# TreeHop: Generate and Filter Next Query Embeddings Efficiently for Multi-hop Question Answering

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#### Abstract

Retrieval-augmented generation (RAG) systems face significant challenges in multihop question answering (MHQA), where complex queries require synthesizing information across multiple document chunks. Existing approaches typically rely on iterative LLM-based query rewriting and routing, resulting in high computational costs due to repeated LLM invocations and multi-stage processes. To address these limitations, we propose TreeHop, an embedding-level framework without the need for LLMs in query refinement. TreeHop dynamically updates query embeddings by fusing semantic information from prior queries and retrieved documents, enabling iterative retrieval through embedding-space operations alone. This method replaces the traditional "Retrieve-Rewrite-Vectorize-Retrieve" cycle with a streamlined "Retrieve-Embed-Retrieve" loop, significantly reducing computational overhead. Moreover, a rule-based stop criterion is introduced to further prune redundant retrievals, balancing efficiency and recall rate. Experimental results show that TreeHop rivals advanced RAG methods across three open-domain MHQA datasets, achieving comparable performance with only 5%-0.4% of the model parameter size and reducing the query latency by approximately 99% compared to concurrent approaches. This makes TreeHop a faster and more cost-effective solution for deployment in a range of knowledge-intensive applications. For reproducibility purposes, codes and data are available here.

# 1 Introduction

Recent breakthroughs in Large Language Models (LLMs) (DeepSeek-AI et al., 2025; OpenAI et al., 2024) have demonstrated their impressive capabilities in understanding queries (Brown et al., 2020; Ouyang et al., 2022) and generating human-like language texts. Nonetheless, LLMs still face significant limitations, particularly in domain-specific (Li et al., 2024; Zhang et al., 2024) or knowledge-intensive (Kandpal et al., 2023) tasks, where they often hallucinate (Zhang et al., 2023) when dealing with queries that exceed their parametric knowledge (Muhlgay et al., 2024). To address this issue, Retrieval-augmented generation (RAG) (Lewis et al., 2021) has undergone rapid development (Gao et al., 2024), leveraging external knowledge bases to retrieve relevant document chunks and integrate them into LLMs, thereby producing more faithful (Khandelwal et al., 2020) and generalizable (Kamalloo et al., 2023) answers.

However, the conventional single-retrieval paradigm of RAG falters in multi-hop question answering (MHQA) scenarios (Yang et al., 2018; Ho et al., 2020; Tang & Yang, 2024; Trivedi et al., 2022), where answers require synthesizing information from multiple document chunks. For instance, consider the query "Who is the grandfather of Donald Trump?" A single retrieval might return a chunk stating "Donald John Trump was born on June 14, 1946..., the fourth child of Fred Trump and Mary Ann Macleod Trump.", but resolving the grandfather requires a follow-up query like "Who is the father of Fred Trump?". This typical multi-hop scenario reveals the need to dynamically compose new query based on information in relevant document chunk. Current methods like query-rewriters (Ma et al., 2023), routers (Zhuang et al., 2024), and iterative loops (Shao et al., 2023) attempt to resolve this by

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https://github.com/allen-li1231/TreeHop-RAG

iteratively refining queries with retrieved information, and drop chunks irrelevant to answering to the query. While these approaches improve retrieval, they introduce computational overhead due to repeated LLM invocations and multi-stage processes, leading to latency and complexity trade-offs.

To addresses these limitations, we propose TreeHop, a framework enabling iterative retrieval through embedding-level updates without requiring LLM rewrites. Inspired by the semantic and structural properties of sentence embeddings (Zhu et al., 2018), TreeHop dynamically generates next-step query embeddings by fusing prior queries and retrieved content embeddings (Step 3, Figure 1). For the aforementioned example, the initial information in query "grandfather of Donald Trump" was substituted with "father of Fred Trump", now encoded directly at the embedding level. This approach collapses the traditional "Retrieve-Rewrite-Vectorize-Retrieve" cycle into a streamlined "Retrieve-Embed-Retrieve" loop, significantly reducing computational costs. TreeHop further introduces two pruning strategies to ensure computational efficiency: *redundancy pruning* terminates paths where the retrieved chunks have been seen in previous iterations, while *layer-wise top-K pruning* retains only the top-ranked retrieval candidates at each step, curbing exponential branch growth (Step 4, Figure 1).

