BRITISH STANDARD

BS ISO 80000-2:2009

Quantities and units

Part 2: Mathematical signs and symbols to be used in the natural sciences and technology

ICS 01.060

British Standards

National foreword

This British Standard is the UK implementation of ISO 80000-2:2009. It supersedes BS ISO 31-11:1992 which is withdrawn.

The UK participation in its preparation was entrusted to Technical Committee SS/7, General metrology, quantities, units and symbols.

A list of organizations represented on this committee can be obtained on request to its secretary.

This publication does not purport to include all the necessary provisions of a contract. Users are responsible for its correct application.

Compliance with a British Standard cannot confer immunity from legal obligations.

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Part 2:
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Grandeurs et unités —

Partie 2: Signes et symboles mathématiques à employer dans les sciences de la nature et dans la technique

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 80000-2 was prepared by Technical Committee ISO/TC 12, *Quantities and units*, in collaboration with IEC/TC 25, *Quantities and units*.

This first edition cancels and replaces ISO 31-11:1992, which has been technically revised. The major technical changes from the previous standard are the following:

— Four clauses have been added, i.e. “Standard number sets and intervals”, “Elementary geometry”, “Combinatorics” and “Transforms”.

ISO 80000 consists of the following parts, under the general title *Quantities and units*:

— *Part 1: General*

— *Part 2: Mathematical signs and symbols to be used in the natural sciences and technology*[1)]

— *Part 3: Space and time*

— *Part 4: Mechanics*

— *Part 5: Thermodynamics*

— *Part 7: Light*

— *Part 8: Acoustics*

— *Part 9: Physical chemistry and molecular physics*

— *Part 10: Atomic and nuclear physics*

— *Part 11: Characteristic numbers*

— *Part 12: Solid state physics*

1) Title to be shortened to read “Mathematics” in the second edition of ISO 80000-2.

IEC 80000 consists of the following parts, under the general title *Quantities and units*:

— *Part 6: Electromagnetism*

— *Part 13: Information science and technology*

— *Part 14: Telebiometrics related to human physiology*

Introduction

Arrangement of the tables

The first column "Item No." of the tables contains the number of the item, followed by either the number of the corresponding item in ISO 31-11 in parentheses, or a dash when the item in question did not appear in ISO 31-11.

The second column "Sign, symbol, expression" gives the sign or symbol under consideration, usually in the context of a typical expression. If more than one sign, symbol or expression is given for the same item, they are on an equal footing. In some cases, e.g. for exponentiation, there is only a typical expression and no symbol.

The third column "Meaning, verbal equivalent" gives a hint on the meaning or how the expression may be read. This is for the identification of the concept and is not intended to be a complete mathematical definition.

The fourth column "Remarks and examples" gives further information. Definitions are given if they are short enough to fit into the column. Definitions need not be mathematically complete.

The arrangement of the table in Clause 16 "Coordinate systems" is somewhat different.

Quantities and units —

Part 2: Mathematical signs and symbols to be used in the natural sciences and technology

1 Scope

ISO 80000-2 gives general information about mathematical signs and symbols, their meanings, verbal equivalents and applications.

The recommendations in ISO 80000-2 are intended mainly for use in the natural sciences and technology, but also apply to other areas where mathematics is used.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 80000-1:—[2] , *Quantities and units — Part 1: General*

3 Variables, functions and operators

Variables such as x, y, etc., and running numbers, such as i in $\Sigma_i\ x_i$ are printed in italic (sloping) type. Parameters, such as a, b, etc., which may be considered as constant in a particular context, are printed in italic (sloping) type. The same applies to functions in general, e.g. f, g.

An explicitly defined function not depending on the context is, however, printed in Roman (upright) type, e.g. sin, exp, ln, Γ. Mathematical constants, the values of which never change, are printed in Roman (upright) type, e.g. $\mathrm{e} = 2{,}718\ 218\ 8\ldots$; $\pi = 3{,}141\ 592\ldots$; $\mathrm{i}^2 = -1$. Well-defined operators are also printed in Roman (upright) style, e.g. div, δ in δx and each d in $\mathrm{d}f/\mathrm{d}x$.

Numbers expressed in the form of digits are always printed in Roman (upright) style, e.g. 351 204; 1,32; 7/8.

The argument of a function is written in parentheses after the symbol for the function, without a space between the symbol for the function and the first parenthesis, e.g. $f(x)$, $\cos(\omega t + \varphi)$. If the symbol for the function consists of two or more letters and the argument contains no operation symbol, such as +, −, ×, · or / , the parentheses around the argument may be omitted. In these cases, there should be a thin space between the symbol for the function and the argument, e.g. int 2,4; sin $n\pi$; arcosh $2A$; Ei x.

If there is any risk of confusion, parentheses should always be inserted. For example, write $\cos(x) + y$; do not write $\cos\ x + y$, which could be mistaken for $\cos(x + y)$.

2) To be published. (Revision of ISO 31-0:1992)

A comma, semicolon or other appropriate symbol can be used as a separator between numbers or expressions. The comma is generally preferred, except when numbers with a decimal comma are used.

If an expression or equation must be split into two or more lines, one of the following methods shall be used.

a) Place the line breaks immediately after one of the symbols $=$, $+$, $-$, $\pm$ or $\mp$, or, if necessary, immediately after one of the symbols $\times$, $\cdot$, or $/$. In this case, the symbol indicates that the expression continues on the next line or next page.

b) Place the line breaks immediately before one of the symbols $=$, $+$, $-$, $\pm$ or $\mp$, or, if necessary, immediately before one of the symbols $\times$, $\cdot$, or $/$. In this case, the symbol indicates that the expression is a continuation of the previous line or page.

The symbol shall not be given twice around the line break; two minus signs could for example give rise to sign errors. Only one of these methods should be used in one document. If possible, the line break should not be inside of an expression in parentheses.

It is customary to use different sorts of letters for different sorts of entities. This makes formulas more readable and helps in setting up an appropriate context. There are no strict rules for the use of letter fonts which should, however, be explained if necessary.

4 Mathematical logic

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-4.1 (*11-3.1*)	$p \wedge q$	conjunction of p and q, p and q	
2-4.2 (*11-3.2*)	$p \vee q$	disjunction of p and q, p or q	This “or” is inclusive, i.e. $p \vee q$ is true, if either p or q, or both are true.
2-4.3 (*11-3.3*)	$\neg p$	negation of p, not p	
2-4.4 (*11-3.4*)	$p \Rightarrow q$	p implies q, if p, then q	$q \Leftarrow p$ has the same meaning as $p \Rightarrow q$. $\Rightarrow$ is the implication symbol.
2-4.5 (*11-3.5*)	$p \Leftrightarrow q$	p is equivalent to q	$(p \Rightarrow q) \wedge (q \Rightarrow p)$ has the same meaning as $p \Leftrightarrow q$. $\Leftrightarrow$ is the equivalence symbol.
2-4.6 (*11-3.6*)	$\forall x \in A \quad p(x)$	for every x belonging to A, the proposition $p(x)$ is true	If it is clear from the context which set A is being considered, the notation $\forall x\ p(x)$ can be used. $\forall$ is the universal quantifier. For $x \in A$, see 2-5.1.
2-4.7 (*11-3.7*)	$\exists x \in A \quad p(x)$	there exists an x belonging to A for which $p(x)$ is true	If it is clear from the context which set A is being considered, the notation $\exists x\ p(x)$ can be used. $\exists$ is the existential quantifier. For $x \in A$, see 2-5.1. $\exists^1 x\ p(x)$ is used to indicate that there is exactly one element for which $p(x)$ is true. $\exists!$ is also used for $\exists^1$.

5 Sets

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-5.1 (*11-4.1*)	$x \in A$	x belongs to A, x is an element of the set A	$A \ni x$ has the same meaning as $x \in A$.
2-5.2 (*11-4.2*)	$y \notin A$	y does not belong to A, y is not an element of the set A	$A \not\ni y$ has the same meaning as $y \notin A$. The negating stroke may also be vertical.
2-5.3 (*11-4.5*)	$\{x_1, x_2, \ldots, x_n\}$	set with elements $x_1, x_2, \ldots, x_n$	Also $\{x_i \mid i \in I\}$, where I denotes a set of subscripts.
2-5.4 (*11-4.6*)	$\{x \in A \mid p(x)\}$	set of those elements of A for which the proposition $p(x)$ is true	EXAMPLE $\{x \in \mathbf{R} \mid x \leqslant 5\}$ If it is clear from the context which set A is being considered, the notation $\{x \mid p(x)\}$ can be used (for example $\{x \mid x \leqslant 5\}$, if it is clear that x is a variable for real numbers).
2-5.5 (*11-4.7*)	card A $\lvert A \rvert$	number of elements in A, cardinality of A	The cardinality can be a transfinite number. See also 2-9.16.
2-5.6 (*11-4.8*)	$\varnothing$	the empty set	
2-5.7 (*11-4.18*)	$B \subseteq A$	B is included in A, B is a subset of A	Every element of B belongs to A. $\subset$ is also used, but see remark to 2-5.8. $A \supseteq B$ has the same meaning as $B \subseteq A$.
2-5.8 (*11-4.19*)	$B \subset A$	B is properly included in A, B is a proper subset of A	Every element of B belongs to A, but at least one element of A does not belong to B. If $\subset$ is used for 2-5.7, then $\subsetneq$ shall be used for 2-5.8. $A \supset B$ has the same meaning as $B \subset A$.
2-5.9 (*11-4.24*)	$A \cup B$	union of A and B	The set of elements which belong to A or to B or to both A and B. $A \cup B = \{x \mid x \in A \vee x \in B\}$
2-5.10 (*11-4.26*)	$A \cap B$	intersection of A and B	The set of elements which belong to both A and B. $A \cap B = \{x \mid x \in A \wedge x \in B\}$
2-5.11 (*11-4.25*)	$\bigcup\limits_{i=1}^{n} A_i$ $A_1 \cup A_2 \cup \ldots \cup A_n$	union of the sets $A_1, A_2, \ldots, A_n$	The set of elements belonging to at least one of the sets $A_1, A_2, \ldots, A_n$ $\bigcup_{i=1}^{n}$, $\bigcup\limits_{i \in I}$ and $\bigcup_{i \in I}$ are also used, where I denotes a set of subscripts.
2-5.12 (*11-4.27*)	$\bigcap\limits_{i=1}^{n} A_i$ $A_1 \cap A_2 \cap \ldots \cap A_n$	intersection of the sets $A_1, \ldots, A_n$	The set of elements belonging to all sets $A_1, A_2, \ldots, A_n$ $\bigcap_{i=1}^{n}$, $\bigcap\limits_{i \in I}$ and $\bigcap_{i \in I}$ are also used, where I denotes a set of subscripts.

