Token-Budget-Aware LLM Reasoning

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Abstract

Reasoning is critical for large language models (LLMs) to excel in a wide range of tasks. While methods like Chain-of-Thought (CoT) reasoning enhance LLM performance by decomposing problems into intermediate steps, they also incur significant overhead in token usage, leading to increased costs. We find that the reasoning process of current LLMs is unnecessarily lengthy and it can be compressed by including a reasonable token budget in the prompt, but the choice of token budget plays a crucial role in the actual compression effectiveness. We then propose a token-budget-aware LLM reasoning framework, which dynamically estimates token budgets for different problems based on reasoning complexity and uses the estimated token budgets to guide the reasoning process. Experiments show that our method effectively reduces token costs in CoT reasoning with only a slight performance reduction, offering a practical solution to balance efficiency and accuracy in LLM reasoning. Code: https://github.com/GeniusHTX/TALE.

"It is not enough to have a good mind; the main thing is to use it well."

René Descartes

1 Introduction

Reasoning plays a crucial role in enabling large language models (LLM) to perform effectively across a wide range of tasks (Zhou et al., 2022; Hao et al., 2023, 2024a). A variety of methods have been proposed to enhance the reasoning capabilities of large language models (Suzgun et al., 2022; Wang et al., 2023; Feng et al., 2023; Yao et al., 2024a; Xie et al., 2024). Among these, Chain-of-Thought (CoT) (Wei et al., 2022) is the most representative

and widely adopted approach. It enhances the reliability of the model's answers by guiding large language models with the prompt "Let's think step by step", encouraging them to decompose the problem into intermediate steps and solve each before arriving at the final answer. Figure 1a and Figure 1b illustrate an intuitive example. Observe that without CoT, the LLM produces incorrect answers to the question. With a CoT-enhanced prompt, the LLM systematically breaks the question into multiple steps and reasons through each step sequentially. By addressing each step incrementally, the LLM eventually arrives at the correct answer.

Although reasoning enhancement approaches such as CoT impressively improve LLM performance, they produce substantial additional overhead, specifically in the form of the increased number of tokens produced (Wei et al., 2022; Feng et al., 2023; Yao et al., 2024a). As shown in Figure 1b, the answer to prompt with CoT has notably higher token costs due to the detailed intermediate reasoning steps included in the output. Such high token costs can lead to significant expenses, including increased computational resource usage and longer running times during the LLM inference phase, ultimately resulting in significant additional monetary and energy costs.

This raises an important question: "Is the reasoning process of current LLMs unnecessarily lengthy, and how can it be compressed?" Nayab et al. (2024) demonstrate that LLM has the potential to follow a length constraint in the prompt. Building on this, we find that including a token budget (see Table 1) in the prompts is a promising approach to compressing the CoT reasoning tokens. However, the choice of token budget plays a crucial role in the actual compression effectiveness. For example, Figure 1d illustrates that including a reasonable token budget (e.g., 50 tokens in this case) in the instructions reduces the token cost in the chain-of-thought (CoT) process from 258 output tokens to

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Pre-print with preliminary results, work in progress.

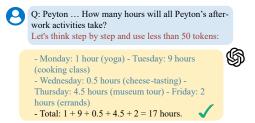
O: Peyton scheduled after-work activities of a onehour yoga class on Monday, a cooking class that lasts 9times as long as Monday's yoga on Tuesday, a half-hour cheese-tasting event on Wednesday, a museum tour that takes half as long as the cooking class on Thursday, and two hours of errands on Friday. How many hours will all Peyton's afterwork activities take? Peyton's after-work activities will take 12.5 hours. X (a) Direct answering (15 output tokens). Q: Peyton ... How many hours will all Peyton 's afterwork activities take? Let's think step by step: Let's break down activities step by step: 1. Monday: Yoga class lasts 1 hour Total for Monday: 1 hour 2. Tuesday: Cooking class lasts 9 times as long as Monday's yoga class. - Cooking class duration = 9 * 1 hour = 9 hours Total for Tuesday: 9 hours 3. Wednesday: Cheese-tasting event lasts half an hour. Total for Wednesday: 0.5 hours 4. Thursday: Museum tour is half the cooking class. Museum tour duration = 0.5 * 9 hours = 4.5 hours Total for Thursday: 4.5 hours 5. Friday: Errands take 2 hours - Total for Friday: 2 hours Now, let's add up all the hours from each day: Monday: 1 hour - Tuesday: 9 hours Wednesday: 0.5 hours - Thursday: 4.5 hours - Friday: 2 hours Now, we sum these amounts 1 + 9 + 0.5 + 4.5 + 2 = 17 hours. Therefore, the total time is 17 hours. (b) Vanilla CoT (258 output tokens). O: Pevton ... How many hours will all Pevton's after-work activities take? Let's think step by step and use less than 10 tokens Let's break down each activity: •Monday: Yoga class = 1 hour. •Tuesday: Cooking class = 9 times Monday's yoga, so $9 \times 1 = 9$ hours. •Wednesday: Cheese-tasting event = 0.5 hours. •Thursday: Museum tour = Half of Tuesday's cooking class, so $9 \div 2 = 4.5$ hours. •Friday: Errands = 2 hours. Now, let's add them up:

(c) CoT with an unreasonable budget (157 output tokens).

1+9+0.5+4.5+2=171+9+0.5+4.5+2=

Peyton's after-work activities will take 17 hours

171+9+0.5+4.5+2=17



(d) CoT with an reasonable budget (86 output tokens).

Figure 1: Examples of different problem solving paradigms. The reasoning processes are highlighted.

86 output tokens, while still enabling the LLM to arrive at the correct answer. However, when the token budget is set to a different smaller value (e.g., 10 tokens), the output token reduction is less effective, resulting in 157 output tokens—nearly twice

as many as with a 50-token budget. In other words, when the token budget is relatively small, LLMs often fail to follow the given token budget. In such cases, the actual token usage significantly exceeds the given budget—even much larger than the token costs with larger token budgets. We refer to this phenomenon as the "Token Elasticity" in the CoT process with token budgeting. To address this, the optimal token budget for a specific LLM and a particular question can be searched by gradually reducing the budget specified in the prompt, identifying the smallest token budget that achieves both the correct answer and the lowest actual token cost.

Based on the above observations and analysis, we designed a prototype for token-budget-aware reasoning in large language models (LLMs). Our approach leverages the token budget to guide the reasoning process, dynamically allocating different token budgets to problems based on an estimation of their reasoning complexity. We call our method TALE (Token-Budget-Aware LLM rEasoning). For a given problem and a specific LLM, TALE first estimates an appropriate token budget and then uses it to guide the reasoning process. We discuss different implementations of TALE in Section 5. Experiment results show that TALE significantly reduces token costs in LLM chain-of-thought (CoT) reasoning while largely maintaining the correctness of the answers. On average, TALE achieves a 68.64% reduction in token usage while maintaining accuracy with less than 5% decrease.

2 Related Work

LLM Reasoning. Reasoning in LLMs has seen substantial advancements through techniques that generate intermediate steps, enabling more accurate and effective performance across diverse domains (Wu et al., 2022; Yang et al., 2022; Zhou et al., 2022; Sun et al., 2024; OpenAI, 2024c). Various LLM reasoning techniques are proposed to improve the LLM performance. Chen et al. (2024) formulates reasoning as sampling from a latent distribution and optimizing it via variational approaches. Ho et al. (2022) utilizes LLM as reasoning teachers, improving the reasoning abilities of smaller models through knowledge distillation. Among them, Chain-of-Thought (CoT) prompting has emerged as a key technique for improving LLM reasoning by breaking problems into intermediate steps, enabling better performance on multiple

2023; Feng et al., 2024). Extensions of CoT include self-consistency, which aggregates multiple reasoning paths to improve robustness (Wang et al., 2022), and Tree-of-Thoughts, which explores reasoning steps in a tree-like structure for more complex tasks (Yao et al., 2024b). Reflexion introduces iterative refinement, where the model critiques and updates its intermediate steps (Shinn et al., 2024). **Token Cost of LLM.** Although the above methods enhance reasoning accuracy, they often increase token usages, posing challenges to efficiency (Wang et al., 2024; Chiang and Lee, 2024; Bhargava et al., 2023). Consequently, it is important to mitigate token consumption while maintaining the model performance. To address this issue, Li et al. (2021) introduces a multi-hop processing technique designed to filter out irrelevant reasoning. While effective, this approach is limited to traditional neural networks, such as PALM (Bi et al., 2020), and lacks adaptability to large language models (LLMs). Zheng et al. (2024) aims to improve LLM inference speed by predicting response lengths and applying a scheduling algorithm to enhance efficiency. However, it is constrained to scheduling level, and it does not reduce the actual token costs. Hao et al. (2024b) reduces token usage by substituting decoded text tokens with continuous latent tokens. However, its application is currently restricted to small-scale, early language models like GPT-2 (Radford et al., 2019). Additionally, it significantly impacts reasoning accuracy, resulting in over a 20% relative accuracy reduction on benchmarks such as GSM8K (Cobbe et al., 2021).

