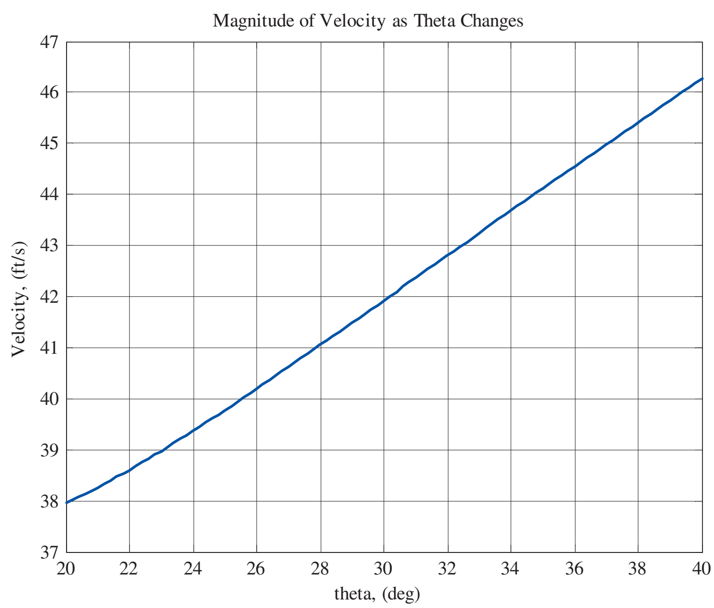
Where:

$$v_{Px} = v_B - \dot{r}\cos\theta + r\dot{\theta}\sin\theta$$
$$v_{Py} = \dot{r}\sin\theta + r\dot{\theta}\cos\theta$$
$$a_r = \ddot{r} - r\dot{\theta}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

Magnitude of velocity:

$$\mathbf{v}_P = \sqrt{v_{Px}^2 + v_{Py}^2}$$
$$\mathbf{a}_P = \sqrt{a_r^2 + a_\theta^2}$$

(*a*) Plot of Velocity as a function of theta:

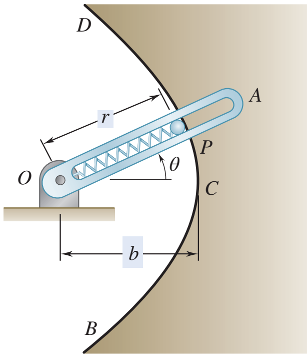



(*b*) Magnitude of Acceleration at t = 5s:

$$r = 600 - \frac{1}{8}(5)^{5/2} = 593.012 \text{ m}$$
$$\dot{r} = \frac{5}{16}(5)^{3/2} = -3.494 \text{ m/s}$$
$$\ddot{r} = \frac{15}{32}(5)^{1/2} = -1.048 \text{ m/s}^2$$
$$a_r = (\ddot{r} - r\dot{\theta}^2) = -1.771 \text{ m/s}^2$$
$$a_\theta = (r\ddot{\theta} + 2\dot{r}\dot{\theta}) = -0.2439 \text{ m/s}^2$$
$$\mathbf{a}_P = \sqrt{-1.771^2 + -0.2439^2}$$

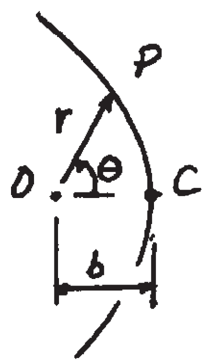
$$a_P = 1.787 \text{ m/s}^2 \blacktriangleleft$$

## PROBLEM 11.165




As rod $OA$ rotates, pin $P$ moves along the parabola $BCD$. Knowing that the equation of this parabola is $r = 2b(1+\cos\theta)$ and that $\theta = kt$, determine the velocity and acceleration of $P$ when (a) $\theta = 0$, (b) $\theta = 90°$.

## SOLUTION




$$r = \frac{2b}{1+\cos kt} \qquad \theta = kt$$

$$\dot{r} = \frac{2bk\sin kt}{(1+\cos kt)^2} \qquad \theta = k \qquad \ddot{\theta} = 0$$

$$\ddot{r} = \frac{2bk}{(1+\cos kt)^4}[(1+\cos kt)^2 k\cos kt + (\sin kt)2(1+\cos kt)(k\sin kt)]$$

(a) <u>When $\theta = kt = 0$</u>:

$$r = b \quad \dot{r} = 0 \quad \ddot{r} = \frac{2bk}{(2)^4}[(2)^2 k(1) + 0] = \frac{1}{2}bk^2$$

$$\theta = 0 \quad \dot{\theta} = k \quad \ddot{\theta} = 0$$

$$v_r = \dot{r} = 0 \quad v_\theta = r\dot{\theta} = bk$$

$$\mathbf{v} = bk\ \mathbf{e}_\theta \blacktriangleleft$$

$$\left.\begin{aligned} a_r &= \ddot{r} - r\dot{\theta}^2 = \frac{1}{2}bk^2 - bk^2 = -\frac{1}{2}bk^2 \\ a_\theta &= r\dot{\theta} + 2\dot{r}\dot{\theta} = b(0) + 2(0) = 0 \end{aligned}\right\}$$

$$\mathbf{a} = -\frac{1}{2}bk^2\mathbf{e}_r \blacktriangleleft$$

(b) <u>When $\theta = kt = 90°$</u>:

$$r = 2b \quad \dot{r} = 2bk \quad \ddot{r} = \frac{2bk}{19}[0 + 2k] = 4bk^2$$

$$\theta = 90° \quad \dot{\theta} = k \quad \ddot{\theta} = 0$$

$$v_r = \dot{r} = 2bk \quad v_\theta = r\dot{\theta} = 2bk$$

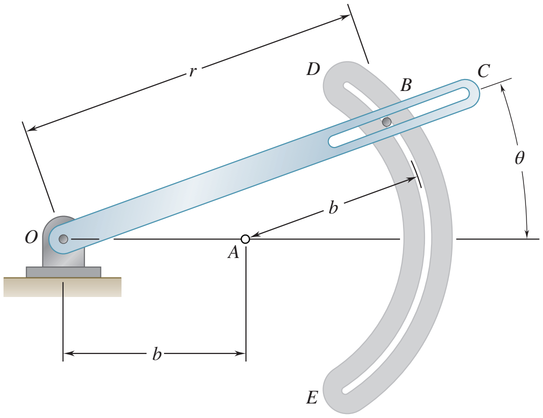
$$\mathbf{v} = 2bk\mathbf{e}_r + 2bk\mathbf{e}_\theta \blacktriangleleft$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4bk^2 - 2bk^2 = 2bk^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2b(0) + 2(2bk)k = 4bk^2$$

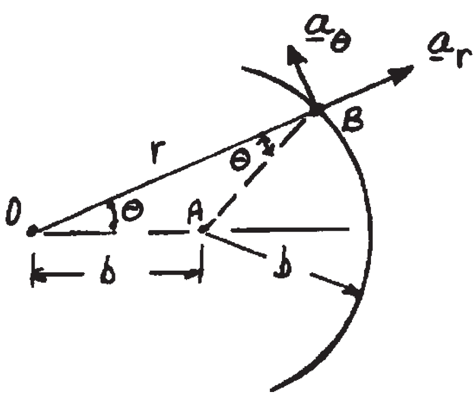
$$\mathbf{a} = 2bk^2\mathbf{e}_r + 4bk^2\mathbf{e}_\theta \blacktriangleleft$$

## PROBLEM 11.166




The pin at $B$ is free to slide along the circular slot $DE$ and along the rotating rod $OC$. Assuming that the rod $OC$ rotates at a constant rate $\dot{\theta}$, ($a$) show that the acceleration of pin $B$ is of constant magnitude, ($b$) determine the direction of the acceleration of pin $B$.

## SOLUTION




From the sketch:

$$r = 2b\cos\theta$$
$$\dot{r} = -2b\sin\theta\,\dot{\theta}$$

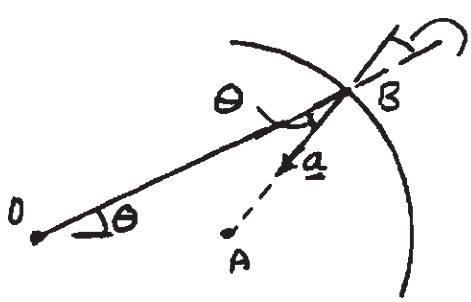
Since $\dot{\theta} = \text{constant}$, $\ddot{\theta} = 0$

$$\ddot{r} = -2b\cos\theta\,\dot{\theta}^2$$
$$a_r = \ddot{r} - r\dot{\theta}^2 = -2b\cos\theta\,\dot{\theta}^2 - (2b\cos\theta)\dot{\theta}^2$$
$$a_r = -4b\cos\theta\,\dot{\theta}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (2b\cos\theta)(0) + 2(-2b\sin\theta)\dot{\theta}^2$$
$$a_\theta = -4b\sin\theta\,\dot{\theta}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = 4b\dot{\theta}^2\sqrt{(-\cos\theta)^2 + (-\sin\theta)^2}$$
$$a = 4b\dot{\theta}^2$$

Since both $b$ and $\dot{\theta}$ are constant, we find that

$a = \text{constant}$ ◀




$$\gamma = \tan^{-1}\frac{a_\theta}{a_r} = \tan^{-1}\left(\frac{-4b\sin\theta\,\dot{\theta}^2}{-4b\cos\theta\,\dot{\theta}^2}\right)$$
$$\gamma = \tan^{-1}(\tan\theta)$$
$$\gamma = \theta$$

Thus, **a** is directed toward $A$ ◀

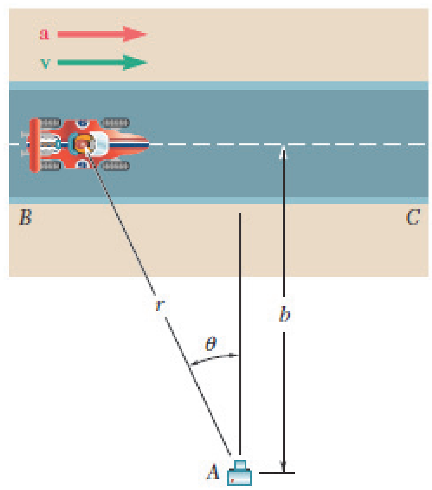


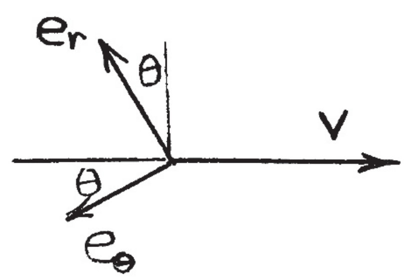

Fig. P11.169

## PROBLEM 11.167

To study the performance of a racecar, a high-speed camera is positioned at Point $A$. The camera is mounted on a mechanism which permits it to record the motion of the car as the car travels on straightway $BC$. Determine (*a*) the speed of the car in terms of $b$, $\theta$, and $\dot{\theta}$, (*b*) the magnitude of the acceleration in terms of $b$, $\theta$, $\dot{\theta}$, and $\ddot{\theta}$.

## SOLUTION

Velocity, sketch the directions of the vectors $\boldsymbol{v}$ and $\mathbf{e}_\theta$.




$$\mathbf{v}_\theta = \boldsymbol{v} \cdot \mathbf{e}_\theta = -v\cos\theta$$

But
$$\mathbf{v}_\theta = r\dot{\theta}$$

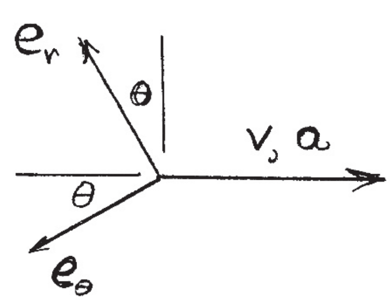
Hence,
$$r\dot{\theta} = -v\cos\theta$$

But from geometry,
$$r = \frac{b}{\cos\theta}$$

$$\frac{b\dot{\theta}}{\cos\theta} = -v\cos\theta \qquad \text{or} \qquad v = -\frac{b\dot{\theta}}{\cos^2\theta}$$

Speed is the absolute value of $v$.

$$v = \left|\frac{b\dot{\theta}}{\cos^2\theta}\right| \blacktriangleleft$$




Acceleration, from geometry,
$$r = \frac{b}{\cos\theta}$$

Differentiating with respect to time,
$$\dot{r} = \frac{b\sin\theta\dot{\theta}}{\cos^2\theta}$$

## PROBLEM 11.167 (CONTINUED)

Transverse component of acceleration

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{b\ddot{\theta}}{\cos\theta} + \frac{2b\dot{\theta}^2 \sin\theta}{\cos^2\theta} \tag{1}$$

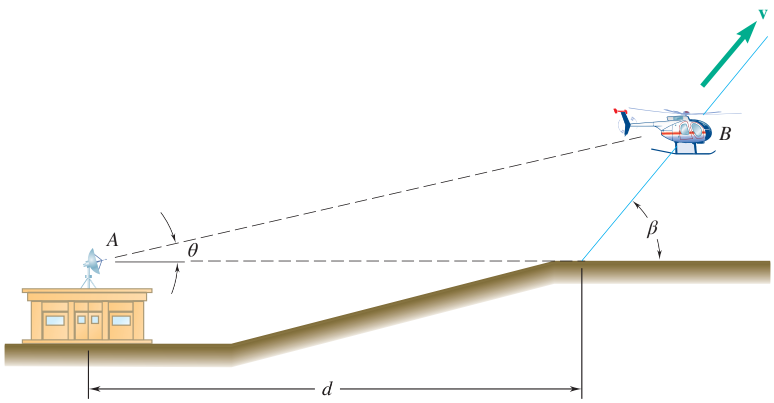
Sketch the directions of the vectors $\boldsymbol{a}$ and $\mathbf{e}_\theta$ referring to the figure above,

$$a_\theta = \boldsymbol{a} \cdot \boldsymbol{e}_\theta = -a\cos\theta \tag{2}$$

Matching from (1) and (2) and solving for $a$,

$$\begin{aligned} a &= -\frac{b\ddot{\theta}}{\cos^2\theta} - \frac{2b\dot{\theta}^2 \sin\theta}{\cos^3\theta} \\ &= -\frac{b}{\cos^2\theta}\left(\ddot{\theta} + 2\dot{\theta}^2 \tan\theta\right) \end{aligned}$$

Since magnitude of a is sought, $$|a| = \frac{b}{\cos^2\theta}\left|\ddot{\theta} + 2\dot{\theta}^2 \tan\theta\right| \blacktriangleleft$$



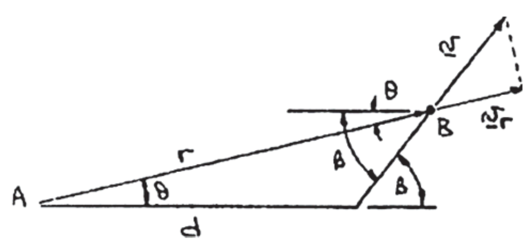

## PROBLEM 11.168

After taking off, a helicopter climbs in a straight line at a constant angle $\beta$. Its flight is tracked by radar from Point $A$. Determine the speed of the helicopter in terms of $d$, $\beta$, $\theta$, and $\dot{\theta}$.

