Lab # 2: Index of Refraction

A. Complete this:

YOUR NAME Neil Sawhney **Slot #** ________

PARTNER'S NAME Shahreef Al Hossain

PLEASE CHECK THE CIRCLE NEXT TO YOUR LAB SECTION:

- ◯ **A, Prof Yecko, Mon 1–3 PM**
- ◉ **C, Prof Yecko, Wed 10 AM–12**
- ◯ **E, Prof Webb-Mack, Tue 9–11 AM**
- ◯ **B, Prof Webb-Mack, Tue 2–4 PM**
- ◯ **D, Prof Corn-Agostini, Thu 9 AM–11 AM**
- ◯ **F, Prof Yecko, Fri 1 PM–3 PM**

B. Read and sign Academic Integrity Statement:

I hereby attest that I have not given or received any unauthorized assistance on this assignment.

Neil Sawhney

Sign here

C. Grading rubric:

CATEGORY AND VERY BRIEF GRADING COMMENTS...........	PTS AVAILABLE	PTS EARNED
Purpose	1	
Data	3	
Explanation of Errors	3	
Calculations	3	
Results	2	
Conclusion	1	
Answers	2	
Total	15	

Lab #2: Index of Refraction

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Contents

1. Purpose

If the refractive index of two mediums are known, the angle at which any incident light refracts at these boundaries can be calculated. The purpose of this lab is to determine the refractive index of Cooper Union's tap water using two separate methods, Pfund's method, and Snell's law.

2. Data

2.1 Part A: Pfund's Method

Measurement #	Thickness (mm)	Instrumental Error(...)	Random Error(...)
1	2.226	0.0071	0.0149
2	2.200		
3	2.181		
4	2.200		
5	2.210		
6	2.197		
Thickness =	2.202	+/-	0.015

Figure 2.1: Petri Dish Thickness as measured with a micrometer (mm)

Measurement #	radius	# of lines	Grid increment	Instrumental Error(...)	Random Error(...)
1	23.36	15	1.557	0.0141	0.0145
2	6.16	4	1.540		
3	17.14	11	1.558		
4	9.14	6	1.523		
5	12.30	8	1.538		
6	15.28	10	1.528		
Grid increment	1.54	+/-	0.015		

Figure 2.2: Polar Grid Increment as measured with a vernier caliper (mm)

Measurement #	# of rings	Instrumental Error(...)	Random Error(...)
1	4.0	0.7071	0.0816
2	4.1		
3	4.1		
4	3.9		
5	4.1		
6	4.0		
# of rings	4.0	+/-	0.707
Diameter =	6.214	+/-	0.710

Figure 2.3: Ring diameter without liquid (mm)

Measurement #	# of rings	Instrumental Error(...)	Random Error(...)
1	11.4	0.7071	0.1265
2	11.5		
3	11.2		
4	11.2		
5	11.2		
6	11.3		
# of rings	11.3	+/-	0.707
Diameter =	17.410	+/-	0.726

Figure 2.4: Ring diameter with liquid (mm)