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-5.13 (*11-4.28*)	$A \setminus B$	difference of A and B, A minus B	The set of elements which belong to A but not to B. $A \setminus B = \{x \mid x \in A \wedge x \notin B\}$ $A - B$ should not be used. $\complement_A B$ is also used. $\complement_A B$ is mainly used when B is a subset of A, and the symbol A may be omitted if it is clear from the context which set A is being considered.
2-5.14 (*11-4.30*)	(a, b)	ordered pair a, b, couple a, b	$(a, b) = (c, d)$ if and only if $a = c$ and $b = d$. If the comma can be mistaken as the decimal sign, then the semicolon (;) or a stroke ($\vert$) may be used as separator.
2-5.15 (*11-4.31*)	$(a_1, a_2, \ldots, a_n)$	ordered n-tuple	See remark to 2-5.14.
2-5.16 (*11-4.32*)	$A \times B$	Cartesian product of the sets A and B	The set of ordered pairs (a, b) such that $a \in A$ and $b \in B$. $A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$
2-5.17 (—)	$\prod_{i=1}^{n} A_i$ $A_1 \times A_2 \times \ldots \times A_n$	Cartesian product of the sets $A_1, A_2, \ldots, A_n$	The set of ordered n-tuples $(x_1, x_2, \ldots, x_n)$ such that $x_1 \in A_1, x_2 \in A_2, \ldots, x_n \in A_n$. $A \times A \times \ldots \times A$ is denoted by A^n, where n is the number of factors in the product.
2-5.18 (*11-4.33*)	id_A	identity relation on A, diagonal of $A \times A$	id_A is the set of all pairs (x, x) where $x \in A$. If the set A is clear from the context, the subscript A can be omitted.

6 Standard number sets and intervals

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-6.1 (*11.4.9*)	$\mathbf{N}$	the set of natural numbers, the set of positive integers and zero	$\mathbf{N} = \{0, 1, 2, 3, \ldots\}$ $\mathbf{N}^* = \{1, 2, 3, \ldots\}$ Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{N}_{>5} = \{n \in \mathbf{N} \mid n > 5\}$ The symbols N and $\mathbb{N}$ are also used.
2-6.2 (*11.4.10*)	$\mathbf{Z}$	the set of integers	$\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ $\mathbf{Z}^* = \{n \in \mathbf{Z} \mid n \neq 0\}$ Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{Z}_{\geqslant -3} = \{n \in \mathbf{Z} \mid n \geqslant -3\}$ The symbol $\mathbb{Z}$ is also used.
2-6.3 (*11.4.11*)	$\mathbf{Q}$	the set of rational numbers	$\mathbf{Q}^* = \{r \in \mathbf{Q} \mid r \neq 0\}$ Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{Q}_{<0} = \{r \in \mathbf{Q} \mid r < 0\}$ The symbols Q and $\mathbb{Q}$ are also used.
2-6.4 (*11.4.12*)	$\mathbf{R}$	the set of real numbers	$\mathbf{R}^* = \{x \in \mathbf{R} \mid x \neq 0\}$ Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{R}_{\geqslant 0} = \{x \in \mathbf{R} \mid x \geqslant 0\}$ The symbols R and $\mathbb{R}$ are also used.
2-6.5 (*11.4.13*)	$\mathbf{C}$	the set of complex numbers	$\mathbf{C}^* = \{z \in \mathbf{C} \mid z \neq 0\}$ The symbols $\mathbb{C}$ and $\mathbb{C}$ are also used.
2-6.6 (—)	$\mathbf{P}$	the set of prime numbers	$\mathbf{P} = \{2, 3, 5, 7, 11, 13, 17, \ldots\}$ The symbols $\mathbb{P}$ and $\mathbb{P}$ are also used.
2-6.7 (*11.4.14*)	$[a, b]$	closed interval from a included to b included	$[a, b] = \{x \in \mathbf{R} \mid a \leqslant x \leqslant b\}$
2-6.8 (*11.4.15*)	$(a, b]$	left half-open interval from a excluded to b included	$(a, b] = \{x \in \mathbf{R} \mid a < x \leqslant b\}$ The notation $]a, b]$ is also used.
2-6.9 (*11.4.16*)	$[a, b)$	right half-open interval from a included to b excluded	$[a, b) = \{x \in \mathbf{R} \mid a \leqslant x < b\}$ The notation $[a, b[$ is also used.
2-6.10 (*11.4.17*)	(a, b)	open interval from a excluded to b excluded	$(a, b) = \{x \in \mathbf{R} \mid a < x < b\}$ The notation $]a, b[$ is also used.
2-6.11 (—)	$(-\infty, b]$	closed unbounded interval up to b included	$(-\infty, b] = \{x \in \mathbf{R} \mid x \leqslant b\}$ The notation $]-\infty, b]$ is also used.

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-6.12 (—)	$(-\infty, b)$	open unbounded interval up to b excluded	$(-\infty, b) = \{x \in \mathbf{R} \mid x < b\}$ The notation $]-\infty, b[$ is also used.
2-6.13 (—)	$[a, +\infty)$	closed unbounded interval onward from a included	$[a, +\infty) = \{x \in \mathbf{R} \mid a \leqslant x\}$ The notations $[a, \infty[$, $[a, +\infty[$ and $[a, \infty)$ are also used.
2-6.14 (—)	$(a, +\infty)$	open unbounded interval onward from a excluded	$(a, +\infty) = \{x \in \mathbf{R} \mid a < x\}$ The notations $]a, +\infty[$, $]a, \infty[$ and (a, ∞) are also used.

7 Miscellaneous signs and symbols

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-7.1 (*11-5.1*)	$a = b$	a is equal to b	The symbol $\equiv$ may be used to emphasize that a particular equality is an identity. See also 2-7.18.
2-7.2 (*11-5.2*)	$a \neq b$	a is not equal to b	The negating stroke may also be vertical.
2-7.3 (*11-5.3*)	$a := b$	a is by definition equal to b	EXAMPLE $\boldsymbol{p} := m\boldsymbol{v}$, where $\boldsymbol{p}$ is momentum, m is mass and $\boldsymbol{v}$ is velocity. The symbols $=_{\text{def}}$ and $\stackrel{\text{def}}{=}$ are also used.
2-7.4 (*11-5.4*)	$a \triangleq b$	a corresponds to b	EXAMPLES When $E = kT$, then 1 eV $\triangleq$ 11 604,5 K When 1 cm on a map corresponds to a length of 10 km, one may write 1 cm $\triangleq$ 10 km. The correspondence is not symmetric.
2-7.5 (*11-5.5*)	$a \approx b$	a is approximately equal to b	It depends on the user whether an approximation is sufficiently good. Equality is not excluded.
2-7.6 (*11-7.7*)	$a \simeq b$	a is asymptotically equal to b	EXAMPLE $\dfrac{1}{\sin(x-a)} \simeq \dfrac{1}{x-a}$ as $x \to a$ (For $x \to a$, see 2-7.16.)
2-7.7 (*11-5.6*)	$a \sim b$	a is proportional to b	The symbol $\sim$ is also used for equivalence relations. The notation $a \propto b$ is also used.
2-7.8 (—)	$M \cong N$	M is congruent to N, M is isomorphic to N	M and N are point sets (geometrical figures). This symbol is also used for isomorphisms of mathematical structures.
2-7.9 (*11-5.7*)	$a < b$	a is less than b	
2-7.10 (*11-5.8*)	$b > a$	b is greater than a	
2-7.11 (*11-5.9*)	$a \leqslant b$	a is less than or equal to b	
2-7.12 (*11-5.10*)	$b \geqslant a$	b is greater than or equal to a	
2-7.13 (*11-5.11*)	$a \ll b$	a is much less than b	It depends on the user whether a is sufficiently small as compared to b.
2-7.14 (*11-5.12*)	$b \gg a$	b is much greater than a	It depends on the user whether b is sufficiently great as compared to a.

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-7.15 (*11-5.13*)	∞	infinity	This symbol does not denote a number but is often part of various expressions dealing with limits. The notations $+\infty$, $-\infty$ are also used.
2-7.16 (*11-7.5*)	$x \to a$	x tends to a	This symbol occurs as part of various expressions dealing with limits. a may be also ∞, $+\infty$, or $-\infty$.
2-7.17 (—)	$m \mid n$	m divides n	For integers m and n: $\exists\, k \in \mathbf{Z}\ m \cdot k = n$
2-7.18 (—)	$n \equiv k \bmod m$	n is congruent to k modulo m	For integers n, k and m: $m \mid (n-k)$ See also 2-7.1.
2-7.19 (*1-5.14*)	$(a+b)$ $[a+b]$ $\{a+b\}$ $\langle a+b \rangle$	parentheses square brackets braces angle brackets	It is recommended to use only parentheses for grouping, since brackets and braces often have a specific meaning in particular fields. Parentheses can be nested without ambiguity.

8 Elementary geometry

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-8.1 (*11-5.15*)	AB ∥ CD	the straight line AB is parallel to the straight line CD	It is written g ∥ h if g and h are the straight lines determined by the points A and B, and the points C and D, respectively. AB//CD is also used.
2-8.2 (*11-5.16*)	AB ⊥ CD	the straight line AB is perpendicular to the straight line CD	It is written g ⊥ h if g and h are the straight lines determined by the points A and B, and the points C and D, respectively. In a plane, the straight lines must intersect.
2-8.3 (—)	∢ABC	angle at vertex B in the triangle ABC	The angle is not oriented, it holds that ∢ABC = ∢CBA and $0 \leqslant$ ∢ABC $\leqslant \pi$ rad.
2-8.4 (—)	$\overline{\mathrm{AB}}$	line segment from A to B	The line segment is the set of points between A and B on the straight line AB.
2-8.5 (—)	$\overrightarrow{\mathrm{AB}}$	vector from A to B	If $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{CD}}$ then B, seen from A, is in the same direction and distance as D is, seen from C. It does not follow that A = C and B = D.
2-8.6 (—)	$d(\mathrm{A}, \mathrm{B})$	distance between points A and B	The distance is the length of the line segment $\overline{\mathrm{AB}}$ and also the magnitude of the vector $\overrightarrow{\mathrm{AB}}$.

9 Operations

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-9.1 (*11-6.1*)	$a + b$	a plus b	This operation is named addition. The symbol + is the addition symbol.
2-9.2 (*11-6.2*)	$a - b$	a minus b	This operation is named subtraction. The symbol – is the subtraction symbol.
2-9.3 (*11-6.3*)	$a \pm b$	a plus or minus b	This is a combination of two values into one expression.
2-9.4 (*11-6.4*)	$a \mp b$	a minus or plus b	$-(a \pm b) = -a \mp b$
2-9.5 (*11-6.5*)	$a \cdot b$ $a \times b$ $a\ b$ ab	a multiplied by b, a times b	This operation is named multiplication. The symbol for multiplication is a half-high dot (·) or a cross (×). Either may be omitted if no misunderstanding is possible. See also 2-5.16, 2-5.17, 2-17.11, 2-17.12, 2-17.23 and 2-17.24 for the use of the dot and cross in various products.
2-9.6 (*11-6.6*)	$\frac{a}{b}$ a/b	a divided by b	$\frac{a}{b} = a \cdot b^{-1}$ See also ISO 80000-1:—, 7.1.3. For ratios, the symbol : is also used. EXAMPLE The ratio of height h to breadth b of an A4 sheet is $h : b = \sqrt{2}$. The symbol ÷ should not be used.
2-9.7 (*11-6.7*)	$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$, sum of $a_1, a_2, \ldots, a_n$	The notations $\sum_{i=1}^{n} a_i$, $\sum_i a_i$, $\sum_i a_i$ and $\sum a_i$ are also used.
2-9.8 (*11-6.8*)	$\prod_{i=1}^{n} a_i$	$a_1 \cdot a_2 \cdot \ldots \cdot a_n$, product of $a_1, a_2, \ldots, a_n$	The notations $\prod_{i=1}^{n} a_i$, $\prod_i a_i$, $\prod_i a_i$ and $\prod a_i$ are also used.
2-9.9 (*11-6.9*)	a^p	a to the power p	The verbal equivalent of a^2 is a squared; the verbal equivalent of a^3 is a cubed.
2-9.10 (*11-6.10*)	$a^{1/2}$ $\sqrt{a}$	a to the power 1/2, square root of a	If $a \geqslant 0$, then $\sqrt{a} \geqslant 0$. The symbol $\surd a$ should be avoided. See remark to 2-9.11.
2-9.11 (*11-6.11*)	$a^{1/n}$ $\sqrt[n]{a}$	a to the power $1/n$, nth root of a	If $a \geqslant 0$, then $\sqrt[n]{a} \geqslant 0$. The symbol ${}^n\surd a$ should be avoided. If the symbol ${}^n\surd$ or $\surd$ acts on a composite expression, parentheses shall be used to avoid ambiguity.

