Exploratory Data Analysis

1. EDA (Exploratory Data Analysis by J.Tukey).

The basic idea behind EDA methodology is to learn how to explore data and find valuable information, structures and relationships among variables. Find the structure of the majority of the data but also detect exceptional observations, rare events. The old fashioned thinking was to compile a list of procedures (such as Proc's in SAS) and to decide which one was suitable for your data so in fact you are "fitting" the data to a suitable "procedure". With EDA we take the opposite approach, we try to devise procedures that are best suitable for understanding the information that is contained in our data.

If you would like to learn in depth about debate that took place in the 1950's about the transformation of statistics and data analysis that was taking place at than time please read the article "The future of Data Analysis" J W Tukey (1952), Annals of Probability & Statistics.

We are going to start with how to look at one sample that is a single set of observations that are alike and that are the outcomes of a variable or measurement.

The simplest way to represent the data is using a stem-and-leaf display

2. STEM AND LEAF DISPLAYS (Tukey).

When we work with sample of numbers the first thing that we need to understand is that the variation that really matters resides only in 2-3 significant digits. If we find a set of numbers such as the list below with 13 digits we should try to reduce them to summary of 2-3 digits which contain the significant variation among them. The numbers below represent the time in seconds required by the speed of light to cover the distance between two towers that are approximately 1.5 miles apart. Since the units of measurements are seconds, all the observations start with 0.0000078. We call this the common digits. Then we are left with three significant digits and the remaining residual of two digits.

Example: 0.000007884482 is split into common + significant + residual

0.0000078 844 82

The significant digits 844 are then split into STEM + LEAF or 84 4.

Figure 1. Decomposing a sample of numbers.

If we find a set of numbers such as	they can be decomposed common +	stem	+ leaf	+ residual
0.000007884482	0.0000078	84	4	82
0.000007888237	0.0000078	88	2	37
0.000007884282	0.0000078	84	2	82
0.000007883634	0.0000078	83	6	34
0.000007885782	0.0000078	85	7	82
0.000007884150	0.0000078	84	1	50

2.1 Rounding. When we delete the residual digits we should not just truncate the numbers but rather should round the previous significant digit. The first four number on Figure 1 are rounded to 84 5, 88 2, 84 3, 83 6 and 85 8. A good question is what to do with 841 50, should we round it downwards or upwards as many text books recommend? If we do so, we may get a slight bias so the best is to alternate by rounding to even numbers. Hence 84 1 50 rounds to 84 2.

2.2 Constructing a STEM AND LEAF diagram.

Put the stems in one column and the leaves are added in rows. Stems: 83 to 88

```
83  6
84  235
85  8
86
87
88  2
```

The above display appears reasonable but there are alternative ways to display a stem and leaf graph. Since one digit can take 10 possible values 0,…,9 it can be split into equal subgroups in two ways: (i) 0-1 2-3 4-5 6-7 8-9 or (ii) 0-4 5-9. Using (ii) The previous display becomes:

```
83  6
84  23
84  5
85
85  8
86
86
87
87
88  2
```

which appears too dispersed. We could have made the split for the stem one digit to the left and rounding the other digits we would have 8 4, 8 8, 8 4, 84, 8 6, 8

8f 444 or 8* 444
8s 6 8 68
8e 8

Note that '.', t, f, s, e and '.' after the stems on the left indicate the ranges 0-1 2-3 4-5 6-7 8-9. On the right hand side stems * indicates 0-4 and otherwise is 5-9

Examples of stem-and-leaf displays

Example in R: In R we use the ***stem*** function. The number of stems can be altered using the scale argument:

```
> data(faithful)
> attach(faithful)
> stem(waiting,scale=1)
The decimal point is 1 digit(s) to the right of the |
  4 | 3
  4 | 5556666677788899999
  5 | 00000111111222223333333444444444
  5 | 555555666677788889999999
  6 | 00000022223334444
  6 | 555667899
  7 | 0000111112333333444444
  7 | 55555555666666666777777777777888888888888889999999999
  8 | 00000000111111111111222222222223333333333333334444444444
  8 | 55555566666677888888999
  9 | 00000012334
  9 | 6
```

Example in SAS.

In SAS the stem and leaf display is part of PROC UNIVARIATE.

This is an example of the way SAS implements stem and leaf displays. Notice that the order of the stems is reversed compared to the previous graphs.

```
OPTIONS PS=55 LS=80;
DATA CRIME;
INFILE 'crime.dat';
INPUT MURDER RAPE ROBBERY ASSAULT BURGLARY LARCENY
        AUTOTHFT REGION $;
run;
proc univariate plot;
var murder;
run;
```

```
Stem Leaf                  #            Boxplot
 15 3                      1               |
 14 6                      1               |
 13                                        |
 12 27                     2               |
 11 156778                 6               |
 10 17                     2               |
  9 24456                  5            +-----+
  8 488                    3            |     |
  7 79                     2            |     |
  6 2269                   4            *--+--*
  5 3779                   4            |     |
```

```
4 6688                    4              |     |
3 022455568               9              +-----+
2 000                     3                 |
1 359                     3                 |
0 5                       1                 |
```

3. Depth, Ranks, Order of data values

One simple derivation that can be calculated from data is the rank. Ranks tell the position of each individual value in the sample and hence for a sample of *n* observations the ranks are values between 1 and n. When the observations are tied the midrank or median of ranks is assigned.

The numbers 3, 5, 2, 8, 5, 4, 5, 8 yield the ranks 2 4 1 6.5 4 5 6.5.

Depth (tukey) of an observation if the minimum between the ascending and descending rank or the rank from the closest end. An observation with low depth is near the extremes of the sample where as an observation with high depth is toward the center of the sample.

Example:

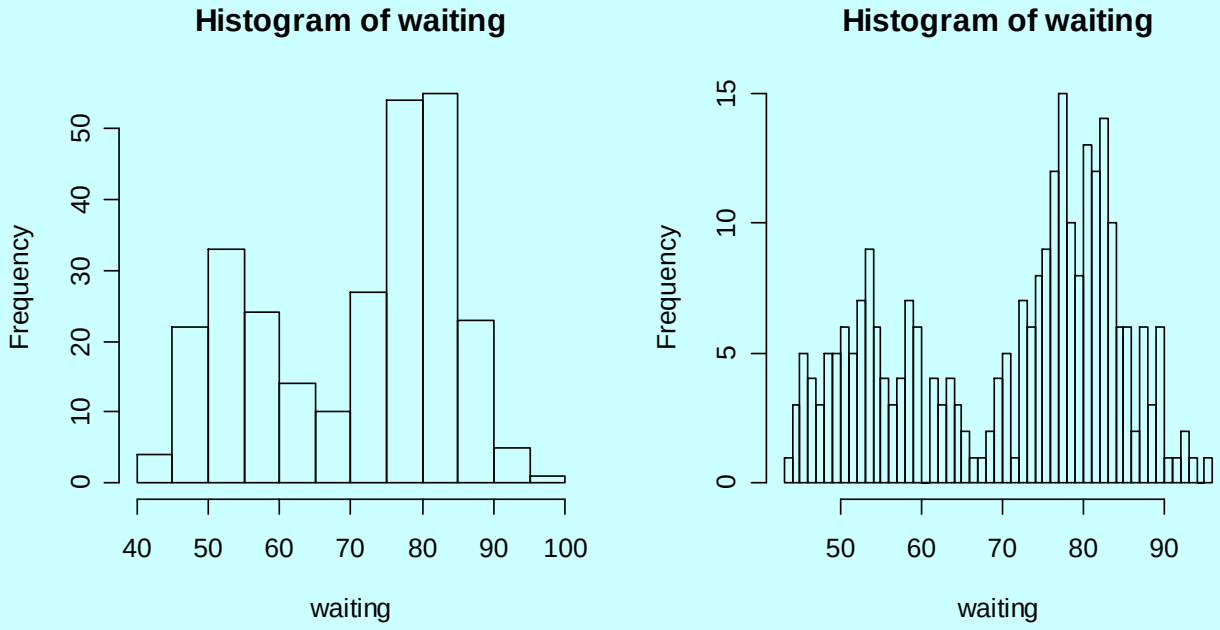
```
> depth = function(x) pmin( rank(x), length(x) +1-rank(x))
> cbind(waiting,depth=depth(waiting), rank=rank(waiting))
      waiting depth  rank
[1,]      79  97.5 175.5
[2,]      54  49.0  49.0
[3,]      74 123.5 123.5 ….
```

4. Histogram

Histograms are less informative than stem and leaf plots because the leaf information is lost. However as the sample size gets larger the leaf information is less important and the histogram becomes a simpler graph and hence superior to the stem and leaf.

The following example is from the Old Faithful geyser at Yellowstone park and it shows a histogram of the waiting times between eruptions. The interesting feature of this graph is the bimodal nature of the data which suggests interesting geophysical properties.

```
> par(mfrow=c(1,2))
> hist(waiting)
> hist(waiting,50)
```



The main steps for constructing a histogram are:

(i) Decide on a suitable histogram range that reduces the data range to a "pretty" interval. In the waiting example if the data range is [42,98] the histogram range could be [40,100].

(ii) Decide on one of the two, the number of intervals or the interval size, and build the histogram intervals. There are many methods that have been devised for this purpose. See below for the details of the different approaches.

(iii) Tabulate the data into a frequency table where the groups are the histogram intervals.

(iv) Finally draw the histogram.

4.1 CHOOSING THE NUMBER OF LINES OF A STEM & LEAF OR A HISTOG

$L = [10 * LOG_{10}(n)]$

where $[x]$ is the integer part of x.

EXAMPLE: If n = 25 $L = [10 * \log_{10}(25)] = [10 * 1.39794] = 13$

4.2 CLASS WIDTH: Round(RANGE / L)

The range for ROBBERY IS 435.7 so 435.7 / 13 = 33.5

We choose width=50.

The range for MURDER IS 13.3 so 13.3 / 13 = 1.02

We choose width= 1 or 2.

4.3 OTHER RULES

Sturges: L = [1+ $\log_2(n)$]

RootN: L = [2 sqrt(n)]

Table 1. comparing Log_{10}, RootN and Stsurges rules.

N	LOG_{10}	ROOTN	STURGES
10	10.00	6.32	4.32
20	13.01	8.94	5.32
30	14.77	10.95	5.90
40	16.02	12.64	6.32
50	16.98	14.14	6.64
60	17.78	15.49	6.90
70	18.45	16.73	7.12
80	19.03	17.88	7.32
90	19.54	18.97	7.49
100	20.00	20.00	7.64
200	23.01	28.28	8.64
300	24.77	34.64	9.22
400	26.02	40.00	9.64
500	26.98	44.72	9.96
N	LOG_{10}	ROOTN	STURGES
16	12.04	8.00	5.00
32	15.05	11.31	6.00
64	18.06	16.00	7.00
128	21.07	22.62	8.00
256	24.08	32.00	9.00

DOANE: CORRECTION FOR SKEWNESS

LOG2(1 + $skewness/\sigma(skewness)$)

$$skewness = \mu_3 / \sigma^3$$

$$\sigma(skewness) = \sqrt{\frac{6(n-2)}{(n+1)(n+3)}}$$

4.4 ANOTHER APPROACH: RULES FOR INTERVAL WIDTH: h

Scotts Rule: $h = \frac{3.49 \times s}{\sqrt[3]{n}}$

where s is a resistant estimate of σ

Freedman & Diaconis $h = 1.66 \times s \times \sqrt[3]{Log_e(n)/n}$

Freedman & Diaconis $h^* = \frac{2 \times IQR}{\sqrt[3]{n}}$

Table 2. comparing Freedman & Diaconis *h* Vs Scott's rule (times *s*).

n	F-D	SCOTT
10	1.017	1.619
20	0.881	1.285
30	0.803	1.123
40	0.749	1.020
50	0.709	0.947
60	0.678	0.891
70	0.652	0.846
80	0.630	0.809
90	0.611	0.778
100	0.595	0.751
200	0.494	0.596
300	0.443	0.521
400	0.409	0.473
500	0.384	0.439
N	F-D	SCOTT
16	0.925	1.385
32	0.791	1.099
64	0.667	0.872
128	0.557	0.692
256	0.462	0.549

5. LETTER VALUES

Letter values are statistics that describe the tails or the shape of our sample. The idea is that our sample can be described as a center and tails. The center is measured by the sample mean or sample median and the shape is described by either symmetric or skewed and by normal or heavy tails.

