Lab # 1: Statistics of Measurement

A. Complete this:

YOUR NAME Neil Sowhey

PARTNER'S NAME (PARTNER'S SECTION) Shahreer Al Hossain

(Partners are optional Fall 2020)

PLEASE CHECK THE CIRCLE NEXT TO YOUR LAB SECTION:

- ◯ A, Prof Yecko, Mon 1–3 PM
- ◯ B, Prof Webb-Mack, Tue 2–4 PM
- ✓ C, Prof Yecko, Wed 11 AM–1 PM
- ◯ D, Prof Corn-Agostini, Thu 9 AM–11 AM
- ◯ E, Prof Webb-Mack, Tue 9–11 AM
- ◯ F, Prof Yecko, Fri 1 PM–3 PM

Ah so this is why you were late haha

B. Read and sign Academic Integrity Statement:

I hereby attest that I have not given or received any unauthorized assistance on this assignment.

Neil Sowhey

Sign here

C. Grading rubric:

CATEGORY AND VERY BRIEF GRADING COMMENTS...........	PTS AVAILABLE	PTS EARNED
Purpose	1	
Data	3	
Calculations	3	
Results	3	
Conclusion	1	
Answers	4	
Total	15	

Lab #1: Statistics of Measurement

Neil Sawhney

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Contents

1. Purpose

A degree of uncertainty is present in any all measurements. The purpose of this lab is to investigate the ways in which uncertainty propagates through calculations and become familiar with the many means of statistical reasoning. It was decided to investigate both larger and smaller samples, to experiment with a wider range of statistical analysis. For the smaller set of data, mean values with the appropriately propagated error was compared to determine if the sample was a specific alloy of aluminum. For the larger set of data, a t-test was performed to verify or reject a null hypothesis.

2. Data

2.1 Part A: Effect of instrumental uncertainty in the final result

Linear Scale

Right Reading (mm)	Left Reading (mm)	Diameter (mm)
60.9	10.0	50.9
70.9	20.0	50.9
65.6	15.0	50.6
71.7	21.0	50.7
80.9	30.5	50.4
87.6	37.0	50.6
Instrumental Error = $\pm$ 0.707		
Random Error = $\pm$ 0.19		
Mean = 50.683 ± 0.71		

Table 2.1: Diameter of the cylindrical disc

Right Reading (mm)	Left Reading (mm)	Thickness (mm)
13.1	10.0	3.1
24.2	21.0	3.2
35.1	32.0	3.1
53.9	50.5	3.4
76.4	73.0	3.4
83.1	80.0	3.1
Instrumental Error = $\pm$ 0.707		
Random Error = $\pm$ 0.15		
Mean = 3.22 ± 0.71		

Table 2.2: Thickness of the cylindrical disc

Vernier Caliper

Diameter (mm)	Corrected Diameter (mm)
51.10	51.10
51.06	51.06
51.02	51.02
50.82	50.82
51.10	51.10
51.18	51.18
Zero Error = 0.0	
Random Error = $\pm$ 0.12	
Instrumental Error = $\pm$ 0.014	
Mean = 51.05 ± 0.123	

Table 2.3: Diameter of the cylindrical disc

Thickness (mm)	Corrected Thickness (mm)
3.36	3.36
3.20	3.20
3.16	3.16
3.26	3.26
3.22	3.22
3.16	3.16
Zero Error = 0.0	
Random Error = $\pm$ 0.076	
Instrumental Error = $\pm$ 0.014	
Mean = 3.227 ± 0.076	

Table 2.4: Thickness of the cylindrical disc

2.2 Part B: Density of aluminum

Vernier Caliper

Length (mm)	Corrected Length (mm)
54.34	54.34
54.34	54.34
54.34	54.34
54.30	54.30
54.32	54.32
54.36	54.36
Zero Error = 0.0	
Instrumental Error = $\pm$ 0.014	
Random Error = $\pm$ 0.021	
Mean = 54.333 ± 0.021	