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-9.12 (*11-6.14*)	$\bar{x}$ $\langle x \rangle$ $\bar{x}_a$	mean value of x, arithmetic mean of x	Mean values obtained by other methods are the - harmonic mean denoted by subscript h, - geometric mean denoted by subscript g, - quadratic mean, often called "root mean square", denoted by subscript q or rms. The subscript may only be omitted for the arithmetic mean. In mathematics $\bar{x}$ is also used for the complex conjugate of x; see 2-14.6.
2-9.13 (*11-6.13*)	sgn a	signum a	For real a: $\operatorname{sgn} a = \begin{cases} 1 \text{ if } a > 0 \\ 0 \text{ if } a = 0 \\ -1 \text{ if } a < 0 \end{cases}$ See also item 2-14.7.
2-9.14 (—)	inf M	infimum of M	Greatest lower bound of a non-empty set of numbers bounded from below.
2-9.15 (—)	sup M	supremum of M	Smallest upper bound of a non-empty set of numbers bounded from above.
2-9.16 (*11-6.12*)	$\|a\|$	absolute value of a, modulus of a, magnitude of a	The notation abs a is also used. Absolute value of real number a. Modulus of complex number a; see 2-14.4. Magnitude of vector $\boldsymbol{a}$; see 2-17.4. See also 2-5.5.
2-9.17 (*11-6.17*)	$\lfloor a \rfloor$	floor a, the greatest integer less than or equal to the real number a	The notation ent a is also used. EXAMPLES $\lfloor 2{,}4 \rfloor = 2$ $\lfloor -2{,}4 \rfloor = -3$
2-9.18 (—)	$\lceil a \rceil$	ceil a, the least integer greater than or equal to the real number a	"ceil" is an abbreviation of the word "ceiling". EXAMPLES $\lceil 2{,}4 \rceil = 3$ $\lceil -2{,}4 \rceil = -2$
2-9.19 (—)	int a	integer part of the real number a	int a = sgn $a \cdot \lfloor \|a\| \rfloor$ EXAMPLES int(2,4) = 2 int(–2,4) = –2
2-9.20 (—)	frac a	fractional part of the real number a	frac a = a – int a EXAMPLES frac(2,4) = 0,4 frac(–2,4) = –0,4

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-9.21 (—)	min(a, b)	minimum of a and b	The operation generalizes to more numbers and to sets of numbers. However, an infinite set of numbers need not have a smallest element.
2-9.22 (—)	max(a, b)	maximum of a and b	The operation generalizes to more numbers and to sets of numbers. However, an infinite set of numbers need not have a greatest element.

10 Combinatorics

In this clause, n and k are natural numbers, with $k \leqslant n$.

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-10.1 (*11-6.15*)	$n!$	factorial	$n! = \prod_{k=1}^{n} k = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n \ (n > 0)$ $0! = 1$
2-10.2 (—)	$a^{\underline{k}}$ $[a]_k$	falling factorial	$a^{\underline{k}} = a \cdot (a-1) \cdot \ldots \cdot (a-k+1) \ (k > 0)$ $a^{\underline{0}} = 1$ a may be a complex number. For a natural number n: $n^{\underline{k}} = \frac{n!}{(n-k)!}$
2-10.3 (—)	$a^{\overline{k}}$ $(a)_k$	rising factorial	$a^{\overline{k}} = a \cdot (a+1) \cdot \ldots \cdot (a+k-1) \ (k > 0)$ $a^{\overline{0}} = 1$ a may be a complex number. For a natural number n: $n^{\overline{k}} = \frac{(n+k-1)!}{(n-1)!}$ $(a)_k$ is called Pochhammer symbol in the theory of special functions. In combinatorics and statistics, however, the same symbol is often used for the falling factorial.
2-10.4 (*11-6.16*)	$\binom{n}{k}$	binomial coefficient	$\binom{n}{k} = \frac{n!}{k!(n-k)!} \quad (0 \leqslant k \leqslant n)$
2-10.5 (—)	B_n	Bernoulli numbers	$\mathrm{B}_n = -\frac{1}{n+1} \sum_{k=0}^{n-1} \binom{n+1}{k} \mathrm{B}_k \ (n > 0)$ $\mathrm{B}_0 = 1$ $\mathrm{B}_1 = -1/2$, $\mathrm{B}_{2n+3} = 0$
2-10.6 (*11-6.16*)	C_n^k	number of combinations without repetition	$\mathrm{C}_n^k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$
2-10.7 (—)	${}^{\mathrm{R}}\mathrm{C}_n^k$	number of combinations with repetition	${}^{\mathrm{R}}\mathrm{C}_n^k = \binom{n+k-1}{k}$
2-10.8 (—)	V_n^k	number of variations without repetition	$\mathrm{V}_n^k = n^{\underline{k}} = \frac{n!}{(n-k)!}$ The term "permutation" is used when $n = k$.
2-10.9 (—)	${}^{\mathrm{R}}\mathrm{V}_n^k$	number of variations with repetition	${}^{\mathrm{R}}\mathrm{V}_n^k = n^k$

11 Functions

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-11.1 (*11-7.1*)	$f, g, h, \ldots$	functions	A function assigns to any argument in its domain a unique value in its range.
2-11.2 (*11-7.2*)	$f(x)$ $f(x_1, \ldots, x_n)$	value of function f for argument x or for argument $(x_1, \ldots, x_n)$, respectively	A function having a set of n-tuples as its domain is an n-place function.
2-11.3 (—)	$f: A \to B$	f maps A into B	The function f has domain A and range included in B.
2-11.4 (—)	$f: x \mapsto T(x)$, $x \in A$	f is the function that maps any $x \in A$ to $T(x)$	$T(x)$ is a defining term denoting the value of the function f for the argument x. Since $f(x) = T(x)$, the defining term is often used as a symbol for the function f. EXAMPLE $f: x \mapsto 3x^2 y, x \in [0; 2]$ f is the function (depending on the parameter y) defined on the stated interval by the term $3x^2 y$.
2-11.5 (—)	$x \xrightarrow{f} y$	$f(x) = y$, f maps x onto y	EXAMPLE $\pi \xrightarrow{\cos} -1$
2-11.6 (*11-7.3*)	$f\big\vert_a^b$ $f(\ldots, u, \ldots)\big\vert_{u=a}^{u=b}$	$f(b) - f(a)$ $f(\ldots, b, \ldots) - f(\ldots, a, \ldots)$	This notation is used mainly when evaluating definite integrals.
2-11.7 (*11-7.4*)	$g \circ f$	composite function of f and g, g circle f	$(g \circ f)(x) = g(f(x))$ In the composite $g \circ f$, the function g is applied after function f has been applied.
2-11.8 (*11-7.6*)	$\lim\limits_{x \to a} f(x)$ $\lim_{x \to a} f(x)$	limit of $f(x)$ as x tends to a	$f(x) \to b$ as $x \to a$ may be written for $\lim_{x \to a} f(x) = b$. Limits "from the right" $(x > a)$ and "from the left" $(x < a)$ are denoted by $\lim_{x \to a+} f(x)$ and $\lim_{x \to a-} f(x)$, respectively.
2-11.9 (*11-7.8*)	$f(x) = \mathrm{O}(g(x))$	$f(x)$ is big-O of $g(x)$, $\lvert f(x)/g(x) \rvert$ is bounded from above in the limit implied by the context, $f(x)$ is of the order comparable with or inferior to $g(x)$	The symbol "=" here is used for historical reasons and does not have the meaning of equality, because transitivity does not apply. EXAMPLE $\sin x = \mathrm{O}(x)$, when $x \to 0$
2-11.10 (*11-7.9*)	$f(x) = \mathrm{o}(g(x))$	$f(x)$ is little-o of $g(x)$, $f(x)/g(x) \to 0$ in the limit implied by the context, $f(x)$ is of the order inferior to $g(x)$	The symbol "=" here is used for historical reasons and does not have the meaning of equality, because transitivity does not apply. EXAMPLE $\cos x = 1 + \mathrm{o}(x)$, when $x \to 0$

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-11.11 (*11-7.10*)	Δf	delta f, finite increment of f	Difference of two function values implied by the context. EXAMPLES $\Delta x = x_2 - x_1$ $\Delta f = f(x_2) - f(x_1)$
2-11.12 (*11-7.11*)	$\frac{\mathrm{d}f}{\mathrm{d}x}$ $\mathrm{d}f/\mathrm{d}x$ f'	derivative of f with respect to x	Only to be used for functions of one variable. $\frac{\mathrm{d}f(x)}{\mathrm{d}x}$, $\mathrm{d}f(x)/\mathrm{d}x$, $f'(x)$ and $\mathrm{D}f$ are also used. If the independent variable is time t, $\dot{f}$ is also used for f'.
2-11.13 (*11-7.12*)	$\left(\frac{\mathrm{d}f}{\mathrm{d}x}\right)_{x=a}$ $(\mathrm{d}f/\mathrm{d}x)_{x=a}$ $f'(a)$	value of the derivative of f for $x = a$	
2-11.14 (*11-7.13*)	$\frac{\mathrm{d}^n f}{\mathrm{d}x^n}$ $\mathrm{d}^n f/\mathrm{d}x^n$ $f^{(n)}$	nth derivative of f with respect to x	Only to be used for functions of one variable. $\frac{\mathrm{d}^n f(x)}{\mathrm{d}x^n}$, $\mathrm{d}^n f(x)/\mathrm{d}x^n$, $f^{(n)}(x)$ and $\mathrm{D}^n f$ are also used. f'' and f''' are also used for $f^{(2)}$ and $f^{(3)}$, respectively. If the independent variable is time t, $\ddot{f}$ is also used for f''.