TreeHop employs a gated cross-attention mechanism (Vaswani et al., 2023) to effectively focus on extracting salient information from retrieved chunks, making the model effective while parameterizing with only 25 million parameters. Trained with contrastive learning (Chen et al., 2020; Wu et al., 2022b), TreeHop is capable of achieving a performance comparable to computationally intensive multi-hop methods across three benchmarks while maintaining significantly lower latency. Remarkably, TreeHop reduces retrieval latency by 99% compared to LLM-based methods while sacrificing only 4.8% of the recall rate at maximum, even outperforming some advanced systems by 4.1% in deeper retrieval iterations.

In summary, this work makes three key contributions:

- A novel embedding-updating mechanism that replaces LLM-driven iterative query rewrites with lightweight neural operations, enabling linear computational complexity for MHQA tasks.
- An efficient rule-based stopping criterion that controls branching factor growth while maintaining performance in retrieval iteration.
- Empirical validation demonstrating superior efficiency-accuracy trade-offs in three MHQA
  tasks. Our approach bridges the gap between computational efficiency and retrieval effectiveness, offering a scalable solution for diverse knowledge-intensive applications.

## 2 Preliminaries

# 2.1 Multi-hop Retrieval-Augmented Generation

The Retrieval Augmented Generation (RAG) (Lewis et al., 2021; Gao et al., 2024) fundamentally enhances the capabilities of LLMs by retrieving pertinent documents from an external knowledge base, which is made possible through the calculation of semantic similarity between user's query and document chunks. Building upon RAG, multi-hop variants have been proposed to tackle more complex tasks, such as multi-hop question answering (MHQA). Notable approaches include iterative retrieval methods (Shao et al., 2023), where the knowledge base is repeatedly searched based on the initial query and generated text, providing a more comprehensive information retrieval. Other approaches revolve around employing cooperative language models as query-rewriters (Ma et al., 2023), routers (Manias et al., 2024) or both (Zhuang et al., 2024). These models generate new queries for document chunk retrieval and filter out irrelevant chunks, ensuring the most relevant information is retained. It is worth noting that they all mentioned solutions utilize one or multiple transformer model variants (Vaswani et al., 2023; Reimers & Gurevych, 2019) for enhanced retrieval, which induces additional computational cost and significantly increases system latency.

# 2.2 Sentence Representation Learning and Contrastive Learning

Sentence representation learning, a technique for training retrieval model in the realm of RAG, refers to the task of encoding sentences into fixed-dimensional embeddings. Early approaches

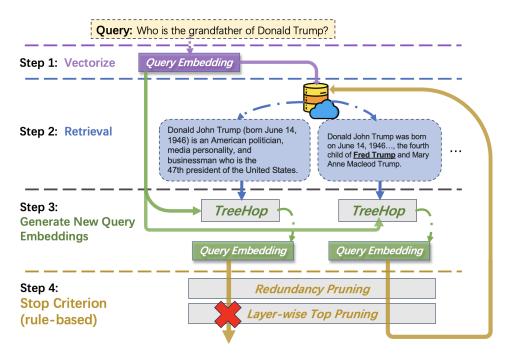


Figure 1: The TreeHop model utilizes query and content chunk embeddings to generate new query embeddings, which are subsequently filtered with similarity and ranking thresholds, thereby streamlines the conventional "Retrieve-Rewrite-Vectorize-Retrieve" into a "Retrieve-Embed-Retrieve" loop.

extended word-level techniques like word2vec (Mikolov et al., 2013) to sentences, such as Skip-Though (Kiros et al., 2015) and FastSent citephill-etal-2016-learning, which learned unsupervised sentence embeddings by optimizing sequential or semantic coherence objectives. Subsequent work leveraged pre-trained language models like BERT (Reimers & Gurevych, 2019), extracting sentence embeddings via the [CLS] token or mean pooling of contextualized token representations (Reimers & Gurevych, 2019; Li et al., 2020; Su et al., 2021).