tasks (Wei et al., 2022; Lyu et al., 2023; Li et al.,

3 Token Redundancy in LLM Reasoning

Token Budget. Previous research Nayab et al. (2024) demonstrates that LLM has the potential to follow a length constraint in the prompt. Table 1 shows the difference between the vanilla CoT and the CoT with token budget. For instance, by including a token budget (50 tokens) within the prompt, as illustrated in Figure 1d, the LLM adjusts the length of its output (86 output tokens), trying to align with the specified budget. This indicates that LLMs have a certain capability in following prompts with an explicit token budget.

Token Redundancy Phenomenon. We find that providing a reasonable token budget can significantly reduce the token cost during reasoning. As shown in Figure 1d, including a token budget in

Table 1: Illustrations of the vanilla CoT prompt and the token-budget-aware prompt.

Prompt method	Content			
Vanilla CoT	Let's think step by step:			
CoT with Token Budget	Let's think step by step and use less than budget tokens:			
Example	Let's think step by step and use less than 50 tokens:			

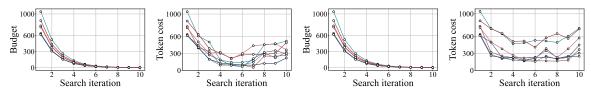
the instructions reduces the token cost in the chainof-thought (CoT) process by several times, but the LLM still gets the correct answer. Our results in Figure 2 and Table 2 also confirm there are a large number of redundant tokens in the reasoning process of the state-of-the-art LLMs.

Causes of Token Redundancy in LLM Reasoning. A possible explanation for this token redundancy is that during the post-training phase, such as the RLHF process (Ouyang et al., 2022), annotators might favor more detailed responses from LLMs, marking them as preferred. As a result, the model learns to associate longer, more detailed responses with alignment to human preferences and tends to produce such outputs during reasoning. However, in many scenarios, we primarily need LLMs to provide the correct answer and make accurate decisions, rather than elaborate extensively with detailed explanations. This motivates the need to eliminate redundant tokens in the LLM reasoning process in many cases.

4 Searching Optimal Token Budget

As demonstrated in Figure 1, different token budgets have different effects. Therefore, it is natural to investigate the following question: "How to search the optimal token budget for a specific question and a particular LLM?"

Vanilla Method for Optimal Budget Search. An intuitive method is finding the minimal needed tokens as the budget, ensuring that the LLM can still produce correct and accurate responses within this constraint. To find the minimal token budget required for each question, we utilize a binary search-based minimal budget search algorithm. Algorithm 1 showcases the details. Before initiating the search process, we first apply the vanilla CoT to generate an answer for each question, as illustrated in Figure 1b. The number of tokens in the resulting answer is then calculated and designated as the right boundary for search, denoted by right. The function isFeasible is used to determine the fea-



(a) GPT-4o-mini budget search. (b) GPT-4o-mini token cost. (c) Yi-lightning budget search. (d) Yi-lightning token cost.

Figure 2: Token elasticity phenomenon. The x-axis denotes the budget search iteration. The y-axis denotes the searched budget (Figure 2a and Figure 2c) or the real token costs for each searched budget (Figure 2b and Figure 2d). Different colors denote different samples randomly selected from MathBench-College (Liu et al., 2024). The token cost is significantly lower in a reasonable token budget range. When the token budget is smaller than the reasonable range, the token cost gradually increases.

Algorithm 1 Budget Search

return β

12:

Input: feasibility checking function is Feasible, a large language model \mathcal{M} , a given question \boldsymbol{x} and the ground truth label y

```
Output: searched budget \beta
  1: function SEARCH(isFeasible,\mathcal{M}, x, y)
          right \leftarrow the actual token costs of \mathcal{M} with
     vanilla CoT prompt on x
 3:
          \beta \leftarrow |(0 + right)/2|
  4:
          \beta_0 \leftarrow right
          while True do
  5:
               if is Feasible (\mathcal{M}, \boldsymbol{x}, y, \beta_0, \beta) then
  6:

    □ Update the searched budget

                    \beta \leftarrow |(0 + right)/2|
  7:
                  > Record previous searched budget
  8:
                    \beta_0 \leftarrow right

    □ Update the search range

                    right \leftarrow \beta
  9:
               else
 10:
                    break
11:
```