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-11.15 (*11-7.14*)	$\frac{\partial f}{\partial x}$ $\partial f / \partial x$ $\partial_x f$	partial derivative of f with respect to x	Only to be used for functions of several variables. $\frac{\partial f(x, y, \ldots)}{\partial x}$, $\partial f(x, y, \ldots) / \partial x$, $\partial_x f(x, y, \ldots)$ and $D_x f(x, y, \ldots)$ are also used. The other independent variables may be shown as subscripts, e.g. $\left(\frac{\partial f}{\partial x}\right)_{y \ldots}$. This partial-derivative notation is extended to derivatives of higher order, e.g. $\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$ $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$ Other notations, e.g. $f_{xy} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$, are also used.
2-11.16 (*11-7.15*)	$\mathrm{d} f$	total differential of f	$\mathrm{d} f(x, y, \ldots) = \frac{\partial f}{\partial x} \mathrm{d}x + \frac{\partial f}{\partial y} \mathrm{d}y + \ldots$
2-11.17 (*11-7.16*)	δf	infinitesimal variation of f	
2-11.18 (*11-7.17*)	$\int f(x) \mathrm{d}x$	indefinite integral of f	
2-11.19 (*11-7.18*)	$\int_a^b f(x) \mathrm{d}x$	definite integral of f from a to b	This is the simple case of a function defined on an interval. Integration of functions defined on more general domains may also be defined. Special notations, e.g. $\int_C$, $\int_S$, $\int_V$, $\oint$, are used for integration over a curve C, a surface S, a three-dimensional domain V, and a closed curve or surface, respectively. Multiple integrals are also denoted $\iint$, $\oiint$, etc.
2-11.20 (—)	$⨍_a^b f(x) \mathrm{d}x$	Cauchy principal value of the integral of f with f singular at c	$\lim_{\delta \to 0+} \left(\int_a^{c-\delta} f(x) \mathrm{d}x + \int_{c+\delta}^b f(x) \mathrm{d}x \right)$ where $a < c < b$
2-11.21 (—)	$⨍_{-\infty}^{\infty} f(x) \mathrm{d}x$	Cauchy principal value of the integral of f	$\lim_{a \to \infty} ⨍_{-a}^{a} f(x) \mathrm{d}x$

12 Exponential and logarithmic functions

Complex arguments can be used, in particular for the base e.

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-12.1 (*11-8.2*)	e	base of natural logarithm	$\mathrm{e} := \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 2{,}718\,281\,8\ldots$
2-12.2 (*11-8.1*)	a^x	a to the power of x, exponential function to the base a of argument x	See also 2-9.9.
2-12.3 (*11-8.3*)	e^x $\exp x$	e to the power of x, exponential function to the base e of argument x	See 2-14.5.
2-12.4 (*11-8.4*)	$\log_a x$	logarithm to the base a of argument x	$\log x$ is used when the base does not need to be specified.
2-12.5 (*11-8.5*)	$\ln x$	natural logarithm of x	$\ln x = \log_\mathrm{e} x$ $\log x$ shall not be used in place of $\ln x$, $\lg x$, $\mathrm{lb}\, x$, or $\log_\mathrm{e} x$, $\log_{10} x$, $\log_2 x$.
2-12.6 (*11-8.6*)	$\lg x$	decimal logarithm of x, common logarithm of x	$\lg x = \log_{10} x$ See remark to 2-12.5.
2-12.7 (*11-8.7*)	$\mathrm{lb}\, x$	binary logarithm of x	$\mathrm{lb}\, x = \log_2 x$ See remark to 2-12.5.

13 Circular and hyperbolic functions

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-13.1 (*11-9.1*)	π	ratio of the circumference of a circle to its diameter	$\pi = 3{,}141\ 592\ 6\ldots$
2-13.2 (*11-9.2*)	$\sin x$	sine of x	$\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$, $\sin x = x - x^3/3! + x^5/5! - \ldots$ $(\sin x)^n$, $(\cos x)^n$, etc., are often written $\sin^n x$, $\cos^n x$, etc.
2-13.3 (*11-9.3*)	$\cos x$	cosine of x	$\cos x = \sin(x + \pi/2)$
2-13.4 (*11-9.4*)	$\tan x$	tangent of x	$\tan x = \sin x / \cos x$ tg x should not be used.
2-13.5 (*11-9.5*)	$\cot x$	cotangent of x	$\cot x = 1/\tan x$ ctg x should not be used.
2-13.6 (*11-9.6*)	$\sec x$	secant of x	$\sec x = 1/\cos x$
2-13.7 (*11-9.7*)	$\csc x$	cosecant of x	$\csc x = 1/\sin x$ cosec x is also used.
2-13.8 (*11-9.8*)	$\arcsin x$	arc sine of x	$y = \arcsin x \Leftrightarrow x = \sin y$, $-\pi/2 \leqslant y \leqslant \pi/2$ The function arcsin is the inverse of the function sin with the restriction mentioned above.
2-13.9 (*11-9.9*)	$\arccos x$	arc cosine of x	$y = \arccos x \Leftrightarrow x = \cos y$, $0 \leqslant y \leqslant \pi$ The function arccos is the inverse of the function cos with the restriction mentioned above.
2-13.10 (*11-9.10*)	$\arctan x$	arc tangent of x	$y = \arctan x \Leftrightarrow x = \tan y$, $-\pi/2 < y < \pi/2$ The function arctan is the inverse of the function tan with the restriction mentioned above. arctg x should not be used.
2-13.11 (*11-9.11*)	$\operatorname{arccot} x$	arc cotangent of x	$y = \operatorname{arccot} x \Leftrightarrow x = \cot y$, $0 < y < \pi$ The function arccot is the inverse of the function cot with the restriction mentioned above. arcctg x should not be used.
2-13.12 (*11-9.12*)	$\operatorname{arcsec} x$	arc secant of x	$y = \operatorname{arcsec} x \Leftrightarrow x = \sec y$, $0 \leqslant y \leqslant \pi$, $y \neq \pi/2$ The function arcsec is the inverse of the function sec with the restriction mentioned above.

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-13.13 (*11-9.13*)	arccsc x	arc cosecant of x	$y = \operatorname{arccsc} x \Leftrightarrow x = \csc y$, $-\pi/2 \leqslant y \leqslant \pi/2$, $y \neq 0$ The function arccsc is the inverse of the function csc with the restriction mentioned above. arccosec x should be avoided.
2-13.14 (*11-9.14*)	sinh x	hyperbolic sine of x	$\sinh x = \dfrac{e^x - e^{-x}}{2}$ $\sinh x = x + x^3/3! + \ldots$ sh x should be avoided.
2-13.15 (*11-9.15*)	cosh x	hyperbolic cosine of x	$\cosh^2 x = \sinh^2 x + 1$ ch x should be avoided.
2-13.16 (*11-9.16*)	tanh x	hyperbolic tangent of x	$\tanh x = \sinh x / \cosh x$ th x should be avoided.
2-13.17 (*11-9.17*)	coth x	hyperbolic cotangent of x	$\coth x = 1/\tanh x$
2-13.18 (*11-9.18*)	sech x	hyperbolic secant of x	$\operatorname{sech} x = 1/\cosh x$
2-13.19 (*11-9.19*)	csch x	hyperbolic cosecant of x	$\operatorname{csch} x = 1/\sinh x$ cosech x should be avoided.
2-13.20 (*11-9.20*)	arsinh x	inverse hyperbolic sine of x, area hyperbolic sine of x	$y = \operatorname{arsinh} x \Leftrightarrow x = \sinh y$ The function arsinh is the inverse of the function sinh. arsh x should be avoided.
2-13.21 (*11-9.21*)	arcosh x	inverse hyperbolic cosine of x, area hyperbolic cosine of x	$y = \operatorname{arcosh} x \Leftrightarrow x = \cosh y$, $y \geqslant 0$ The function arcosh is the inverse of the function cosh with the restriction mentioned above. arch x should be avoided.
2-13.22 (*11-9.22*)	artanh x	inverse hyperbolic tangent of x, area hyperbolic tangent of x	$y = \operatorname{artanh} x \Leftrightarrow x = \tanh y$ The function artanh is the inverse of the function tanh. arth x should be avoided.
2-13.23 (*11-9.23*)	arcoth x	inverse hyperbolic cotangent of x, area hyperbolic cotangent of x	$y = \operatorname{arcoth} x \Leftrightarrow x = \coth y$, $y \neq 0$ The function arcoth is the inverse of the function coth with the restriction mentioned above.
2-13.24 (*11-9.24*)	arsech x	inverse hyperbolic secant of x, area hyperbolic secant of x	$y = \operatorname{arsech} x \Leftrightarrow x = \operatorname{sech} y$, $y \geqslant 0$ The function arsech is the inverse of the function sech with the restriction mentioned above.
2-13.25 (*11-9.25*)	arcsch x	inverse hyperbolic cosecant of x, area hyperbolic cosecant of x	$y = \operatorname{arcsch} x \Leftrightarrow x = \operatorname{csch} y$, $y \geqslant 0$ The function arcsch is the inverse of the function csch with the restriction mentioned above. arcosech x should be avoided.

14 Complex numbers

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-14.1 (*11-10.1*)	i j	imaginary unit	$\mathrm{i}^2 = \mathrm{j}^2 = -1$ i is used in mathematics and in physics, j is used in electrotechnology.
2-14.2 (*11-10.2*)	Re z	real part of z	$z = x + \mathrm{i}\,y$ where x and y are real numbers. $x = \mathrm{Re}\,z$ and $y = \mathrm{Im}\,z$.
2-14.3 (*11-10.3*)	Im z	imaginary part of z	See 2-14.2.
2-14.4 (*11-10.4*)	$\lvert z \rvert$	modulus of z	$\lvert z \rvert = \sqrt{x^2 + y^2}$ where $x = \mathrm{Re}\,z$ and $y = \mathrm{Im}\,z$. See also 2-9.16.
2-14.5 (*11-10.5*)	arg z	argument of z	$z = r\,\mathrm{e}^{\mathrm{i}\varphi}$ where $r = \lvert z \rvert$ and $\varphi = \arg z$, $-\pi < \varphi \leqslant \pi$ i.e. $\mathrm{Re}\,z = r\cos\varphi$ and $\mathrm{Im}\,z = r\sin\varphi$.
2-14.6 (*11-10.6*)	$\bar{z}$ z^*	complex conjugate of z	$\bar{z}$ is mainly used in mathematics, z^* mainly in physics and engineering.
2-14.7 (*11-10.7*)	sgn z	signum z	$\mathrm{sgn}\,z = z / \lvert z \rvert = \exp(\mathrm{i}\arg z)$ $\quad (z \neq 0)$ $\mathrm{sgn}\,z = 0$ for $z = 0$ See also item 2-9.13.

15 Matrices

Matrices are usually written with boldface italic capital letters and their elements with thin italic lower case letters, but other typefaces may also be used.