## SOLUTION

From the diagram

$$\frac{r}{\sin(180° - \beta)} = \frac{d}{\sin(\beta - \theta)}$$




or $$d\sin\beta = r(\sin\beta\cos\theta - \cos\beta\sin\theta)$$

or $$r = d\frac{\tan\beta}{\tan\beta\cos\theta - \sin\theta}$$

Then $$\dot{r} = d\tan\beta\frac{-(-\tan\beta\sin\theta - \cos\theta)}{(\tan\beta\cos\theta - \sin\theta)^2}\dot{\theta}$$

$$= d\dot{\theta}\tan\beta\frac{\tan\beta\sin\theta + \cos\theta}{(\tan\beta\cos\theta - \sin\theta)^2}$$

From the diagram

$$v_r = v\cos(\beta - \theta) \qquad \text{where} \qquad v_r = \dot{r}$$

Then

$$d\dot{\theta}\tan\beta\frac{\tan\beta\sin\theta + \cos\theta}{(\tan\beta\cos\theta - \sin\theta)^2} = v(\cos\beta\cos\theta + \sin\beta\sin\theta)$$

$$= v\cos\beta(\tan\beta\sin\theta + \cos\theta)$$

or $$v = \frac{d\dot{\theta}\tan\beta\sec\beta}{(\tan\beta\cos\theta - \sin\theta)^2} \blacktriangleleft$$

Alternative solution.

We have $$v^2 = v_r^2 + v_\theta^2 = (\dot{r})^2 + (r\dot{\theta})^2$$

## PROBLEM 11.168 (Continued)

Using the expressions for $r$ and $\dot{r}$ from above

$$v = \left[ d\dot{\theta} \tan\beta \frac{\tan\beta \sin\theta + \cos\theta}{(\tan\beta\cos\theta - \sin\theta)^2} \right]^2$$

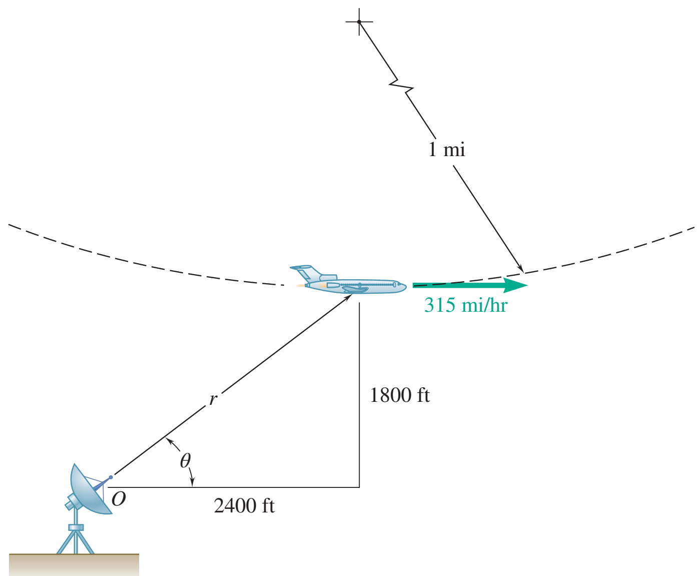
or

$$v = \pm \frac{d\dot{\theta}\tan\beta}{(\tan\beta\cos\theta - \sin\theta)} \left[ \frac{(\tan\beta\sin\theta + \cos\theta)^2}{(\tan\beta\cos\theta - \sin\theta)^2} + 1 \right]^{1/2}$$

$$= \pm \frac{d\dot{\theta}\tan\beta}{(\tan\beta\cos\theta - \sin\theta)} \left[ \frac{(\tan^2\beta + 1}{(\tan\beta\cos\theta - \sin\theta)^2} \right]^{1/2}$$

Note that as $\theta$ increases, the helicopter moves in the indicated direction. Thus, the positive root is chosen.

$$v = \frac{d\dot{\theta}\tan\beta\sec\beta}{(\tan\beta\cos\theta - \sin\theta)^2} \blacktriangleleft$$




## PROBLEM 11.169

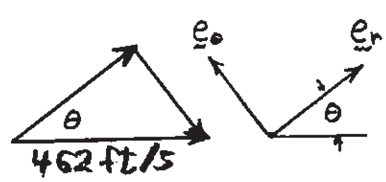
At the bottom of a loop in the vertical plane, an airplane has a horizontal velocity of 315 mi/h and is speeding up at a rate of 10 ft/s$^2$. The radius of curvature of the loop is 1 mi. The plane is being tracked by radar at $O$. What are the recorded values of $\dot{r}, \ddot{r}, \dot{\theta}$ and $\ddot{\theta}$ for this instant?

## SOLUTION

Geometry. The polar coordinates are

$$r = \sqrt{(2400)^2 + (1800)^2} = 3000 \text{ ft} \qquad \theta = \tan^{-1}\left(\frac{1800}{2400}\right) = 36.87°$$

Velocity Analysis.




$$\mathbf{v} = 315 \text{ mi/h} = 462 \text{ ft/s} \longrightarrow$$

$$v_r = 462\cos\theta = 369.6 \text{ ft/s}$$

$$v_\theta = -462\sin\theta = -277.2 \text{ ft/s}$$

$$v_r = \dot{r} \qquad \dot{r} = 370 \text{ ft/s} \blacktriangleleft$$

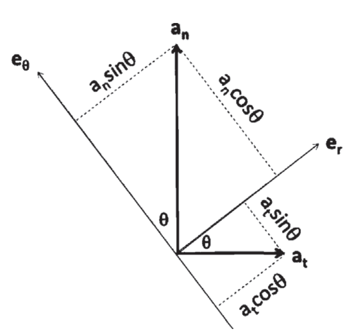
$$v_\theta = r\dot{\theta} \qquad \dot{\theta} = \frac{v_\theta}{r} = -\frac{277.2}{3000}$$

$$\dot{\theta} = -0.0924 \text{ rad/s} \blacktriangleleft$$

Acceleration analysis.

$$a_t = 10 \text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(462)^2}{5280} = 40.425 \text{ ft/s}^2$$



## PROBLEM 11.169 (CONTINUED)

$$a_r = a_t \cos\theta + a_n \sin\theta = 10\cos 36.87° + 40.425\sin 36.87° = 32.225 \text{ ft/s}^2$$

$$a_\theta = a_t \sin\theta + a_n \cos\theta = 10\sin 36.87° + 40.425\cos 36.87° = 26.34 \text{ ft/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \qquad \ddot{r} = a_r + r\dot{\theta}^2$$

$$\ddot{r} = 32.255 + (3000)(-0.0924)^2$$

$\ddot{r} = 57.9 \text{ ft/s}^2$ ◀

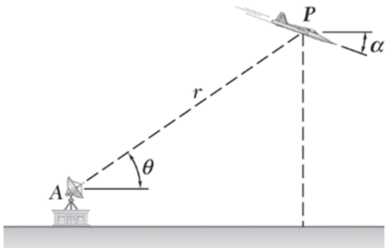
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$\ddot{\theta} = \frac{a_\theta}{r} - \frac{2\dot{r}\dot{\theta}}{r}$$

$$= \frac{26.34}{3000} - \frac{(2)(369.6)(-0.0924)}{3000}$$

$\ddot{\theta} = 0.0315 \text{ rad/s}^2$ ◀

## PROBLEM 11.170




An airplane passes over a radar tracking station at A and continues to fly due east. When the plane is at P, the distance and angle of elevation of the plane are, respectively, $r = 12{,}600\text{ ft}$ and $\theta = 31.2^\circ$. Two seconds later the radar station sights the plane at $r = 13{,}600\text{ ft}$ and $\theta = 28.3^\circ$. Determine approximately the speed and the angle of dive $\alpha$ of the plane during the 2-s interval.

## SOLUTION

Changes in values over the interval $\Delta r = 13600 - 12600 = 1000\text{ ft}$

$$\Delta\theta = 28.3^\circ - 31.2^\circ = -2.9^\circ = -5.0615 \times 10^{-2}\text{ rad}$$

$$\Delta t = 2s$$

Average rates of change. $$\dot{r} = \frac{\Delta r}{\Delta t} = \frac{1000}{2} = 500\text{ ft/s}$$

$$\dot{\theta} = \frac{\Delta\theta}{\Delta t} = \frac{-5.0615 \times 10^{-2}}{2} = -2.5307 \times 10^{-2}\text{ rad/s}$$

Mean values. $$r = \frac{12600 + 13600}{2} = 13100\text{ ft}$$

$$\theta = \frac{31.2^\circ + 28.3^\circ}{2} = 29.75^\circ$$

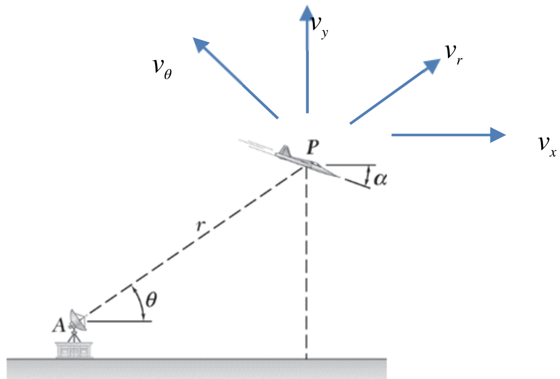
Velocity components. $$v_r = \dot{r} = 500\text{ ft/s}$$

$$v_\theta = r\dot{\theta} = (13100)\left(-2.5307 \times 10^{-2}\right) = -331.53\text{ ft/s}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(500)^2 + (-331.53)^2} = 600\text{ ft/s}$$

$v = 409\text{ mi/h}$ ◀

## PROBLEM 11.170 (CONTINUED)



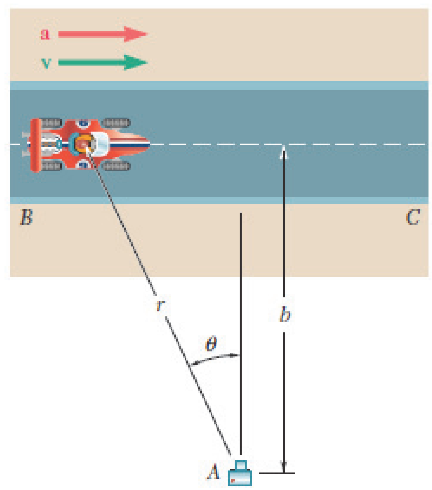

$$v_x = v_r\cos\theta - v_\theta\sin\theta$$

$$= 500\cos 29.75^\circ - (-331.53)\sin 29.75^\circ = 598.61 \text{ ft/s}$$

$$v_y = v_r\sin\theta + v_\theta\cos\theta$$

$$= 500\sin 29.75^\circ + ((-331.53)\cos 29.75^\circ) = -39.73 \text{ ft/s}$$

$$\tan\alpha = \frac{-v_y}{v_x} = \frac{39.73}{598.61} = 0.06636 \qquad \alpha = 3.80^\circ \blacktriangleleft$$



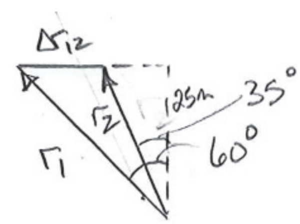

## PROBLEM 11.171

For the racecar of Problem 11.167, it was found that it took 0.4 s for the car to travel from the position $\theta = 60°$ to the position $\theta = 35°$. Knowing that $b = 25$ m, determine the average speed of the car during the 0.4-s interval.