sibility of a budget. A budget is considered feasible here if the CoT prompt with that budget preserves the correctness of the answer. Given the feasibility function, large language model \mathcal{M} , question xand label yas the input, Algorithm 1 first calculates the right boundary of search (line 2). With 0 as the left boundary, the current possible budget β is computed as the midpoint of 0 and right (line 3). We use β_0 to record the previously searched budget (line 4). While the current β is feasible, the algorithm updates β by recalculating the midpoint (line 7) and adjusts the search bounds accordingly to narrow the range (line 9). Once the loop ends, the final budget β is returned as the searched result (line 12). Algorithm 1 is designed to find the minimal budget efficiently. However, we observe that the minimal

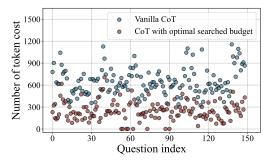


Figure 3: The effects of optimal searched budget. CoT, with our optimal searched budget, reduces the token costs significantly without influencing the accuracy. We conduct it on MathBench-College (Liu et al., 2024).

budget required to produce a correct answer is not

necessarily the optimal budget. When the budget is unreasonably small, the actual token cost often exceeds that of cases where a larger budget is used. Observation of Token Elasticity. During our minimal budget search process, we observe a "token elasticity" phenomenon as we approach the minimal budget. Specifically, as Algorithm 1 progresses, we aim to identify the minimal budget that still ensures the answer's correctness. However, we find that if the budget is reduced beyond a certain range, the token cost increases, indicating that further reductions in the budget lead to increasing token consumption. Figure 2 showcases the evidence. The x-axis represents the iterations of the budget binary search, with the budget values decreasing progressively. The y-axis in Figure 2b and Figure 2d show the corresponding token costs at each budget search iteration. We can observe that when the searched budget crosses a range for reasonable budgets, the token costs begin to gradually increase. Figure 1c also shows an example. As observed, when a small token budget (e.g., 10 tokens) is used, the real token cost is significantly higher compared to scenarios where a reasonable

token budget is allocated (i.e., Figure 1d).

Algorithm 2 Greedy Feasibility Function

Input: a large language model \mathcal{M} , a question x and the ground truth label y, previous and current budget: β_0 , β

Output: True if the budget satisfies the requirements, False otherwise

- 1: **function** is Feasible $(\mathcal{M}, \boldsymbol{x}, y, \beta_0, \beta)$
- 2: $t, t_0 \leftarrow \text{gets the actual token costs under budgets of } \beta \text{ and } \beta_0$
- 3: if $\mathcal{M}(\boldsymbol{x}, \beta) == y$ and $t < t_0$ then
- 4: **return** True
- 5: return False



Figure 4: The workflow of TALE. Given a question, TALE first estimates the token budget using a budget estimator. It then crafts a token-budget-aware prompt by combining the question with the estimated budget. Finally, the prompt is input to the LLM to generate the answer as the final output.

Token Elasticity based Optimal Budget Search.

The token elasticity observation shows that while a minimal budget may keep the correctness of the answer, it does not necessarily minimize the token cost. Figure 1c and Figure 1d illustrate an intuitive example. To address this, we enhance Algorithm 1 by incorporating a greedy search strategy aimed at finding the optimal budget that simultaneously minimizes token cost and preserves answer correctness. Specifically, we introduce an additional constraint to the isFeasible condition. Beyond ensuring correctness, the updated budget must result in a lower token cost compared to the previously searched budget. Algorithm 2 outlines the feasibility function employed during the search process. Initially, the actual token cost is computed for both the current and previously evaluated budgets (line 2). Next, feasibility is assessed based on two criteria: the correctness of the answer and a reduction in token cost following a greedy strategy (line 3). The search process is terminated if either condition fails.

5 Methodology

5.1 Overview

Based on the above observations and analysis, we designed our method TALE for token-budget-

aware reasoning in LLMs. Figure 4 provides an overview of TALE's workflow. TALE aims to elaborate a token-budget-aware prompt that achieves performance comparable to vanilla CoT while reducing token costs. To strike this balance, TALE follows a two-phase approach: budget estimation and prompt construction. The observation of token elasticity, as discussed in Section 4, suggests that only an appropriate budget located in a reasonable range can effectively reduce token costs and preserve LLM performance simultaneously. The optimal budget searched by Algorithm 1 and Algorithm 2 is located in such reasonable budget range and achieves satisfying trade-off between token costs and LLM performance. In this case, TALE first estimates a reasonable token budget that is close to the searched optimal budget for the given question. Using the estimated token budget, TALE then crafts a token-budget-aware prompt and then feeds it into LLM to generate the final answer. Figure 7 illustrates an example of TALE.