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-15.1 (*11-11.1*)	$\boldsymbol{A}$ $\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$	matrix $\boldsymbol{A}$ of type m by n	$\boldsymbol{A}$ is the matrix with the elements $a_{ij} = (\boldsymbol{A})_{ij}$. m is the number of rows and n is the number of columns. $\boldsymbol{A} = (a_{ij})$ is also used. Square brackets are also used instead of parentheses.
2-15.2 (—)	$\boldsymbol{A} + \boldsymbol{B}$	sum of matrices $\boldsymbol{A}$ and $\boldsymbol{B}$	$(\boldsymbol{A} + \boldsymbol{B})_{ij} = (\boldsymbol{A})_{ij} + (\boldsymbol{B})_{ij}$ The matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ must have the same number of columns and rows.
2-15.3 (—)	$x\,\boldsymbol{A}$	product of scalar x and matrix $\boldsymbol{A}$	$(x\,\boldsymbol{A})_{ij} = x\,(\boldsymbol{A})_{ij}$
2-15.4 (*11-11.2*)	$\boldsymbol{AB}$	product of matrices $\boldsymbol{A}$ and $\boldsymbol{B}$	$(\boldsymbol{AB})_{ik} = \sum_j (\boldsymbol{A})_{ij}(\boldsymbol{B})_{jk}$ The number of columns of $\boldsymbol{A}$ must be equal to the number of rows of $\boldsymbol{B}$.
2-15.5 (*11-11.3*)	$\boldsymbol{E}$ $\boldsymbol{I}$	unit matrix	A square matrix for which $(\boldsymbol{E})_{ik} = \delta_{ik}$. See 2-17.9.
2-15.6 (*11-11.4*)	$\boldsymbol{A}^{-1}$	inverse of a square matrix $\boldsymbol{A}$	$\boldsymbol{AA}^{-1} = \boldsymbol{A}^{-1}\boldsymbol{A} = \boldsymbol{E}$
2-15.7 (*11-11.5*)	$\boldsymbol{A}^{\mathrm{T}}$	transpose matrix of $\boldsymbol{A}$	$(\boldsymbol{A}^{\mathrm{T}})_{ik} = (\boldsymbol{A})_{ki}$
2-15.8 (*11-11.6*)	$\overline{\boldsymbol{A}}$ $\boldsymbol{A}^*$	complex conjugate matrix of $\boldsymbol{A}$	$(\overline{\boldsymbol{A}})_{ik} = \overline{(\boldsymbol{A})_{ik}}$ $\overline{\boldsymbol{A}}$ is used in mathematics, $\boldsymbol{A}^*$ in physics and electrotechnology.
2-15.9 (*11-11.7*)	$\boldsymbol{A}^{\mathrm{H}}$	Hermitian conjugate matrix of $\boldsymbol{A}$	$\boldsymbol{A}^{\mathrm{H}} = (\overline{\boldsymbol{A}})^{\mathrm{T}}$ The term "adjoint matrix" is also used. $\boldsymbol{A}^*$ and $\boldsymbol{A}^+$ are also used for $\boldsymbol{A}^{\mathrm{H}}$.
2-15.10 (*11-11.8*)	det $\boldsymbol{A}$ $\begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}$	determinant of a square matrix $\boldsymbol{A}$	
2-15.11 (—)	rank $\boldsymbol{A}$	rank of matrix $\boldsymbol{A}$	The rank of matrix $\boldsymbol{A}$ is the number of the linearly independent rows of $\boldsymbol{A}$. It is also equal to the number of linearly independent columns.

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-15.12 (*11-11.9*)	tr $\boldsymbol{A}$	trace of a square matrix $\boldsymbol{A}$	tr $\boldsymbol{A} = \sum_i (\boldsymbol{A})_{ii}$
2-15.13 (*11-11.10*)	$\Vert\boldsymbol{A}\Vert$	norm of matrix $\boldsymbol{A}$	The norm of matrix $\boldsymbol{A}$ is a number characterizing this matrix and undergoing the triangle inequality: if $\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{C}$, then $\Vert\boldsymbol{A}\Vert + \Vert\boldsymbol{B}\Vert \geqslant \Vert\boldsymbol{C}\Vert$. Different matrix norms are used.

16 Coordinate systems

Item No.	Coordinates	Position vector and its differential	Name of coordinates	Remarks
2-16.1 (*11-12.1*)	x, y, z	$\boldsymbol{r} = x\boldsymbol{e}_x + y\boldsymbol{e}_y + z\boldsymbol{e}_z$ $\mathrm{d}\boldsymbol{r} = \mathrm{d}x\, \boldsymbol{e}_x + \mathrm{d}y\, \boldsymbol{e}_y + \mathrm{d}z\, \boldsymbol{e}_z$	Cartesian coordinates	x_1, x_2, x_3 for the coordinates and $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$ for the base vectors are also used. This notation easily generalizes to n-dimensional space. $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, $\boldsymbol{e}_z$ form an orthonormal right-handed system. See Figures 1 and 4. For the base vectors, $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ are also used.
2-16.2 (*11-12.2*)	ρ, φ, z	$\boldsymbol{r} = \rho\, \boldsymbol{e}_\rho + z\, \boldsymbol{e}_z$ $\mathrm{d}\boldsymbol{r} = \mathrm{d}\rho\, \boldsymbol{e}_\rho + \rho\, \mathrm{d}\varphi\, \boldsymbol{e}_\varphi + \mathrm{d}z\, \boldsymbol{e}_z$	cylindrical coordinates	$\boldsymbol{e}_\rho(\varphi)$, $\boldsymbol{e}_\varphi(\varphi)$, $\boldsymbol{e}_z$ form an orthonormal right-handed system. See Figure 2. If $z = 0$, then ρ and φ are the polar coordinates.
2-16.3 (*11-12.3*)	r, ϑ, φ	$\boldsymbol{r} = r\, \boldsymbol{e}_r$ $\mathrm{d}\boldsymbol{r} =$ $\mathrm{d}r\, \boldsymbol{e}_r + r\, \mathrm{d}\vartheta\, \boldsymbol{e}_\vartheta + r \sin\vartheta\, \mathrm{d}\varphi\, \boldsymbol{e}_\varphi$	spherical coordinates	$\boldsymbol{e}_r(\vartheta, \varphi)$, $\boldsymbol{e}_\vartheta(\vartheta, \varphi)$, $\boldsymbol{e}_\varphi(\varphi)$ form an orthonormal right-handed system. See Figure 3.

NOTE If, exceptionally, instead of a right-handed system (see Figure 4), a left-handed system (see Figure 5) is used for certain purposes, this shall be clearly stated to avoid the risk of sign errors.

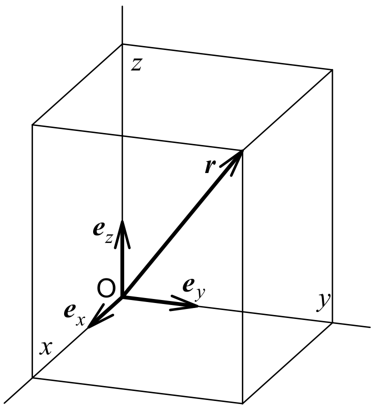


Figure 1 — Right-handed Cartesian coordinate system

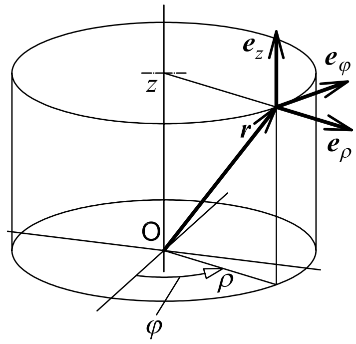


Figure 2 — Right-handed cylindrical coordinate system

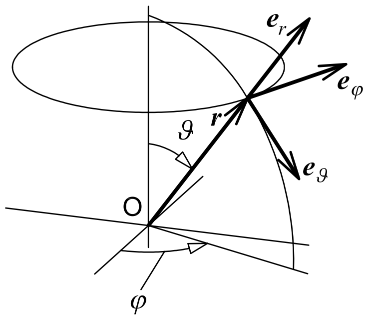
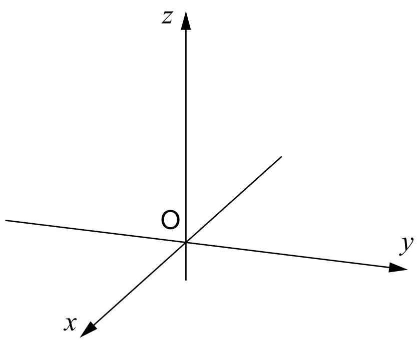
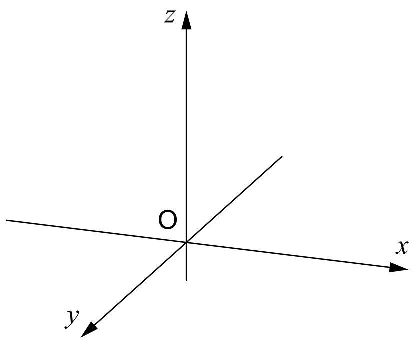


Figure 3 — Right-handed spherical coordinate system



The x-axis is pointing towards the viewer.

Figure 4 — Right-handed coordinate system



The y-axis is pointing towards the viewer.

Figure 5 — Left-handed coordinate system

17 Scalars, vectors and tensors

Scalars, vectors and tensors are mathematical objects that can be used to denote certain physical quantities and their values. They are as such independent of the particular choice of a coordinate system, whereas each component of a vector or a tensor and each component vector and component tensor depend on that choice.

It is important to distinguish between the components of a vector $\boldsymbol{a}$ with respect to some base vectors, i.e. the quantity values a_x, a_y and a_z, and the “component vectors”, i.e. $a_x\boldsymbol{e}_x$, $a_y\boldsymbol{e}_y$ and $a_z\boldsymbol{e}_z$. Care should be taken not to confuse components of a vector and component vectors. Components of a vector are often called coordinates of a vector.

The Cartesian components of the position vector are equal to the Cartesian coordinates of the point given by the vector.

Instead of treating each component as a physical quantity value (i.e. a numerical value multiplied by a unit), the vector could be written as a numerical value vector multiplied by a unit. All units are scalars.

EXAMPLE

$\boldsymbol{F}$ = (3 N, –2 N, 5 N) = (3, –2, 5) N (in Cartesian coordinates)

where

$\boldsymbol{F}$	is a force;
3 N	is the first component, e.g. F_x, of the vector $\boldsymbol{F}$ with numerical value 3 and unit N (the other components being –2 N and 5 N);
(3, –2, 5)	is a numerical value vector and N the unit.

The same considerations apply to tensors of second and higher orders.

In this clause, only Cartesian (orthonormal) coordinates in ordinary space are considered. The more general cases requiring covariant and contravariant representations are not treated here. The Cartesian coordinates are denoted either by x, y, z or by x_1, x_2, x_3. In the latter case, subscripts i, j, k, l, each ranging from 1 to 3, are used, and the following summation convention is often used:

> if such a subscript appears twice in a term, summation over the range of this subscript is understood, and the sign Σ may be omitted.

A scalar is a tensor of zero order and a vector is a tensor of the first order.

Vectors and tensors are often represented by general symbols for their components, e.g. a_i for a vector, T_{ij} for a tensor of the second order, and a_ib_j for a dyadic product.

The abbreviation “cycl” stands for cyclic permutation of the components and the subscripts. Instead of writing three similar component equations, it is enough to write only one and the two others follow from cycl, cycl.

