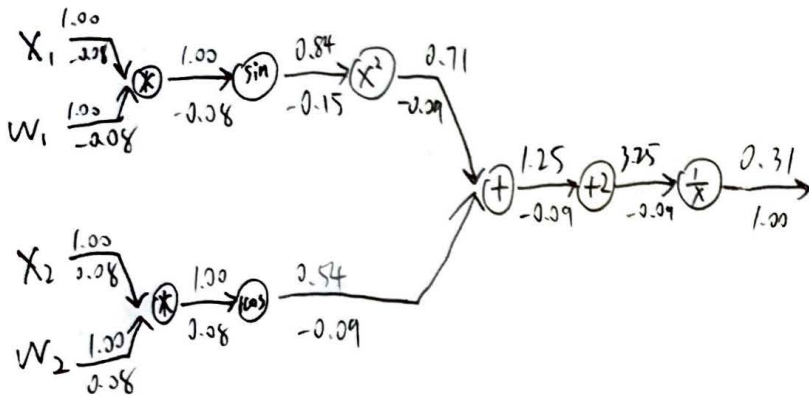


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HW2:

Problem 1: $f(x) = \frac{1}{2 + \sin^2(x_1 w_1) + \cos(x_2 w_2)}$



With inputs: $x_1 = 1.00$, $w_1 = 1.00$, $x_2 = 1.00$, $w_2 = 1.00$,

Gradients with respect to x_1 : -0.08

$$w_1 = -0.08$$

$$x_2 = 0.08$$

$$w_2 = 0.08$$

$$f(x) = \frac{1}{x} \quad \frac{df}{dx} = -\frac{1}{x^2} \quad (1.00) \left(\frac{-1}{3.25^2} \right) = -0.09$$

$$f(x) = 1 + x \quad \frac{df}{dx} = 1 \quad (-0.09)(1) = -0.09$$

$$f(x, y) = x + y \quad \frac{df}{dx} = 1, \frac{df}{dy} = 1 \quad (-0.09)(1) = -0.09$$

$$f(x) = x^2 \quad \frac{df}{dx} = 2x \quad (-0.09)(2 \times 0.84) = -0.15$$

$$f(x) = \sin x \quad \frac{df}{dx} = \cos x \quad (-0.15)(\cos(1.00)) = -0.08$$

$$f(x, y) = xy \quad \frac{df}{dx} = y, \frac{df}{dy} = x \quad (-0.08)(1.00) = -0.08$$

$$f(x) = \cos x \quad \frac{df}{dx} = -\sin x \quad (-0.09)(-\sin(1.00)) = 0.08$$

$$f(x, y) = xy \quad \frac{df}{dx} = y, \frac{df}{dy} = x \quad (0.08)(1.00) = 0.08$$

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Problem 2:

$$f(x, w) = ||\sigma(Wx)||^2, W: 3 \times 3, X: 3 \times 1$$

$$= \sum_{i=1}^n [\sigma(W \cdot x)]_i^2$$

Diagram illustrating the calculation of $f(x, w)$:

- Weight matrix $W = \begin{bmatrix} 1.00 & 1.00 & 1.00 \\ 0.09 & 0.09 & 0.09 \\ 0.09 & 0.09 & 0.09 \end{bmatrix}$
- Input vector $x = \begin{bmatrix} 1.00 \\ 1.00 \\ 1.00 \end{bmatrix}$
- Intermediate vector: $W \cdot x = \begin{bmatrix} 2.95 \\ 2.95 \\ 2.95 \end{bmatrix}$
- Activation function: $\sigma \left(\begin{bmatrix} 2.95 \\ 2.95 \\ 2.95 \end{bmatrix} \right) = \begin{bmatrix} 1.9 \\ 1.9 \\ 1.9 \end{bmatrix}$
- Final result: $||\sigma(Wx)||^2 = 2.71$

$$q = \sigma(Wx) = \begin{bmatrix} \sigma(W_{1,1}x_1) + \dots + \sigma(W_{1,n}x_n) \\ \vdots \\ \sigma(W_{n,1}x_1) + \dots + \sigma(W_{n,n}x_n) \end{bmatrix}$$

$$f(q) = ||q||^2 = q_1^2 + q_2^2 + \dots + q_n^2$$

$$\frac{\partial f}{\partial q_i} = 2q_i \quad \nabla_q f = 2q \quad (1.00) \left(2 \begin{bmatrix} 2.95 \\ 2.95 \\ 2.95 \end{bmatrix} \right) = \begin{bmatrix} 1.9 \\ 1.9 \\ 1.9 \end{bmatrix}$$

$$m = Wx$$

$$q = \sigma(m)$$

$$\frac{dq}{dm} = \frac{d\sigma(m)}{dm} = \frac{e^{-m}}{(1+e^{-m})^2} = (1 - \sigma(m)) \sigma(m) \quad \left(\begin{bmatrix} 1.9 \\ 1.9 \\ 1.9 \end{bmatrix} \right) \left(\begin{bmatrix} 1 - \sigma(2.95) \\ 1 - \sigma(2.95) \\ 1 - \sigma(2.95) \end{bmatrix} \right) = \begin{bmatrix} 0.09 \\ 0.09 \\ 0.09 \end{bmatrix}$$

$$\frac{\partial m_k}{\partial w_{ij}} = \delta_{k,i} x_j \quad \begin{bmatrix} 0.09 & 0.09 & 0.09 \\ 0.09 & 0.09 & 0.09 \\ 0.09 & 0.09 & 0.09 \end{bmatrix}$$

$$\frac{\partial m_k}{\partial x_i} = W_{ki}$$

$$\begin{bmatrix} 0.27 \\ 0.27 \\ 0.27 \end{bmatrix}$$

$$\text{For input: } W = \begin{bmatrix} 1.00 & 1.00 & 1.00 \\ 0.09 & 0.09 & 0.09 \\ 0.09 & 0.09 & 0.09 \end{bmatrix}$$

$$x = \begin{bmatrix} 1.00 \\ 1.00 \\ 1.00 \end{bmatrix}$$

$$\text{Gradient respect to } W: \begin{bmatrix} 0.09 & 0.09 & 0.09 \\ 0.09 & 0.09 & 0.09 \\ 0.09 & 0.09 & 0.09 \end{bmatrix}$$

$$x = \begin{bmatrix} 0.27 \\ 0.27 \\ 0.27 \end{bmatrix}$$