5.2 Budget Estimation

To estimate an appropriate budget within the reasonable budget range, two possible solutions are taken into consideration: zero-shot-based mechanism and budget regression. We also discuss an approach that internalizes the budget awareness of LLM by fine-tuning it. All approaches focus on developing an estimator that effectively approximates the optimal budget.

Zero-shot Estimator. For zero-shot mechanism to predict the reasonable budget, TALE leverages the reasoning LLM itself as an estimator. Before querying the LLM with a question, TALE first prompts the LLM to estimate the number of output tokens needed to answer the question. Figure 5 illustrates the budget estimation prompt. The key intuition behind this is the human-like thinking paradigm. When presented with a mathematical question, although it may take humans a few minutes to calculate the answer, they can typically estimate the time or effort required to solve it after just briefly reviewing the question. For instance, when presented with a question from primary school arithmetic and another from college-level calculus, a human may not immediately provide the answers. Still, it is easy to infer that the former can be solved in seconds, while the latter requires a significantly longer time, even with only a brief glance. RQ2 in Section 6 showcases the performance of budget estimation. Observe that a large amount of the estimated bud-



Task: Analyze the given question and estimate the minimum number of tokens required to generate a complete and accurate response. Please Give the response by strictly following this format: [[budget]], for example, Budget: [[12]].

Figure 5: The prompt for zero-shot estimator.

gets are around the optimal searched budget and achieve competitive performance.

Regression Estimator. For regression-based estimator, we aim to train/fine-tune another LLM $f(\theta)$ to serve as the estimator, such that $f(\theta)$ estimates the optimal token budget given a specific LLM and a particular question. Given $\mathcal{D} = \{(\boldsymbol{p}, \boldsymbol{x}_i, \beta_i^*)\}_{i=1}^N$, p is the instruction prompt, x_i is a question, β_i^* is our searched optimal budget (searched by Algorithm 1 and Algorithm 2) for x and N is the dataset size. We assign the instruction prompt p as "Estimate the token budget for the following question". Next, we initialize $f(\theta)$ using a pre-trained LLM, such as LLaMA 3-8B (AI@Meta, 2024). Then, we craft the target output y_i for x_i using β_i^* . For example, given a β_0^* as 14, the corresponding target output y_0 is "The token budget is 14". We aim to make the model output as close as possible to the target output y_i by minimizing loss function, which can be formalized as:

$$\theta^* = \arg\min_{\theta} \sum_{i=1}^{N} \mathcal{L}\left(f_{\theta}(\boldsymbol{p}_i, \boldsymbol{x}_i), y_i\right)$$
 (1)

where θ denotes the model parameters and θ^* the optimized parameters. The loss function is defined as follows:

$$\mathcal{L} = -\sum_{i=1}^{N} \log \mathbb{P}(y_i \mid \boldsymbol{p}_i, \boldsymbol{x}_i; \theta)$$
 (2)

Through Equation 2, model parameters θ are optimized to maximize the probability $P(y_i \mid p_i, x_i; \theta)$ of the target output y_i given the input x_i and instruction prompt p across all N training samples. **Token-Budget Awareness Internalization.** To obtain an LLM with token awareness, we fine-tune the LLM to internalize the budget estimation into the inference process and produce token-efficient reasoning responses. Specifically, we fine-tune the LLM $\mathcal{M}(\theta)$ so that it generates token-budget-aware answers. This process is divided into two key stages: target output generation and LLM fine-tuning. In the target output generation stage, we craft the target output y_i by prompting $\mathcal{M}(\theta)$ with a

Chain-of-Thought (CoT) prompt that incorporates our searched optimal token budget. The prompt is formatted as follows:

"Let's think step by step and use less than β_i^* tokens:"

where β_i^* is the searched optimal budget for the given question x_i (see search process in Algorithm 1 and Algorithm 2). Figure 1d illustrates an example. The resulting LLM output, constrained by the token budget specified in the prompt, is taken as the crafted target output y_i . In the LLM fine-tuning stage, we train the LLM $\mathcal{M}(\theta)$ using the crafted target outputs from the first stage. The instruction prompt for fine-tuning is standardized as follows:

The training sample for fine-tuning consists of the given question x_i paired with the crafted target output y_i . The training objective aligns with the loss function described in Equation 2 where (x, y_i) indicates the training sample and p_i means the above instruction prompt. During fine-tuning, the LLM is encouraged to internalize the token budget constraint and prefer a compact reasoning process, following the target outputs generated in the first stage. This two-stage process ensures that the LLM produces concise yet correct responses, effectively balancing reasoning quality with token efficiency during inference.