To further improve the performance, contrastive learning emerged as a powerful paradigm for learning discriminative sentence representations (Zhang et al., 2020; Carlsson et al., 2021; Giorgi et al., 2021; Yan et al., 2021; Kim et al., 2021). A cornerstone in this space is SimCSE (Gao et al., 2021), which employs InfoNCE (van den Oord et al., 2019) to maximize agreement between augmented views of the same sentence. The loss function is defined as:

$$\mathcal{L}_{\text{infoNCE}} = -\sum_{i=1}^{N} \log \frac{e^{sim(f(x_i), f(x_i)')/\tau}}{\sum_{j=1}^{N} e^{sim(f(x_i), f(x_j)')/\tau}},$$
(1)

where N is the batch size,  $\tau$  is a temperature hyperparameter and  $sim(f(x_i), f(x_j)') = \frac{f(x_i) \top f(x_j)'}{\|f(x_i)\| \cdot \|f(x_j)'\|}$  is the cosine similarity used in this work.  $f(\cdot)$  is the sentence representation encoder,  $x_i$  and  $x_i'$  are a paired semantically related sentences derived from positive set  $\mathcal{D} = \{(x_i, x_i')\}_{i=1}^m$ .

Additionally, SimCSE applied a dropout as a data augmentation strategy, inspired many following works. Meanwhile, DiffCSE (Chuang et al., 2022) introduces equivariant transformations to ensure invariance to input perturbations, while PCL (Wu et al., 2022a) leverages diverse augmentation strategies to reduce bias in negative sampling. InfoCSE (Wu et al., 2022c) learns sentence representations with the ability to reconstruct the original sentence fragments, RankCSE (Liu et al., 2023) further introduce a listwise ranking objectives for learning effective sentence representations.

# 3 The Proposed Method: TreeHop

Our proposed model, TreeHop, is designed to generate the next query embedding by integrating previous query embeddings and retrieved content embeddings. This approach streamlines the

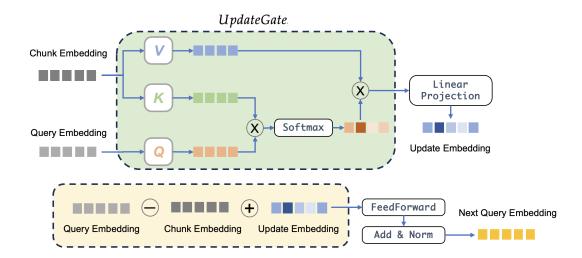


Figure 2: The model architecture of TreeHop. The *UpdateGate*, using cross-attention, updates embeddings via selectively incorporating new information from chunk embeddings. The output is combined with the difference between the previous query and chunk embeddings to form the next query embedding.

conventional iterative "Retrieve-Rewrite-Vectorize-Retrieve" process inherent in RAG systems into a more efficient "Retrieve-Embed-Retrieve" workflow, reducing both system latency and computational overhead. Furthermore, we have optimized the architecture to achieve high retrieval performance while ensuring a compact parameter size. In the following sections, we first formally define the problem, then detail the model architecture and stopping criterion that contribute to its computational efficiency and effectiveness. Finally, we explain the construction of the training data.

# 3.1 Problem Formulation

At retrieval iteration r, given query embedding q, a set of the top K document chunk embeddings,  $\mathcal{T} = \{c^i\}_{i=1}^K$ , is retrieved using the retriever g(q,K). The TreeHop model then generates the corresponding next query embedding set  $\mathcal{Q}_1 = \{q^i_{r+1}\}_{i=1}^K$  for the subsequent hop retrieval.

$$q_{r+1}^i = \text{TreeHop}(q_r, c_r^i), c_r^i \in \mathcal{T}_r$$
 (2)

Please refer to Figure 1 for detailed TreeHop inference steps with the stop criterion included in subsection 3.3.

# 3.2 Model Architecture

TreeHop's architecture is tailored to be effective in performance while maintain a small parameter size. The core of TreeHop's query update is the UpdateGate (see Figure 2), which modulates how information from prior queries q and retrieved chunks c is retained or discarded. The intuition behind is that we only need to remove information presents in both embeddings, and update information yet to be further retrieved from the retrieved chunks to form a new query embedding.