Start with a sample of data $X_1,...,X_n$ and an sorted version given by the order statistics $X_{(1)},...,X_{(n)}$. We also construct the set of ranks $R_1,...,R_n$ and the set of depths $D_1,...,D_n$. Then we define the usual statistics.

MEDIAN

$$\text{Median} = \begin{cases} (X_{(k)} + X_{(k+1)})/2 & \text{if } n = 2k \\ X_{(k+1)} & \text{if } n = 2k+1 \end{cases}$$

The median is a simple way to measure center that is not affected by skewness and heavy tails. But it is not perfect since it has a high standard error relatively to the sample mean and when the sample size increases from even to odd may not decrease standard error.

FOURTHS

Most statistics texts talk about the quartiles as the 25^{th} and 75^{th} percentiles of the sample. This is complicated because if you have a sample of 37 observations the definition of 25^{th} percentile is quite arbitrary. Tukey proposed the idea of fourth which is the median of the half samples delimited by the sample median.

$$\text{depth(Fourth)} = (\text{[depth(Median)]} + 1)/2$$

5-NUMBER SUMMARIES

The five number summary is defined as the combination of the minimum, lower fourth, median, upper fourth and maximum of the sample. These five numbers give a basic description of the sample which includes information about center and shape. The relative position of the fourths and extremes might suggest symmetry or skewness or heavy tails.

LETTER VALUES

More accurate measures of skewness or shape can be extracted by calculating more extreme percentiles of the data such as eighths or sixteenths and so on. To calculate the eights use the following definition based on depth depth(Eight) = ([depth(FOURTH)] + 1)/2 and more in general (Next depth) = ([Prev depth] +1)/2. This way we can go futher and further on the tails. These values are called letter values and we use the following letters as Tags : M F E D C B A Z Y X

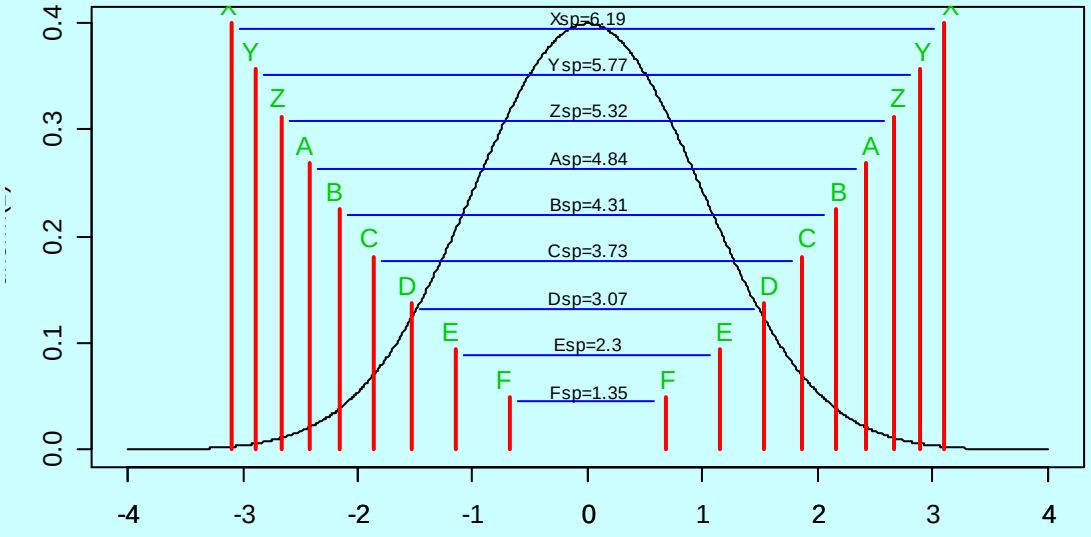
We also introduce the concept of midvalues and spread: Midvalue = (Upper LV + Lower LV)/2 and Spread = Upper LV - Lower LV

Example: Fourth Spread = (Upper Fourth - Lower Fourth)

NICE FORM LETTER VALUE DISPLAYS

```
M     Depth of the Median              Median
F     Depth of Fourth        Lower Fourth   Upper Fourth    Mid   Spread
1                            Lower Extreme  Upper Extreme   Mid   Spread
```

Figure 1. Letter values for the Standard Normal Distribution



The letter value display is then compared to what you would expect from normal distribution and the shape is described by pointing out departures from normality. Figure 1 shows the letter values, mid values and spreads for a standard normal population and Table 3 gives the list of upper letter values.

Table 3. Letter Values for the standard Normal

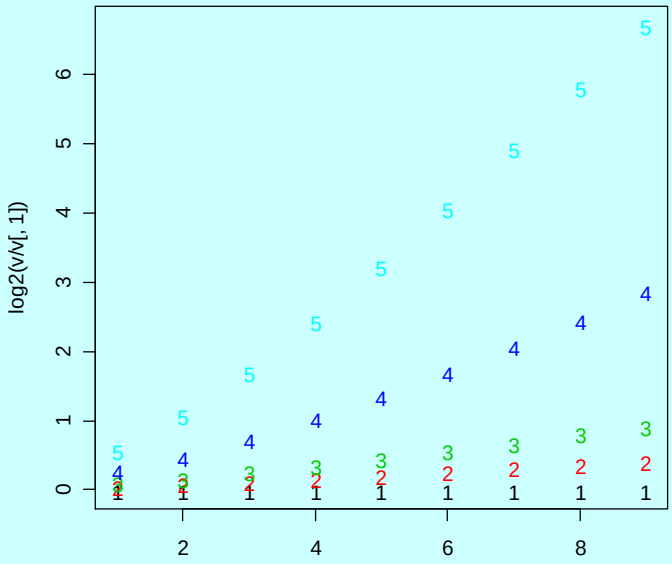
F	E	D	C	B	A	Z	Y	X
0.67	1.15	1.53	1.86	2.15	2.42	2.66	2.89	3.10

If the letter spreads of the sample are proportional to the corresponding normal letter-spreads then we can assume that the sample is approximately normal. However if the sample letter spread grow faster than the normal spreads then we found evidence of heavy tails. An example of heavy tailed distribution is the t-distribution with k degrees of freedom. For small values of k the t distribution has very heavy tails whereas for large k the tails are like a normal distribution. One way to If we are able to match the spreads to a particular k then we will be able to evaluate how heavy are the tails of the data.

Table 4. Letter Spreads for the T-10, T5, T-2 and T-1 and Ratios by std Normal spreads

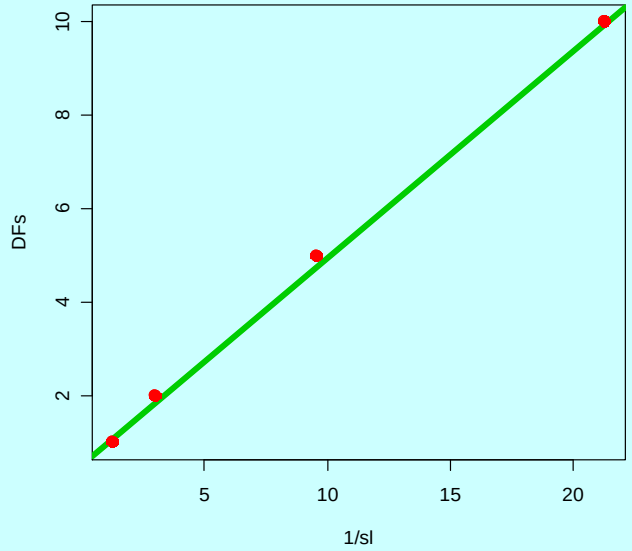
	NORMAL	T-10	RAT	T-5	RAT	T-2	RAT	T-1	RAT
F_{SPREAD}	1.35	1.40	1.04	1.45	1.08	1.63	1.21	2.00	1.48
E_{SPREAD}	2.30	2.44	1.06	2.60	1.13	3.21	1.39	4.83	2.10
D_{SPREAD}	3.07	3.35	1.09	3.68	1.20	5.11	1.67	10.05	3.28
C_{SPREAD}	3.73	4.19	1.13	4.78	1.28	7.62	2.05	20.31	5.45
B_{SPREAD}	4.31	5.01	1.16	5.93	1.38	11.05	2.56	40.71	9.45
A_{SPREAD}	4.84	5.82	1.20	7.19	1.49	15.81	3.27	81.47	16.85
Z_{SPREAD}	5.32	6.63	1.25	8.57	1.61	22.49	4.23	162.97	30.63
Y_{SPREAD}	5.77	7.46	1.29	10.12	1.75	31.91	5.53	325.95	56.48
X_{SPREAD}	6.19	8.32	1.34	11.85	1.91	45.19	7.29	651.90	105.24

Figure 3. $\log_2$(spread ratios of T with df=10,5,2,1 over Normal spreads) Vs -$\log_2$(Tail Prob)



Estimated slopes: 0.047 0.1046 0.328 0.782

Figure 3. DF approximation: 1/2 + 1/(2.25*slope)



R-CODE

```
probs= 2^-(2:10) # tail prob for 9 letter values
u = -qnorm(probs) # Letter values for Standard Normal
x = c(t(cbind(-u,-u,NA,u,u,NA))) # Lines for the graph
yy = seq(0.05,0.4,length=9)      # Lines for the graph
y = c(t(cbind(0,yy,NA,0,yy,NA))) # Lines for the graph
x1 = c(t(cbind(-u+0.07,u-0.09,NA))) # Lines for the graph
y1 = c(t(cbind(yy-0.005,yy-0.005,NA))) # Lines for the
graph
# Graph commands start here
```

```
par(cex=0.8)              # Set character size at 80%
plot(z <- ((-400):400)/100,dnorm(z),type="l",xlab="")
#Plots
#                                  the bell curve
lines(x,y,lwd=2,col=2)  # draws vertical lines
L=c("F","E","D","C","B","A","Z","Y","X") # vector of
letters
text(-u-.05,yy+0.02,L,col=3)     # Draws letters
text(u+.03,yy+0.02,L,col=3)      # Draws letters
lines(x1,y1,col=4)               # Draw lines
par(cex=0.6)                          # lower character size
text(0,yy+0.006,paste(L,"sp=",round(2*u,2),sep=""))
                                 #text over lines
v = qnorm(probs)                      # normal quantiles
v = -2*cbind( v,qt(probs,df=10),qt(probs,df=5),
qt(probs,df=2),qt(probs,df=1)) # combine it with t
quantiles
                               # for df= 10,5,2,1
matplot(1:9,log2(v/v[,1]))        #  Plot all the ratios
lsfit(1:9,log2(v/v[,1])[,2])$coef[2] ->sl    # calculate
lsfit(1:9,log2(v/v[,1])[,3])$coef[2] ->sl[2] # individual
lsfit(1:9,log2(v/v[,1])[,4])$coef[2] ->sl[3] # slopes for
lsfit(1:9,log2(v/v[,1])[,5])$coef[2] ->sl[4] # all lines

lsfit(1/sl,c(10,5,2,1))$coef    # line fit
plot(1/sl,c(10,5,2,1),pch=16,col=2,ylab="DFs") # graph of
abline(0.5, 1/2.25,col=3,lwd=2)              # fit
round(1/2 + 1/(sl*2.25))  # final estimate of df's from
line
```