Table 2.5: Length of the rectangular object

Width (mm)	Corrected Width (mm)
33.26	33.26
33.28	33.28
33.26	33.26
33.26	33.26
33.22	33.22
33.26	33.26
Zero Error = 0.0	
Instrumental Error = $\pm$ 0.014	
Random Error = $\pm$ 0.020	
Mean = 33.26 ± 0.020	

Table 2.6: Width of the rectangular object

Micrometer

Thickness (mm)	Corrected Thickness (mm)
3.115	3.115
3.095	3.095
3.100	3.100
3.102	3.102
3.162	3.162
3.113	3.113
Zero Error = -0.008	
Instrumental Error = $\pm$ 0.0071	
Random Error = $\pm$ 0.025	
Mean = 3.11 ± 0.025	

Table 2.7: Thickness of the rectangular object

Thickness (mm)	Corrected Thickness (mm)
3.189	3.197
3.191	3.199
3.152	3.160
3.169	3.177
3.150	3.158
3.154	3.162
Zero Error = -0.008	
Instrumental Error = $\pm$ 0.0071	
Random Error = $\pm$ 0.019	
Mean = 3.176 ± 0.0187	

Table 2.8: Thickness of the cylindrical disc

Digital Balance

Mass (g)	Corrected Mass (g)
17.4	17.4
17.4	17.4
17.4	17.4
17.4	17.4
17.4	17.4
17.4	17.4
Zero Error = 0.0	
Instrumental Error = $\pm$ 0.05	
Random Error = $\pm$ 0	
Mean = 17.40 ± 0.050	

Table 2.9: Mass of the cylindrical disc

Mass (g)	Corrected Mass (g)
17.4	17.4
17.4	17.4
17.4	17.4
17.4	17.4
17.4	17.4
17.4	17.4
Zero Error = 0.0	
Instrumental Error = $\pm$ 0.05	
Random Error = $\pm$ 0	
Mean = 17.40 ± 0.050	

Table 2.10: Mass of the rectangular object

2.3 Part C: Normal distribution and hypothesis testing

Micrometer

Bottle 24		Bottle 7	
Length (mm)	Corrected Length (mm)	Length (mm)	Corrected Length (mm)
13.927	13.927	14.877	14.877
13.408	13.408	13.491	14.491
14.421	14.421	14.889	14.889
14.458	14.458	14.873	14.873
14.399	14.399	14.391	14.391
14.245	14.245	14.490	14.490
14.410	14.410	14.934	14.934
13.961	13.961	14.484	15.484
14.430	14.430	14.123	14.123
14.150	14.150	13.968	13.968
14.532	14.532	13.942	13.942
14.430	14.430	15.700	15.700
14.440	14.440	13.820	13.820
14.371	14.371	14.388	15.388
14.141	14.141	14.509	14.509
13.813	13.813	14.431	15.431
14.442	14.442	13.849	13.849
14.437	14.437	14.495	14.495
14.750	14.750	15.529	15.529
Zero Error = 0.0		Zero Error = 0.0	
Random Error = $\pm$ 0.19		Random Error = $\pm$ 0.19	
SDOM = $\pm$ 0.071		SDOM = $\pm$ 0.071	
Mean = 14.272 ± 0.071		Mean = 14.483 ± 0.130	

Table 2.11: Lengths of bottle contents

3. Calculations

3.1 Part A: Effect of instrumental uncertainty in the final result

3.1.1 Volume of rectangular disc measured with linear scale

$$Volume = \pi r^2 h \tag{3.1}$$

From Table 2.1 and 2.2:

$$Volume = \pi \left(\frac{Diameter}{2}\right)^2 (Thickness)$$

$$\bar{v} = \pi \left(\frac{50.683}{2}\right)^2 (3.22) = 6496.364\ mm^3$$

Propagation of uncertainty formula for independent error:

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2 ... \left(\frac{\partial f}{\partial z}\delta z\right)^2} \tag{3.2}$$