## SOLUTION

From the diagram:

$$\Delta r_{12} = 25 \tan 60° - 25 \tan 35°$$
$$= 25.796 \text{ m}$$




$$v_{avg} = \frac{\Delta r_{12}}{\Delta t_{12}}$$

Now

$$v_{avg} = \frac{25.796 \text{ m}}{0.4 \text{ s}}$$
$$v_{avg} = 46.490 \text{ m/s}$$

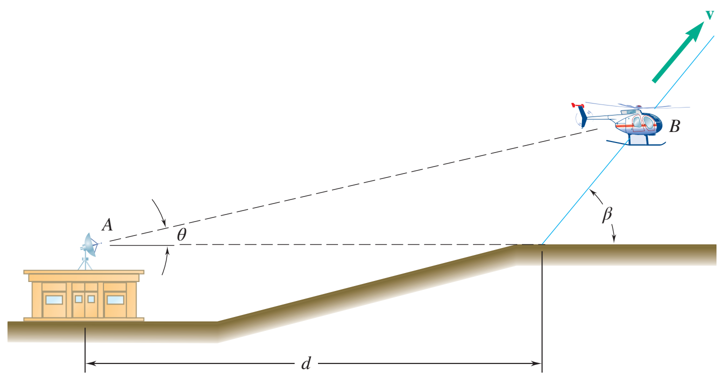
or

$$v_{avg} = 232.2 \text{ km/h} \blacktriangleleft$$

## PROBLEM 11.172

For the helicopter of Problem 11.168, it was found that when the helicopter was at $B$, the distance and the angle of elevation of the helicopter were $r = 3000$ ft and $\theta = 20°$, respectively. Four seconds later, the radar station sighted the helicopter at $r = 3320$ ft and $\theta = 23.1°$. Determine the average speed and the angle of climb $\beta$ of the helicopter during the 4-s interval.

**PROBLEM 11.168** After taking off, a helicopter climbs in a straight line at a constant angle $\beta$. Its flight is tracked by radar from Point $A$. Determine the speed of the helicopter in terms of $d$, $\beta$, $\theta$, and $\dot{\theta}$.

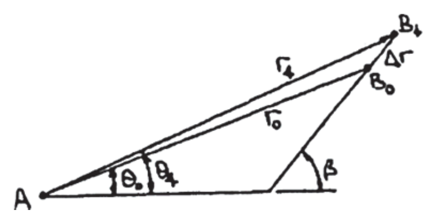



## SOLUTION

We have

$$r_0 = 3000 \text{ ft} \qquad \theta_0 = 20°$$

$$r_4 = 3320 \text{ ft} \qquad \theta_4 = 23.1°$$




From the diagram:

$$\Delta r^2 = 3000^2 + 3320^2 - 2(3000)(3320)\cos(23.1° - 20°)$$

or

$$\Delta r = 362.70 \text{ ft}$$

Now

$$v_{\text{ave}} = \frac{\Delta r}{\Delta t} = \frac{362.70 \text{ ft}}{4 \text{ s}} = 90.675 \text{ ft/s}$$

or

$$v_{\text{ave}} = 61.8 \text{ mi/h} \blacktriangleleft$$

Also,

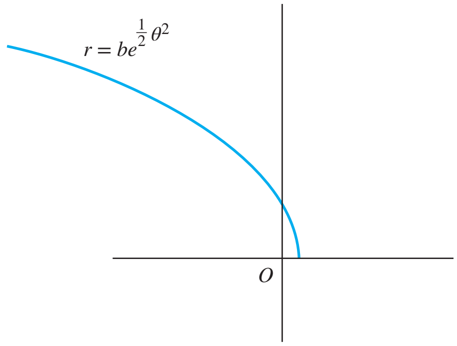
$$\Delta r \cos\beta = r_4 \cos\theta_4 - r_0 \cos\theta_0$$

or

$$\cos\beta = \frac{3320\cos 23.1° - 3000\cos 20°}{362.70}$$

or

$$\beta = 49.7° \blacktriangleleft$$




## PROBLEM 11.173

A particle moves along the spiral shown. Determine the magnitude of the velocity of the particle in terms of $b,\ \theta,$ and $\dot{\theta}$.

## SOLUTION

Given: $r = be^{\frac{1}{2}\theta^2}$

Take time derivative $\dot{r} = be^{\frac{1}{2}\theta^2}\theta\dot{\theta}$

Radial and Transverse Components

$$v_r = \dot{r} = be^{\frac{1}{2}\theta^2}\theta\dot{\theta}$$

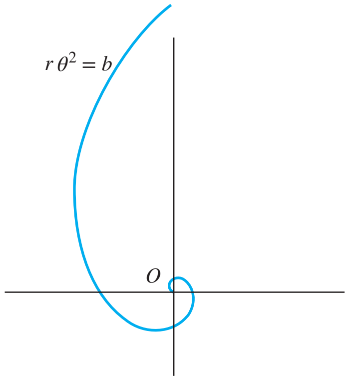
$$v_\theta = r\dot{\theta} = be^{\frac{1}{2}\theta^2}\dot{\theta}$$

Magnitude of Velocity

$$v^2 = v_r^2 + v_\theta^2$$

$$= \left(be^{\frac{1}{2}\theta^2}\right)^2 \left(\theta^2 + 1\right)\dot{\theta}^2$$

$$v = \left|be^{\frac{1}{2}\theta^2}(\theta^2 + 1)^{1/2}\dot{\theta}\right| \blacktriangleleft$$




## PROBLEM 11.174

A particle moves along the spiral shown. Determine the magnitude of the velocity of the particle in terms of $b$, $\theta$, and $\dot{\theta}$.

## SOLUTION

Given: $r = \dfrac{b}{\theta^2}$

Take time derivative $\dot{r} = \dfrac{2b}{\theta^3}\dot{\theta}$

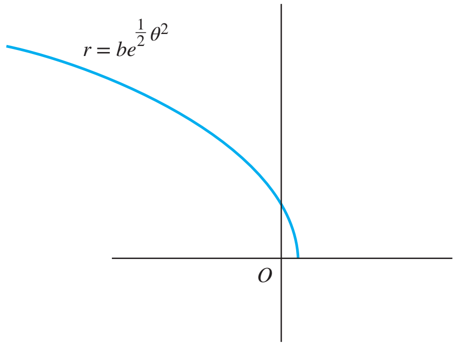
Radial and Transverse Components $v_r = \dot{r} = -\dfrac{2b}{\theta^3}\dot{\theta}$

$$v_\theta = r\dot{\theta} = \frac{b}{\theta^2}\dot{\theta}$$

Magnitude of Velocity

$$\begin{aligned} v^2 &= v_r^2 + v_\theta^2 \\ &= \frac{4b^2}{\theta^6}\dot{\theta}^2 + \frac{b^2}{\theta^4}\dot{\theta}^2 \\ &= \frac{b^2}{\theta^6}(4+\theta^2)\dot{\theta}^2 \end{aligned}$$

$$v = \left|\frac{b}{\theta^3}\left(4+\theta^2\right)^{1/2}\dot{\theta}\right| \blacktriangleleft$$




## PROBLEM 11.175

A particle moves along the spiral shown. Knowing that $\dot{\theta}$ is constant and denoting this constant by $\omega$, determine the magnitude of the acceleration of the particle in terms of $b, \theta$, and $\dot{\theta}$.

## SOLUTION

Given:

$$r = be^{\frac{1}{2}\theta^2}$$

$$\dot{\theta} = \omega \qquad \text{and} \qquad \ddot{\theta} = 0$$

Take time derivatives

$$\dot{r} = be^{\frac{1}{2}\theta^2}\,\theta\dot{\theta}$$

$$\ddot{r} = be^{\frac{1}{2}\theta^2}\left[(\theta\dot{\theta})^2 + \dot{\theta}^2 + \theta\ddot{\theta}\right]$$

Radial Component of Acceleration

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$= be^{\frac{1}{2}\theta^2}\left[(\theta\dot{\theta})^2 + \dot{\theta}^2 + \theta\ddot{\theta} - \dot{\theta}^2\right]$$

$$= be^{\frac{1}{2}\theta^2}\left[(\theta\dot{\theta})^2 + \theta\ddot{\theta}\right]$$

Transverse Component of Acceleration

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$= be^{\frac{1}{2}\theta^2}\,\ddot{\theta} + be^{\frac{1}{2}\theta^2}\,\theta\dot{\theta}^2$$

$$= be^{\frac{1}{2}\theta^2}\left[\ddot{\theta} + 2\theta\dot{\theta}^2\right]$$

Substitute Given Values

$$\dot{\theta} = \omega \qquad \text{and} \qquad \ddot{\theta} = 0$$

$$a_r = be^{\frac{1}{2}\theta^2}(\theta\omega)^2 \qquad \text{and} \qquad a_\theta = be^{\frac{1}{2}\theta^2}\left(2\theta\omega^2\right)$$

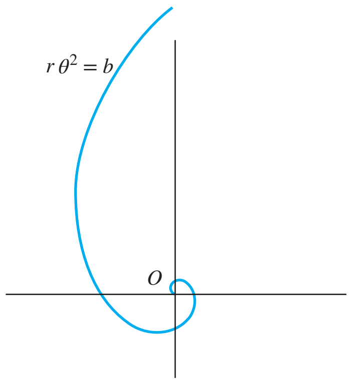
Magnitude of Acceleration

$$a^2 = a_r^2 + a_\theta^2$$

$$= \left(be^{\frac{1}{2}\theta^2}\right)^2\left(\theta^4 + 4\theta^2\right)\omega^4$$

$$a = be^{\frac{1}{2}\theta^2}\,\theta(\theta^2 + 4)^{\frac{1}{2}}\,\omega^2 \blacktriangleleft$$

## PROBLEM 11.176




A particle moves along the spiral shown. Knowing that $\dot{\theta}$ is constant and denoting this constant by $\omega$, determine the magnitude of the acceleration of the particle in terms of $b$, $\theta$, and $\dot{\theta}$.

## SOLUTION

Given:
$$r = \frac{b}{\theta^2},\ \dot{\theta} = \omega \text{ and } \ddot{\theta} = 0$$

Take time derivatives
$$\dot{r} = -\frac{2b}{\theta^3}\dot{\theta}$$
$$\ddot{r} = \frac{2b}{\theta^3}\ddot{\theta} + \frac{6b}{\theta^4}\dot{\theta}^2$$

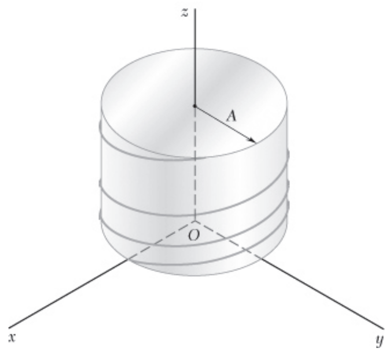
Radial Component of Acceleration
$$a_r = \ddot{r} - r\dot{\theta}^2$$
$$= -\frac{2b}{\theta^3}\ddot{\theta} + \frac{6b}{\theta^4}\dot{\theta}^2 - \frac{b}{\theta^2}\dot{\theta}^2$$
$$= \frac{b}{\theta^4}\left(-2\theta\ddot{\theta} + 6\dot{\theta}^2 - \theta^2\dot{\theta}^2\right)$$

Transverse Component of Acceleration
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$
$$= \frac{b}{\theta^2}\ddot{\theta} + (2)\left(-\frac{2b}{\theta^3}\right)\dot{\theta}^2$$
$$= \frac{b}{\theta^3}\left(\theta\ddot{\theta} - 4\dot{\theta}^2\right)$$

Substitute Given Values
$$a_r = \frac{b}{\theta^4}(6-\theta^2)\omega^2 \quad \text{and} \quad a_\theta = -\frac{4b}{\theta^3}\omega^2$$

Magnitude of Acceleration
$$a^2 = a_r^2 + a_\theta^2$$
$$= \frac{b^2}{\theta^8}\left(36 - 12\theta^2 + \theta^4\right)\omega^4 + \frac{16b^2}{\theta^6}\omega^4$$
$$= \frac{b^2}{\theta^8}\left(36 + 4\theta^2 + \theta^4\right)\omega^4$$

$$a = \frac{b}{\theta^4}\left(36 + 4\theta^2 + \theta^4\right)^{1/2}\omega^2 \blacktriangleleft$$




## PROBLEM 11.177

The motion of a particle on the surface of a right circular cylinder is defined by the relations $R = A,\ \theta = 2\pi t,$ and $z = At^2/4,$ where A is a constant. Determine the magnitudes of the velocity and acceleration of the particle at any time $t$.

## SOLUTION

In cylindrical coordinates. $\quad R = A, \quad \theta = 2\pi t, \quad z = \dfrac{At^2}{4}$

Differentiating with respect to time,

$$\dot{R} = 0, \quad \dot{\theta} = 2\pi, \quad \dot{z} = \frac{At}{2}$$
$$\ddot{R} = 0, \quad \ddot{\theta} = 0, \quad \ddot{z} = \frac{At}{2}$$

Velocity vector: $\quad v_r = \dot{R} = 0, \quad v_\theta = R\dot{\theta} = 2\pi A, \quad v_z = \dot{z} = \dfrac{At}{2}$

$$v^2 = v_R^2 + v_\theta^2 + v_z^2 = 0 + 4\pi^2 A^2 + \frac{1}{4}A^2 t^2$$

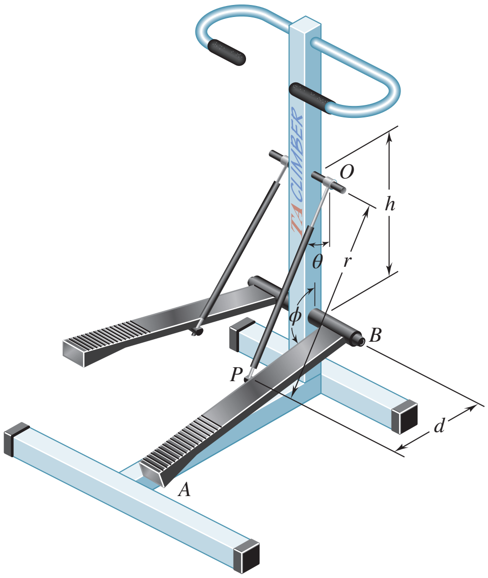
$$v = \frac{1}{2}A\sqrt{16\pi^2 + t^2} \blacktriangleleft$$

Acceleration vector: $\quad a_r = \ddot{R} - R\dot{\theta}^2 = 0 - 4\pi^2 A$

$$a_\theta = R\ddot{\theta} + 2\dot{R}\dot{\theta} = 0, \quad a_z = \ddot{z} = A/2$$

$$a^2 = a_R^2 + a_\theta^2 + a_z^2 = 16\pi^4 A^2 + 0 + \frac{1}{4}A^2$$

$$a = \frac{1}{2}A\sqrt{64\pi^4 + 1} \blacktriangleleft$$



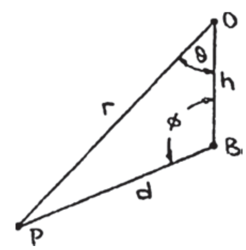

## PROBLEM 11.178

Show that $\dot{r} = h\dot{\phi}\sin\theta$ knowing that at the instant shown, step $AB$ of the step exerciser is rotating counterclockwise at a constant rate $\dot{\phi}$.