LETTER VALUE DISPLAYS USING SAS: "PROC IML" CODE

We write a program in SAS PROC IML. This is more like a programming language.

```
/* This is just an example of PROC IML*/
OPTIONS LS=80 PS=55;
PROC IML;
START LETVAL;
/* DATA IS IN X, M IS THE NUMBER OF LETTER VALUES*/
M = 5;
N = NROW(X);
/* SORT X */
A = X;
```

```
X[RANK(X),]= A ;
/* SET THE TABLE OF LETTER VALUES */
LV = REPEAT(0,M,4);

/* D IS THE LENGTH OF THE SUBSAMPLE FOR THE LV */
D = N;
DO I=1 TO M;
     IF(D > 0) THEN DO;

/* CREATE THE RIGHT SUBSET */
           B = X[1:D,];
/* SORT THE RIGHT SUBSET */
           A = B;
           B[RANK(B),]=A;
/* CREATE THE LEFT  SUBSET */
           C = X[(N-D+1):N,];
/* SORT THE LEFT  SUBSET */
           A = C;
           C[RANK(C),]=A;
/* THE MEDIANS*/
           DD = INT((D+1)/2);
         IF( D = (2*DD -1)) THEN DO;
           LV[I,1] = B[DD];
           LV[I,3] = C[DD];
         END;
         ELSE DO;
           LV[I,1] = (B[DD]+B[DD+1])/2;
           LV[I,3] = (C[DD]+C[DD+1])/2;
         END;
/*CALCULATE THE MID VALUES*/
        LV[I, 2] = (LV[I, 1] + LV[I, 3])/2;
/*CALCULATE THE SPREAD*/
        LV[I, 4] = LV[I, 3] - LV[I, 1];
        D = DD;
     END;
END;
FINISH;

/*DEFINE X*/
X = {42, 37, 37, 28, 18, 18, 19, 20, 15, 14, 14, 13, 11, 12, 8, 7, 8, 8,
9, 15, 15};

/* RUN THE CODE */
RUN LETVAL;

/* PRINT THE TABLE */
R = {"M" "F" "E" "D" "C"} ;
C = {"Lower" "Mid" "Upper" "Spread"};
PRINT LV[ ROWNAME=R COLNAME=C];
QUIT;
```

OUTPUT FROM SAS:

LV	Lower	Mid	Upper	Spread
M	77.5	77.5	77.5	0
F	7	68.5	130	123
E	3	81.5	160	157
D	2	84.75	167.5	165.5
C	1	88	175	174

LETTER VALUE DISPLAYS USING R: LETTER VALUE FUNCTION.

```
letval <- function(x, k = 4) {
    LV <- c("M", "F", "E", "D", "C", "B", "A", "Z", "Y", "X", "W")
    out <- array(NA, c(k, 4))
    lx <- rx <- sort(x)
    dimnames(out) <- list(LV[1:k],c("LOWER","UPPER","MID","SPREAD"))
    for(i in 1:k) {
           out[i, 1:2] <- c(median(lx), median(rx))
           nn <- (length(lx) + 1)/2
           lx <- lx[1:nn]
           rx <- rev(rev(rx)[1:nn])
          }
          out[, 3] <- (out[, 1] + out[, 2])/2
          out[, 4] <- out[, 2] - out[, 1]
          out
}
```

A more complicated version:

```
letval2 <- function(x, k = 4) {
 LV <- c("M", "F", "E", "D", "C", "B", "A", "Z", "Y", "X", "W")
 out <- array(NA, c(k, 6))
 lx <- rx <- sort(x)
 dimnames(out) <- list(LV[1:k],c("LOWER","UPPER","DEPTH","MID","SPREAD","TAIL"))
 for(i in 1:k) {
  out[i, 1:2] <- c(median(lx), median(rx))
  nn <- (length(lx) + 1)/2
  lx <- lx[1:nn]
  rx <- rev(rev(rx)[1:nn])
  out[i, 3] <- nn
 }
 out[, 4] <- (out[, 1] + out[, 2])/2
 out[, 5] <- out[, 2] - out[, 1]
 out[, 6] <- c(0, out[-1, 5]/2/qnorm(1 - 1/2^(2:k)))
 out
}
```

OUTPUT FROM R:

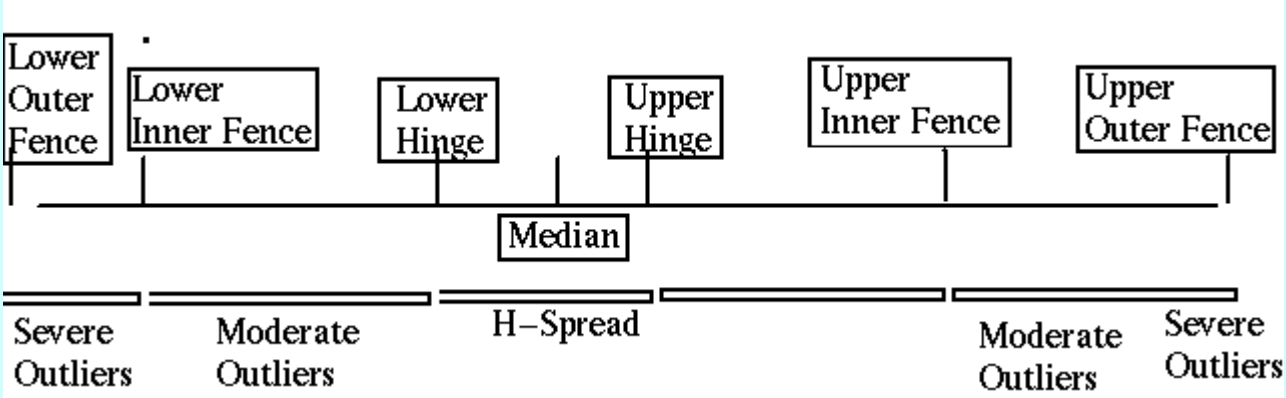
```
> data(stackloss)
> letval2(stack.loss,7)

    LOWER UPPER DEPTH   MID SPREAD      TAIL
M  15.0  15.0  11.0 15.00    0.0  0.000000
F  11.0  19.0   6.0 15.00    8.0  5.930409
E   8.0  32.5   3.5 20.25   24.5 10.648939
D   8.0  37.0   2.0 22.50   29.0  9.451669
C   7.5  39.5   1.5 23.50   32.0  8.589535
B   7.0  42.0   1.0 24.50   35.0  8.124892
A   7.0  42.0   1.0 24.50   35.0  7.238706
```

6. Box-Plots

6.1 Box Plots and sample comparisons

BOX PLOTS

- **MEDIAN:** Center of the data
- **HINGES:**
  More or less like quartiles. In fact you do not need to learn this definition if you do not wish to. Just use the definition of quartile.
- **HSPREAD = upper hinge - lower hinge. (Similar to IQR)**
- **INNER FENCES:**
  - Lower inner fence = lower hinge - 1.5 Hspread
  - Upper inner fence = upper hinge + 1.5 Hspread
- **OUTER FENCES :**
  - Lower outer fence = lower hinge - 3 Hspread
  - Upper outer fence = Upper hinge + 3 Hspread
- **ADJACENT VALUS**
  closest point to the inner fences toward the median.



Simple example:

```
data  6, 9, 12, 13, 13, 15, 15, 17, 27
depth 1, 2,  3,  4,  5,  4,  3,  2,  1
H-spread = 15-12 =3
Inner fences = 12 -3*1.5, 15+3*1.5 = 7.5, 19.5
outer fences = 3, 24
adjacent values = 9, 17
```

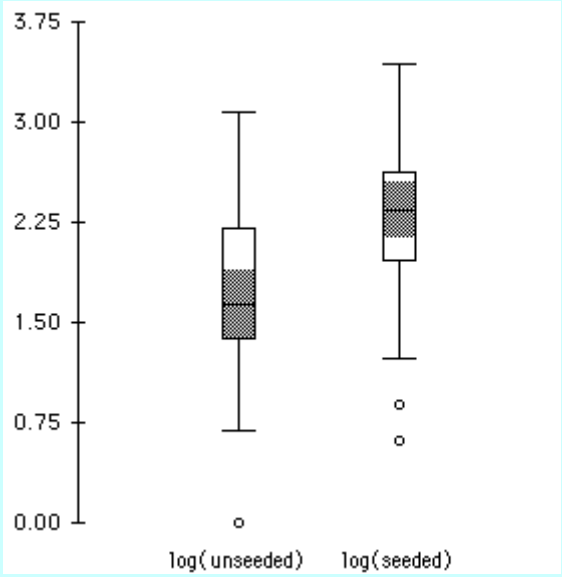
Example: Clouds were randomly seeded or not with silver nitrate. Rainfall amounts were recorded from the clouds. The purpose of the experiment was to determine if cloud seeding increases rainfall. The rainfall distributions are more

nearly symmetric after a log transformation. The log transformation also makes the variance of the two groups more nearly equal.

After a log transformation, a pooled t-test may be appropriate. (Without a transformation it is neither appropriate (failing both the normality and equal variance assumptions) nor significant at .05.) Without transforming, a Mann-Whitney U test would be appropriate.

A boxplot of rainfall for the two groups of clouds is helpful.

Image: Side by side boxplots of the two logged variables.

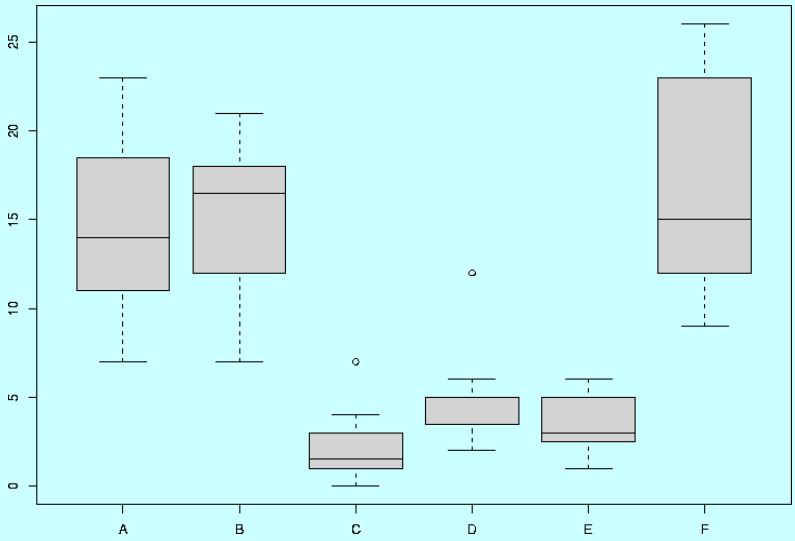


Example on R:

The function boxplot produces boxplots in R

Data: Count of insects after spraying by spray factor.

```
data(InsectSprays)
boxplot(count~spray,data=InsectSprays,col="lightgray")
```



6.2 SAS PROGRAM USING PROC GPLOT:

In SAS there are several ways to get boxplots

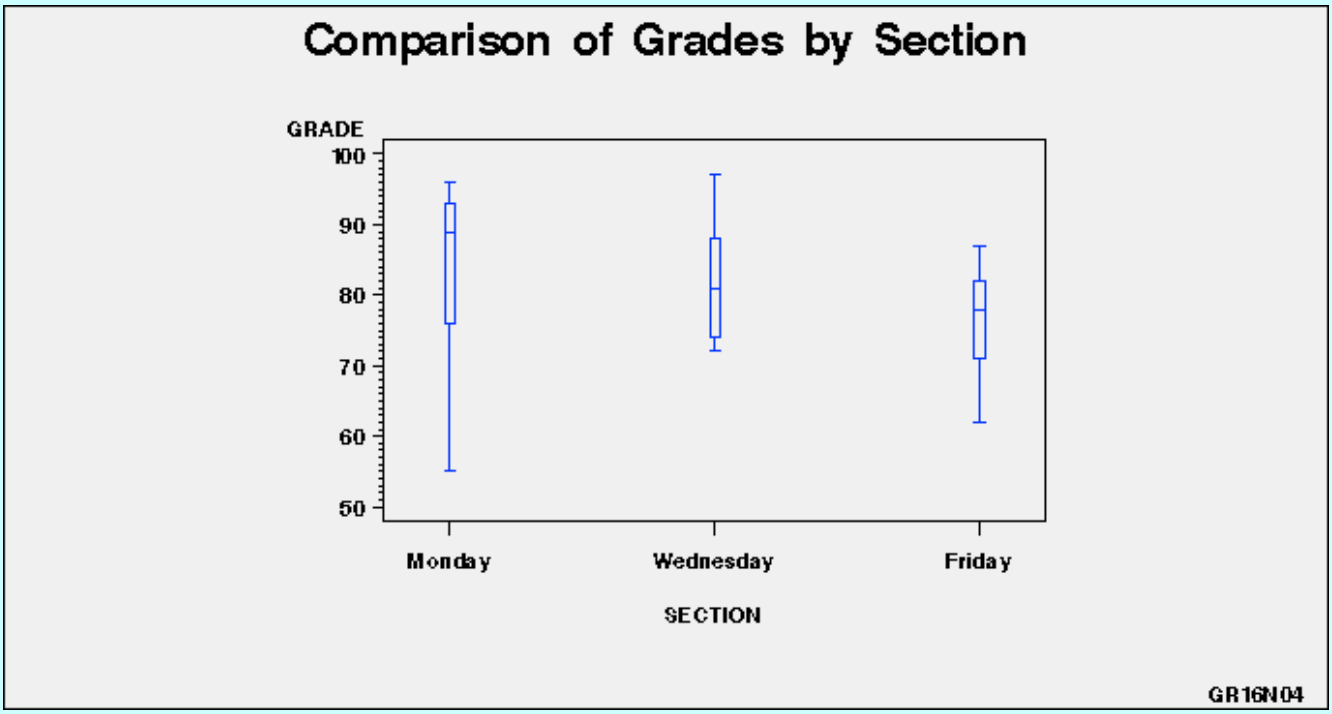
a. **Use Proc Boxplot;**
b. **Interactive data analysis menu**
c. **Use proc gplot with the symbol option**