2.2 Part B: Snell's Law

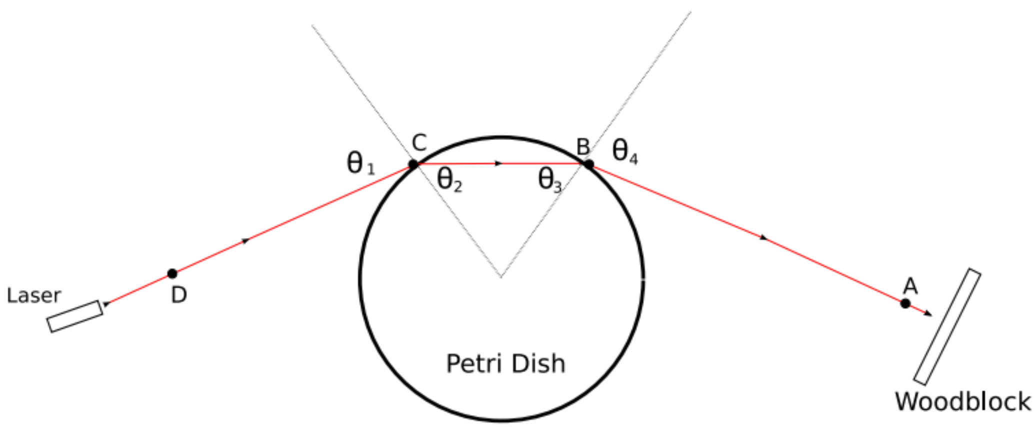

Figure 2.5: Snell's law angles reference

Trial #	Incident Angle $\theta1$	Refracted Angle $\theta2$	Incident Angle $\theta3$	Refracted Angle $\theta4$	Instrumental Error(...)
1	75.4	57	58	76	0.7071
2	35.9	18.2	17.2	37.7	
3	48.5	31.2	31.5	41.5	

Figure 2.6: Snell's law measured angles (degrees)

3. Calculations

3.1 Part A: Pfund's Method

3.1.1 Diameter of ring sample calculation

The diameter from table 2.4 and 2.3 is the product of the interpolated number of rings and the grid increment from table 2.2

$$\text{Diameter error} = \sqrt{\left(\frac{\text{instrumental error calipers}}{Diameter}\right)^2 + \frac{\text{random error grid increment}}{\text{Grid Increment}}^2} * \text{number of rings}$$

$$\text{Diameter error} = \sqrt{\left(\frac{0.707}{17.410}\right)^2 + \frac{0.015}{1.54}^2} * 17.410 = 0.726$$

3.1.2 Derivation of n_{glass}

Question1:

$$n_1 sin\theta_1 = n_2 sin\theta_2$$
$$\frac{sin\theta_1}{sin\theta_2} = \frac{n_2}{n_1}$$

$\theta_1 = \theta_c$ when $\theta_2 = 90$
index of refraction of air $= n_2 = 1$

$$sin\theta_c = \frac{1}{n_{glass}} = \frac{d}{4} * \frac{1}{\sqrt{(\frac{d}{4})^2 + t^2}}$$

$$\frac{1}{n_{glass}} = \frac{d}{4} * \frac{1}{\sqrt{(\frac{d^2}{16}) + t^2}} = \frac{d}{\sqrt{16 * ((\frac{d^2}{16}) + t^2)}}$$

$$n_{glass} = \frac{\sqrt{d^2 + 16t^2}}{d} \tag{3.1}$$

3.1.3 Index of refraction of the glass

From Eq 3.1

$$n_{glass} = \frac{\sqrt{d^2 + 16t^2}}{d}$$

From Table 2.1 and 2.3:

$$n_{glass} = \frac{\sqrt{6.214^2 + 16 * 2.202^2}}{6.214} = 1.735$$

Propagation of uncertainty formula for independent error:

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2 ... \left(\frac{\partial f}{\partial z}\delta z\right)^2} \tag{3.2}$$

If we substitute from 3.1 into 3.2 we get:

$$\delta n_{glass} = \sqrt{\left(\frac{\partial n_{glass}}{\partial d}\delta d\right)^2 + \left(\frac{\partial n_{glass}}{\partial t}\delta t\right)^2}$$

$$\delta n_{glass} = \sqrt{\left(-\frac{16 * t^2}{d^2\sqrt{d^2 + 16t^2}} * \delta d\right)^2 + \left(\frac{16 * t}{d\sqrt{d^2 + 16t^2}} * \delta t\right)^2}$$

After plugging in values from 2.1, and 2.3:

$$\delta n_{glass} = \sqrt{\left(-\frac{16 * 2.202^2}{6.214^2\sqrt{6.214^2 + 16 * 2.202^2}} * 0.710\right)^2 + \left(\frac{16 * 2.202}{6.214\sqrt{6.214^2 + 16 * 2.202^2}} * 0.015\right)^2} = 0.133$$

$$n_{glass} = 1.74 \pm 0.13 \tag{3.3}$$

3.1.4 Derivation of n_{liquid}

Question2:

$$n_1 sin\theta_1 = n_2 sin\theta_2$$

$$\frac{sin\theta_1}{sin\theta_2} = \frac{n_2}{n_1}$$

$\theta_1 = \theta_c$ when $\theta_2 = 90$
$n_2 = n_{glass}$

$$sin\theta_c = \frac{n_{liquid}}{n_{glass}} = \frac{d}{\sqrt{d^2 + 16t^2}}$$

$$n_{liquid} = \frac{n_{glass}d}{\sqrt{d^2 + 16t^2}} \tag{3.4}$$

Question4:
It's important to note that Pfund's method wont work with liquids of all n. If the following is true:

$$\left|\frac{n_{liquid}}{n_{glass}}\right| \geq 1$$

θ_c does not exist, and therefore total internal reflection will not occur and so a second ring will not form.

3.1.5 Index of refraction of the liquid

From Eq 3.4

$$n_{liquid} = \frac{n_{glass} d}{\sqrt{d^2 + 16t^2}}$$

From Table 2.1 and 2.4 and eq 3.1:

$$n_{liquid} = \frac{1.735 * 17.410}{\sqrt{17.410^2 + 16 * 2.202^2}} = 1.548$$

If we substitute from 3.4 into 3.2 we get:

$$\delta n_{liquid} = \sqrt{\left(\frac{\partial n_{liquid}}{\partial d}\delta d\right)^2 + \left(\frac{\partial n_{liquid}}{\partial t}\delta t\right)^2 + \left(\frac{\partial n_{liquid}}{\partial n_{glass}}\delta n_{glass}\right)^2}$$

$$\delta n_{liquid} = \sqrt{\left(\frac{16 * n_{glass} * t^2}{(d^2 + 16 * t^2)\sqrt{d^2 + 16 * t^2}} * \delta d\right)^2 + \left(\frac{-16 * n_{glass} * t * d}{(d^2 + 16 * t^2)^{3/2}} * \delta t\right)^2 + \left(\frac{d}{\sqrt{d^2 + 16 * t^2}} * \delta n_{glass}\right)^2}$$

After plugging in values from 3.1, 2.1, and 2.4:

$$\delta n_{liquid} = 0.119$$

$$n_{liquid} = 1.55 \pm 0.12 \tag{3.5}$$

3.2 Part B: Snell's Law

3.2.1 Index of refraction of liquid general formula

$$n_1 sin\theta_1 = n_2 sin\theta_2$$

$$n_2 = \frac{n_1 * sin\theta_1}{sin\theta_2}$$

Assuming the index of refraction of air is 1:

$$n_2 = \frac{sin\theta_1}{sin\theta_2} \tag{3.6}$$

If we sub 3.6 into 3.2 we get:

$$\delta n_2 = \sqrt{\left(\frac{\partial n_2}{\partial \theta_1}\delta\theta_1\right)^2 + \left(\frac{\partial n_2}{\partial \theta_2}\delta\theta_2\right)^2}$$

now we solve partials:

$$\delta n_2 = \sqrt{(cos\theta_1 csc\theta_2 * \delta\theta_1)^2 + \left(\frac{-sin\theta_1 cot\theta_2}{sin\theta_2}\delta\theta_2\right)^2} \tag{3.7}$$

3.2.2 Index of refraction of liquid for trial 1

After plugging in theta 1 and 2 from 2.6 into 3.6 and 3.7:

$$n_2 = \frac{sin(75.4)}{sin(57)} = 1.154$$

$$\delta n_2 = \sqrt{(cos(75.4)csc(57) * (0.707))^2 + \left(\frac{-sin(75.4)cot(57)}{sin(57)}(0.707)\right)^2} = 0.571$$