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-17.1 (*11-13.1*)	$\boldsymbol{a}$ $\vec{a}$	vector $\boldsymbol{a}$	An arrow above the letter symbol can be used instead of bold face type to indicate a vector.
2-17.2 (—)	$\boldsymbol{a}+\boldsymbol{b}$	sum of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$	$(\boldsymbol{a}+\boldsymbol{b})_i = a_i + b_i$
2-17.3 (—)	$x\boldsymbol{a}$	product of a number, scalar, or component x and vector $\boldsymbol{a}$	$(x\boldsymbol{a})_i = xa_i$
2-17.4 (*11-13.2*)	$\lvert\boldsymbol{a}\rvert$, a	magnitude of the vector $\boldsymbol{a}$, norm of the vector $\boldsymbol{a}$	$\lvert\boldsymbol{a}\rvert = \sqrt{a_x^2 + a_y^2 + a_z^2}$ $\lVert \boldsymbol{a} \rVert$ is also used. See also 2-9.16.
2-17.5 (—)	$\boldsymbol{0}$ $\vec{0}$	zero vector	The zero vector has magnitude 0.
2-17.6 (*11-13.3*)	$\boldsymbol{e_a}$	unit vector in the direction of $\boldsymbol{a}$	$\boldsymbol{e_a} = \boldsymbol{a}/\lvert\boldsymbol{a}\rvert, \quad \boldsymbol{a} \neq \boldsymbol{0}$ $\boldsymbol{a} = \lvert\boldsymbol{a}\rvert\boldsymbol{e_a}$
2-17.7 (*11-13.4*)	$\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z$ $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$	unit vectors in the directions of the Cartesian coordinate axes	$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are also used.
2-17.8 (*11-13.5*)	a_x, a_y, a_z a_i	Cartesian coordinates of vector $\boldsymbol{a}$, Cartesian components of vector $\boldsymbol{a}$	$\boldsymbol{a} = a_x\,\boldsymbol{e}_x + a_y\,\boldsymbol{e}_y + a_z\,\boldsymbol{e}_z$ $a_x\,\boldsymbol{e}_x$ etc., are the component vectors. If it is clear from the context which are the base vectors, the vector can be written $\boldsymbol{a} = (a_x, a_y, a_z)$. $a_x = \boldsymbol{a}\cdot\boldsymbol{e}_x$, cycl, cycl $\boldsymbol{r} = x\boldsymbol{e}_x + y\boldsymbol{e}_y + z\boldsymbol{e}_z$ is the position vector (radius vector) of the point with coordinates x, y, z.
2-17.9 (*11-7.19*)	δ_{ik}	Kronecker delta symbol	$\delta_{ik} = \begin{cases} 1 \text{ for } i = k \\ 0 \text{ for } i \neq k \end{cases}$
2-17.10 (*11-7.20*)	ε_{ijk}	Levi-Civita symbol	$\varepsilon_{123} = \varepsilon_{231} = \varepsilon_{312} = 1$ $\varepsilon_{132} = \varepsilon_{321} = \varepsilon_{213} = -1$ All other ε_{ijk} are equal to 0.
2-17.11 (*11-13.6*)	$\boldsymbol{a}\cdot\boldsymbol{b}$	scalar product of $\boldsymbol{a}$ and $\boldsymbol{b}$	$\boldsymbol{a}\cdot\boldsymbol{b} = a_xb_x + a_yb_y + a_zb_z$ $\boldsymbol{a}\cdot\boldsymbol{b} = \sum_i a_ib_i$ $\boldsymbol{a}\cdot\boldsymbol{a} = \boldsymbol{a}^2 = \lvert\boldsymbol{a}\rvert^2 = a^2$ In special fields, $(\boldsymbol{a}, \boldsymbol{b})$ is also used.
2-17.12 (*11-13.7*)	$\boldsymbol{a}\times\boldsymbol{b}$	vector product of $\boldsymbol{a}$ and $\boldsymbol{b}$	The coordinates, in a right-handed Cartesian coordinate system, are $(\boldsymbol{a}\times\boldsymbol{b})_x = a_yb_z - a_zb_y$, cycl, cycl. $(\boldsymbol{a}\times\boldsymbol{b})_i = \sum_j\sum_k \varepsilon_{ijk}a_jb_k$ See 2-17.10.

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-17.13 (*11-13.8*)	∇ $\vec{\nabla}$	nabla operator	$\nabla = \boldsymbol{e}_x \frac{\partial}{\partial x} + \boldsymbol{e}_y \frac{\partial}{\partial y} + \boldsymbol{e}_z \frac{\partial}{\partial z} =$ $\sum_i \boldsymbol{e}_i \frac{\partial}{\partial x_i}$ The operator is also called “del operator”.
2-17.14 (*11-13.9*)	$\nabla \varphi$ **grad** φ	gradient of φ	$\nabla \varphi = \sum_i \boldsymbol{e}_i \frac{\partial \varphi}{\partial x_i}$ Writing the operator grad in thin face should be avoided.
2-17.15 (*11-13.10*)	$\nabla \cdot \boldsymbol{a}$ div $\boldsymbol{a}$	divergence of $\boldsymbol{a}$	$\nabla \cdot \boldsymbol{a} = \sum_i \frac{\partial a_i}{\partial x_i}$
2-17.16 (*11-13.11*)	$\nabla \times \boldsymbol{a}$ **rot** $\boldsymbol{a}$	rotation of $\boldsymbol{a}$	The coordinates are $(\nabla \times \boldsymbol{a})_x = \frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}$, cycl, cycl The operator curl and thin face rot shall be avoided. $(\nabla \times \boldsymbol{a})_i = \sum_j \sum_k \varepsilon_{ijk} \frac{\partial a_k}{\partial x_j}$ See 2-17.10.
2-17.17 (*11-13.12*)	∇^2 Δ	Laplacian	$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$
2-17.18 (*11-13.13*)	$\square$	D'Alembertian	$\square = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}$
2-17.19 (*11-13.14*)	$\boldsymbol{T}$ $\vec{\vec{T}}$	tensor $\boldsymbol{T}$ of the second order	Two arrows above the letter symbol can be used instead of bold face sans serif type to indicate a tensor of the second order.
2-17.20 (*11-13.15*)	$T_{xx}, T_{xy}, \ldots, T_{zz}$ $T_{11}, T_{12}, \ldots, T_{33}$	Cartesian coordinates of tensor $\boldsymbol{T}$, Cartesian components of tensor $\boldsymbol{T}$	$\boldsymbol{T} = T_{xx}\boldsymbol{e}_x\boldsymbol{e}_x + T_{xy}\boldsymbol{e}_x\boldsymbol{e}_y + \ldots + T_{zz}\boldsymbol{e}_z\boldsymbol{e}_z$, are the component tensors. If it is clear from the context which are the base vectors, the tensor can be written $\boldsymbol{T} = \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix}$
2-17.21 (*11-13.16*)	$\boldsymbol{ab}$ $\boldsymbol{a} \otimes \boldsymbol{b}$	dyadic product, tensor product of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$	tensor of the second order with coordinates $(\boldsymbol{ab})_{ij} = a_i b_j$
2-17.22 (*11-13.17*)	$\boldsymbol{T} \otimes \boldsymbol{S}$	tensor product of two tensors $\boldsymbol{T}$ and $\boldsymbol{S}$ of the second order	tensor of the fourth order with coordinates $(\boldsymbol{T} \otimes \boldsymbol{S})_{ijkl} = T_{ij} S_{kl}$

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-17.23 (*11-13.18*)	$\boldsymbol{T} \cdot \boldsymbol{S}$	inner product of two tensors ***T*** and ***S*** of the second order	tensor of the second order with coordinates $(\boldsymbol{T} \cdot \boldsymbol{S})_{ik} = \sum_j T_{ij} S_{jk}$
2-17.24 (*11-13.19*)	$\boldsymbol{T} \cdot \boldsymbol{a}$	inner product of a tensor ***T*** of the second order and a vector ***a***	vector with coordinates $(\boldsymbol{T} \cdot \boldsymbol{a})_i = \sum_j T_{ij} a_j$
2-17.25 (*11-13.20*)	$\boldsymbol{T} : \boldsymbol{S}$	scalar product of two tensors ***T*** and ***S*** of the second order	scalar quantity $\boldsymbol{T} : \boldsymbol{S} = \sum_i \sum_j T_{ij} S_{ji}$

18 Transforms

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-18.1 (—)	$\mathcal{F} f$	Fourier transform of f	$(\mathcal{F} f)(\omega) = \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}\omega t} f(t)\,\mathrm{d}t \qquad (\omega \in \mathbf{R})$ This is often denoted by $\mathcal{F}(\omega)$. $(\mathcal{F} f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}\omega t} f(t)\,\mathrm{d}t$ is also used.
2-18.2 (—)	$\mathcal{L} f$	Laplace transform of f	$(\mathcal{L} f)(s) = \int_{0}^{\infty} \mathrm{e}^{-st} f(t)\,\mathrm{d}t \qquad (s \in \mathbf{C})$ This is often denoted by $\mathcal{L}(s)$. The two-sided Laplace transform is also used, defined by the same formula, but with minus infinity instead of zero.
2-18.3 (—)	$\mathfrak{Z}(a_n)$	Z transform of (a_n)	$\mathfrak{Z}(a_n) = \sum_{n=0}^{\infty} a_n z^{-n} \qquad (z \in \mathbf{C})$ $\mathfrak{Z}$ is an operator operating on a sequence (a_n) and not a function of a_n. The two-sided Z transform is also used, defined by the same formula, but with minus infinity instead of zero.
2-18.4 (*11-7.22*)	$\mathrm{H}(x)$ $\varepsilon(x)$	Heaviside function, unit step function	$\mathrm{H}(x) = \begin{cases} 1 & \text{for } x \geqslant 0 \\ 0 & \text{for } x < 0 \end{cases}$ $\mathrm{U}(x)$ is also used. $\vartheta(t)$ is used for the unit step function of time. EXAMPLE $(\mathrm{LH})(s) = 1/s \qquad (\mathrm{Re}\, s > 0)$
2-18.5 (*11-7.21*)	$\delta(x)$	Dirac delta distribution, Dirac delta function	$\int_{-\infty}^{+\infty} \varphi(t)\delta(t-x)\mathrm{d}t = \varphi(x)$ $\mathrm{H}' = \delta$ The name unit pulse is also used. EXAMPLE $\mathrm{L}\,\delta = 1$ See also 2-18.6 and IEC 60027-6:2006, item 2.01.
2-18.6 (*11-7.23*)	$f * g$	convolution of f and g	$(f * g)(x) = \int_{-\infty}^{\infty} f(y) g(x-y)\,\mathrm{d}y$

19 Special functions

The conventions used in this clause are: a, b, c, z, w, v are complex numbers, x is a real number, and k, l, m, n, are natural numbers.

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-19.1 (—)	γ C	Euler constant	$\gamma = \lim\limits_{n\to\infty}\left(\sum\limits_{k=1}^{n}\frac{1}{k} - \ln n\right) = 0{,}577\ 215\ 6...$
2-19.2 (*11-14.19*)	$\Gamma(z)$	gamma function	$\Gamma(z)$ is a meromorphic function with poles at 0, −1, −2, −3, … $\Gamma(z) = \int\limits_0^\infty t^{z-1}\mathrm{e}^{-t}\,\mathrm{d}t$ (Re $z > 0$) $\Gamma(n+1) = n!$ ($n \in \mathbf{N}$)
2-19.3 (*11-14.23*)	$\zeta(z)$	Riemann zeta function	$\zeta(z)$ is a meromorphic function with one pole at $z = 1$. $\zeta(z) = \sum\limits_{n=1}^{\infty}\frac{1}{n^z}$ (Re $z > 1$)
2-19.4 (*11-14.20*)	$\mathrm{B}(z,w)$	beta function	$\mathrm{B}(z,w) = \int\limits_0^1 t^{z-1}(1-t)^{w-1}\mathrm{d}t$ (Re $z > 0$, Re $w > 0$) $\mathrm{B}(z,w) = \Gamma(z)\,\Gamma(w)/\Gamma(z+w)$ $\frac{1}{(n+1)\ \mathrm{B}(k+1, n-k+1)} = \binom{n}{k}$ $(k \leqslant n)$
2-19.5 (*11-14.21*)	Ei x	exponential integral	$\mathrm{Ei}\,x = \fint\limits_{-\infty}^{x} \frac{\mathrm{e}^t}{t}\,\mathrm{d}t$ For $\fint$, see 2-11.20.
2-19.6 (—)	li x	logarithmic integral	$\mathrm{li}\,x = \int\limits_0^x \frac{1}{\ln t}\mathrm{d}t$ $(0 < x < 1)$ $\mathrm{li}\,x = \fint\limits_0^x \frac{1}{\ln t}\,\mathrm{d}t$ $(x > 1)$ For $\fint$, see 2-11.20.
2-19.7 (—)	Si z	sine integral	$\mathrm{Si}\,z = \int\limits_0^z \frac{\sin t}{t}\ \mathrm{d}t$ $\mathrm{si}\,z = -\frac{\pi}{2} + \mathrm{Si}\,z$ is called the complementary sine integral.