6 Evaluation

In this section, we provide the **preliminary results of the zero-shot estimator version** of our method TALE. This project is ongoing and we will update more results soon. Three state-of-the-art LLMs (i.e., GPT-4o (OpenAI, 2024b), GPT-4o-mini (OpenAI, 2024a), Yi-lightning (Wake et al., 2024)) are involved in the experiments. Our evaluation is surrounding around the following research questions(RQs):

RQ1. How effective is TALE in reducing token costs while maintaining LLM performance?

RQ2. How effective is TALE to estimate the token budget given a question?

RQ3. How general is TALE across different state-of-the-art LLMs?

6.1 Experiment Setup

Metrics. The target of TALE is to balance the LLM performance and extra redundant token costs.

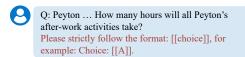


Figure 6: The instruction prompt used to format the LLM output on multiple-choice questions.

Specifically, TALE seeks to minimize token consumption while maintaining comparable LLM performance. To evaluate the LLM performance, three most challenging mathematical datasets are taken into consideration: GSM8K (Cobbe et al., 2021), GSM8K-Zero (Chiang and Lee, 2024), and Math-Bench (Liu et al., 2024). GSM8K-Zero, derived from the GSM8K dataset, specifically targets the analysis of over-reasoning and redundancy in LLM-generated outputs. In short, GSM8K-Zero is designed so that the answers are embedded within the questions themselves. LLMs can easily generate correct responses without complicated additional reasoning or redundant calculations.

Accuracy (Acc). This metric is calculated as the following: $Accuracy = \frac{1}{N} \sum_{i=1}^{N} \mathbb{I}\{\mathcal{M}(\boldsymbol{x}_i) = y_i\}$, where $(\boldsymbol{x}_i, y_i) \in \mathcal{X}$. \boldsymbol{x}_i is the math question from dataset \mathcal{X} and y_i the ground truth answer. $\mathcal{M}(\cdot)$ returns the answer for a given question. $\mathbb{I}\{\cdot\}$ represents an indicator function. This function evaluates whether the inside given condition holds. Specifically, it returns 1 if the condition is true and 0 if the condition is false. For a better evaluation, we format the LLM output by crafting an elaborate instruction detailed in Figure 6.

Number of Output Tokens. We evaluate the token costs by calculating the average output token consumption for each specific task. The output token costs are measured as follows: Number of Output Tokens $= \frac{1}{N} \sum_{i=1}^{N} \mathbb{T}(\mathcal{M}(\boldsymbol{x}_i))$, where \boldsymbol{x}_i represents the given question, and \mathbb{T} is a function that measures the number of tokens. Intuitively, the more output tokens, the higher the costs incurred by \mathcal{M} . To evaluate costs more precisely, we calculate the average expense per sample for querying the LLM. The total token expense includes both input and output tokens used during the query process.

6.2 RQ1. Effectiveness of TALE.

Table 2 presents a comparison of TALE and other prompt engineering methods, including "Directly Answering" and "Vanilla CoT", in their effectiveness for seven different datasets. The results are evaluated regarding ACC, Number of Output Tokens (Output Tokens for short), and Expense. A

well-designed prompt engineering approach should induce the LLM to generate a correct response with as few tokens as possible. TALE consistently demonstrates significant improvements in efficiency while maintaining competitive accuracy. Directly answering achieves the lowest output tokens (14.57 on average) and expenses (25.37 on average) but has the lowest accuracy (52.31% on average). Vanilla CoT achieves the highest accuracy (83.75% on average) but incurs a significant token cost (461.25 on average) and expenses (289.78 on average). TALE demonstrates a balanced trade-off between performance and cost. It achieves competitive accuracy (81.03%) while reducing token costs (32% of Vanilla CoT) and expenses (41% of Vanilla CoT) significantly. For accuracy, notably, on GSM8K, TALE even improves accuracy to 84.46%, surpassing Vanilla CoT, demonstrating its ability to adapt well to complex reasoning tasks while remaining efficient. For output tokens on GSM8K-Zero, TALE achieves an impressive reduction in output token costs from 252.96 to 22.67 while maintaining high accuracy (98.72%), showcasing its capability to optimize reasoning efficiency in such tasks. For expenses, TALE demonstrates its cost-effectiveness, reducing expenses from 78.58 to 18.62 while achieving reasonable accuracy (73.67% vs 75.00%) on MathBench-Arithmetic. TALE demonstrates that incorporating token-budget awareness allows for a significant reduction in token costs and monetary expenses without a major compromise in accuracy. TALE reduces output token costs by 68.64% on average, making it a more efficient solution for budget-constrained reasoning tasks while retaining competitive performance. These results highlight TALE's generalizability across tasks with varying complexity, demonstrating its potential to scale in real-world scenarios while managing computational and financial resources effectively.