This is code from SAS that generates a boxplot using option c.

```
/*************************************************************/
/*           S A S   S A M P L E   L I B R A R Y             */
/*                                                           */
/*     NAME: GR16N04                                         */
/*    TITLE: Creating a Box Plot - GR16N04                   */
/*  PRODUCT: GRAPH                                           */
/*   SYSTEM: ALL                                             */
/*     KEYS: graphics symbol gplot interpol box reference    */
/*    PROCS: GPLOT                                           */
/*     DATA: INTERNAL                                        */
/*                                                           */
/*      REF: SAS/GRAPH REFERENCE GUIDE                       */
/*     MISC:                                                 */
/*                                                           */
/*************************************************************/

   /* set the graphics environment */
goptions reset=global gunit=pct border
         ftext=swissb htitle=6 htext=3;
   /* create the data set GRADES */
data grades;
   input section $ grade @@;
   cards;
A 74 A 89 A 91 A 76 A 87 A 93 A 93 A 96 A 55
B 72 B 72 B 84 B 81 B 97 B 78 B 88 B 90 B 74
C 62 C 74 C 71 C 87 C 68 C 78 C 80 C 85 C 82
;
run;
   /* define title and footnote */
title 'Comparison of Grades by Section';
footnote j=r 'GR16N04  ';
   /* define symbol characteristics */
symbol interpol=boxt10   /* box plot              */
       cv=red            /* plot symbol color     */
       co=blue           /* box and whisker color */
       width=6           /* line width            */
       value=square      /* plot symbol           */
       height=4;         /* symbol height         */
   /* define axis characteristics */
axis1 value=('Monday' 'Wednesday' 'Friday')
      offset=(5,5)
      length=50;
   /* generate plot */
proc gplot data=grades;
```

```
   plot grade*section / haxis=axis1
                        vaxis=50 to 100 by 10
                        frame
                        des='GR16N04-1';
run;
quit;
```

Image: Boxplots of Grades By Section



Go back to the Cloud seed data and see if it is of without the use of log transformation

```
 /*************************************************************/
 /*           S A S   S A M P L E   L I B R A R Y             */
 /*     NAME: GR16N04                                         */
 /*    TITLE: Creating a Box Plot - GR16N04                   */
 /*  PRODUCT: GRAPH                                           */
 /*   SYSTEM: ALL                                             */
 /*     KEYS: graphics symbol gplot interpol box reference    */
 /*    PROCS: GPLOT                                           */
 /*     DATA: INTERNAL                                        */
 /*      REF: SAS/GRAPH REFERENCE GUIDE                       */
 /*************************************************************/
   /* set the graphics environment */
goptions reset=global gunit=pct border
         ftext=swissb htitle=6 htext=3;
   /* create the data set GRADES */
data clouds;
   input Clouds $ Rain   ;
   cards;
U  1202.6
S  2745.6
U  830.1
S   1697.8
U  372.4
```

```
S   1656.0
U  345.5
S   978.0
U  321.2
S   703.4
U  244.3
S   489.1
U  163.0
S   430.0
U  147.8
S   334.1
U  95.0
S    302.8
U  87.0
S    274.7
U  81.2
S    274.7
U  68.5
S    255.0
U  47.3
S    242.5
U  41.1
S    200.7
U  36.6
S    198.6
U  29.0
S    129.6
U  28.6
S    119.0
U  26.3
S    118.3
U  26.1
S    115.3
U  24.4
S    92.4
U  21.7
S    40.6
U  17.3
S    32.7
U  11.5
S    31.4
U  4.9
S     17.5
U  4.9
S     7.7
U  1.0
S     4.1
;
run;
 proc print;
   /* define title and footnote */
title 'Comparison of Rain Level for seeded and unseeded clouds';
footnote j=r 'GR16N04  ';

   /* define symbol characteristics */
symbol interpol=boxt10   /* box plot              */
       cv=red            /* plot symbol color     */
       co=blue           /* box and whisker color */
       width=6           /* line width            */
       value=square      /* plot symbol           */
       height=4;         /* symbol height         */
```

```
   /* define axis characteristics */
axis1 value=('Unsided Cloud' 'Seeded Cloud')
      offset=(15,15)
      length=50;

   /* generate plot */
proc gplot data=clouds;
   plot rain*clouds/ haxis=axis1
                     vaxis=50 to 100 by 10
                     frame
                     des='GR16N04-1';
run;
quit;
```

Image: Boxplots of Rain by Seeded

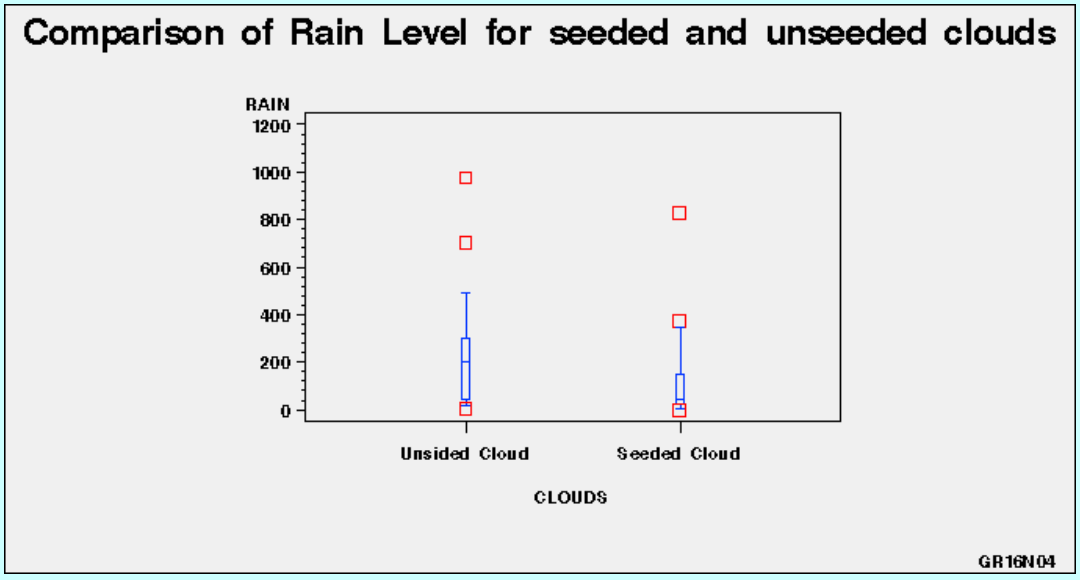
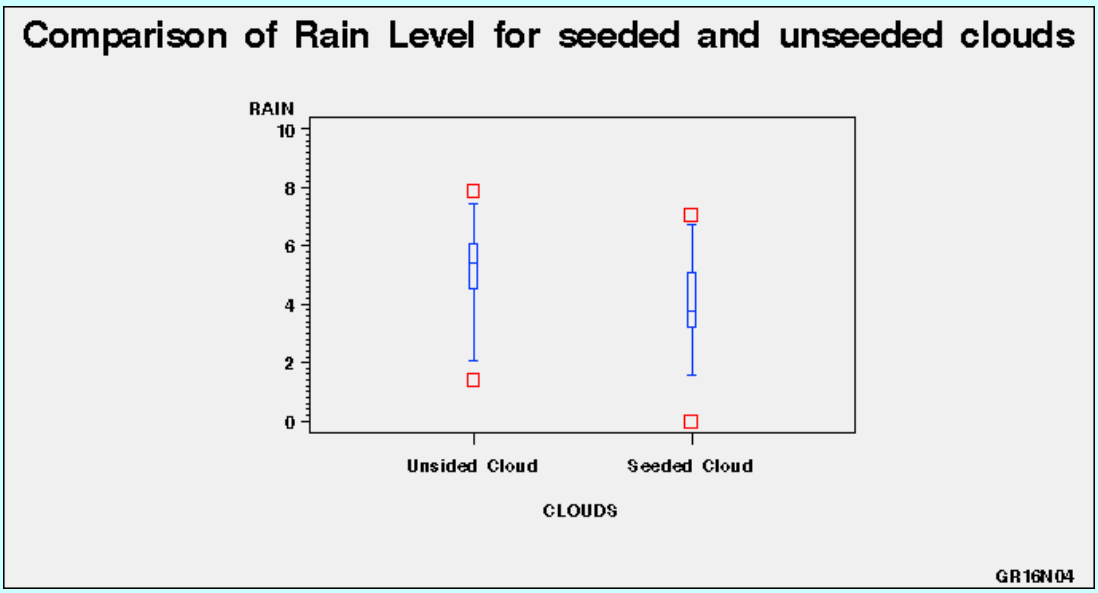


Image: Boxplots of Log(Rain) by Seeded



6.3 SAS PROGRAM USING PROC BOXPLOT:

```
data clouds;
   set clouds ;
   if Clouds eq 'U' then cld = 1;
   if Clouds eq 'S' then cld = 2;
run;

proc sort data=clouds; by cld;
proc boxplot;
plot rain*cld;
run;
```

7. THE USE OF POWER TRANSFORMATIONS

7.1 Definition of Power Transformation:

if p = 0 log(x)
if p > 0 x^p
if p < 0 $- x^p$

7.2. SPREAD VS LEVEL PLOT

Compare several datasets with boxplots.

Problem: Sometimes the boxplots are hard to compare because the spread of measurements may depend on the level. That is large values have large spread and small values have small spread.

Solution: Use power transformations to make the scales of the samples more comparable.

