Computer Engineering Technology – Computing Science

Course: Numerical Computing – CST8233
Term: Fall 2021

Assignment #3

Due Date: Friday, December 10th, 2021 before 11:59 pm.
Earning: This assignment worth 8% of your final course mark.

The objective of this assignment is to learn how to use menus to prompt users with different options. Also, you will learn how to solve first order Ordinary Differential Equations (ODE) using well known methods; namely, Euler's and Runge-Kutta 4th Order Methods.

Submission:

Please submit ONE file that contains R script.

Task:

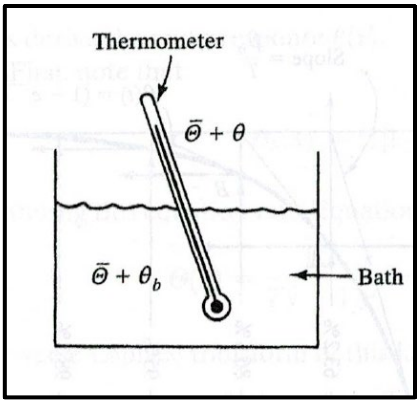
For the thin, glass-walled mercury thermometer system shown in Figure 1, assume that the temperature of the bath changes based on certain chemical process occurring between two substances reacting with each other inside the bath. It is found that the equation that describes this process is given as follows:

$$\frac{d\theta(t)}{dt} + 2\theta(t) = \cos 4t$$

It can be found that the actual solution of the response of the thermometer, $\theta(t)$, is given by the following equation:

$$\theta(t) = 0.1 \cos 4t + 0.2 \sin 4t + 2.9\, e^{-2t}$$

The ODE given above can be solved using many numerical methods, such as Euler's and Runge-Kutta 2nd Order Methods.



1. Write an R program that computes the solution $\theta(t)$ using Euler's Method. For this step, use the following information: $h = 0.8, 0.2, 0.05$, $\theta_0 = 3$ ℃, $0 \leq t \leq 2$ second. Find the discrete values of $\theta(t)$ at each h step value.

2. Modify the previous code and implement Runge-Kutta 4th method to solve the same ODE using the following information: $h = 0.8, 0.2, 0.05$, $\theta_0 = 3$ ℃, $0 \leq t \leq 2$ second. Find the discrete values of $\theta(t)$ at each h step value.

3. Calculate the relative error of the resultant solution at each time for each h step. Your output of your code should show a table that shows the exact temperature, the estimated temperature, and the relative error. The user will choose one method and one step size.

Example Output

The output of the code should look like below. The results of test case when using Euler's and Runge-Kutta for $h = 0.2$ are shown in the table below.

```
>> Choose the method for solving the ODE:
1. Euler's Method
2. Runge-Kutta 4th Order Method

>> 1
>> Choose step size "h" (0.8, 0.2, 0.05)
>> 0.2

Time(second)    Exact Temp(C)   Estimated Temp(C)      Percentage Error(%)
0.2             2.157           2.000                  7.28
0.4             1.500           1.339                  10.71
0.6             0.935           0.798                  14.66
0.8             0.474           0.331                  30.13
1.0             0.176           -0.001                 100.54
1.2             0.073           -0.131                 280.86
1.4             0.128           -0.061                 148.01
1.6             0.241           0.118                  50.86
1.8             0.299           0.270                  9.76
2.0             0.236           0.283                  19.89
```

```
>> Choose the method for solving the ODE:
1. Euler’s Method
2. Runge-Kutta 4th Order Method

>> 2
>> Choose step size “h” (0.8, 0.2, 0.05)
>> 0.2
```

Time(second)	Exact Temp(C)	Estimated Temp(C)	Percentage Error(%)
0.2	2.157	2.157	0.01
0.4	1.500	1.500	0.02
0.6	0.935	0.935	0.03
0.8	0.474	0.474	0.06
1.0	0.176	0.176	0.15
1.2	0.073	0.073	0.34
1.4	0.128	0.128	0.15
1.6	0.241	0.241	0.05
1.8	0.299	0.299	0.02
2.0	0.236	0.236	0.00

```
>> Choose the method for solving the ODE:

1. Euler’s Method
2. Runge-Kutta 4th Order Method
```