6.3 RQ2. Effectiveness of Budget Estimation.

In this RQ, we evaluate the effectiveness of the budget estimation performance. An ideal estimated budget should be located around the optimal searched budget and in the bottom area of Figure 2. We further define such an area as the ideal budget range and give the formalized definition in Section A.1. A good budget should be located in the ideal budget range. Two metrics are taken into consideration: *in-range accuracy* and *out-of-range distance*. In-range accuracy determines whether

Table 2: Comparison of TALE (Zero-shot Estimator Version) and other prompt engineering methods. "Directly Answering" means prompting LLM without any reasoning process. "Vanilla CoT" means the vanilla CoT prompting with budget. The model used in our evaluation is GPT-4o-mini (OpenAI, 2024a). Observe that TALE achieves an average accuracy (ACC) of 80.22%, with an average output token cost of 138.53 and an average expense of 118.46. TALE reduces output token costs by 67%, lowers expenses by 59%, and maintains competitive performance compared to the vanilla CoT approach. ACC \uparrow , Output Tokens \downarrow , Expense $(10^{-5} \$ / \text{sample}) \downarrow$.

Dataset	Directly Answering			Vanilla CoT			TALE (Ours)		
	ACC ↑	Output Tokens ↓	Expense ↓	ACC ↑	Output Tokens ↓	Expense ↓	ACC ↑	Output Tokens ↓	Expense ↓
GSM8K	28.29%	12.46	39.43	81.35%	318.10	541.09	84.46%	77.26	279.84
GSM8K-Zero	97.21%	18.85	91.69	99.50%	252.96	886.79	98.72%	22.67	276.12
MathBench-Arithmetic	59.67%	41.10	9.78	75.00%	313.51	78.58	73.67%	39.60	18.62
MathBench-Middle	33.33%	5.00	3.58	84.67%	553.93	68.22	79.33%	238.14	42.95
MathBench-High	51.33%	5.00	4.07	84.00%	653.24	82.44	80.00%	254.82	47.61
MathBench-College	44.00%	5.00	3.68	78.00%	675.78	81.56	70.00%	259.85	45.60
Average	52.31%	14.57	25.37	83.75%	461.25	289.78	81.03%	148.72	118.46

Table 3: The generalization of TALE (Zero-shot Estimator Version) across different LLMs. Yi-lightning (Wake et al., 2024), GPT-40-mini (OpenAI, 2024a) and GPT-40 (OpenAI, 2024b) are taken into consideration. We conduct the evaluation on MathBench-College. ACC \uparrow , Output Tokens \downarrow , Expense (10^{-5} \$ / sample) \downarrow .

LLM	Directly Answering			Vanilla CoT			TALE (Ours)		
	ACC ↑	Output Tokens ↓	Expense ↓	ACC ↑	Output Tokens ↓	Expense ↓	ACC ↑	Output Tokens ↓	Expense ↓
Yi-lightning	66.67%	80.01	3.09	79.33%	998.10	21.55	76.67%	373.52	17.25
GPT-4o-mini	44.00%	5.00	3.68	78.00%	675.78	81.56	70.00%	259.85	45.60
GPT-40	57.33%	5.00	61.34	84.00%	602.29	1359.42	80.00%	181.61	759.95

the predicted budget $\hat{\beta}$ falls within the ideal budget range W_k^* . Mathematically, it can be expressed as:

$$\mathbb{I}\{\hat{\beta} \in W_k^*\} = \begin{cases} 1, & \text{if } \hat{\beta} \in W_k^*, \\ 0, & \text{otherwise.} \end{cases}$$