2.1 Method for calculating p for the power transformation:

- We will call d_F to the fourths spread.
- Use equation $d_F = c\ \text{Median}^b$
- Then take logs
- $\log(d_F) = c' + b \log(\text{Median})$
- and calculate b using linear regression.
- Then the desired power is p=1-b

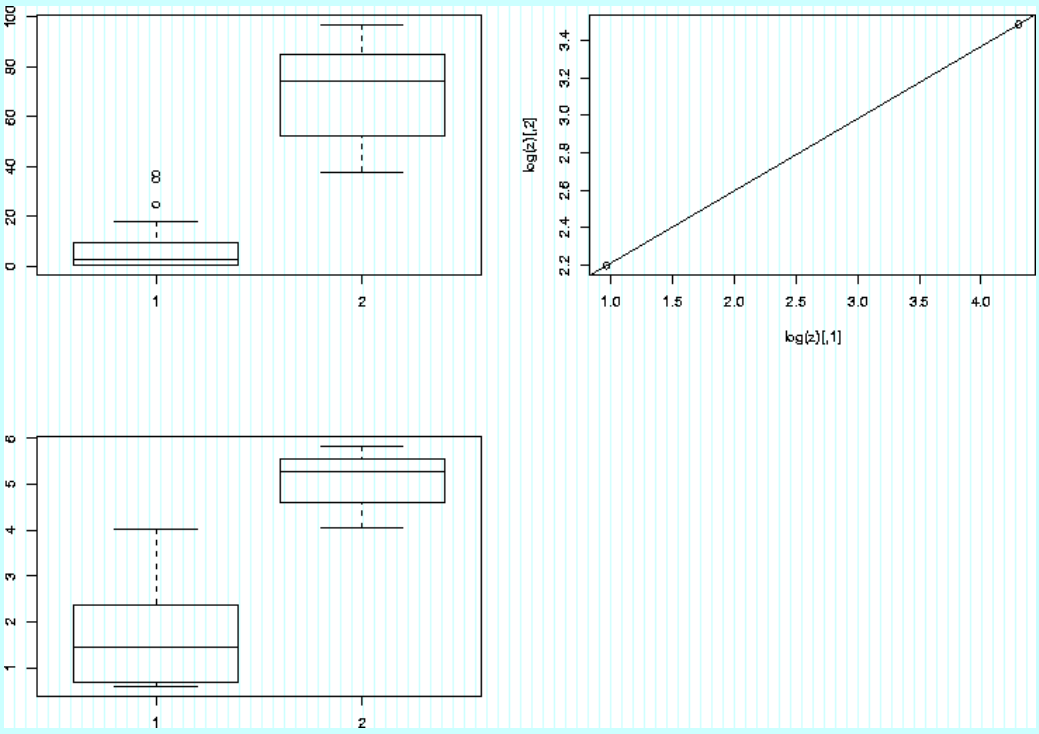
7.3 SAS Example:

EXAMPLE CODE IN SAS

OUTPUT FILE

7.4 R:

```
SprVsLevel = function(u1,u2,...) {
 x = list(u1,u2,...)
 z <- sapply(x, function(z){u=letval2(z);c(u[1,1],u[2,5])})
 z <- t(z)
 b <- lsfit(log(z[, 1]), log(z[, 2]))$coef
 boxplot(x)
 plot(log(z))
 abline(b)
 lz <- sapply(x, function(u, uu) u^uu, uu=1-b[2])
 if( is.matrix(lz))
 boxplot(data.frame(lz)) else boxplot(lz)
 1 - b[2]
}

letval2 = function(x, k = 4)  {
 LV <- c("M", "F", "E", "D", "C", "B", "A", "Z", "Y", "X", "W")
 out <- array(NA, c(k, 6))
 lx <- rx <- sort(x)
dimnames(out)=list(LV[1:k],c("LOWER","UPPER","DEPTH","MID","SPREAD","TAIL"))
 for(i in 1:k) {
  out[i, 1:2] <- c(median(lx), median(rx))
  nn <- (length(lx) + 1)/2
  lx <- lx[1:nn]
  rx <- rev(rev(rx)[1:nn])
  out[i, 3] <- nn
 }
 out[, 4] <- (out[, 1] + out[, 2])/2
 out[, 5] <- out[, 2] - out[, 1]
 out[, 6] <- c(0, out[-1, 5]/2/qnorm(1 - 1/2^(2:k)))
 out
}
x<-c(0.250,0.375,0.250,0.250,0.250,0.250,1.250,3.250,2.000,1.375,34.750,
0.625,37.125,12.000,18.125,5.000,10.000,5.250,4.875,0.375,7.000,1.000,0.250,
9.375,7.125,24.750)
y<-c(85.000,68.875,97.000,83.500,65.625,52.500,51.000,77.950,85.125,66.625,
87.625,94.750,58.875,49.650,72.250,48.625,69.250,82.750,52.125,89.500,65.12
5,80.375,47.875,86.125,79.375,37.875,76.000,49.250,86.125,88.000)
par(mfrow=c(2,2))
   SprVsLevel(x,y)
SprVsLevel = function(u1,...) {
if(is.list(u1) ) x = u1 else   x = list(u1,...)
 z <- sapply(x, function(z){u=letval2(z);c(u[1,1],u[2,5])})
 z <- t(z)
 b <- lsfit(log(z[, 1]), log(z[, 2]))$coef
 boxplot(x)
 plot(log(z))
 abline(b)
 lz <- sapply(x, function(u, uu) u^uu, uu=1-b[2])
 if( is.matrix(lz))
 boxplot(data.frame(lz)) else boxplot(lz)
 1 - b[2]
}
            0.6144924
```



8. TRANSFORMING FOR SYMMETRY

Problem: We have a sample of observations that shows moderate to strong skewness.

Solution: Use power transformations to make the sample more symmetric.

8.1 Method for calculating p for the power transformation:

For several letter values define

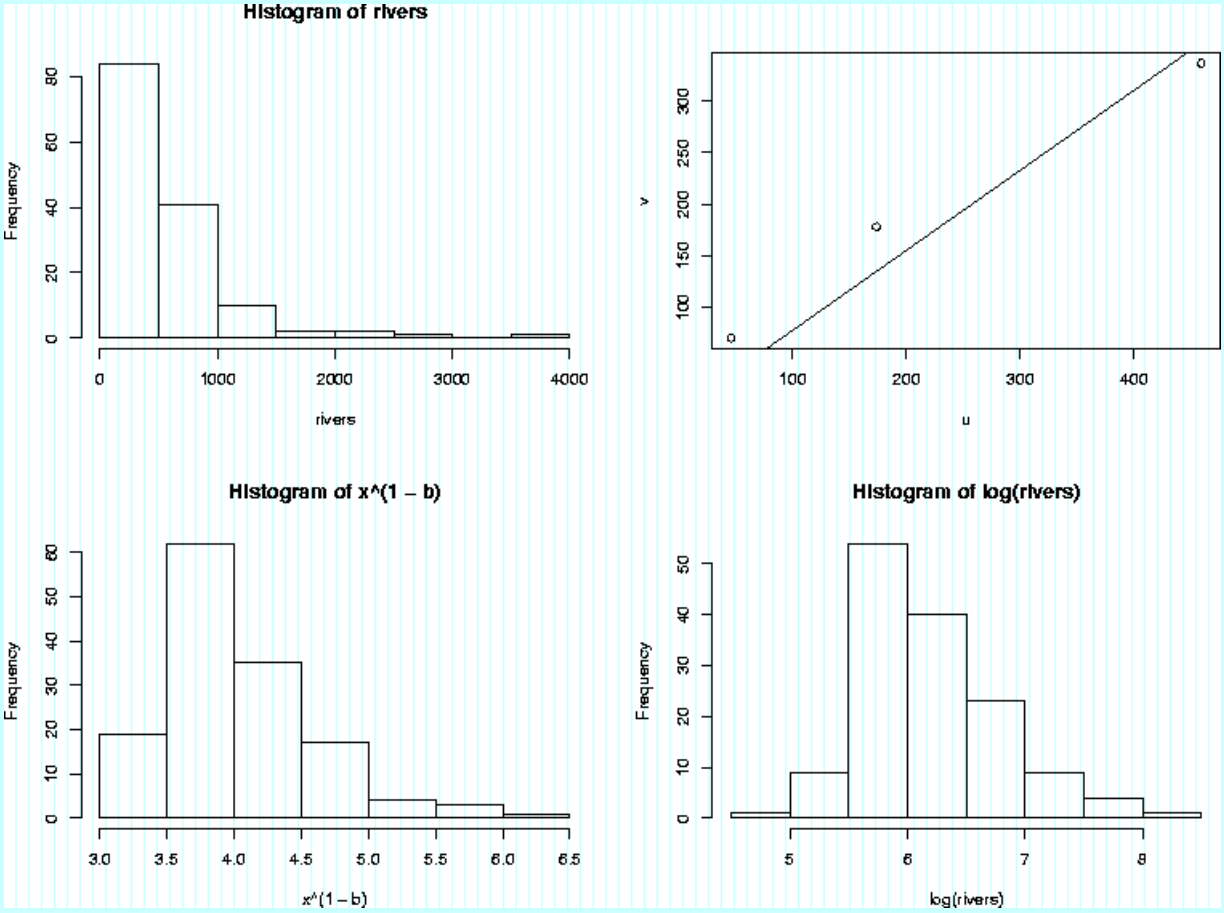
$$Y = ((X_U - M)^2 - (M - X_L)^2)/(4M)$$
$$Z = (X_U + X_L)/2 - M$$

- **Plot Y VS Z**
- **b is the slope of Y vs Z**
- **If the result is linear then p = 1-b is the power for the transformation:**
- **$T(X) = X^p$ or maybe $k X^p$**

9.2 R:

```
letval2 <- function(x, k = 4)
{
  LV <- c("M", "F", "E", "D", "C", "B", "A", "Z", "Y", "X", "W")
  out <- array(NA, c(k, 6))
```

```
  lx <- rx <- sort(x)
  dimnames(out) <- list(LV[1:k],c("LOWER","UPPER","DEPTH","MID","SPREAD","TAIL"))
  for(i in 1:k) {
    out[i, 1:2] <- c(median(lx), median(rx))
    nn <- (length(lx) + 1)/2
    lx <- lx[1:nn]
    rx <- rev(rev(rx)[1:nn])
    out[i, 3] <- nn
   }
  out[, 4] <- (out[, 1] + out[, 2])/2
  out[, 5] <- out[, 2] - out[, 1]
  out[, 6] <- c(0, out[-1, 5]/2/qnorm(1 - 1/2^(2:k)))
  out
}
sym <- function(x, k = 7) {
    z <- letval2(x, k)
M <- z[1,1]
    u <- ((z[-1,2]-M)^2 + (z[-1,1]-M)^2)/(4*M)
    v <- z[-1, 4] - M
    plot(u, v)
    b <- lsfit(u, v, int = F)$coef
    abline(0, b)
    hist(x^(1 - b))
    1 - b
}
par(mfrow=c(2,2))
data(rivers)
hist(rivers)
sym(rivers,4)
hist(log(rivers))
```



9. One way and Two way ANOVA

ONE-WAY ANOVA:

Linear Model: $x_{gi} = \mu_g + \varepsilon_{gi}$, $g=1,\ldots,k \;\; i=1,\ldots,n_g$

Assumptions: Errors ε_{gi} are (i) independent (ii) normally distributed (iii) with equal variances.

Fit: **Data = group effect + residual**

Hypothesis Testing: **All means are equal Vs some are not equal. ANOVA**

(a) $H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$

(b) $H_a : \mu_i \neq \mu_j$

LINEAR MODEL FOR TWO WAY TABLE:

This is a typical dataset where we have a response and two factors.
Exampe:

	B1	B2	B3
clarion	32.7	32.3	31.5
clinton	32.1	29.7	29.1
knox	35.7	35.9	33.1
o'neill	36.0	34.2	31.2
compost	31.8	28.0	29.2
wabash	38.2	37.8	31.9
webster	32.5	31.1	29.7

Fitted model: Data = Tot Mean + Row Effect + Col Eff + Residual

	B1	B2	B3	row effect
clarion	-1.052	-0.024	1.076	-0.390
clinton	0.214	-0.757	0.543	-2.257
knox	-0.786	0.843	-0.057	2.343
o'neill	0.614	0.243	-0.857	1.243
compost	0.548	-1.824	1.276	-2.890
wabash	0.648	1.676	-2.324	3.410
webster	-0.186	-0.157	0.343	-1.457
col eff	1.586	0.157	-1.743	32.557

SAS DOES IT IN A DIFFERENT WAY, IT SETS THE EFFECT FOR ONE OF THE GROUPS AS ZERO.

		ESTIMATE		TVALUE	PVALUE	STD ERROR
INTERCEPT		29.3571	B	34.96	0.0001	0.839703
TYPE						
clarion		1.0667	B	1.02	0.3285	1.047294
clinton		-0.800	B	-0.76	0.4597	1.047294
knox		3.800	B	3.63	0.0035	1.047294
o'neill		2.700	B	2.58	0.0242	1.047294
compost		-1.433	B	-1.37	0.1962	1.047294
wabash		4.867	B	4.65	0.0006	1.047294
webster		0.000	B	.	.	.
BLOCK	1	3.3285	B	4.85	0.0004	0.685615
	2	1.900	B	2.77	0.0169	0.685615
	3	0.000	B	.	.	.