If we substitute from (3.1) into (3.2) we get:

$$\delta v = \sqrt{\left(\frac{\partial v}{\partial r}\delta r\right)^2 + \left(\frac{\partial v}{\partial h}\delta h\right)^2}$$

$$\delta v = \sqrt{(2\pi r h * \delta r)^2 + (\pi r^2 * \delta h)^2}$$

After plugging in values:

$$\delta v = \sqrt{\left(2\pi \frac{50.683}{2} 3.22 * \frac{0.71}{2}\right)^2 + \left(\pi \frac{50.683}{2}^2 * \frac{0.71}{2}\right)^2} = 738.979\ mm^3 \to 700\ mm^3$$

$$v = 6500 \pm 700 mm^3 \tag{3.3}$$

3.1.2 Volume of rectangular disc measured with vernier caliper

From Table 2.3 and 2.4

$$Volume = \pi \left(\frac{Diameter}{2}\right)^2 (Thickness)$$

$$\bar{v} = \pi \left(\frac{51.05}{2}\right)^2 (3.227) = 6605.114\ mm^3$$

If we substitute from (3.1) into (3.2) we get:

$$\delta v = \sqrt{\left(\frac{\partial v}{\partial r}\delta r\right)^2 + \left(\frac{\partial v}{\partial h}\delta h\right)^2}$$

$$\delta v = \sqrt{(2\pi rh * \delta r)^2 + (\pi r^2 * \delta h)^2}$$

After plugging in values:

$$\delta v = \sqrt{\left(2\pi\frac{51.05}{2}3.227 * \frac{0.123}{2}\right)^2 + \left(\pi\frac{51.05}{2}^2 * \frac{0.076}{2}\right)^2} = 84.039\ mm^3 \rightarrow 80\ mm^3$$

$$v = 6610 \pm 80 mm^3 \tag{3.4}$$

3.2 Part B: Density of aluminum

3.2.1 Density of rectangular disc

$$\rho = \frac{m}{v} = \frac{m}{\text{(Length)(Width)(Thickness)}} \tag{3.5}$$

From tables 2.9, 2.5, 2.6:

$$\rho = \frac{17.4\text{ g}}{(54.333\text{ mm})(33.26\text{ mm})(3.11\text{ mm})} = 3.0960 * 10^{-3}\frac{\text{g}}{\text{mm}^3}$$

If we sub (3.4) into (3.2) we get:

$$\delta\rho = \sqrt{\left(\frac{\partial\rho}{\partial m}\delta m\right)^2 + \left(\frac{\partial\rho}{\partial L}\delta L\right)^2 + \left(\frac{\partial\rho}{\partial W}\delta W\right)^2 + \left(\frac{\partial\rho}{\partial T}\delta T\right)^2}$$

now we solve partials:

$$\delta\rho = \sqrt{\left(\frac{1}{LWT}\delta m\right)^2 + \left(\frac{-m}{L^2WT}\delta L\right)^2 + \left(\frac{-m}{LW^2T}\delta W\right)^2 + \left(\frac{-m}{LWT^2}\delta T\right)^2}$$

After plugging in values:

$$\delta m = 0.050g\ \delta L = 0.021mm\ \delta W = 0.020mm\ \delta T = 0.025mm$$
$$m = 17.40g\ W = 33.26mm\ T = 3.11mm\ L = 54.333mm$$

$$\delta\rho = \sqrt{7.0344 * 10^{-10}\frac{g^2}{mm^9}} = 2.6522 * 10^{-5}\frac{g}{mm^3}$$
$$\rho = (3.096 \pm .027) * 10^{-3}\frac{\text{g}}{\text{mm}^3}$$

3.2.2 Density of cylindrical disc

$$\rho = \frac{m}{v} \tag{3.6}$$

From table 2.9 and equation (3.3):

$$\rho = \frac{17.4 \text{ g}}{6500mm^3} = 2.6778 * 10^{-3} \frac{\text{g}}{\text{mm}^3}$$

If we sub (3.5) into (3.2) we get:

$$\delta\rho = \sqrt{\left(\frac{\partial\rho}{\partial m}\delta m\right)^2 + \left(\frac{\partial\rho}{\partial v}\delta v\right)^2}$$

now we solve partials:

$$\delta\rho = \sqrt{\left(\frac{1}{v}\delta m\right)^2 + \left(\frac{-m}{v^2}\delta v\right)^2}$$