After plugging in theta 4 and 3 from 2.6 into 3.6 and 3.7:

$$n_2 = 1.144$$

$$\delta n_2 = 0.544$$

3.2.3 Index of refraction of liquid for trial 2

After plugging in theta 1 and 2 from 2.6 into 3.6 and 3.7:

$$n_2 = 1.877$$

$$\delta n_2 = 4.434$$

After plugging in theta 4 and 3 from 2.6 into 3.6 and 3.7:

$$n_2 = 2.068$$

$$\delta n_2 = 5.087$$

3.2.4 Index of refraction of liquid for trial 3

After plugging in theta 1 and 2 from 2.6 into 3.6 and 3.7:

$$n_2 = 1.446$$

$$\delta n_2 = 1.915$$

After plugging in theta 4 and 3 from 2.6 into 3.6 and 3.7:

$$n_2 = 1.268$$

$$\delta n_2 = 1.780$$

3.2.5 Index of refraction of liquid average

$$n_2 = \frac{1.154 + 1.144 + 1.877 + 2.068 + 1.446 + 1.268}{6} = 1.492$$

$$\delta n_2 = \frac{1}{6} * \sqrt{0.571^2 + 0.544^2 + 4.434^2 + 5.087^2 + 1.915^2 + 1.780^2} = 1.213$$

$$n_{liquid} = 1.5 \pm 1.2 \tag{3.8}$$

4. Analysis and Results

4.1 Part A: Index of refraction of liquid using Pfund's Method

The index of refraction of the liquid found using Pfund's method is

$$v = 1.55 \pm 0.12mm^3$$

4.2 Part B: Index of refraction of liquid using Snell's Law

The index of refraction of the liquid found using Snell's law is

$$v = 1.5 \pm 1.2mm^3$$

For the three trials, each trial produced two values for the index of refraction of the liquid, for a total of 6 snell values. As can be seen in figure 4.1, All the values agreed with each other. However, there was likely some systematic error such as a shift in the petri dish or laser in trial 2, evident by the abnormally large error.

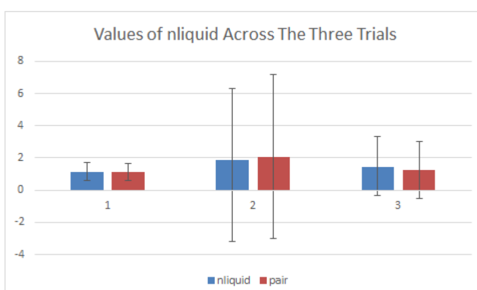

Figure 4.1: Experimental snell values compared

4.3 Part C: Index of refraction compared between Snell's Law, Pfund's method, and literature values

As can be seen in figure 4.2, the value for pfunds method agrees with that of Snells, but doesn't agree with that of the literature value.

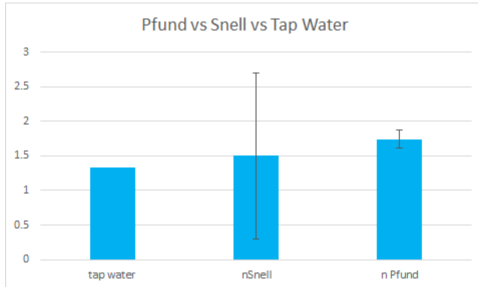

Figure 4.2: Pfund, Snell, and Literature values compared

5. Conclusions

While the value found using Pfund's method did agree with the value found using Snell's, Snell's error bounds are so large that it's an insignificant bench mark. Part of the reason Snell's error bounds are so large is because of the imperfectness of the laser pointer. Instead of a small solid dot, the laser pointer produced a large blurry strip, which got longer as the angle of incidence was increased. This made it almost impossible to precisely place the pins. In addition, the placing of the pins introduced a lot of error that wasn't accounted for since no precise measurement device was used to ensure the pins were perfectly vertical. Aside from these sources of error, the main cause was likely a systematic error such as a shift in the petri dish when measuring as evident by the abnormally large error in specifically trial 2.

Question3:
Snell's law also assumed that the refractive index of the petri dish was perfectly 1, which may not have been the case. Pfunds method was fairly precise, but not accurate to the literature values. It's possible that the tap water used was not perfectly clear or that the refractive index of the petri dish was not uniform.

6. Answered Questions

Click under each question to be brought to the location where the respective question is answered.

6.0.1 Question 1

derivation of n_{glass}

6.0.2 Question 2

derivation of n_{liquid}

6.0.3 Question 3

assumptions and approximations about Snell's law

6.0.4 Question 4

limitation of calculable n values when using Pfund's method