PH-219 Lab Report 4: Geometrical Optics

1 Purpose

The experiment's objective was to determine the focal lengths of two lenses using Bessel's method. A magnification was then derived from these focal lengths, a magnification which was verified with the use of a telescope.

2 Data

Table 1: Approximate Focal Lengths of Lenses

Lens	Focal Length(cm)	Instrumental Error (cm)
Short	5.5	± 0.05
Long	10	

Table 2: Image and Positions of Screen, Lens, and Source - Short Lens

Image	Screen Position (cm)	Lens Position (cm)	Source Position (cm)	Instrument Error (cm)
Small Image	124	116.2	97.7	± 0.07
Large Image	124	108.2	97.7	
D (distance between screen and source) = 26.3 cm				
d (difference between lens positions) = 8 cm				

Table 3: Image and Positions of Screen, Lens, and Source - Long Lens

Image	Screen Position (cm)	Lens Position (cm)	Source Position (cm)	Instrument Error (cm)
Small Image	124	109.9	74	± 0.07
Large Image	124	89.9	74	
D (distance between screen and source) = 50 cm				
d (difference between lens positions) = 20 cm				

Table 4: Lengths of Lines to test Magnification

Shorter Line	28 cm
Longer Line	49 cm

3 Explanation of Error

One key source of error is the difficulty in determining precisely when an object is in focus. This means the distance could've been read when the image was not precisely in focus, leading to a calculated focal length. Another possible source of error is the difficulty in verifying the line segments looked exactly of the same length through the lens. A third possible source of error is the thin lens approximation, as that could overestimate the approximate focal length of the lens.

4 Calculations

Focal Length:

$$\begin{aligned} f &= \frac{D^2 - d^2}{4D} \\ &= \frac{26.3cm^2 - 8.0cm^2}{4(23.6cm} \\ &= \frac{627.69}{105.2} \\ &= 5.96cm \end{aligned}$$

Linear Scale Error Propagation:

$$\begin{aligned} \delta\mathbf{D} &= \sqrt{(0.05)^2 + (0.05^2)} \\ &= 0.07cm \end{aligned}$$

Error Propagation of Focal Length:

$$\begin{aligned}
\delta\mathbf{f} &= |\frac{\partial f}{\partial D} * \delta D| + |\frac{\partial f}{\partial d} * \delta d| \\
&= |\frac{D^2 + d^2}{4D^2} * \delta D| + | - \frac{d}{2D} * \delta d| \\
&= |\frac{(26.3)^2 + (8.0)^2}{4(26.3)^2} * 0.07| + |\frac{(8.0)}{2(26.3)} * 0.25| \\
&= 0.06 \rightarrow 0.1cm
\end{aligned}$$

Magnification:

$$\begin{aligned}
m_\theta &= -\frac{f_o}{f_e} \\
&= -\frac{10.5}{5.966} \\
&= -1.76
\end{aligned}$$

Error Propagation of Calculated Angular Magnification:

$$\begin{aligned}
\delta\mathbf{m}_\theta &= \sqrt{(\frac{\partial m_\theta}{\partial f_{obj}} * \delta f_{obj})^2 + (\frac{\partial m_\theta}{\partial f_{eye}} * \delta f_{eye})^2} \\
&= \sqrt{(-\frac{1}{f_{eye}} * 0.03)^2 + (\frac{f_{obj}}{f_{eye}^2} * 0.03)^2} \\
&= \sqrt{(\frac{1}{5.9} * 0.03)^2 + (\frac{10.5}{5.9^2} * 0.03)^2} \\
&= 0.024 \rightarrow 0.02
\end{aligned}$$

Angular Magnification (Experiment):

$$\begin{aligned}
\mathbf{m_{exp}} &= -\frac{l_{short}}{l_{long}} \\
&= -\frac{49.0cm}{28.0cm} = -1.75
\end{aligned}$$

Error Propagation of Experimental Angular Magnification:

$$
\begin{aligned}
\delta \mathbf{m_{exp}} &= \sqrt{(\frac{\partial m_{exp}}{\partial l_{long}} * \delta l_{long})^2 + (\frac{\partial m_{exp}}{\partial l_{short}} * \delta l_{short})^2} \\
&= \sqrt{(-\frac{1}{l_{short}} * 0.07)^2 + (\frac{l_{long}}{l_{short}^2} * 0.07)^2} \\
&= \sqrt{(\frac{1}{28.0} * 0.07)^2 + (\frac{49.0}{28.0^2} * 0.07)^2} \\
&= 0.005 \rightarrow 0.01
\end{aligned}
$$

5 Results

Table 5: Part A: Calculated Focal Lengths of Lenses

Lens	Focal Length(cm)
Short	5.9 ± 0.1
Long	10.5 ± 0.1

Table 6: Part B: Calculated Angular Magnification

Angular Magnification, m_θ	-1.76 ± 0.02
Experimental Angular Magnification	-1.75 ± 0.01

6 Conclusion

Bessel's Method was used to calculate the focal length of two lenses, which were then compared to the approximate focal lengths. The short focal lens was calculated to have a focal length of 5.9cm with an error of 0.1cm, well away from the approximate focal length of 5.5cm, even with . The long lens was calculated to have a focal length of 10.5cm with an error of 0.1cm, also well off from the approximate focal length of 10cm.

7 Answers to Questions in the Write Up

Question 1: Derive this equation

$$f = \frac{D^2 - d^2}{4D}$$

and explain why we need the subsidiary condition of D>4f
Answer:

$$\begin{aligned}
D &= 2p + i \\
P &= \frac{D - d}{2} \\
i &= \frac{D + d}{2} \\
\frac{1}{f} &= \frac{2}{D - d} + \frac{2}{D + d} \\
\frac{1}{f} &= \frac{2(D + d) + 2(D - d)}{(D + d)(D - d)} \\
f &= \frac{(D + d)(D - d)}{2(D + d) + 2(D - d)} \\
f &= \frac{(D + d)(D - d)}{4D} \\
f &= \frac{D^2 - d^2}{4D}
\end{aligned}$$

If D is less than 4F, d becomes complex and gains an imaginary component, which is not possible in the real world

$$
\begin{aligned}
f &= \frac{D^2 - d^2}{4D} \\
4F &= D - \frac{d^2}{D} \\
\frac{d^2}{D} &= D - 4F \\
d^2 &= D(D - 4F) \\
d &= \sqrt{D(D - 4F)}
\end{aligned}
$$

Question 2: To get the approximate focal length of the lens we approximated the distance from the lens to the source to be infinite when it was actually finite (but large). Use the thin lens equation to show that this will slightly overestimate the focal length of the lens.
Answer:
If P approaches infinity, i approaches f. However, as P never actually reaches infinity, the focal length will be overestimated due to 1/P being small but not 0.

$$
\begin{aligned}
Theoretical: \lim_{p \to \infty} \frac{1}{p} + \frac{1}{i} &= \lim_{p \to \infty} \frac{1}{f} \\
Real: \frac{1}{p} + \frac{1}{i} &= \frac{1}{f} \\
\frac{i + p}{ip} &= \frac{1}{f} \\
\frac{ip}{i + p} &= f
\end{aligned}
$$