## SOLUTION

From the diagram

$$r^2 = d^2 + h^2 - 2dh\cos\phi$$




Then $$2r\dot{r} = 2dh\dot{\phi}\sin\phi$$

Now $$\frac{r}{\sin\phi} = \frac{d}{\sin\theta}$$

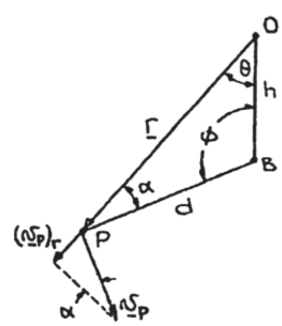
or $$r = \frac{d\sin\phi}{\sin\theta}$$

Substituting for $r$ in the expression for $\dot{r}$

$$\left(\frac{d\sin\phi}{\sin\theta}\right)\dot{r} = dh\dot{\phi}\sin\phi$$

or $$\dot{r} = h\dot{\phi}\sin\theta \qquad \text{Q.E.D.}$$ ◀

<u>Alternative solution</u>.




First note $$\alpha = 180° - (\phi + \theta)$$

Now $$\mathbf{v} = \mathbf{v}_r + \mathbf{v}_\theta = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$

With $B$ as the origin

$$v_P = d\dot{\phi} \qquad (d = \text{constant} \Rightarrow \dot{d} = 0)$$

## PROBLEM 11.178 (CONTINUED)

With $O$ as the origin $\qquad (v_P)_r = \dot{r}$

where $\qquad (v_P)_r = v_P \sin\alpha$

Then $\qquad \dot{r} = d\dot{\phi}\sin\alpha$

Now $\qquad \dfrac{h}{\sin\alpha} = \dfrac{d}{\sin\theta}$

or $\qquad d\sin\alpha = h\sin\theta$

substituting $\qquad \dot{r} = h\phi\sin\theta \qquad$ Q.E.D.

## PROBLEM 11.179

The three-dimensional motion of a particle is defined by the cylindrical coordinates $R = A/(t+1)$, $\theta = Bt$, and $z = Ct/(t+1)$. Determine the magnitudes of the velocity and acceleration when (a) $t = 0$, (b) $t = \infty$.

## SOLUTION

Differentiating with respect to time,

$$\dot{R} = -\frac{A}{(t+1)^2}, \qquad \dot{\theta} = B, \qquad \dot{z} = \frac{C(t+1) - Ct}{(t+1)^2} = \frac{C}{(1+t)^2}$$

$$\ddot{R} = \frac{2A}{(t+1)^3}, \qquad \ddot{\theta} = 0, \qquad \ddot{z} = -\frac{2C}{(1+t)^3}$$

*(a)* Let $t = 0$ and use the above equations to get,

$$R = A, \qquad \theta = 0, \qquad z = 0$$

$$\dot{R} = -A, \qquad \dot{\theta} = B, \qquad \dot{z} = C$$

$$\ddot{R} = 2A, \qquad \ddot{\theta} = 0, \qquad \ddot{z} = -2C$$

For velocity we get, $\quad v_R = \dot{R} = -A, \qquad v_\theta = R\dot{\theta} = AB, \qquad v_z = \dot{z} = C$

So, $\quad v^2 = v_R^2 + v_\theta^2 + v_z^2 = A^2 + A^2B^2 + C^2 \qquad v = \sqrt{A^2 + A^2B^2 + C^2}$ ◀

Also for acceleration we get,

$$a_R = \ddot{R} - R\dot{\theta}^2 = 2A - AB^2 \qquad a_R^2 = 4A^2 - 4A^2B^2 + A^2B^4$$

$$a_\theta = R\ddot{\theta} + 2\dot{R}\dot{\theta} = 0 - 2AB \quad \text{and} \quad a_\theta^2 = 4A^2B$$

$$a_z = \ddot{z} = -2c \quad \text{and} \quad a_z^2 = 4C^2$$

So $\quad a^2 = a_R^2 + a_\theta^2 + a_z^2 = 4A^2 + A^2B^4 + 4C^2$

$$a = \sqrt{4A^2 + A^2B^4 + 4C^2} \ ◀$$

*(b)* Let $t = \infty$ and use the above equations to get,

$$R = 0, \qquad \theta = \infty, \qquad z = C, \qquad \dot{R} = 0, \qquad \dot{\theta} = B, \qquad \dot{z} = 0,$$

$$\ddot{R} = 0, \qquad \ddot{\theta} = 0, \qquad \ddot{z} = 0$$

## PROBLEM 11.179 (CONTINUED)

$$v_r = \dot{R} = 0, \qquad v_\theta = R\dot{\theta} = 0, \qquad v_z = \dot{z} = 0, \qquad v = 0 \blacktriangleleft$$

$$a_r = \ddot{R} - R\dot{\theta}^2 = 0, \qquad a_\theta = R\ddot{\theta} - R\dot{\theta}^2 = 0, \qquad a_z = \ddot{z} = 0, \qquad a = 0 \blacktriangleleft$$

## PROBLEM 11.180*

For the conic helix of Problem 11.95, determine the angle that the osculating plane forms with the $y$ axis.

**PROBLEM 11.95** The three-dimensional motion of a particle is defined by the position vector $\mathbf{r} = (Rt \cos \omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt \sin \omega_n t)\mathbf{k}$. Determine the magnitudes of the velocity and acceleration of the particle. (The space curve described by the particle is a conic helix.)

## SOLUTION

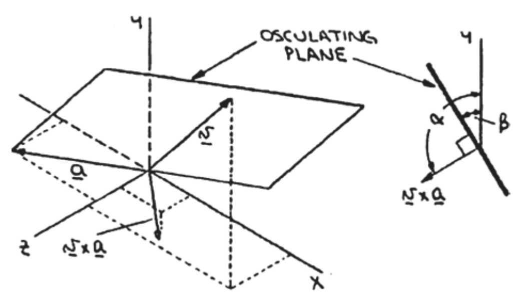
First note that the vectors **v** and **a** lie in the osculating plane.

Now $$\mathbf{r} = (Rt\cos\omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt\sin\omega_n t)\mathbf{k}$$

Then $$\mathbf{v} = \frac{dr}{dt} = R(\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{i} + c\mathbf{j} + R(\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}$$

and $$\begin{aligned}\mathbf{a} &= \frac{dv}{dt}\\ &= R(-\omega_n \sin\omega_n t - \omega_n \sin\omega_n t - \omega_n^2 t\cos\omega_n t)\mathbf{i}\\ &\quad + R(\omega_n \cos\omega_n t + \omega_n \cos\omega_n t - \omega_n^2 t\sin\omega_n t)\mathbf{k}\\ &= \omega_n R[-(2\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{i} + (2\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{k}]\end{aligned}$$

It then follows that the vector $(\mathbf{v}\times\mathbf{a})$ is perpendicular to the osculating plane.




$$(\mathbf{v}\times\mathbf{a}) = \omega_n R\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k}\\ R(\cos\omega_n t - \omega_n t\sin\omega_n t) & c & R(\sin\omega_n t + \omega_n t\cos\omega_n t)\\ -(2\sin\omega_n t + \omega_n t\cos\omega_n t) & 0 & (2\cos\omega_n t - \omega_n t\sin\omega_n t)\end{vmatrix}$$

$$\begin{aligned}&= \omega_n R\{c(2\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{i} + R[-\sin\omega_n t + \omega_n t\cos\omega_n t)(2\sin\omega_n t + \omega_n t\cos\omega_n t)\\ &\quad - (\cos\omega_n t - \omega_n t\sin\omega_n t)(2\cos\omega_n t - \omega_n t\sin\omega_n t)]\mathbf{j} + c(2\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}\\ &= \omega_n R\left[c(2\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{i} - R\left(2 + \omega_n^2 t^2\right)\mathbf{j} + c(2\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}\right]\end{aligned}$$

## PROBLEM 11.180* (CONTINUED)

The angle $\alpha$ formed by the vector $(\mathbf{v}\times\mathbf{a})$ and the $y$ axis is found from

$$\cos\alpha = \frac{(\mathbf{v}\times\mathbf{a})\cdot\mathbf{j}}{|(\mathbf{v}\times\mathbf{a})||\mathbf{j}|}$$

Where $$|\mathbf{j}| = 1$$

$$(\mathbf{v}\times\mathbf{a})\cdot\mathbf{j} = -\omega_n R^2\left(2+\omega_n^2 t^2\right)$$

$$|(\mathbf{v}\times\mathbf{a})| = \omega_n R\Big[c^2(2\cos\omega_n t - \omega_n t\sin\omega_n t)^2 + R^2\left(2+\omega_n^2 t^2\right)^2$$
$$+c^2(2\sin\omega_n t + \omega_n t\cos\omega_n t)^2\Big]^{1/2}$$
$$= \omega_n R\left[c^2\left(4+\omega_n^2 t^2\right) + R^2\left(2+\omega_n^2 t^2\right)^2\right]^{1/2}$$

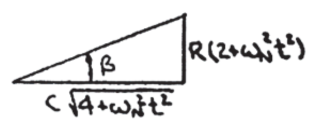
Then
$$\cos\alpha = \frac{-\omega_n R^2\left(2+\omega_n^2 t^2\right)}{\omega_n R\left[c^2\left(4+\omega_n^2 t^2\right) + R^2\left(2+\omega_n^2 t^2\right)^2\right]^{1/2}}$$
$$= \frac{-R\left(2+\omega_n^2 t^2\right)}{\left[c^2\left(4+\omega_n^2 t^2\right) + R^2\left(2+\omega_n^2 t^2\right)^2\right]^{1/2}}$$

The angle $\beta$ that the osculating plane forms with $y$ axis (see the above diagram) is equal to

$$\beta = \alpha - 90^\circ$$

Then
$$\cos\alpha = \cos(\beta + 90^\circ) = -\sin\beta$$

$$-\sin\beta = \frac{-R\left(2+\omega_n^2 t^2\right)}{\left[c^2\left(4+\omega_n^2 t^2\right) + R^2\left(2+\omega_n^2 t^2\right)^2\right]^{1/2}}$$




Then
$$\tan\beta = \frac{R\left(2+\omega_n^2 t^2\right)}{c\sqrt{4+\omega_n^2 t^2}}$$

or
$$\beta = \tan^{-1}\left[\frac{R\left(2+\omega_n^2 t^2\right)}{c\sqrt{4+\omega_n^2 t^2}}\right] \blacktriangleleft$$

## PROBLEM 11.181*

Determine the direction of the binormal of the path described by the particle of Problem 11.96 when ($a$) $t = 0$, ($b$) $t = \pi/2\,\text{s}$.

## SOLUTION

Given:

$$\mathbf{r} = (At\cos t)\mathbf{i} + \left(A\sqrt{t^2+1}\right)\mathbf{j} + (Bt\sin t)\mathbf{k}$$

$$r - \text{ft},\ t - \text{s};\ A - 3,\ B - 1$$

First note that $\mathbf{e}_b$ is given by

$$\mathbf{e}_b = \frac{\mathbf{v}\times\mathbf{a}}{|\mathbf{v}\times\mathbf{a}|}$$

Now

$$\mathbf{r} = (3t\cos t)\mathbf{i} + \left(3\sqrt{t^2+1}\right)\mathbf{j} + (t\sin t)\mathbf{k}$$

Then

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$

$$= 3(\cos t - t\sin t)\mathbf{i} + \frac{3t}{\sqrt{t^2+1}}\mathbf{j} + (\sin t + t\cos t)\mathbf{k}$$

and

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 3(-\sin t - \sin t - t\cos t)\mathbf{i} + 3\frac{\sqrt{t^2+1} - t\left(\frac{t}{t^2+1}\right)}{t^2+1}\mathbf{j}$$

$$+ (\cos t + \cos t - t\sin t)\mathbf{k}$$

$$= -3(2\sin t + t\cos t)\mathbf{i} + \frac{3}{\left(t^2+1\right)^{3/2}}\mathbf{j} + (2\cos t - t\sin t)\mathbf{k}$$

($a$) At $t = 0$:

$$\mathbf{v} = (3\ \text{ft/s})\mathbf{i}$$

$$\mathbf{a} = (3\ \text{ft/s}^2)\mathbf{j} + (2\ \text{ft/s}^2)\mathbf{k}$$

Then

$$\mathbf{v}\times\mathbf{a} = 3\mathbf{i}\times(3\mathbf{j}+2\mathbf{k})$$

$$= 3(-2\mathbf{j}+3\mathbf{k})$$

and

$$|\mathbf{v}\times\mathbf{a}| = 3\sqrt{(-2)^2+(3)^2} = 3\sqrt{13}$$

Then

$$\mathbf{e}_b = \frac{3(-2\mathbf{j}+3\mathbf{k})}{3\sqrt{13}} = \frac{1}{\sqrt{13}}(-2\mathbf{j}+3\mathbf{k})$$

$$\cos\theta_x = 0 \qquad \cos\theta_y = -\frac{2}{\sqrt{13}} \qquad \cos\theta_z = \frac{3}{\sqrt{13}}$$

or

$$\theta_x = 90^\circ \qquad \theta_y = 123.7^\circ \qquad \theta_z = 33.7^\circ \quad \blacktriangleleft$$