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-19.8 (—)	$\mathrm{S}(z)$ $\mathrm{C}(z)$	Fresnel integrals	$\mathrm{S}(z)=\int_0^z \sin\left(\frac{\pi}{2}t^2\right)\mathrm{d}t$ $\mathrm{C}(z)=\int_0^z \cos\left(\frac{\pi}{2}t^2\right)\mathrm{d}t$
2-19.9 (*11-14.22*)	$\mathrm{erf}\,x$	error function	$\mathrm{erf}\,x=\frac{2}{\sqrt{\pi}}\int_{-\infty}^{x}\mathrm{e}^{-t^2}\mathrm{d}t$ $\mathrm{erfc}\,x = 1 - \mathrm{erf}\,x$ is called the complementary error function. In statistics, the distribution function $\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_0^x \mathrm{e}^{-t^2/2}\,\mathrm{d}t$ is used.
2-19.10 (*11-14.16*)	$\mathrm{F}(\varphi, k)$	incomplete elliptic integral of the first kind	$\mathrm{F}(\varphi,k)=\int_0^{\varphi}\frac{\mathrm{d}\sigma}{\sqrt{1-k^2\sin^2\sigma}}$ $\mathrm{K}(k) = \mathrm{F}(\pi/2, k)$ is the complete elliptic integral of the first kind (here $0 < k < 1$, $k \in \mathbf{R}$).
2-19.11 (*11-14.17*)	$\mathrm{E}(\varphi, k)$	incomplete elliptic integral of the second kind	$\mathrm{E}(\varphi,k)=\int_0^{\varphi}\sqrt{1-k^2\sin^2\sigma}\,\mathrm{d}\sigma$ $\mathrm{E}(k)=\mathrm{E}\left(\frac{\pi}{2},k\right)$ is the complete elliptic integral of the second kind (here $0 < k < 1$, $k \in \mathbf{R}$).
2-19.12 (*11-14.18*)	$\Pi(n, \varphi, k)$	incomplete elliptic integral of the third kind	$\Pi(n,\varphi,k)=$ $\int_0^{\varphi}\frac{\mathrm{d}\vartheta}{(1+n\sin^2\vartheta)\sqrt{1-k^2\sin^2\vartheta}}$ $\Pi(n,k)=\Pi\left(n,\frac{\pi}{2},k\right)$ is the complete elliptic integral of the third kind (here $0 < k < 1$, $n, k \in \mathbf{R}$).
2-19.13 (*11-14.14*)	$\mathrm{F}(a, b; c; z)$	hypergeometric functions	$\mathrm{F}(a,b;c;z)=\sum_{n=0}^{\infty}\frac{(a)_n(b)_n\,z^n}{(c)_n\,n!}$ $(-c \notin \mathbf{N})$ For $(a)_n$, $(b)_n$ and $(c)_n$, see 2-10.3. Solutions of $z(1-z)y''+[c-(a+b+1)z]y'-aby=0$

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-19.14 (*11-14.15*)	$\mathrm{F}(a;c;z)$	confluent hypergeometric functions	$\mathrm{F}(a;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n}{(c)_n\, n!} z^n \quad (-c \notin \mathbf{N})$ For $(a)_n$ and $(c)_n$, see 2-10.3. Solutions of $zy'' + (c-z)y' - ay = 0$
2-19.15 (*11-14.8*)	$\mathrm{P}_n(z)$	Legendre polynomials	$\mathrm{P}_n(z) = \frac{1}{2^n\, n!} \frac{\mathrm{d}^n}{\mathrm{d}z^n}\left(z^2-1\right)^n \quad (n \in \mathbf{N})$ Solutions of $(1-z^2)y'' - 2zy' + n(n+1)y = 0$
2-19.16 (*11-14.9*)	$\mathrm{P}_n^m(z)$	associated Legendre functions	$\mathrm{P}_n^m(z) = (-1)^m \left(1-z^2\right)^{m/2} \frac{\mathrm{d}^m}{\mathrm{d}z^m} \mathrm{P}_n(z)$ $(m,\ n \in \mathbf{N},\ m \leqslant n)$ Solutions of $(1-z^2)y'' - 2zy' + \left[n(n+1) - \frac{m^2}{1-z^2}\right]y = 0$ The factor $(-1)^m$ follows from the general theory of spherical functions.
2-19.17 (*11-14.10*)	$\mathrm{Y}_l^m(\vartheta,\varphi)$	spherical harmonics	$\mathrm{Y}_l^m(\vartheta,\ \varphi) =$ $\left[\frac{(2l+1)}{4\pi} \frac{(l-\|m\|)!}{(l+\|m\|)!}\right]^{1/2} \times \mathrm{P}_l^{\|m\|}(\cos\vartheta)\, \mathrm{e}^{\mathrm{i}m\varphi}$ $(l, \|m\| \in \mathbf{N};\ \|m\| \leqslant l)$ Solutions of $\frac{1}{\sin\vartheta} \frac{\partial}{\partial\vartheta}\left(\sin\vartheta \frac{\partial y}{\partial\vartheta}\right) + \frac{1}{\sin^2\vartheta} \frac{\partial^2 y}{\partial\varphi^2} +$ $l(l+1)y = 0$
2-19.18 (*11-14.11*)	$\mathrm{H}_n(z)$	Hermite polynomials	$\mathrm{H}_n(z) = (-1)^n\, \mathrm{e}^{z^2} \frac{\mathrm{d}^n}{\mathrm{d}z^n} \mathrm{e}^{-z^2}$ Solutions of $y'' - 2zy' + 2ny = 0 \quad (n \in \mathbf{N})$
2-19.19 (*11-14.12*)	$\mathrm{L}_n(z)$	Laguerre polynomials	$\mathrm{L}_n(z) = \mathrm{e}^z \frac{\mathrm{d}^n}{\mathrm{d}z^n}\left(z^n \mathrm{e}^{-z}\right) \quad (n \in \mathbf{N})$ Solutions of $zy'' + (1-z)y' + ny = 0$
2-19.20 (*11-14.13*)	$\mathrm{L}_n^m(z)$	associated Laguerre polynomials	$\mathrm{L}_n^m(z) = \frac{\mathrm{d}^m}{\mathrm{d}z^m} \mathrm{L}_n(z) \quad (m \in \mathbf{N},\ m \leqslant n)$ Solutions of $zy'' + (m+1-z)y' + (n-m)y = 0$

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-19.21 (—)	$\mathrm{T}_n(z)$	Chebyshev polynomials of the first kind	$\mathrm{T}_n(z) = \cos(n \arccos z)$ $(n \in \mathbf{N})$ Solutions of $(1-z^2)y'' - zy' + n^2 y = 0$
2-19.22 (—)	$\mathrm{U}_n(z)$	Chebyshev polynomials of the second kind	$\mathrm{U}_n(z) = \dfrac{\sin\left[(n+1)\arccos z\right]}{\sin(\arccos z)}$ $(n \in \mathbf{N})$ Solutions of $(1-z^2)y'' - 3zy' + n(n+2)y = 0$
2-19.23 (*11-14.1*)	$\mathrm{J}_\nu(z)$	Bessel functions, cylindrical functions of the first kind	$\mathrm{J}_\nu(z) = \sum_{k=0}^{\infty} \dfrac{(-1)^k (z/2)^{\nu+2k}}{k!\,\Gamma(\nu+k+1)}$ $(\nu \in \mathbf{C})$ Solutions of $z^2 y'' + zy' + (z^2 - \nu^2)y = 0$
2-19.24 (*11-14.2*)	$\mathrm{N}_\nu(z)$	Neumann functions, cylindrical functions of the second kind	$\mathrm{N}_\nu(z) = \dfrac{\mathrm{J}_\nu(z)\cos(\nu\pi) - \mathrm{J}_{-\nu}(z)}{\sin(\nu\pi)}$ $(\nu \in \mathbf{C})$ The right-hand side of this equation is replaced by its limiting value if $\nu \in \mathbf{Z}$. $\mathrm{Y}_\nu(z)$ is also used.
2-19.25 (*11-14.3*)	$\mathrm{H}_\nu^{(1)}(z)$ $\mathrm{H}_\nu^{(2)}(z)$	Hankel functions, cylindrical functions of the third kind	$\mathrm{H}_\nu^{(1)}(z) = \mathrm{J}_\nu(z) + \mathrm{i}\mathrm{N}_\nu(z)$ $\mathrm{H}_\nu^{(2)}(z) = \mathrm{J}_\nu(z) - \mathrm{i}\mathrm{N}_\nu(z)$ $(\nu \in \mathbf{C})$
2-19.26 (*11-14.4*)	$\mathrm{I}_\nu(z)$ $\mathrm{K}_\nu(z)$	modified Bessel functions	$\mathrm{I}_\nu(z) = \mathrm{e}^{-\frac{1}{2}\mathrm{i}\nu\pi}\,\mathrm{J}_\nu\left(\mathrm{e}^{\frac{1}{2}\mathrm{i}\pi} z\right)$ $\mathrm{K}_\nu(z) = \dfrac{\mathrm{i}\pi}{2}\mathrm{e}^{\frac{1}{2}\mathrm{i}\nu\pi}\,\mathrm{H}_\nu^{(1)}\left(\mathrm{e}^{\frac{1}{2}\mathrm{i}\pi} z\right)$ Solutions of $z^2 y'' + zy' - (z^2 + \nu^2)y = 0$
2-19.27 (*11-14.5*)	$\mathrm{j}_l(z)$	spherical Bessel functions	$\mathrm{j}_l(z) = \left(\dfrac{\pi}{2z}\right)^{\frac{1}{2}} \mathrm{J}_{l+1/2}(z)$ $(l \in \mathbf{N})$ Solutions of $z^2 y'' + 2zy' + \left[z^2 - l(l+1)\right]y = 0$
2-19.28 (*11-14.6*)	$\mathrm{n}_l(z)$	spherical Neumann functions	$\mathrm{n}_l(z) = \left(\dfrac{\pi}{2z}\right)^{\frac{1}{2}} \mathrm{N}_{l+1/2}(z)$ $\mathrm{y}_l(z)$ is also used. $(l \in \mathbf{N})$