COMPUTE ANOVA TABLE:

In order to test the significance of row effects or column effects.

Source	DF	Type I SS	F Value	Pr > F
TYPE	6	103.15142	10.45	0.0004
BLOCK	2	39.03714	11.86	0.0014

Source	DF	Type III SS	F Value	Pr > F
TYPE	6	103.15142	10.45	0.0004
BLOCK	2	39.03714	11.86	0.0014

MULTIPLE COMPARISONS:

Once we find that a factor is significant then we need to explore the difference between the corresponding factor levels. We do that using a multiple comparison procedure such as Tukey or Duncan.

Another useful tool is to use ***contrasts*** if we are interested in testing only a few comparisons or in some linear combinations of the parameters.

This is the SAS code and output file

```
options ps=50 ls=70;
   *---------------snapdragon experiment---------------*
   | as reported by stenstrom, 1940, an experiment was |
   | undertaken to investigate how snapdragons grew in |
   | various soils. each soil type was used in three   |
   | blocks.                                           |
   *---------------------------------------------------*;
   data plants;
      input type $ @;
      do block=1 to 3;
         input stemleng @;
         output;
         end;
      cards;
   clarion  32.7 32.3 31.5
   clinton  32.1 29.7 29.1
   knox     35.7 35.9 33.1
   o'neill  36.0 34.2 31.2
   compost  31.8 28.0 29.2
   wabash   38.2 37.8 31.9
   webster  32.5 31.1 29.7
   ;
   proc glm;
      class type block;
      model stemleng=type block; run;
   proc glm order=data;
      class type block;
      model stemleng=type block / solution;
      means type / bon duncan tukey;

*-type-order---clrn-cltn-knox-onel-cpst-wbsh-wstr;
contrast 'compost v others' type -1 -1 -1 -1  6 -1 -1;
contrast 'knox vs oneill'   type  0  0  1 -1  0  0  0;
   run;
```

OUTPUT: Output file from "glm.sas"

INCOMPLETE DESIGNS

```
120 6559 1240    6   71  237   40  689  165  855
202  233  165   62    5  385   40   74   25   36
 15   22   34  129   32   54   23   48   10    1
```

This is the data but with some missing observations:

	B1	B2	B3
clarion	32.7	32.3	NA
clinton	32.1	29.7	29.1
knox	35.7	35.9	33.1

```
o'neill    NA 34.2 31.2
compost  31.8 28.0 29.2
wabash   38.2 37.8 31.9
webster  32.5   NA 29.7
```

```
Row Effects:
    clarion    clinton      knox   o'neill   compost    wabash    webster
 0.05238095 -2.147619 2.452381  0.252381 -2.780952  3.519048 -1.34761
```

```
Column Effects:
          B1        B2        B3
      1.427778 0.3111111 -1.738889
Main Effect:   32.44762
```

So what is SAS going to do?

```
           ESTIMATE     TVALUE     PVALUE  STD ERROR
INTERCEPT  29.372 B      27.73     0.0001     1.0591
TYPE
clarion     0.281 B       0.20     0.8491     1.4390
clinton    -0.969 B      -0.76     0.4682     1.2804
knox        3.630 B       2.84     0.0196     1.2804
o'neill     2.209 B       1.54     0.1591     1.4390
compost    -1.603 B      -1.25     0.2421     1.2804
wabash      4.696 B       3.67     0.0052     1.2804
webster     0.000 B          .          .          .
BLOCK 1     3.455 B       4.16     0.0025     0.8308
        2   2.236 B       2.69     0.0247     0.8308
        3   0.000 B          .
```

We look at the ANOVA table and we see that order matters.

TYPE BEFORE BLOCK

```
Source    DF          Type I SS  F Value    Pr > F
TYPE       6        95.93611111     8.42    0.0028
BLOCK      2        33.76848485     8.89    0.0074
Sourc     DF        Type III SS  F Value    Pr > F
TYPE       6        98.19681818     8.62    0.0026
BLOCK      2        33.76848485     8.89    0.0074
```

BLOCK BEFORE TYPE

```
Source    DF          Type I SS  F Value    Pr > F
BLOCK      2        31.50777778     8.30    0.0091
TYPE       6        98.19681818     8.62    0.0026
Source    DF        Type III SS  F Value    Pr > F
BLOCK      2        33.76848485     8.89    0.0074
TYPE       6        98.19681818     8.62    0.0026
```

THE BASIC STATS

```
Source    DF    Sum of Squares  F Value    Pr > F
Model      8      129.70459596     8.54    0.0021
Error      9       17.08484848
Total     17      146.78944444

R-Square              C.V.      STEMLENG Mean
0.883610          4.238642         32.5055556
```

10. More on Power Transformations

Power transformations:

if p > 0 x^p
if p = 0 log(x)
if p < 0 $- x^p$

In a more formal way

if p = 0 log(x)
Otherwise $(x^p - 1)/p$

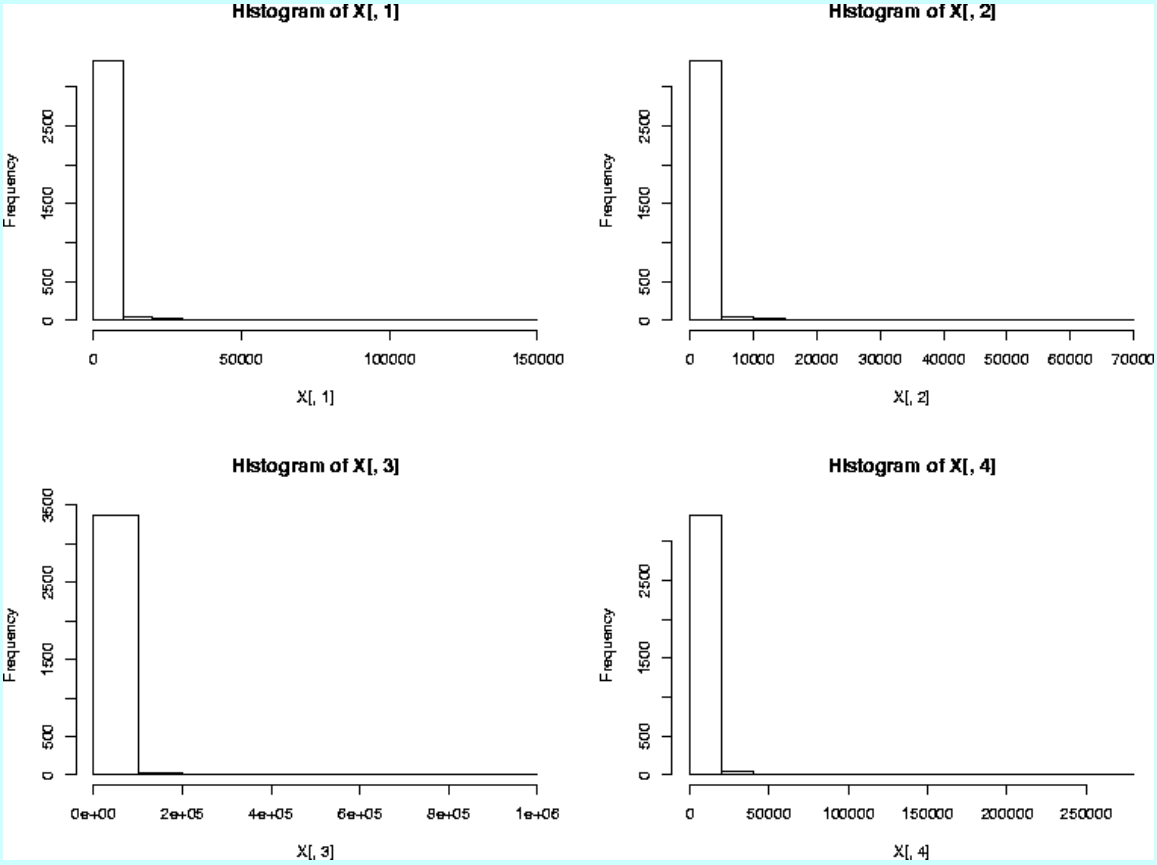
A MORE GENERAL TRANSFORMATION: A + B X^P CAN BE USED TO CHANGE THE LOCATION AND SCALE BUT NOT THE SHAPE.
ANOTHER MORE: A + B $(X- C)^P$

For several letter values define

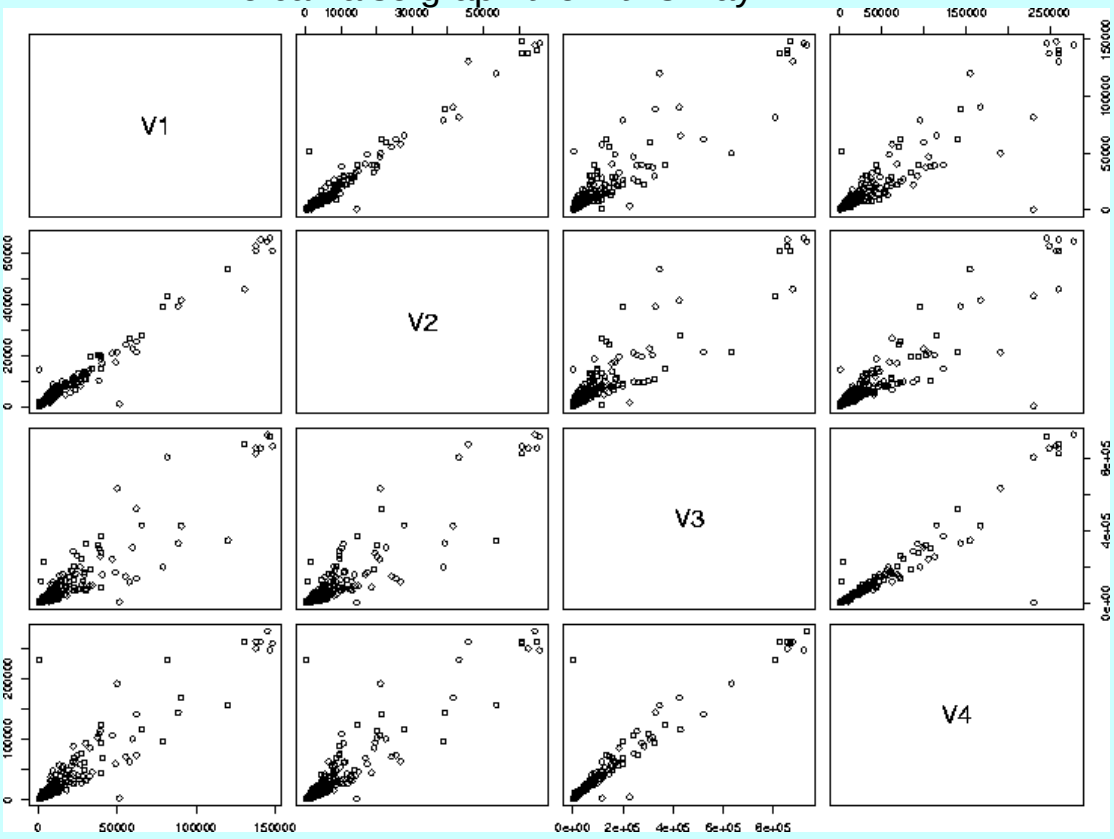
$$Y = ((X_U - M)^2 - (M- X_L)^2)/(4M)$$
$$Z = (X_U + X_L)/2 - M$$

12. **Plot Y VS Z**
13. **b is the slope of Y vs Z**
14. **If the result is linear then p = 1-b is the power for the transformation:**
15. **$T(X) = X^p$ or maybe $k X^p$**

EXAMPLE DNA MICRO ARRAY DATA:
DNA expression data is perhaps the most important tool in drug development and drug discovery. We have here an experiment with four arrays and we want to find out if they differ from other similar sets.