## PROBLEM 11.181* (CONTINUED)

(*b*) At $t = \dfrac{\pi}{2}\,\text{s}$:

$$\mathbf{v} = -\left(\frac{3\pi}{2}\text{ ft/s}\right)\mathbf{i} + \left(\frac{3\pi}{\sqrt{\pi^2+4}}\text{ ft/s}\right)j + (1\text{ ft/s})\mathbf{k}$$

$$\mathbf{a} = -(6\text{ ft/s}^2)\mathbf{i} + \left[\frac{24}{(\pi^2+4)^{3/2}}\text{ ft/s}^2\right]\mathbf{j} - \left(\frac{\pi}{2}\text{ ft/s}^2\right)\mathbf{k}$$

Then

$$\mathbf{v}\times\mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\dfrac{3\pi}{2} & \dfrac{3\pi}{(\pi^2+4)^{1/2}} & 1 \\ -6 & \dfrac{24}{(\pi^2+4)^{3/2}} & -\dfrac{\pi}{2} \end{vmatrix}$$

$$= \left[\frac{3\pi^2}{2(\pi^2+4)^{1/2}} + \frac{24}{(\pi^2+4)^{3/2}}\right]\mathbf{i} - \left(6 + \frac{3\pi^2}{4}\right)\mathbf{j}$$

$$+\left[-\frac{36\pi}{(\pi^2+4)^{3/2}} + \frac{18\pi}{(\pi^2+4)^{1/2}}\right]\mathbf{k}$$

$$= -4.43984\mathbf{i} - 13.40220\mathbf{j} + 12.99459\mathbf{k}$$

and

$$|\mathbf{v}\times\mathbf{a}| = [(-4.43984)^2 + (-13.40220)^2 + (12.99459)^2]^{1/2}$$

$$= 19.18829$$

Then

$$\mathbf{e}_b = \frac{1}{19.1829}(-4.43984\mathbf{i} - 13.40220\mathbf{j} + 12.99459\mathbf{k})$$

$$\cos\theta_x = -\frac{4.43984}{19.18829} \quad \cos\theta_y = -\frac{13.40220}{19.18829} \quad \cos\theta_z = \frac{12.99459}{19.18829}$$

or

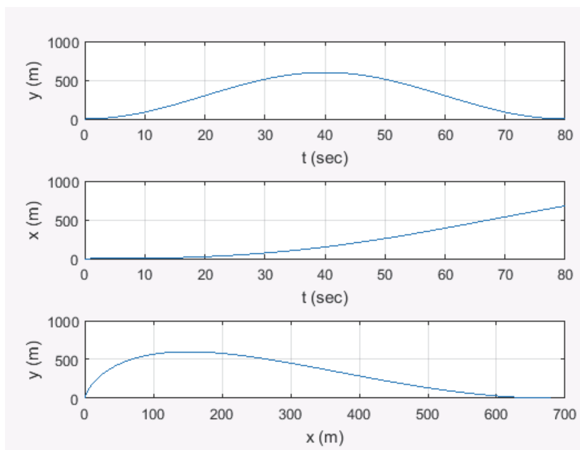
$$\theta_x = 103.4^\circ \qquad \theta_y = 134.3^\circ \qquad \theta_z = 47.4^\circ$$ ◀

## PROBLEM 11.182

Some students are testing their new drone to see if it can safely deliver packages to different departments on campus. Position data can be approximated using the expressions $x(t) = -0.0000225t^4 + 0.003t^3 + 0.01t^2$ and $y(t) = 300(1-\cos(\frac{\pi}{40}t))$, where $x$ and $y$ are expressed in meters and t is expressed in seconds. Knowing that the take-off and landing altitudes are the same, plot the path of the drone and determine (*a*) the duration of the flight, (*b*) its maximum speed in the *x*-direction, (*c*) its maximum altitude and the horizontal distance travelled during the flight.

## SOLUTION

(a) Plot x vs t, y vs t and x vs y




The duration is determined by the time to return to zero elevation

$$y(t) = 300(1-\cos(\tfrac{\pi}{40}t)) = 0$$
$$t = 80\,\text{sec}$$

$t = 80\,\text{sec}$ ◀

The maximum speed in the x direction is determined by differentiating to get velocity and acceleration,

$$x(t) = -0.0000225t^4 + 0.003t^3 + 0.01t^2$$
$$v_x(t) = -(4)0.0000225t^3 + (3)0.003t^2 + (2)0.01t$$
$$a_x(t) = -(3)(4)0.0000225t^2 + (2)(3)0.003t + (2)0.01$$

**PROBLEM 11.182 (CONTINUED)**

(b) Set the acceleration to zero to find the time of the maximum velocity

$$a_x(t) = -(3)(4)0.0000225t^2 + (2)(3)0.003t + (2)0.01 = 0$$
$$t = 67.76\,\text{sec}$$

Evaluate the velocity at this time to get the max velocity

$$v_x(67.76) = -(4)0.0000225(67.76)^3 + (3)0.003(67.76)^2 + (2)0.01(67.76) = 14.67$$

$v_{x\max} = 14.67$ m/s ◀

(c) The max elevation is found by

$$y_{\max} = 300(1-(-1)) = 600$$

$y_{\max} = 600$ m ◀

The max distance is found by evaluating x(t) at 80 seconds,

$$x_{\max} = x(80) = 678$$

$x_{\max} = 678$ m ◀

## PROBLEM 11.183

A drag car starts from rest and starts down the racetrack with and acceleration defined by $a = 50 - 10\,t$, where $a$ and $t$ are in m/s$^2$ and s, respectively. After reaching a speed of 125 m/s, a parachute is deployed to help slow down the dragster. Knowing that this deceleration is defined by the relationship $a = -\,0.02v^2$, where v is the velocity in m/s, determine ($a$) the total time from the beginning of the race until the car slows back down to 10 m/s, ($b$) the total distance the car travels during this time.

## SOLUTION

Given:

$$a_{1-2} = 50 - 10t \text{ m/s}^2 \text{ for } 0 \le v \le 125 \text{ m/s}$$

$$a_{2-3} = -0.02v^2 \text{ after } v = 125 \text{ m/s}$$

Find v(t) as car speeds up

$$a = \frac{dv}{dt} \Rightarrow dv = adt$$

$$\int_0^{v(t)} dv = \int_0^t (50 - 10t)dt$$

$$v(t) = 50t - 5t^2$$

Find time to reach v = 125 m/s

Rearrange and solve for t:

$$125 = 50t - 5t^2$$

$$t_2^2 - 10t_2 + 25 = 0$$

$$(t_2 - 5)^2 = 0$$

$$t_2 = 5 \text{ s}$$

Find distance to reach-125 m/s:

$$dx = vdt$$

$$\int_0^{x_2} dx = \int_0^{t_2} (50t - 5t^2)dt$$

$$x_2 = 25t^2 - \frac{5}{3}t^3\Big|_0^5$$

$$x_2 = 416.7 \text{ m}$$

($a$) Find total time to slow down 10 m/s:

$$a = \frac{dv}{dt} \Rightarrow dt = \frac{dv}{a}$$

$$\int_5^{t_3} dt = -50\int_{125}^{10} \frac{dv}{v^2}$$

$$t_3 - 5 = \frac{50}{v}\Big|_{125}^{10}$$

$t_3 = 9.6$ s ◀

## PROBLEM 11.183 (CONTINUED)

(*b*) Find total distance to slow down to 10 m/s

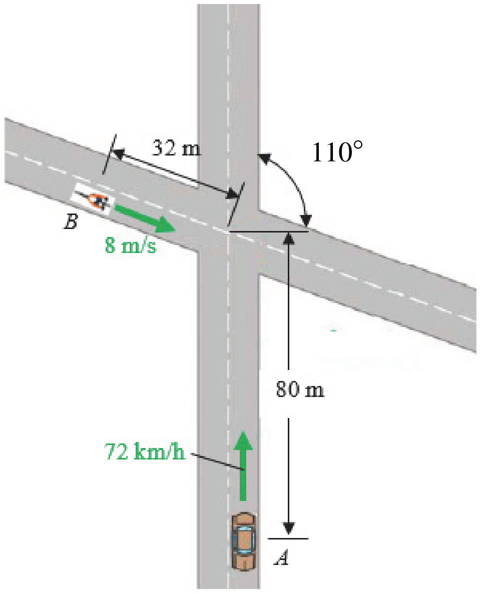
$$adx = vdv \Rightarrow dx\frac{vdv}{a}$$

$$\int_{x_2}^{x_3} dx = -50\int_{v_2}^{v_3} \frac{dv}{v}$$

$$x_3 - 416.7 = -50\ln v\Big|_{125}^{10}$$

$x_3 = 543.0$ m ◀

## PROBLEM 11.184




A driver is travelling at a speed of 72 km/h in car *A* when he looks down to text a friend that he is running late. Just before he looks down, he is 80 m from an intersection, and a bicyclist, *B*, travelling at a constant speed of 8 m/s is 32 m from that same intersection. The light turns red during the 3 s text, and when the driver looks up, he hits the brakes, causing a constant deceleration of 5 m/s$^2$. Determine (*a*) the distance between the car and the bike when the bike reaches the center of the intersection, (*b*) the velocity that the car appears to have to the cyclist at this time, (*c*) where the car stops.

## SOLUTION

Initial car velocity

$$v = 72\text{ km/h} = 20\text{ m/s}$$

Distance of car while texting is $\quad d = vt$

$$d = 20(3) = 60\text{ m}$$

Time for bike to get to intersection

$$t = d / v = (32\text{ m}) / (8\text{ m/s}) = 4\text{ sec}$$

So car has 1 second of deceleration until bike gets to intersection and the distance of car during deceleration

$$s = v_0 t - \frac{1}{2}at^2$$

So $\quad s(1) = 20(1) - \frac{1}{2}(5)1^2 = 17.5\text{ m}$

And $\quad S_{total} = 60 + 17.5 = 77.5\text{ m}$

## PROBLEM 11.184 (CONTINUED)

Distance between car and bike when the bike reaches the center of the intersection

$$100 - 77.5 = 2.5 \text{ m} \qquad 2.5\,\text{m} \blacktriangleleft$$

To find the velocity that the car appears to have to the cyclist at this time

$$v_{car} = 20 - 5(1) = 15 \text{ m/s}$$

$$\mathbf{v_A = v_B + v_{A/B}}$$

$$\mathbf{v_{A/B}} = \mathbf{v_A} - \mathbf{v_B}$$

$$\begin{aligned}\mathbf{v_{A/B}} &= 15\mathbf{i} - 8(\cos\theta\mathbf{i} - \sin\theta\mathbf{j}) \\ &= -7.518\mathbf{i} + (15 + 2.736)\mathbf{j}\end{aligned}$$

$$\mathbf{v_{A/B}} = -7.518\mathbf{i} + (15 + 2.736)\mathbf{j} \text{ m/s} \blacktriangleleft$$

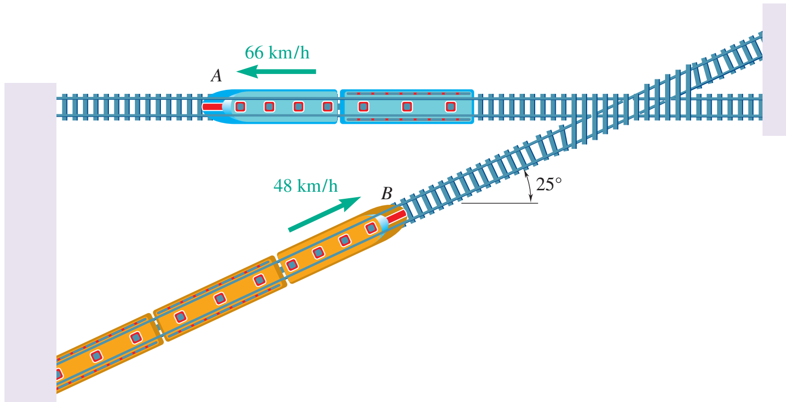
Deceleration time for the car to come to a complete stop

$$\begin{aligned}v &= v_0 - at \\ 0 &= 20 - 5t \\ t &= 4.0 \text{ s}\end{aligned}$$

Braking distance

$$\begin{aligned}s &= v_0 t - \frac{1}{2}at^2 \\ &= 20(4) - \frac{1}{2}(5)4^2 \\ &= 80 - 40 = 40 \text{ m}\end{aligned}$$

Total distance $\qquad 60 + 40 = 100\,\text{m} \qquad 100 \text{ m} \blacktriangleleft$



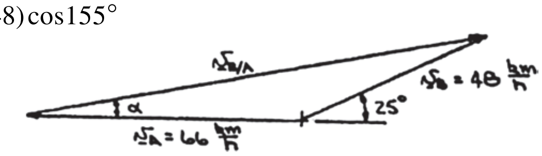

## PROBLEM 11.185

The velocities of commuter trains $A$ and $B$ are as shown. Knowing that the speed of each train is constant and that $B$ reaches the crossing 10 min after $A$ passed through the same crossing, determine ($a$) the relative velocity of $B$ with respect to $A$, ($b$) the distance between the fronts of the engines 3 min after $A$ passed through the crossing.

## SOLUTION

($a$) We have $$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

The graphical representation of this equation is then as shown.