After plugging in values:

$$\delta m = 0.050g \ \delta V = 700mm^3$$

$$m = 17.40g \ v = 6500mm^3$$

$$\delta\rho = \sqrt{8.3167 * 10^{-8} \frac{g^2}{mm^9}} = 2.8839 * 10^{-4} \frac{g}{mm^3}$$

$$\rho = (2.7 \pm 0.3) * 10^{-3} \frac{\text{g}}{\text{mm}^3}$$

3.3 Part C: Normal distribution and hypothesis testing

From table 2.11

H_0: The average length of objects in Bottle 24, $\bar{x}$ will equal the average length of objects in Bottle 7, $\bar{y}$

$$\bar{x} = 14.272$$

$$\bar{y} = 14.483$$

Sample Variance:

$$S_x^2 = \frac{1}{N_x - 1}\sum_{i=1}^{N_x}(x_i - \bar{x})^2 = 0.31$$

$$S_y^2 = \frac{1}{N_y - 1}\sum_{i=1}^{N_y}(y_i - \bar{y})^2 = 0.57$$

T-Test:

$$t = \frac{|\bar{x} - \bar{y}|}{\sqrt{\sigma_{\bar{x}}^2 + \sigma_{\bar{y}}^2}}$$

First we need to find the standard deviation of the means:

$$\sigma_{\bar{x}} = \frac{\text{Sample Variance}}{\sqrt{\# \text{ of elements}}} = \frac{0.31}{\sqrt{19}} = 0.071 \ \sigma_{\bar{y}} = \frac{\text{Sample Variance}}{\sqrt{\# \text{ of elements}}} = \frac{0.57}{\sqrt{19}} = 0.13$$

$$t = \frac{|14.272 - 14.483|}{\sqrt{0.071^2 + 0.13^2}} = 1.42$$

Because the calculated t-value is less than 1.96, there exists a lower bound of 5% chance that the two mean values are unequal.

This verifies the H_0, because if we were to increase our sample size, the discrepancy of the means would approach 0.

3.4 Sample Calculations

$$\text{Corrected Measurement} = \text{Original Measurement} - \text{Zero Error}$$

$$\text{Instrumental Error} = \sqrt{\frac{(\text{Smallest instrument scale})}{2}^2 + \frac{(\text{Smallest instrument scale})}{2}^2}$$

$$\text{Random Error} = \text{Standard Deviation of the sample}$$

4. Results

4.1 Part A: Effect of instrumental uncertainty in the final result

The volume of the cylindrical object, as measured with the linear scale is:

$$v = 6500 \pm 700mm^3$$

The volume of the cylindrical object, as measured with the vernier caliper is:

$$v = 6610 \pm 80mm^3 \tag{4.1}$$

4.2 Part B: Density of aluminum

The Density of the rectangular object as measured with the digital scale, micrometer, and vernier calipers was:

$$\rho = (3.096 \pm .027) * 10^{-3} \frac{\mathrm{g}}{\mathrm{mm}^3}$$

The Density of the cylindrical object as measured with the digital scale, micrometer, and vernier calipers was:

$$\rho = (2.7 \pm 0.3) * 10^{-3} \frac{\mathrm{g}}{\mathrm{mm}^3}$$

As compared to the accepted density of the alloy, of approximately $2.7 * 10^{-3} g/mm^3$ the experimental values were within or reasonably close enough to the accepted value, as can be seen in Figure 4.1

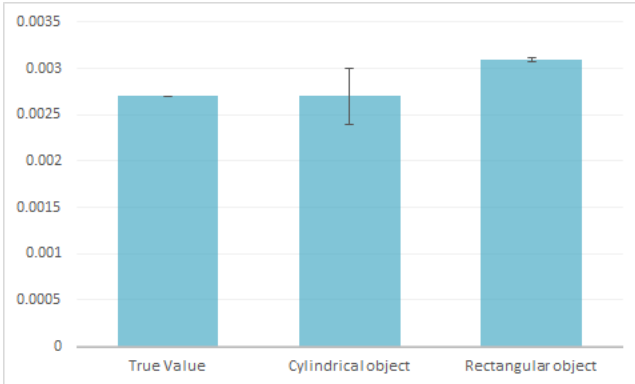

Figure 4.1: Denisty of accepted aluminum alloy 6061 as compared to experimental results