We can also graph them this way:



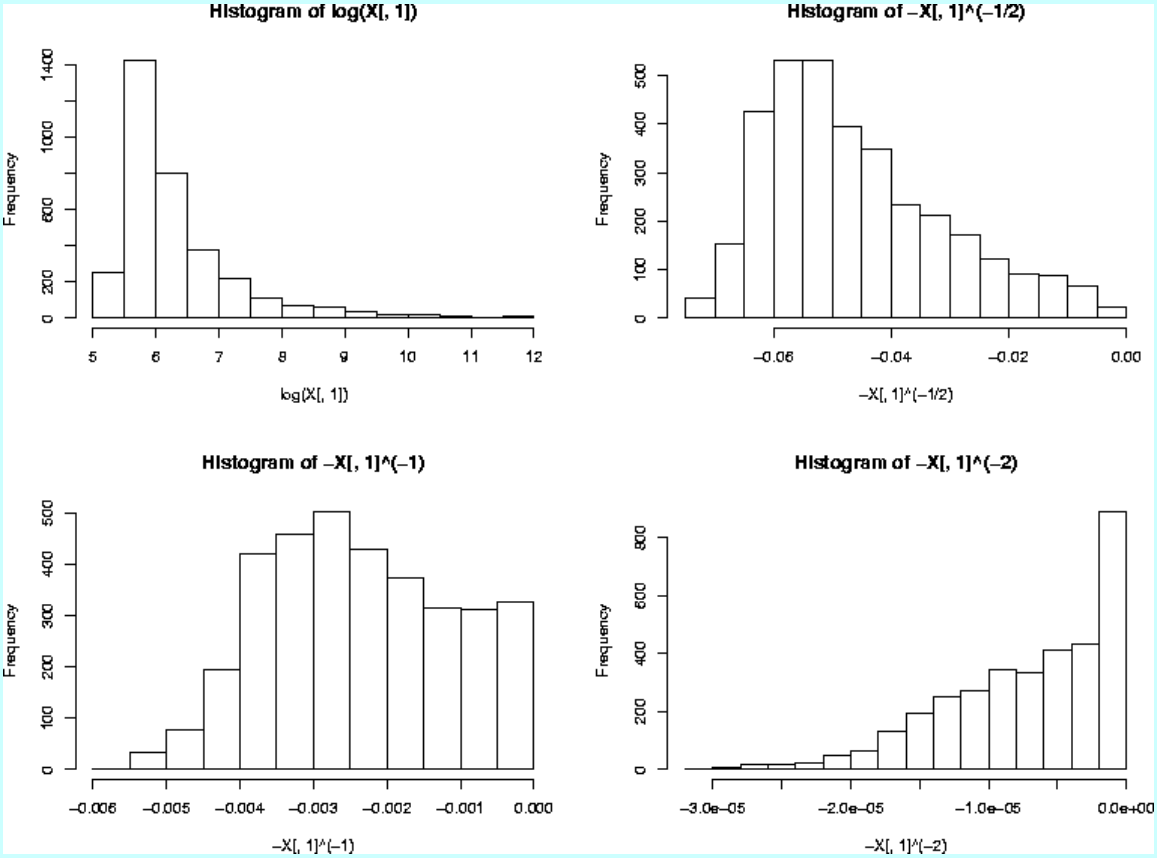
Rweb code:

```
pairs(X)
```

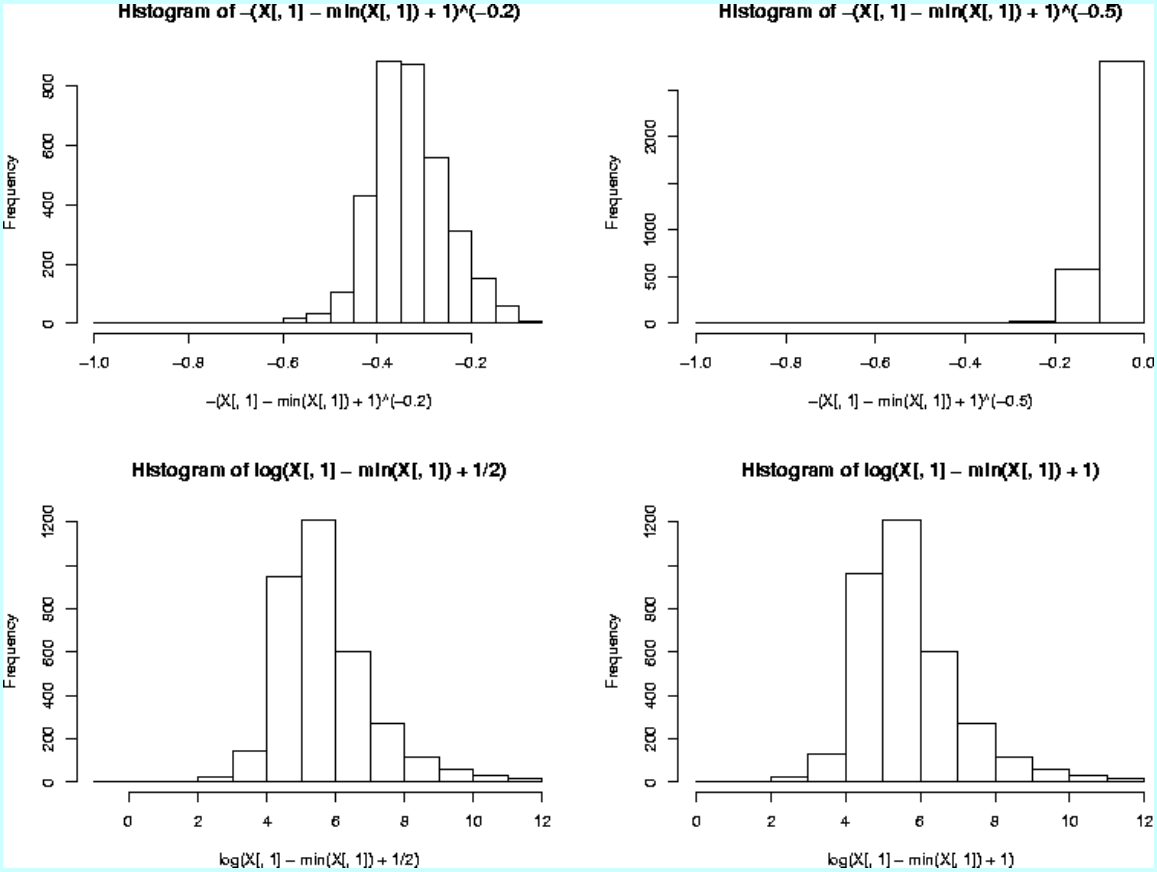
```
par(mfrow=c(2,2))
hist(X[,1])
hist(X[,2])
hist(X[,3])
hist(X[,4])
The data is in :
```
http://www.rci.rutgers.edu/~cabrera/486/micro.txt

Take the first one and try to find a power transformation

```
hist(log(X[,1]),title="log")
hist(- X[,1]^(-1/2),title="p^-1/2")
hist(- X[,1]^(-1),title="p^-1")
hist(- X[,1]^(-2),title="p^-2")
#
```




```
hist(-(X[,1]-min(X[,1])+1)^(-0.2),title="-0.2")
hist(-(X[,1]-min(X[,1])+1)^(-0.5),title="-0.5")
hist(log(X[,1]-min(X[,1])+1/2),title="log")
hist(log(X[,1]-min(X[,1])+1),title="log")
```



11. Resistant Lines

By now you should know that least-squares is a non-robust procedure and when you fit a line to data you should always check for the presence of outliers. One alternative is to use a method for fitting lines that is robust. There is a multitude of robust procedures. The most intuitive is "least median squares" but there are others like the "three group line", median slopes and other.

Least Median S – Least $Median\{(y_i - \beta_0 - \beta_1 x_{i1} - \cdots - \beta_p x_{ip})^2\}$

Quantile S-Trimmed Least Means S $Quantile_p\{(y_i - \beta_0 - \beta_1 x_{i1} - \cdots - \beta_p x_{ip})^2\}$

These methods $TrimMean\{(y_i - \beta_0 - \beta_1 x_{i1} - \cdots - \beta_p x_{ip})^2\}$ minimize the median/quantile/trimmed mean of the square residuals:

Least Squares estimators minimize:

The exact computation of the LMS, LQS and LTS estimators requires solving combinatorial problems that are n^p hard or $O(C_n^{p+1})$. Instead of finding the optimal solution we get an approximated solution by sampling. One such algorithm considers many thousands of subgroups of p+1 points chosen at random and evaluates the LMS (or other) criterion for the hyperplane determined by the group of points. The final estimate is not the optimal one but it might be close enough if the simulation is sufficiently large.

12. Algorithm for Tukey's three group line:

1. Sort the data by x.
2. Divide the data into 3 equal groups by x.
3. Calculate the centers (median) of each group:

$$(x_L, y_L), (x_M, y_M), (x_R, y_R)$$

4. Define

$$b_0 = (y_R - y_L)/(x_R - x_L)$$

5. Define

$$a_0 = (y_L - b_0 x_L + y_M - b_0 x_M + y_R - b_0 x_R) / 3$$

Compare this with the Least Squares Line, which is clearly not resistant because each observation influences the value of the slope and intercept.

SAS PROGRAM

```
options ps=60 ls=80;
GOPTIONS GUNIT=PCT CBACK=BLACK;

proc iml;
start median;
a =u;
u[rank(u),] = a;
d = nrow(u);
DD = INT((D+1)/2);
IF( D = (2*DD -1)) THEN mu = u[DD];
        ELSE mu = (u[DD]+u[DD+1])/2;
finish;
```

$$\sum_{i=1}^{..} (y_i - \beta_0 - \beta_1 x_{i1} - \cdots - \beta_p x_{ip})^2$$

```
START rl;
A = y;
y[RANK(X),]= A ;
A = X;
X[RANK(X),]= A ;
n = nrow(x);
m = int(n/3);
u = x[1:m,];
run median;
xl = mu;

u = y[1:m,];
run median;
yl = mu;

u = x[(m+1):(n-m),];
run median;
xm = mu;

u = y[(m+1):(n-m),];
run median;
ym = mu;

u = x[(n-m+1):n,];
run median;
xr = mu;

u = y[(n-m+1):n,];
run median;
yr = mu;

b =(yr-yl)/(xr-xl);
a = (yl-b*xl + ym-b*xm + yr-b*xr)/3;
estimate = a||b;
cn = {"Intercept" "Slope"};
print estimate[OLNAME=CN];
finish;

xbox={0 100 100 0};
ybox={0 0 100 100};
 year=do(71,86,1);              /* initialize YEAR   */
 price={123.75 128.00 139.75    /* initialize PRICE  */
        155.50 139.750 151.500
        150.575 149.125 159.500
        152.375 147.000 134.125
        138.750 123.625 127.125
        125.50};
y = price`;
x =  year`;
run rl;
```

```
quit;
run;
```

OUTPUT:

```
        ESTIMATE Intercept     Slope
                 229.40076 -1.147727
```

OUTLIERS: Suppose that now we change one observation by error:

```
 price={123.75 128.00 139.75    /* initialize PRICE  */
        155.50 139.750 151.500
        150.575 149.125 159.500
        152.375 147.000 134.125
        138.750 123.625 1027.125 125.50};
```