Then $$v_{B/A}^2 = 66^2 + 48^2 - 2(66)(48)\cos 155°$$

or $$v_{B/A} = 111.366 \text{ km/h}$$

and $$\frac{48}{\sin\alpha} = \frac{111.366}{\sin 155°}$$

or $$\alpha = 10.50°$$

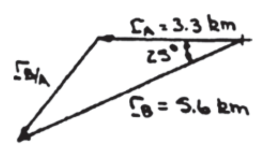
$$v_{B/A} = 111.4 \text{ km/h} \measuredangle\ 10.50° \blacktriangleleft$$

($b$) First note that

at $t = 3$ min, $A$ is $(66 \text{ km/h})\left(\frac{3}{60}\right) = 3.3$ km west of the crossing.

at $t = 3$ min, $B$ is $(48 \text{ km/h})\left(\frac{7}{60}\right) = 5.6$ km southwest of the crossing.

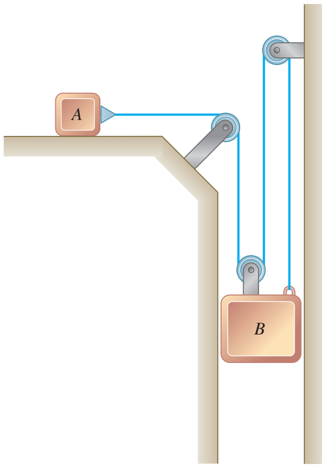
Now $$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$




Then at $t = 3$ min, we have

$$v_{B/A}^2 = 3.3^2 + 5.6^2 - 2(3.3)(5.6)\cos 25°$$

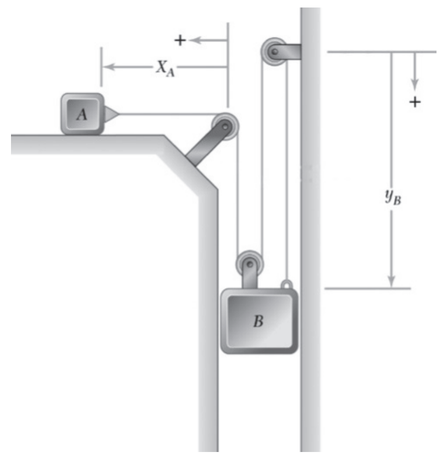
or $$r_{B/A} = 2.96 \text{ km} \blacktriangleleft$$




## PROBLEM 11.186

Knowing that slider block $A$ starts from rest and moves to the left with a constant acceleration of 1 ft/s$^2$, determine ($a$) the relative acceleration of block $A$ with respect to block $B$, ($b$) the velocity of block $B$ after 2 s.

## SOLUTION




Given: $a_A = 1$ ft/s$^2$

From the diagram can write an expression for the length of the cable

$$L = x_A + 3y_B + \text{constants}$$

Differentiate:

$$0 = \dot{x}_A + 3\dot{y}_B$$
$$0 = v_A + 3v_B$$

Differentiate again:

$$0 = a_A + 3a_B$$

Find the acceleration of B:

$$a_B = -\frac{1}{3}a_A$$
$$= -\frac{1}{3}\ \text{ft/s}^2$$

Write accelerations as vectors:

$$\mathbf{a}_A = -1\mathbf{i}\ \text{ft/s}^2$$
$$\mathbf{a}_B = \frac{1}{3}\mathbf{j}\ \text{ft/s}^2$$

($c$) Find relative accelerations:

$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B$$
$$= -1\mathbf{i} - \frac{1}{3}\mathbf{j}\ \text{ft/s}^2$$

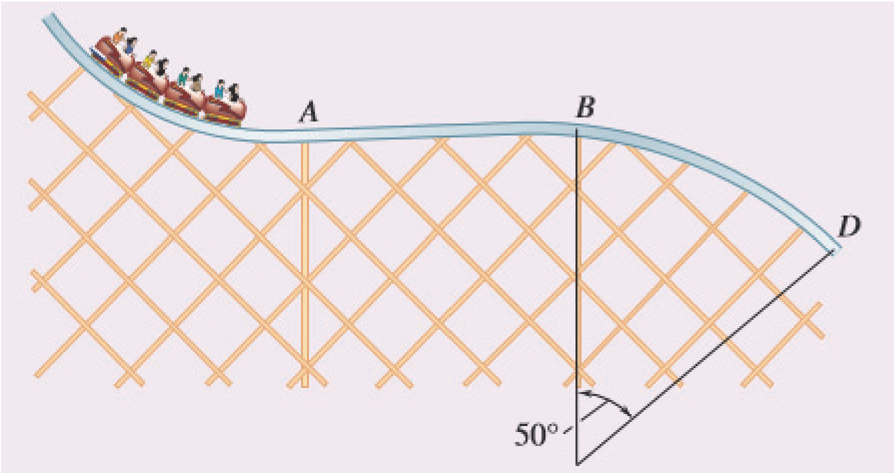
$$\mathbf{a}_{A/B} = 1.054\ \text{ft/s}^2 \ \angle 18.43° \blacktriangleleft$$

($d$) Find speed at t = 2 s:

$$v_B = (v_B)_0 + a_B t$$
$$= 0 + \frac{1}{3}(2)$$
$$= \frac{2}{3}\ \text{m/s}$$

$$v_b = 0.667\ \text{m/s} \blacktriangleleft$$

## PROBLEM 11.187




A roller-coaster car is travelling at a speed of 20 m/s when it passes through point *B*. At that point, it enters a concave down circular section of the track that has a radius of curvature of 60 m. After it reaches point *B*, the car's speed increases at a rate of 4 m/s$^2$. Determine (*a*) the time it takes the car to reach point *D*, (*b*) the *x* and *y* components of the overall acceleration of the car when it reaches *D*.

## SOLUTION

The distance from B to D is found as,

$$x = r\theta = (60\text{m})(50)(\pi/180) = 52.36 \text{ m}$$

For constant acceleration,

$$x = \frac{1}{2}at^2 + v_0 t + x_0$$

Set initial position to zero, and x=52.36 m to find time,

$$52.36 = \frac{1}{2}4t^2 + 20t + 0$$
$$t = 2.154 \text{ sec}$$

$$t = 2.154 \text{ sec} \blacktriangleleft$$

Find the velocity at D,

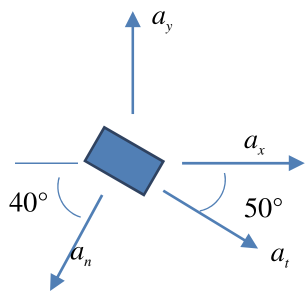
$$v = at + v_0 = 4(2.154) + 20 = 28.62 \text{ m/s}$$

## PROBLEM 11.187 (CONTINUED)

Use n-t coordinates to find the acceleration at D,

$$\mathbf{a} = \dot{v}\hat{\mathbf{e}}_\mathbf{t} + \frac{v^2}{\rho}\hat{\mathbf{e}}_\mathbf{n}$$
$$\mathbf{a} = 4\hat{\mathbf{e}}_\mathbf{t} + \frac{28.62^2}{60}\hat{\mathbf{e}}_\mathbf{n}$$
$$\mathbf{a} = 4\hat{\mathbf{e}}_\mathbf{t} + 13.65\hat{\mathbf{e}}_\mathbf{n}$$

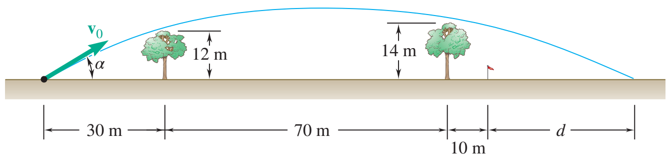
Resolve acceleration from the n-t to x-y coordinates




$$a_x = a_t\cos(50) - a_n\cos(40)$$
$$a_x = 4\cos(50) - 13.65\cos(40)$$
$$a_x = -7.884$$
$$a_y = a_t\sin(50) - a_n\sin(40)$$
$$a_y = 4\sin(50) - 13.65\sin(40)$$
$$a_y = -11.837$$

$a_x = -7.88 \text{ m/s}^2$ ◀

$a_y = -11.84 \text{ m/s}^2$ ◀




## PROBLEM 11.188

A golfer hits a ball with an initial velocity of magnitude $v_0$ at an angle $\alpha$ with the horizontal. Knowing that the ball must clear the tops of two trees and land as close as possible to the flag, determine $v_0$ and the distance $d$ when the golfer uses (*a*) a six-iron with $\alpha = 31°$, (*b*) a five-iron with $\alpha = 27°$.

## SOLUTION

The horizontal and vertical motions are

$$x = (v_0 \cos\alpha)t \qquad or \qquad v_0 = \frac{x}{t\cos\alpha} \tag{1}$$

$$y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2 = x\tan\alpha - \frac{1}{2}gt^2$$

or $$t^2 = \frac{2(x\tan\alpha - y)}{g} \tag{2}$$

At the landing Point $C$: $$y_C = 0, \qquad t = \frac{2v_0 \sin\alpha}{g}$$

And $$x_C = (v_0\cos\alpha)t = \frac{2v_0^2 \sin\alpha\cos\alpha}{g} \tag{3}$$

(*a*) $\alpha = 31°$

To clear tree $A$: $$x_A = 30\,\text{m},\ y_A = 12\,\text{m}$$

From (2), $$t_A^2 = \frac{2(30\tan 31° - 12)}{9.81} = 1.22851\ \text{s}^2, \qquad t_A = 1.1084\ \text{s}$$

From (1), $$(v_0)_A = \frac{30}{1.1084\cos 31°} = 31.58\ \text{m/s}$$

To clear tree $B$: $$x_B = 100\ \text{m}, \qquad y_B = 14\ \text{m}$$

From (2), $$(t_B)^2 = \frac{2(100\tan 31° - 14)}{9.81} = 9.3957\ \text{s}^2, \qquad t_B = 3.0652\ \text{s}$$

From (1), $$(v_0)_B = \frac{100}{3.0652\cos 31°} = 38.06\ \text{m/s}$$

The larger value governs, $$v_0 = 38.06\ \text{m/s}$$ $v_0 = 38.1\ \text{m/s}$ ◀

From (3), $$x_C = \frac{(2)(38.06)^2 \sin 31° \cos 31°}{9.81} = 130.38\ \text{m}$$

$$d = x_C - 110$$ $d = 20.4\ \text{m}$ ◀

## PROBLEM 11.188 (CONTINUED)

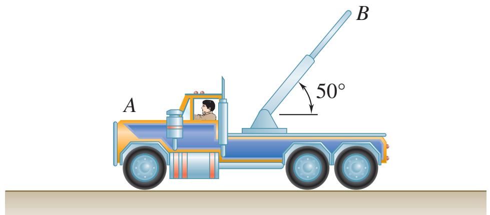
(*b*) $\alpha = 27°$

By a similar calculation, $t_A = 0.81846$ s, $(v_0)_A = 41.138$ m/s,

$t_B = 2.7447$ s, $(v_0)_B = 40.890$ m/s,

$v_0 = 41.138$ m/s $\quad v_0 = 41.1$ m/s ◀

$x_C = 139.56$ m, $\quad d = 29.6$ m ◀




## PROBLEM 11.189

As the truck shown begins to back up with a constant acceleration of $4\ \text{ft/s}^2$, the outer section $B$ of its boom starts to retract with a constant acceleration of $1.6\ \text{ft/s}^2$ relative to the truck. Determine (*a*) the acceleration of section $B$, (*b*) the velocity of section $B$ when $t = 2$ s.

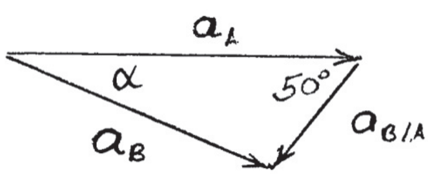
## SOLUTION

For the truck, $\mathbf{a}_A = 4\ \text{ft/s}^2 \longrightarrow$

For the boom, $\mathbf{a}_{B/A} = 1.6\ \text{ft/s}^2 \swarrow 50°$

(*a*) $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Sketch the vector addition.




By law of cosines:

$$a_B^2 = a_A^2 + a_{B/A}^2 - 2a_A a_{B/A} \cos 50°$$

$$= 4^2 + 1.6^2 - 2(4)(1.6)\cos 50°$$

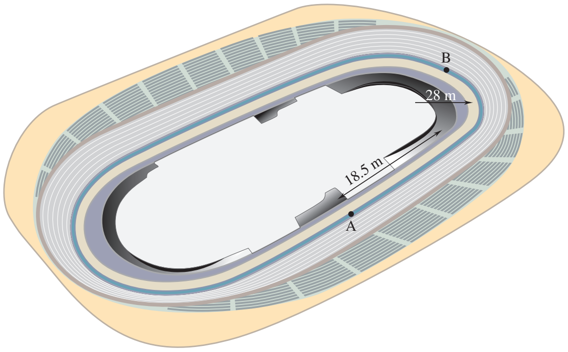
$$a_B = 3.214\ \text{ft/s}^2$$

Law of sines: $\sin\alpha = \dfrac{a_{B/A}\sin 50°}{a_B} = \dfrac{1.6\sin 50°}{3.214} = 0.38131$

$\alpha = 22.4°$, $\quad \mathbf{a}_B = 3.21\ \text{ft/s}^2 \searrow 22.4°$ ◀

(*b*) $\mathbf{v}_B = (v_B)_0 + a_B t = 0 + (3.214)(2)$

$\mathbf{v}_B = 6.43\ \text{ft/s}^2 \searrow 22.4°$ ◀




## PROBLEM 11.190

A velodrome is a specially designed track used in bicycle racing. If a rider starts from rest and accelerates between points $A$ and $B$ according to the relationship $a_t = (11.46 - 0.01878\ v^2)$ m/s$^2$, what is her acceleration at point $B$?