Out-of-range distance quantifies the distance between $\hat{\beta}$ and W_k^* if the predicted budget β^* falls outside the ideal budget range W_k^* . Let $dist(\hat{\beta}, W_k^*)$ represent the distance, defined as:

$$\operatorname{dist}(\hat{\beta}, W_k^*) = \begin{cases} 0, & \text{if } \hat{\beta} \in W_k^*, \\ \min_{\hat{\beta} \in W_k^*} |\hat{\beta} - \beta|, & \text{if } \hat{\beta} \notin W_k^*. \end{cases}$$

Intuitively, a higher in-range accuracy and a lower out-range distance indicate a better estimated budget. During our evaluation, the in-range accuracy is 60.61%, and the out-of-range distance is 109.64. It indicates that more than two-thirds of estimated budgets are located in the ideal range. For those out-of-range samples, they have an offset of 109.64 tokens on average.

6.4 RQ3. Generalization of TALE.

Table 3 demonstrates the generalization of TALE across Yi-lightning (Wake et al., 2024), GPT-4o-mini (OpenAI, 2024a), and GPT-4o (OpenAI, 2024b) on MathBench-College, showing its ability to reduce output tokens and expenses while maintaining competitive accuracy significantly. TALE

achieves substantial token savings, reducing output tokens by 62.6% for Yi-lightning, 61.5% for GPT-4o-mini, and 69.8% for GPT-4o, compared to Vanilla CoT. Expense reductions are equally notable, with costs decreasing from 21.55 to 17.25 for Yi-lightning, 81.56 to 45.60 for GPT-40-mini, and 1359.42 to 759.95 for GPT-40. Despite these cost savings, TALE maintains strong accuracy, achieving 76.67% on Yi-lightning, 70.00% on GPT-40-mini, and 80.00% on GPT-40, comparable to Vanilla CoT. These results highlight TALE's effectiveness in balancing cost efficiency and reasoning performance across diverse LLM architectures. The observed accuracy drop is most significant for GPT-4o-mini. This could be attributed to its smaller number of parameters, which makes it more challenging to answer correctly within a limited response reasoning length.

7 Conclusion

In this paper, we introduce TALE, a framework that reduces token redundancy in Chain-of-Thought (CoT) reasoning by incorporating token budget awareness. TALE dynamically estimates token budgets based on task complexity to guide reasoning, balancing efficiency and accuracy. Experiments show that TALE reduces token usage by 68.9% on average with less than a 5% accuracy loss, outperforming Vanilla CoT in cost-effectiveness while generalizing well across various LLMs.

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A Appendix

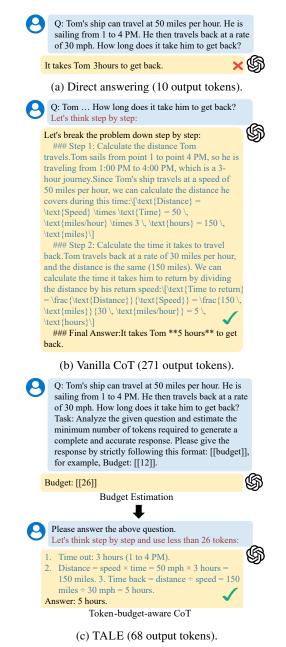


Figure 7: An intuitive example to illustrate the workflow of TALE.

A.1 Definition of Ideal Budget Range

Ideal Budget Range. Based on the observation of token elasticity, a token cost bottom range exists during searching for the optimal budget. In this range, the token costs approach the token cost lowest bound. Before or after the range, the token cost will increase. We define such a bottom range as "ideal budget range". It's worth noting that the budget continuously degrades during the search. Only the token cost rebounds. That's why we refer to this observation as token elasticity. To summarize, ideal budget range is an range that minimizes actual

token consumption. Let $\beta = \{\beta_1, \beta_2, ..., \beta_N\}$ denote all possible budgets that can maintain answer correctness. A rolling window $W \in \beta$ is applied iteratively over β . Let k represent the range size, which is adaptively determined during our evaluation as $\frac{N}{3}$, where N is the total number of possible budgets. A budget range is defined as:

$$W_k(i) = \{ \beta_j \mid i \le j \le i + k - 1 \},$$

 $1 \le i \le |\beta| - k + 1$

The ideal budget range W^* is defined as:

$$W_k^* = \arg\min_i \left(\sum_{\beta_j \in W_k(i)} \mathbb{T}(\beta_j) \right), \quad (3)$$

where \mathbb{T} denote the actual token consumption for a given budget $\beta \in \beta$. We aim to estimate a budget located in the ideal budget ranges without any search process. In that case, TALE obtains the ideal budget within acceptable sacrifice.