Then this is what happens to the LS estimator

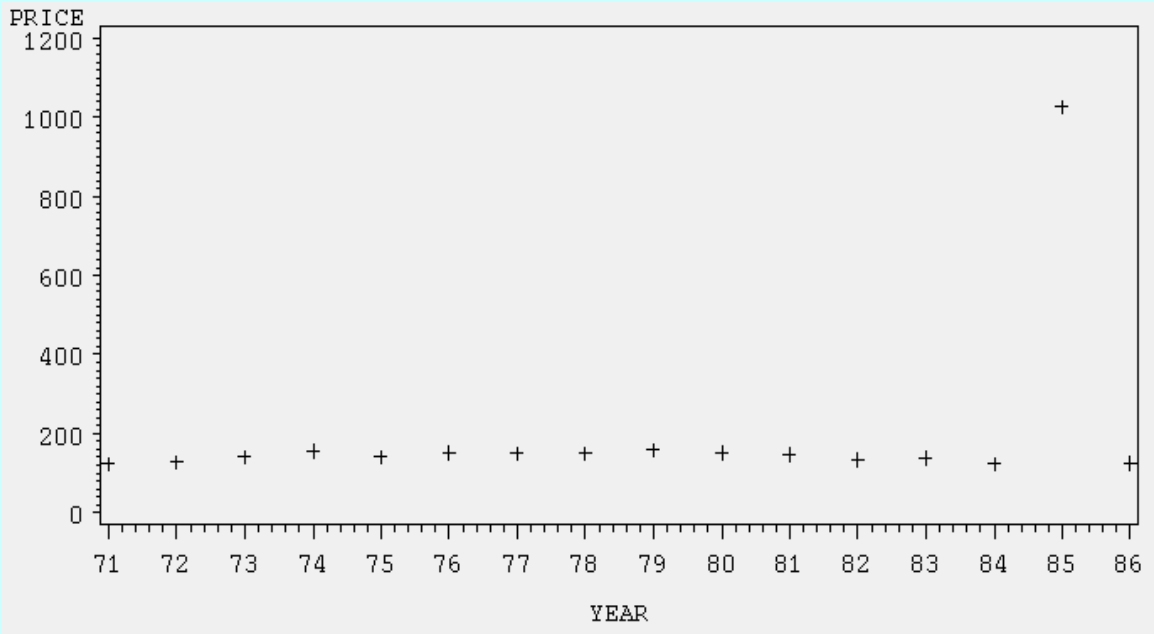
```
      ESTIMATE Intercept     Slope
               181.77955 -0.511364
```

```
data a;
input  year price;
cards;
  71    123.75
  72       128
  73    139.75
  74     155.5
  75    139.75
  76     151.5
  77   150.575
  78   149.125
  79     159.5
  80   152.375
  81       147
  82   134.125
  83    138.75
  84   123.625
  85  1027.125
  86     125.5
;
proc reg; model price = year; run;
```

OUTPUT:

```
Dependent Variable:   PRICE
                        Analysis of Variance
                       Sum of         Mean
Source          DF     Squares       Square      F Value       Prob>F

Model            1  94566.42825  94566.42825        2.058       0.1733
Error           14 643200.39284  45942.88520
C Total         15 737766.82109

      Root MSE     214.34291     R-square       0.1282
      Dep Mean     196.62188     Adj R-sq       0.0659
      C.V.         109.01275
```

```
                          Parameter Estimates
                Parameter     Standard   T for H0:
Variable DF     Estimate         Error  Parameter=0    Prob > |T|
INTERCEP  1  -1112.556103  914.08557756     -1.217     0.2437
YEAR      1     16.677426   11.62437667      1.435     0.1733

To see the outlier you can do a graph of the data:
proc gplot data=a;
plot price*year / vaxis=0 to 1200 by 200 frame;
run;
```



These are the Least squares estimators without the outlier:

```
 INTERCEPT       181.8557
 YEAR             -0.52846
```

Notice that they are very similar to what you got from the resistant line.

13. Scientific discovery: Fitting nonlinear equations to data.

We have two examples that show the way to fit this kind of models

Kepler's Law

```
#  Kepler's data
#  Period^2=k*Distance^3
#  Distance   Period
#     36        88
#     67.25    224.7
#     93       365.3
```

```
#    141.75    687
#    483.8    4332.1

d <- c(36 ,67.25 ,93 ,  141.75 , 483.8) # Distance
p <- c( 88, 224.7 ,365.3 ,687 ,4332.1 ) # Period
plot(d,p,xlab="Distance",ylab="Period",pch="o")
# Plot the data. The graph is inconclusive about linearity of
relationship.
# Plot residuals to check the goodness of the fit. Residuals show pattern.
rr <- lsfit(d,p)
plot(d,rr$resid,xlab="residuals-distance",ylab="period",pch="o")
# Plot log-log to see if the relationship is a power law
dl <- log(d); pl  <-  log(p)
plot(dl,pl,xlab="log-distance",ylab="log-period",pch="o")
abline(lsfit(dl,pl))
rr <- lsfit(dl,pl) # Plot residuals to check the goodness of the fit
plot(dl,rr$resid,xlab="residuals-log-distance",ylab="log-period",pch="o")
dp <- d^3  ; pp <- p^2 # Try several version of Keplers law
plot(dp,pp,xlab="cube-distance",ylab="square-period",pch="o")
abline(lsfit(dp,pp))
rr <- lsfit(dp,pp)
plot(dp,rr$resid,xlab="residuals-cube-distance",ylab="square-
period",pch="o")
dq <- d^(3/2)
plot(dq,p,xlab="distance^3/2",ylab="period",pch="o")
abline(lsfit(dq,p))
rr <- lsfit(dq,p)
plot(dq,rr$resid,xlab="residuals-distance^3/2",ylab="period",pch="o")

========================================================================
Y VS X
                Coef        Std Err       t Value
Intercept    -471.7008    119.6464       -3.942456
x2              9.80239     0.5161943    18.98972

Residual Standard Error = 188.642    Multiple R-Square = 0.99175
N = 5        F Value = 360.61 on 1, 3 df
========================================================================
log(Y) VS log(X)
                Coef       Std Err       t Value
Intercept    -0.896615  6.807606e-4    -1317.078
x2            1.499643  1.426618e-4    10511.87

Residual Standard Error = 2.77212e-4    Multiple R-Square = 1
N = 5        F Value = 110499512 on 1, 3 df
========================================================================
Y^2 VS X^3
                Coef       Std Err      t Value
Intercept    44.72135    4.34842e1    1.028451e0
x2            0.165729  8.58359e-7   1.930766e5

Residual Standard Error = 86.1907    Multiple R-Square = 1
N = 5        F Value = 3.727856e10 on 1, 3 df
Y VS X^3/2
                Coef       Std Err       t Value
```

```
Intercept    0.1130044  5.930967e-2  1.905328e0
x2           0.4070870  1.224807e-5  3.323684e4

Residual Standard Error = 0.1082272    Multiple R-Square = 1
N = 5        F Value = 1.104687e9 on 1, 3 df
```

Boyle's Law P x V = cte

```
     V       P
    29.750  1.0      5.625  5.0      3.000 10.0      1.500 24.0
    19.125  1.5      4.875  6.0      2.625 12.0      1.375 28.0
    14.375  2.0      4.250  7.0      2.250 14.0      1.750 20.0
     9.500  3.0      3.750  8.0      2.000 16.0      1.250 32.0
     7.125  4.0      3.375  9.0      1.875 18.0
boyle <- data.frame(matrix(scan(),ncol=2,byrow=T))
names(boyle) <- c("V","P")
attach(boyle)
par(mfrow=c(2,2),cex=1.07,mar=c(4,4,0.2,0.2),mgp=c(2,1,0))
plot(P,V,pch=16,col=2)  # Plot 1
plot(log(P),log(V),pch=16,col=2) # Plot 2
lsfit(log(P),log(V))$coef
[1]  3.2717004 -0.9152016
#  the slope is close to -1.
Pm1 <- 1/P
rm1 <- lsfit(Pm1,V)
plot(rm1$resid,V,pch=16,col=2) # Plot 3
# See U-pattern. It shows a possible quadratic fit.
rm2 <- lsfit(cbind(Pm1,Pm1^2),V)
plot(rm2$resid,V,pch=16,col=2) # Plot 4
#   Residuals look very good!!
#   We have obtained the fit: V = a + b 1/P + c (1/P)^2 where a , b
and c are
rm2$coef
[1]  0.4127083 26.0636703  3.2543654
#    But the polynomial in the fit equation looks like a tylor
expanssion
#    of something, but what?
#    A possibility will be to expand  1/(P-d) as a
#    polynomial on 1/P near around zero.  We get
#       1/(P-d) = 1/P + d (1/P)^2 + d^2 (1/P)^3 + O( (1/P)^4 )
#    So , fit (1) suggests a possible fit ( Fit 2)
#      y = a1 + b1  1/(P -d)
#    where a1 = a = 0.41  ,  d = c/b =  0.12, and b1=26.06
#
nls(formula= V~ a1 + b1/(P-d),data=boyle, start=c(a1=0,b1=1,d=0))
Nonlinear regression mo1del
  model:  V ~ a1 + b1/(P - d)
   data:  boyle
        a1         b1          d
 0.4016758 26.2392374  0.1055646
 residual sum-of-squares:  0.05783763
# We conclude  d= 0.1055646
```

```
par(mfrow=c(2,2))
d <-  0.1055646
rmd1 <- lsfit(1/(P-d),V)
d <-  0.12
rmd2 <- lsfit(1/(P-d),V)
plot(rmd1$resid,P)
plot(rmd2$resid,P)
boxplot(rmd1$resid,rm2$resid,rmd2$resid,rm1$resid)
```

14. Multiple Linear Regression procedures

This is the standard model in many statistical problems. The data contains several predictors and one or more responses.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + ... + \beta_P x_P + \text{Error}$$

We calculate the least squares estimators of the regression parameters. We define fitted values and residuals the usual way:

$$y_i = b_0 + b_1 x_{i1} + ... + b_p x_{ip} + r_i$$

In practice we have a group of predictors that are candidates for the regression equation but we need to determine which ones are appropriate.

There maybe collinearities among the predictors

Most datasets contain a number of errors, bad observation or simply unusual observations which we call OUTLIERS and LEVERAGE POINTS. Regression diagnostics must be used to find out if there are any outliers on the dataset.

Example:

This example comes from Urban Housing in a US city in 1997. The cases correspond to urban units each of the size of a neighborhood. In each unit we observe a response that gives the increase in property taxes for the residential housing units in that neighborhood. The predictors are several demographic variables describing the type unit plus four variables given expenditures in transportation and in roads. The question is to try to asses the impact of expenditures in transportation and roads on property values. This dataset like many other contains a number of outlier and leverage points.

- First we plot the data and we run a regression to check if there are any outliers.

- The graphs are concentrated in the four variables measuring expenditure, but in practice one should plot the response vs all of the predictors.
- The options PR in the model statement produce an output containing the residuals and Cook's D statistic. You must eyeball the numbers and find the corresponding outliers.

```
libname mylib spss 'Reg.por';
options linesize=70 pagesize=55;

data a;
format  IN80BO IN80C IN90C INBBID INBP90 INCENP INCODE INCR80 INCR90
INEMP INMAIN INMAIN2 INMAIN3 INNV INO INREST INT79 INT80 INT90 INV90
INVAL SEV79 SEV80 SEVR80 SFAMV VAPT79 VAPT80 VCOM79 VCOM80 VFRM79
VFRM80 VIND79 VIND80 VRES79 VRES80 VTOT79 VTOT80 VVAL79 VVAL80
comma8. NAMEV1 $CHAR200.;
set mylib._first_;
if _N_ < 100;
run;

proc plot;
plot ravlchm*(roadacc roadcap transacc transcap);
run;

proc reg data=a;
model  ravlchm = hhinc black hslds setr vac indexr pstu
            roadacc roadcap transacc transcap/P R COLLIN;
```

Output file from "reg.sas"

Then we find that observations 14 and 13 are outliers so we may omit them for now. We repeat the regression again and we find that #2 is also an outlier.

```
proc reg data=a;
model  ravlchm = hhinc black hslds setr vac indexr pstu
           roadacc roadcap transacc transcap/P R COLLIN;
output out=b p=pred r=res;

proc plot data=b;
plot res*pred res*(roadacc roadcap transacc transcap);
run;

proc reg data=a;
model  ravlchm = hhinc black hslds setr vac indexr pstu
           roadacc roadcap transacc transcap/selection=adjrsq;

proc reg data=a;
model  ravlchm = hhinc black hslds vac indexr roadacc
                  roadcap transacc transcap;
```

Output file from "reg1.sas"

Then we repeat and we find new outliers and so on.

Output file from "reg2.sas"

On the other hand the robust procedure yields the output

```
Intercept      HHINC      BLACK      HSLDS      SETR       VAC
 -72.54527 0.00780025 -0.1025825 0.1485328 -36.75587 0.1651357
      INDEXR      PSTU   ROADACC
 -0.001151889 4.031584 5.032116
   ROADCAP TRANSACC TRANSCAP
  48.80187 1.165922 166.9255
```