## SOLUTION

Given: $$a_t = (11.46 - 0.01878v^2)\,\text{m/s}^2$$

Distance traveled between points A and B:

$$\begin{aligned} s_B &= 18.5 + r\theta\text{m} \\ &= 18.5 + 28 * \frac{\pi}{2}\text{m} \\ &= 62.48\ \text{m} \end{aligned}$$

Find velocity at point B:

$$ads = vdv \Rightarrow ds = \frac{vdv}{a}$$

$$\int_0^{62.48} ds = \int_0^{v_B} \frac{vds}{11.46 - 0.01878v^2}$$

$$62.48 = \frac{1}{-(2)0.01878}\ln(11.46 - 0.01878v^2)\Big|_0^{v_B}$$

$$-2.347 = \ln(11.46 - 0.01878v_B^2) - \ln(11.46)$$

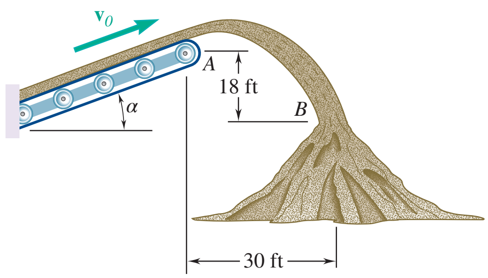
$$e^{0.09211} = 11.46 - 0.01878v_B^2$$

$$v_B = 23.49\ \text{m/s}$$

Acceleration:

$$\begin{aligned} \mathbf{a}_B &= a_t\mathbf{e}_t + \frac{v_B^2}{\rho}\mathbf{e}_n \\ &= (11.46 - 0.01878v_B^2)\mathbf{e}_t + \frac{(23.49)^2}{28}\mathbf{e}_n \end{aligned}$$

$$\mathbf{a}_B = 1.097\mathbf{e}_t + 19.71\mathbf{e}_n\ \text{m/s}^2 \blacktriangleleft$$




## PROBLEM 11.191

Sand is discharged at $A$ from a conveyor belt and falls onto the top of a stockpile at $B$. Knowing that the conveyor belt forms an angle $\alpha = 25°$ with the horizontal, determine (*a*) the speed $v_0$ of the belt, (*b*) the radius of curvature of the trajectory described by the sand at Point $B$.

## SOLUTION

The motion is projectile motion. Place the origin at Point $A$. Then $x_0 = 0$ and $y_0 = 0$.

The coordinates of Point $B$ are $x_B = 30$ ft and $y_B = -18$ ft.

Horizontal motion:

$$v_x = v_0 \cos 25° \tag{1}$$

$$x = v_0 t \cos 25° \tag{2}$$

Vertical motion:

$$v_y = v_0 \sin 25° - gt \tag{3}$$

$$y = v_0 \sin 25° - \frac{1}{2} g t^2 \tag{4}$$

At Point $B$, Eq. (2) gives

$$v_0 t_B = \frac{x_B}{\cos 25°} = \frac{30}{\cos 25°} = 33.101 \text{ ft}$$

Substituting into Eq. (4),

$$-18 = (33.101)(\sin 25°) - \frac{1}{2}(32.2) t_B^2$$

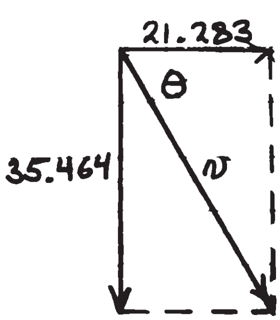
$$t_B = 1.40958 \text{ s}$$

(*a*) Speed of the belt.

$$v_0 = \frac{v_0 t_B}{t_B} = \frac{33.101}{1.40958} = 23.483$$

$v_0 = 23.4$ ft/s ◀

Eqs. (1) and (3) give




$$v_x = 23.483 \cos 25° = 21.283 \text{ ft/s}$$

$$v_y = (23.483) \sin 25° - (32.2)(1.40958) = -35.464 \text{ ft/s}$$

$$\tan\theta \frac{-v_y}{v_x} = 1.66632 \qquad \theta = 59.03°$$

$$v = 41.36 \text{ ft/s}$$

## PROBLEM 11.191 (CONTINUED)

Components of acceleration.

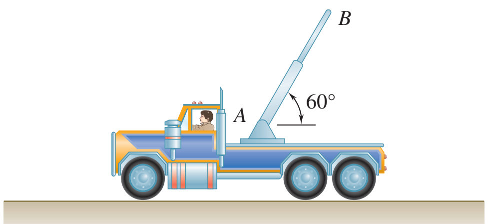
$$\mathbf{a} = 32.2 \text{ ft/s}^2 \downarrow \qquad a_t = 32.2\sin\theta$$

$$a_n = 32.2\cos\theta = 32.2\cos 59.03° = 16.57 \text{ ft/s}^2$$

(*b*) Radius of curvature at *B*. $\quad a_n = \dfrac{v^2}{\rho}$

$$\rho = \frac{v^2}{a_n} = \frac{(41.36)^2}{16.57}$$

$\rho = 103.2$ ft ◀




## PROBLEM 11.192

The end Point $B$ of a boom is originally 5 m from fixed Point $A$ when the driver starts to retract the boom with a constant radial acceleration of $\ddot{r} = -1.0 \text{ m/s}^2$ and lower it with a constant angular acceleration $\ddot{\theta} = -0.5 \text{ rad/s}^2$. At $t = 2$ s, determine (*a*) the velocity of Point $B$, (*b*) the acceleration of Point $B$, (*c*) the radius of curvature of the path.

## SOLUTION

Radial motion.

$$r_0 = 5 \text{ m}, \quad \dot{r}_0 = 0, \quad \ddot{r} = -1.0 \text{ m/s}^2$$

$$r = r_0 + \dot{r}_0 t + \frac{1}{2}\ddot{r}t^2 = 5 + 0 - 0.5t^2$$

$$\dot{r} = \dot{r}_0 + \ddot{r}t - 0 - 1.0t$$

At $t = 2$ s,

$$r = 5 - (0.5)(2)^2 = 3 \text{ m}$$

$$\dot{r} = (-1.0)(2) = -2 \text{ m/s}$$

Angular motion.

$$\theta_0 = 60° = \frac{\pi}{3} \text{ rad}, \quad \dot{\theta}_0 = 0, \quad \ddot{\theta} = -0.5 \text{ rad/s}^2$$

$$\theta = \theta_0 + \dot{\theta}_0 + \frac{1}{2}\ddot{\theta}t^2 = \frac{\pi}{3} + 0 - 0.25t^2$$

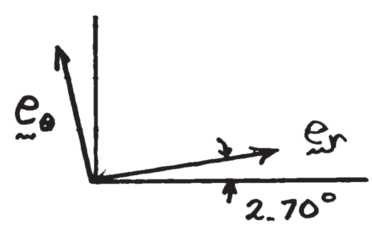
$$\dot{\theta} = \dot{\theta}_0 + \ddot{\theta}t = 0 - 0.5t$$

At $t = 2$ s,

$$\theta = \frac{\pi}{3} + 0 - (0.25)(2)^2 = 0.047198 \text{ rad} = 2.70°$$

$$\dot{\theta} = -(0.5)(2) = -1.0 \text{ rad/s}$$

Unit vectors $\mathbf{e}_r$ and $\mathbf{e}_\theta$.




(*a*) Velocity of Point $B$ at $t = 2$ s.

$$\mathbf{v}_B = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$

$$= -(2 \text{ m/s})\mathbf{e}_r + (3 \text{ m})(-1.0 \text{ rad/s})\mathbf{e}_\theta$$

$$\mathbf{v}_B = (-2.00 \text{ m/s})\mathbf{e}_r + (-3.00 \text{ m/s})\mathbf{e}_\theta \blacktriangleleft$$

$$\tan\alpha = \frac{v_\theta}{v_r} = \frac{-3.0}{-2.0} = 1.5 \qquad \alpha = 56.31°$$

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-2)^2 + (-3)^2} = 3.6055 \text{ m/s}$$

## PROBLEM 11.192 (CONTINUED)

Direction of velocity.

$$\mathbf{e}_t = \frac{\mathbf{v}}{v} = \frac{-2\mathbf{e}_r - 3\mathbf{e}_\theta}{3.6055} = -0.55470\mathbf{e}_r - 0.83205\mathbf{e}_\theta$$

$$\theta + \alpha = 2.70 + 56.31° = 59.01°$$

$$\mathbf{v}_B = 3.61 \text{ m/s} \measuredangle 59.0° \blacktriangleleft$$

(*b*) Acceleration of Point *B* at $t = 2$ s.

$$\mathbf{a}_B = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$
$$= [-1.0 - (3)(-1)^2]\mathbf{e}_r + [(3)(-0.5) + (2)(-1.0)(-0.5)]\mathbf{e}_\theta$$

$$\mathbf{a}_B = (-4.00 \text{ m/s}^2)\mathbf{e}_r + (2.50 \text{ m/s}^2)\mathbf{e}_\theta \blacktriangleleft$$

$$\tan\beta = \frac{a_\theta}{a_r} = \frac{2.50}{-4.00} = -0.625 \qquad \beta = -32.00°$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-4)^2 + (2.5)^2} = 4.7170 \text{ m/s}^2$$

$$\theta + \beta = 2.70° - 32.00° = -29.30°$$

$$\mathbf{a}_B = 4.72 \text{ m/s}^2 \measuredangle 29.3° \blacktriangleleft$$

Tangential component: $\mathbf{a}_t = (\mathbf{a} \cdot \mathbf{e}_t)\mathbf{e}_t$

$$\mathbf{a}_t = (-4\mathbf{e}_r + 2.5\mathbf{e}_\theta) \cdot (-0.55470\mathbf{e}_r - 0.83205\mathbf{e}_\theta)\mathbf{e}_t$$
$$= [(-4)(-0.55470) + (2.5)(-0.83205)]\mathbf{e}_t$$
$$= (0.138675 \text{ m/s}^2)\mathbf{e}_t = 0.1389 \text{ m/s}^2 \measuredangle 59.0°$$

Normal component: $\mathbf{a}_n = \mathbf{a} - \mathbf{a}_t$

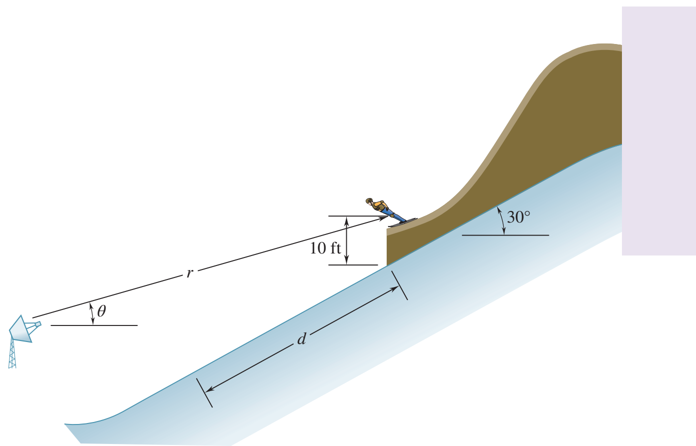
$$\mathbf{a}_n = -4\mathbf{e}_r + 2.5\mathbf{e}_\theta - (0.138675)(-0.55470\mathbf{e}_r - 0.83205\mathbf{e}_\theta)$$
$$= (-3.9231 \text{ m/s}^2)\mathbf{e}_r + (2.6154 \text{ m/s}^2)\mathbf{e}_\theta$$

$$a_n = \sqrt{(3.9231)^2 + (2.6154)^2} = 4.7149 \text{ m/s}^2$$

(*c*) Radius of curvature of the path.

$$a_n = \frac{v^2}{\rho}$$

$$\rho = \frac{v^2}{a_n} = \frac{(3.6055 \text{ m/s}^2)}{4.7149 \text{ m/s}} \qquad \rho = 2.76 \text{ m} \blacktriangleleft$$




## PROBLEM 11.193

A telemetry system is used to quantify kinematic values of a ski jumper immediately before she leaves the ramp. According to the system $r = 500$ ft, $\dot{r} = -105$ ft/s, $\ddot{r} = -10$ ft/s$^2$, $\theta = 25°$, $\dot{\theta} = 0.07$ rad/s, $\ddot{\theta} = 0.06$ rad/s$^2$, Determine (*a*) the velocity of the skier immediately before she leaves the jump, (*b*) the acceleration of the skier at this instant, (*c*) the distance of the jump $d$ neglecting lift and air resistance.