4.3 Part C: Normal distribution and hypothesis testing

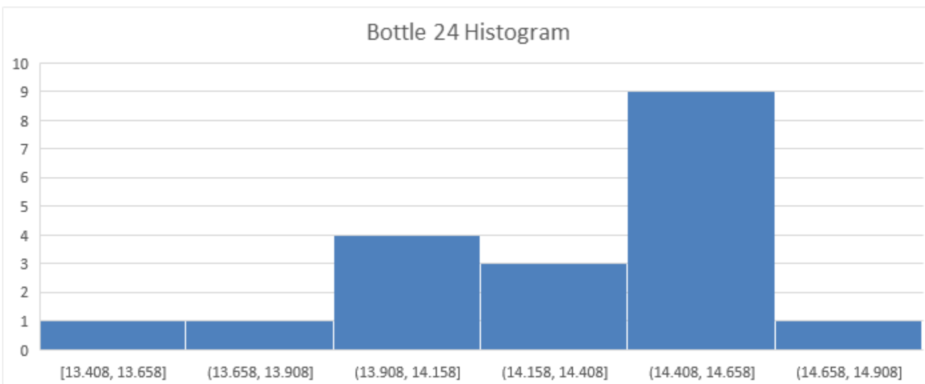

Figure 4.2: Lengths of bottle 24 contents, grouped with a bin width of 0.35 mm

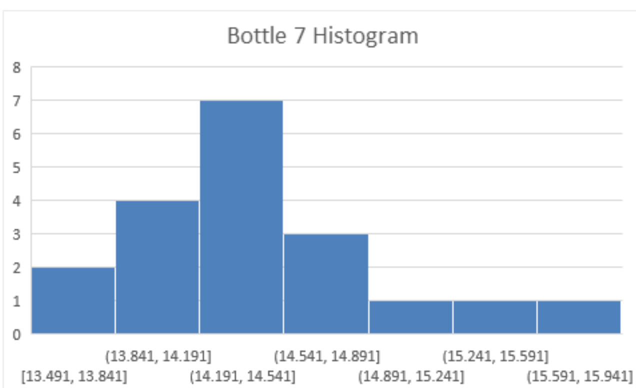

Figure 4.3: Lengths of bottle 7 contents, grouped with a bin width of 0.25 mm

Neither of the sets followed a normal distribution, as can be seen in Figure 4.2 and Figure 4.3. Bottle 24 was slightly shifted left, and Bottle 7 was slightly shifted right.

The null hypothesis was verified with a t-test value less than 1.96:

$$t = 1.42$$

Therefore, the Null hypothesis is supported with a lower bound of 5% significance.

5. Conclusions

Most of the data collected was accurate, and compared well to the expected values. However, the error bars in Figure 4.1 for the rectangular object don't quite extend far enough to encapsulate the true value. It does however overlap with the error bars for the cylindrical object. This could imply that there are some impurities in the metal, or that scratches, nicks, and chips, may have altered the distance measurements.

6. Answered Questions

6.0.1 Question 1

The better instrument did indeed correspond to a reduction in error proportional to the ratio of instrumental error

6.0.2 Question 2

in Figure 4.1 for the rectangular object don't quite extend far enough to encapsulate the true value. It does however overlap with the error bars for the cylindrical object. This could imply that there are some impurities in the metal, or that scratches, nicks, and chips, may have altered the distance measurements.

6.0.3 Question 3

Neither of the sets followed a normal distribution, as can be seen in Figure 4.2 and Figure 4.3. Bottle 24 was slightly shifted left, and Bottle 7 was slightly shifted right.

6.0.4 Question 4

The null hypothesis was verified with a t-test value less than 1.96:

$$t = 1.42$$

Therefore, the Null hypothesis is supported with a lower bound of 5% significance.