

17.

因为 $\text{var}(X) = \text{var}(Y)$, 所以

$$\begin{aligned}
 \text{cov}(X+Y, X-Y) &= \text{cov}(X+Y, X) - \text{cov}(X+Y, Y) \\
 &= \text{cov}(X, X) + \text{cov}(Y, X) - \text{cov}(X, Y) - \text{cov}(Y, Y) \\
 &= \text{cov}(X, X) - \text{cov}(Y, Y) \\
 &= \text{var}(X) - \text{var}(Y) \\
 &= 0
 \end{aligned}$$

所以 $X+Y, X-Y$ 不相关。

证毕。

18.

由题意得: $\text{cov}(W, X) = \text{cov}(W, Y) = \text{cov}(W, Z) = \text{cov}(X, Y) = \text{cov}(X, Z) = \text{cov}(Y, Z) = 0$, 所以:

$$\begin{aligned}
 \rho(R, S) &= \frac{\text{cov}(R, S)}{\sqrt{\text{var}(R)\text{var}(S)}} \\
 &= \frac{\text{cov}(W+X, X+Y)}{\sqrt{\text{var}(W+X)\text{var}(X+Y)}} \\
 &= \frac{\text{cov}(W, X) + \text{cov}(W, Y) + \text{cov}(X, X) + \text{cov}(X, Y)}{\sqrt{(\text{var}(W) + \text{var}(X) + 2\text{cov}(W, X))(\text{var}(X) + \text{var}(Y) + 2\text{cov}(X, Y))}} \\
 &= \frac{\text{var}(X)}{\sqrt{(\text{var}(W) + \text{var}(X))(\text{var}(X) + \text{var}(Y))}} \\
 &= \frac{1}{2}
 \end{aligned}$$

$$\text{同理 } \rho(S, T) = \frac{1}{2}$$

19.

$$\begin{aligned}
 \rho(X, Y) &= \frac{\text{cov}(X, Y)}{\sqrt{\text{var}(X)\text{var}(Y)}} \\
 &= \frac{\text{cov}(X, a+bX+cX^2)}{\sqrt{\text{var}(X)\text{var}(a+bX+cX^2)}} \\
 &= \frac{\text{cov}(X, bX) + \text{cov}(X, cX^2)}{\sqrt{\text{var}(X)(\text{var}(a+bX) + \text{var}(cX^2) + 2\text{cov}(a+bX, cX^2))}} \\
 &= \frac{b \times \text{var}(X) + c \times \text{cov}(X, X^2)}{\sqrt{\text{var}(X)(a^2 \text{var}(X) + c^2 \text{var}(X^2) + 2bc \times \text{cov}(X, X^2))}} \\
 &= \frac{b}{\sqrt{a+2c^2}}
 \end{aligned}$$

20.

略。

21.

略。