Item No.	Sign, symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-19.29 (*11-14.7*)	$\mathrm{h}_l^{(1)}(z)$ $\mathrm{h}_l^{(2)}(z)$	spherical Hankel functions	$\mathrm{h}_l^{(1)}(z) = \mathrm{j}_l(z) + \mathrm{i}\,\mathrm{n}_l(z) = \left(\frac{\pi}{2z}\right)^{\frac{1}{2}} \mathrm{H}_{l+\frac{1}{2}}^{(1)}(z)$ $\mathrm{h}_l^{(2)}(z) = \mathrm{j}_l(z) - \mathrm{i}\,\mathrm{n}_l(z) = \left(\frac{\pi}{2z}\right)^{\frac{1}{2}} \mathrm{H}_{l+\frac{1}{2}}^{(2)}(z)$ Modified spherical Bessel functions (analogous to 2-19.26) can be defined and are denoted by $\mathrm{i}_l(z)$ and $\mathrm{k}_l(z)$ respectively.
2-19.30 (—)	$\mathrm{Ai}(z)$ $\mathrm{Bi}(z)$	Airy functions	$\mathrm{Ai}(z) = \frac{1}{3}\sqrt{z}\left[\mathrm{I}_{-\frac{1}{3}}(w) - \mathrm{I}_{\frac{1}{3}}(w)\right]$ $\mathrm{Bi}(z) = \sqrt{\frac{z}{3}}\left[\mathrm{I}_{-\frac{1}{3}}(w) + \mathrm{I}_{\frac{1}{3}}(w)\right]$ where $w = \frac{2}{3}z^{3/2}$. Entire solutions of $y'' - zy = 0$

Annex A
(normative)

Clarification of the symbols used

ISO/IEC 10646 provides names of all symbols, together with a variety of encodings to be used when these symbols or letters are present in machine communication. The main purpose of ISO/IEC 10646 is to provide unambiguous identification of a letter or symbol. It is beyond the scope of ISO 80000-2 to describe fully the facilities and concepts of ISO/IEC 10646.

The Unicode Consortium has published an equivalent specification [3]. However, the Unicode Consortium specification [3] also adds properties of characters (such as whether they are digits, or upper/lower case equivalents, etc). These properties are not important to this International Standard.

For the purposes of the following tables, both ISO/IEC 10646 and Reference [3] give identical specifications, commonly referred to as "Unicode characters".

ISO/IEC 10646 and Unicode were developed as an extension of the alphabet covered by what is known as "ASCII" (Basic Latin). There are a number of possible encodings of the symbols and characters, of which the most popular are a fixed 32-bit encoding, a fixed 16-bit encoding – not all characters – and a so-called UTF-8 encoding that is of variable length, but results in ASCII characters encoding in a single octet.

It is unfortunately the case that many symbols (such as LATIN SMALL LETTER V and GREEK SMALL LETTER NU) have almost identical glyphs in most (but not all) publication fonts.

This normative annex is provided to clarify precisely the symbol that is used in the body of ISO 80000-2, irrespective of the font used.

Table A.1 has four columns.

— The first column gives the row reference to the item in the body of ISO 80000-2 in which the symbol is used. (Heading: "**Item No.**".)

— The second column repeats the symbol in the same font, position, format and size that is used in the body of ISO 80000-2. (Heading: "**Sign, symbol**".)

— The third column gives the name allocated in ISO/IEC 10646 and Unicode [3] (they are identical). Examples are "N-ARY PRODUCT" (as opposed to "GREEK CAPITAL PI") or "N-ARY SUMMATION" (as opposed to "GREEK CAPITAL SIGMA"). [Heading: "**Name of symbol (see ISO/IEC 10646)**".] It is recognized that some of the names in ISO/IEC 10646 are not consistent with current practice and use of the associated symbol and, in particular, differ from the usage in ISO 80000-2.

— The fourth column gives (for easy reference and further clarification) the 16-bit hexadecimal code assigned to the symbol by both ISO/IEC 10646 and Reference [3]. UTF-8 (a variable length encoding, which is only one octet for characters in the ASCII character set) and 32-bit encodings – and others – are also possible. [Heading: "**Hexadecimal value of symbol** (**see ISO/IEC 10646**)".]

NOTE In other parts of the ISO 80000 and IEC 80000 series, the "style" (upright or *italic*, not bold or **bold**, or ***bold italic***) also has semantic significance, and has to be adhered to. However, in ISO 80000-2, all symbols are upright, and there is no column in the following table for such information.

There is additional information on symbols for use in mathematics in Reference [4].

Table A.1

Item No.	Sign, symbol	Name of symbol (see ISO/IEC 10646)	Hexadecimal value of symbol (See ISO/IEC 10646)
2-4.1	∧	LOGICAL AND	2227
2-4.2	∨	LOGICAL OR	2228
2-4.3	¬	NOT SIGN	00AC
2-4.4	⇒	RIGHTWARDS DOUBLE ARROW	21D2
2-4.5	⇔	LEFT RIGHT DOUBLE ARROW	21D4
2-4.6	∀	FOR ALL	2200
2-4.7	∃	THERE EXISTS	2203
2-5.1	∈	ELEMENT OF	2208
2-5.2	∉	NOT AN ELEMENT OF	2209
2-5.4	|	VERTICAL LINE	007C
2-5.5	|	VERTICAL LINE	007C
2-5.6	∅	EMPTY SET	2205
2-5.7	⊆	SUBSET OF OR EQUAL TO	2286
2-5.8	⊂	SUBSET OF	2282
2-5.9	∪	UNION	222A
2-5.10	∩	INTERSECTION	2229
2-5.11	⋃	N-ARY UNION	22C3
2-5.12	⋂	N-ARY INTERSECTION	22C2
2-5.13	∖	SET MINUS	2216
2-5.13	∁	COMPLEMENT	2201
2-5.16	×	MULTIPLICATION SIGN	00D7
2-5.17	∏	N-ARY PRODUCT	220F
2-6.1	ℕ	DOUBLE-STRUCK CAPITAL N	2115
2-6.2	ℤ	DOUBLE-STRUCK CAPITAL Z	2124
2-6.3	ℚ	DOUBLE-STRUCK CAPITAL Q	211A
2-6.4	ℝ	DOUBLE-STRUCK CAPITAL R	211D
2-6.5	ℂ	DOUBLE-STRUCK CAPITAL C	2102
2-6.6	ℙ	DOUBLE-STRUCK CAPITAL P	2119
2-7.1	=	EQUALS SIGN	003D
2-7.2	≠	NOT EQUAL TO	2260
2-7.3	≔	COLON EQUALS	2254
2-7.3	≝	EQUAL TO BY DEFINITION	225D
2-7.4	≙	CORRESPONDS TO	2259
2-7.5	≈	APPROXIMATELY EQUAL TO	2248

Table A.1 (*continued*)

Item No.	**Sign, symbol**	**Name of symbol** (see ISO/IEC 10646)	**Hexadecimal value of symbol** (See ISO/IEC 10646)
2-7.6	≃	ASYMPTOTICALLY EQUAL TO	2243
2-7.7	∼	TILDE OPERATOR	223C
2-7.7	∝	PROPORTIONAL TO	221D
2-7.8	≅	CONGRUENT TO	2245
2-7.9	<	LESS-THAN SIGN	003C
2-7.10	>	GREATER-THAN SIGN	003E
2-7.11	⩽	LESS-THAN OR EQUAL TO	2264
2-7.12	⩾	GREATER-THAN OR EQUAL TO	2265
2-7.13	≪	MUCH LESS-THAN	226A
2-7.14	≫	MUCH GREATER-THAN	226B
2-7.15	∞	INFINITY	221E
2-7.16	→	RIGHTWARDS ARROW	2192
2-7.17	∣	DIVIDES	2223
2-7.18	≡	IDENTICAL TO	2261
2-7.19	⟨	MATHEMATICAL LEFT ANGLE BRACKET	27E8
2-7.19	⟩	MATHEMATICAL RIGHT ANGLE BRACKET	27E9
2-8.1	∥	PARALLEL TO	2225
2-8.2	⊥	PERPENDICULAR	27C2
2-8.3	∢	ANGLE	2222
2-9.1	+	PLUS SIGN	002B
2-9.2	−	MINUS SIGN	2212
2-9.3	±	PLUS-MINUS SIGN	00B1
2-9.4	∓	MINUS-PLUS SIGN	2213
2-9.5	⋅	DOT OPERATOR	22C5
2-9.5	×	MULTIPLICATION SIGN	00D7
2-9.6	/	SOLIDUS	002F
2-9.7	∑	N-ARY SUMMATION	2211
2-9.8	∏	N-ARY PRODUCT	220F
2-9.10	√	SQUARE ROOT	221A
2-9.12	⟨	MATHEMATICAL LEFT ANGLE BRACKET	27E8
2-9.12	⟩	MATHEMATICAL RIGHT ANGLE BRACKET	27E9
2-9.16			VERTICAL LINE	007C

Table A.1 (*continued*)

Item No.	**Sign, symbol**	**Name of symbol** (see ISO/IEC 10646)	**Hexadecimal value of symbol** (See ISO/IEC 10646)
2-9.17	⌊	LEFT FLOOR	230A
2-9.17	⌋	RIGHT FLOOR	230B
2-9.18	⌈	LEFT CEILING	2308
2-9.18	⌉	RIGHT CEILING	2309
2-11.3	→	RIGHTWARDS ARROW	2192
2-11.4	↦	RIGHTWARDS ARROW FROM BAR	21A6
2-11.7	∘	RING OPERATOR	2218
2-11.11	∆	INCREMENT	2206
2-11.12	′	PRIME	2032
2-11.15	∂	PARTIAL DIFFERENTIAL	2202
2-11.16	d	LATIN SMALL LETTER D	0064
2-11.17	δ	GREEK SMALL LETTER DELTA	03B4
2-11.18	∫	INTEGRAL	222B
2-11.19	∬	DOUBLE INTEGRAL	222C
2-11.19	∮	CONTOUR INTEGRAL	222E
2-11.19	∯	SURFACE INTEGRAL	222F
2-11.20	⨍	FINITE PART INTEGRAL	2A0D
2-17.11	⋅	DOT OPERATOR	22C5
2-17.12	×	MULTIPLICATION SIGN	00D7
2-17.13	∇	NABLA	2207
2-17.17	∆	INCREMENT	2206
2-17.18	□	WHITE SQUARE	25A1
2-17.21	⊗	CIRCLED TIMES	2297
2-18.1	ℱ	SCRIPT CAPITAL F	2131
2-18.2	ℒ	SCRIPT CAPITAL L	2112
2-18.3	ℨ	BLACK-LETTER CAPITAL Z	2128
2-18.6	∗	ASTERISK OPERATOR	2217

Bibliography

[1] ISO/IEC 10646:2003, *Information technology — Universal Multiple-Octet Coded Character Set (UCS)*

[2] IEC 60027-6:2006, *Letter symbols to be used in electrical technology — Part 6: Control technology*

[3] The Unicode Standard, Version 5.0:2007. *The Unicode Consortium*. (Reading, MA, Addison-Wesley)

[4] Unicode Technical Report #25 — Unicode support for mathematics. *The Unicode Consortium*. URL http://www.unicode.org/reports/tr25

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