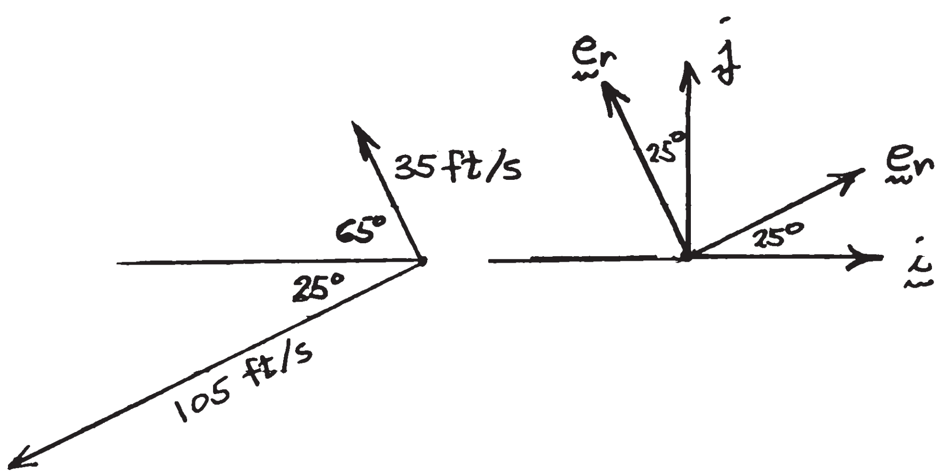
## SOLUTION

(*a*) <u>Velocity of the skier</u>. $\left(r = 500 \text{ ft}, \quad \theta = 25°\right)$

$$\mathbf{v} = v_r\mathbf{e}_r + v_\theta\mathbf{e}_\theta = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$
$$= (-105 \text{ ft/s})\mathbf{e}_r + (500 \text{ ft})(0.07 \text{ rad/s})\mathbf{e}_\theta$$

$$\mathbf{v} = (-105 \text{ ft/s})\,\mathbf{e}_r + (35 \text{ ft/s})\,\mathbf{e}_\theta \quad \blacktriangleleft$$

<u>Direction of velocity</u>:




$$\mathbf{v} = (-105\cos 25° - 35\cos 65°)\,\mathbf{i} + (35\sin 65° - 105\sin 25°)\,\mathbf{j}$$
$$= (-109.95 \text{ ft/s})\,\mathbf{i} + (-12.654 \text{ ft/s})\,\mathbf{j}$$

$$\tan\alpha = \frac{v_y}{v_x} = \frac{-12.654}{-109.95} \qquad \alpha = 6.565°$$

$$v = \sqrt{(105)^2 + (35)^2} = 110.68 \text{ ft/s}$$

$$v = 110.7 \text{ ft/s} \;\swarrow 6.57° \quad \blacktriangleleft$$

## PROBLEM 11.193 (CONTINUED)

(*b*) <u>Acceleration of the skier</u>.

$$\mathbf{a} = a_r\mathbf{e}_r + a_\theta\mathbf{e}_\theta = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

$$a_r = -10 - (500)(0.07)^2 = -12.45 \text{ ft/s}^2$$

$$a_\theta = (500)(0.06) + (2)(-105)(0.07) = 15.30 \text{ ft/s}^2$$

$$\mathbf{a} = \left(-12.45 \text{ ft/s}^2\right)\mathbf{e}_r + \left(15.30 \text{ ft/s}^2\right)\mathbf{e}_\theta \blacktriangleleft$$

$$\mathbf{a} = (-12.45)\left(\mathbf{i}\cos 25° + \mathbf{j}\sin 25°\right) + (15.30)\left(-\mathbf{i}\cos 65° + \mathbf{j}\sin 65°\right)$$

$$= \left(-17.750 \text{ ft/s}^2\right)\mathbf{i} + \left(8.6049 \text{ ft/s}^2\right)\mathbf{j}$$

$$\tan\beta = \frac{a_y}{a_x} = \frac{8.6049}{-17.750} \qquad \beta = -25.9°$$

$$a = \sqrt{(12.45)^2 + (15.30)^2} = 19.725 \text{ ft/s}^2$$

$$a = 19.73 \text{ ft/s}^2 \nearrow 25.9° \blacktriangleleft$$

(*c*) <u>Distance of the jump *d*</u>.

Projectile motion. Place the origin of the *xy*-coordinate system at the end of the ramp with the *x*-coordinate horizontal and positive to the left and the *y*-coordinate vertical and positive downward.

Horizontal motion: (Uniform motion)

$$x_0 = 0$$

$$\dot{x}_0 = 109.95 \text{ ft/s} \qquad \text{(from part } a\text{)}$$

$$x = x_0 + \dot{x}_0 t = 109.95t$$

Vertical motion: (Uniformly accelerated motion)

$$y_0 = 0$$

$$\dot{y}_0 = 12.654 \text{ ft/s} \qquad \text{(from part } a\text{)}$$

$$\ddot{y} = 32.2 \text{ ft/s}^2$$

$$y = y_0 + \dot{y}_0 t + \frac{1}{2}\ddot{y}t^2 = 12.654t - 16.1t^2$$

At the landing point,

$$x = d\cos 30° \qquad (1)$$

$$y = 10 + d\sin 30° \quad \text{or} \quad y - 10 = d\sin 30° \qquad (2)$$

## PROBLEM 11.193 (CONTINUED)

Multiply Eq. (1) by sin 30° and Eq. (2) by cos 30° and subtract

$$x \sin 30° - (y - 10)\cos 30° = 0$$

$$(109.95t)\sin 30° - (12.654t + 16.1t^2 - 10)\cos 30° = 0$$

$$-13.943t^2 + 44.016t + 8.6603 = 0$$

$$t = -0.1858 \text{ s} \quad \text{and} \quad 3.3427 \text{ s}$$

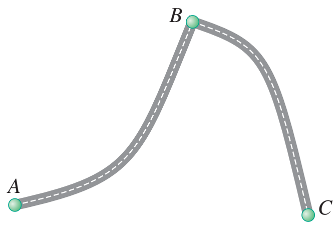
Reject the negative root.

$$x = (109.95 \text{ ft/s})(3.3427 \text{ s}) = 367.53 \text{ ft}$$

$$d = \frac{x}{\cos 30°}$$

$d = 424$ ft ◀

## PROBLEM 11.CQ1




A bus travels the 100 miles between $A$ and $B$ at 50 mi/h and then another 100 miles between $B$ and $C$ at 70 mi/h. The average speed of the bus for the entire 200-mile trip is:

($a$) more than 60 mi/h

($b$) equal to 60 mi/h

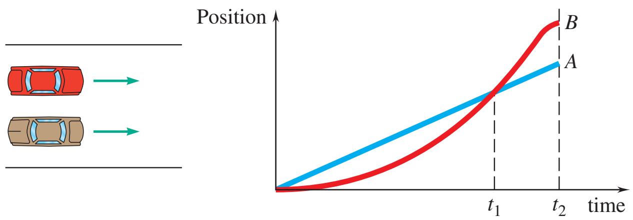
($c$) less than 60 mi/h

## SOLUTION

The time required for the bus to travel from $A$ to $B$ is 2 h and from $B$ to $C$ is $100/70 = 1.43$ h, so the total time is 3.43 h and the average speed is $200/3.43 = 58$ mph.

Answer: ($c$) ◀

## PROBLEM 11CQ2




Two cars $A$ and $B$ race each other down a straight road. The position of each car as a function of time is shown. Which of the following statements are true (more than one answer can be correct)?

(*a*) At time $t_2$ both cars have traveled the same distance

(*b*) At time $t_1$ both cars have the same speed

(*c*) Both cars have the same speed at some time $t < t_1$

(*d*) Both cars have the same acceleration at some time $t < t_1$

(*e*) Both cars have the same acceleration at some time $t_1 < t < t_2$

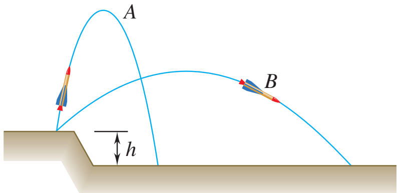
## SOLUTION

The speed is the slope of the curve, so answer c) is true.

The acceleration is the second derivative of the position. Since $A$'s position increases linearly the second derivative will always be zero. The second derivative of curve $B$ is zero at the pont of inflection which occurs between $t_1$ and $t_2$.

Answers: (*c*) and (*e*) ◀

## PROBLEM 11.CQ3




Two model rockets are fired simultaneously from a ledge and follow the trajectories shown. Neglecting air resistance, which of the rockets will hit the ground first?

($a$) $A$

($b$) $B$

($c$) They hit at the same time.

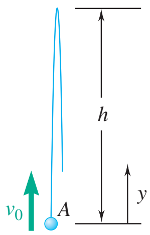
($d$) The answer depends on $h$.

## SOLUTION

The motion in the vertical direction depends on the initial velocity in the $y$-direction. Since $A$ has a larger initial velocity in this direction it will take longer to hit the ground.

Answer: ($b$) ◀

## PROBLEM 11.CQ4




Ball *A* is thrown straight up. Which of the following statements about the ball are true at the highest point in its path?

(*a*) The velocity and acceleration are both zero.

(*b*) The velocity is zero, but the acceleration is not zero.

(*c*) The velocity is not zero, but the acceleration is zero.

(*d*) Neither the velocity nor the acceleration are zero.

## SOLUTION

At the highest point the velocity is zero. The acceleration is never zero.

Answer: (*b*) ◀

## PROBLEM 11.CQ5

Ball $A$ is thrown straight up with an initial speed $v_0$ and reaches a maximum elevation $h$ before falling back down. When $A$ reaches its maximum elevation, a second ball is thrown straight upward with the same initial speed $v_0$. At what height, $y$, will the balls cross paths?

(*a*) $y = h$

(*b*) $y > h/2$

(*c*) $y = h/2$

(*d*) $y < h/2$

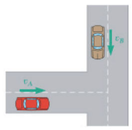
(*e*) $y = 0$

## SOLUTION

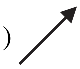
When the ball is thrown up in the air it will be constantly slowing down until it reaches its apex, at which point it will have a speed of zero. So, the time it will take to travel the last half of the distance to the apex will be longer than the time it takes for the first half. This same argument can be made for the ball falling from the maximum elevation. It will be speeding up, so the first half of the distance will take longer than the second half. Therefore, the balls should cross above the half-way point.

Answer: (*b*) ◀

## PROBLEM 11.CQ6

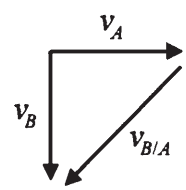


Two cars are approaching an intersection at constant speeds as shown. What velocity will car $B$ appear to have to an observer in car $A$?

($a$)  ($b$)  ($c$)  ($d$)  ($e$) 

## SOLUTION

Since $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$ we can draw the vector triangle and see

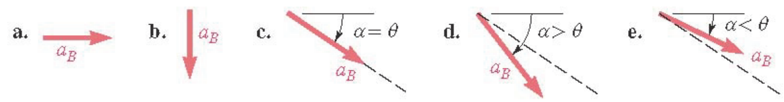


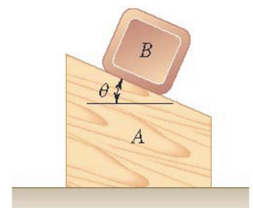

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

Answer: ($e$) ◀

## PROBLEM 11.CQ7

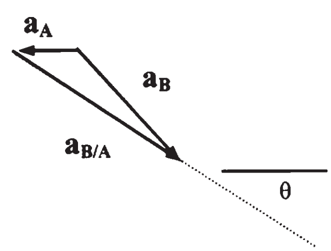
Blocks $A$ and $B$ are released from rest in the positions shown. Neglecting friction between all surfaces, which figure below best indicates the direction $\alpha$ of the acceleration of block $B$?



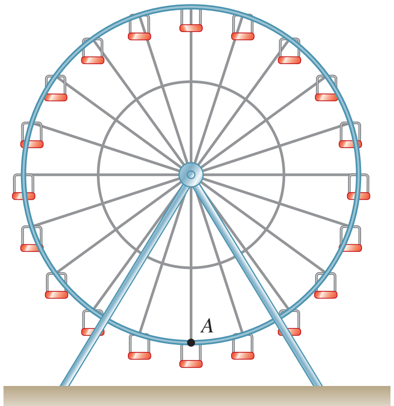

## SOLUTION

Since $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$ we get




Answer: ($d$) ◀

## PROBLEM 11.CQ8




The Ferris wheel is rotating with a constant angular velocity $\omega$. What is the direction of the acceleration of Point $A$?

($a$) →

($b$) ↑

($c$) ↓

($d$) ←

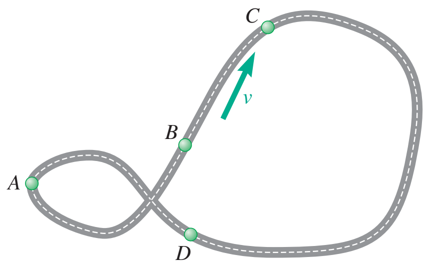
($e$) The acceleration is zero.

## SOLUTION

The tangential acceleration is zero since the speed is constant, so there will only be normal acceleration pointed upwards.

Answer: ($b$) ◀

## PROBLEM 11.CQ9




A racecar travels around the track shown at a constant speed. At which point will the racecar have the largest acceleration?

($a$) A

($b$) B

($c$) C

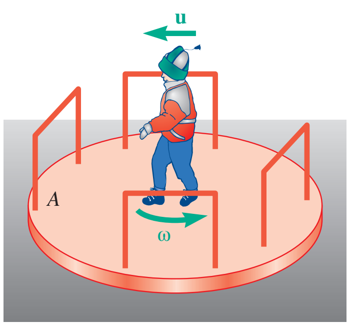
($d$) The acceleration will be zero at all the points.

## SOLUTION

The tangential acceleration is zero since the speed is constant, so there will only be normal acceleration. The normal acceleration will be maximum where the radius of curvature is a minimum that is at Point A.

Answer: ($a$) ◀

## PROBLEM 11.CQ10




A child walks across merry-go-round $A$ with a constant speed $u$ relative to $A$. The merry-go-round undergoes fixed axis rotation about its center with a constant angular velocity $\omega$ counterclockwise.When the child is at the center of $A$, as shown, what is the direction of his acceleration when viewed from above.

(*a*) $\longrightarrow$

(*b*) $\longleftarrow$

(*c*) $\uparrow$

(*d*) $\downarrow$

(*e*) The acceleration is zero.

## SOLUTION

Polar coordinates are most natural for this problem, that is,

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta \tag{1}$$

From the information given, we know $\ddot{r} = 0,\ \ddot{\theta} = 0,\ r = 0,\ \dot{\theta} = \omega,\ r = -u.$ When we substitute these values into (1), we will only have a term in the $-\theta$ direction.

Answer: (*d*) ◀