TreeHop
$$(q_r, c_r^i) = q_r - c_r^i + UpdateGate(q_r, c_r^i)$$
 (3)

The term  $q_r - c_r^i$  suppresses semantic overlap between the current query and chunk embeddings. This prevents redundant retrieval of information already captured. (e.g., if the chunk confirms "Fred Trump is Donald's father," the model avoids re-searching for Fred Trump in subsequent hops). The  $UpdateGate(q_r,c_r^i)$  selectively incorporates new information from  $c_r^i$  to form the next query. We implement cross-attention mechanism (Vaswani et al., 2023) for UpdateGate.

```
Input: Initial user query text x, embedding model f(\cdot) that generates embeddings for input text, total
number of hops N \ge 1, retriever g(\cdot) that takes one query embedding and outputs top K document chunks
\{c^i\}_{i=1}^K and respective embeddings \{v^i\}_{i=1}^K, cosine similarity scores \{s^i\}_{i=1}^K.
Output: Retrieved document chunk set C.
 1: C = \emptyset
     Q = \{f(x)\} // Query embedding set
 2: for r \in \{1, ..., N\} do
 4:
        for q in Q do
 5:
           // Retrieve and iterate over top K document chunks, embeddings and similarity scores
           for c, v, s in g(q_i, K) do
 6:
 7:
              if c \notin \mathcal{C} then
 8:
                 // Include only distinct chunks.
                  \mathcal{S} \leftarrow [q, c, v, s]
 9:
        t = \text{TopSimilarityScore}(\mathcal{S}, K) // Get layer-wise K-th similarity score t from set \mathcal{S}.
10:
        for q, c, v, s in S do
           if s \geq t then
11:
12:
               C \leftarrow c
               Q \leftarrow \mathbf{TreeHop}(q, v)
```

Figure 3: Multihop inference steps and rule-based stop criterion for TreeHop.

$$UpdateGate(q,c) = CrossAttn_{u}(q,c)$$

$$= \operatorname{softmax}(\frac{Q_{u}(q) \odot K_{u}(c)}{\sqrt{d}}) \odot V_{u}(c)$$
(4)

Where d is the number of embedding dimension,  $Q_u$ ,  $K_u$  and  $V_u$  are three weight and bias matrices for UpdateGate. Information to be maintained in the chunk embeddings is selectively extracted through comparing the information in  $q_r$  and  $c_r^i$ . This architectural design is based on empirical experiments for improving the model performance (see subsection 5.1 for ablation details).

# 3.3 Stopping Criterion

The TreeHop iteratively generates query embedding for every retrieved document chunk, this risks excessive computational costs if every query proceeds to subsequent hops. Unchecked, this approach could lead to an exponential increase in retrieved chunks  $(O(n^r))$ , degrading efficiency without proportional gains in accuracy. To address this, we introduce a set of rule-based stop criterion that dynamically prunes irrelevant or redundant retrieval branches to ensure only promising paths advance.

**Redundancy Pruning** We terminate branches where the document chunk has been retrieved in the previous iterations, as depicted in line 8, Figure 3.

**Layer-wise Top-**K **Pruning** At each retrieval layer, we retain only the top-K chunks with highest similarity scores across all generated query embeddings. This reduces the branching factor from  $O(n^r)$  to O(nr) by focusing computation on the most promising paths, as shown in line 12, Figure 3.

### 3.4 Train Data Construction

To train the TreeHop model, we require a dataset that explicitly captures the multi-step knowledge retrieval process for MHQA. The 2WikiMultiHop dataset (Ho et al., 2020) provides an ideal foundation due to its explicit decomposition of complex questions into intermediate steps, with each step linked to corresponding evidence chunks from Wikipedia. However, not all question types in this dataset are equally suitable for our purposes, the following processes are implemented to clean the dataset:

**Question Type Selection** We focus on inference, compositional and bridge comparison questions, as they strictly require the model to synthesize information across multiple hops (e.g., deriving a

grandfather's identity by first retrieving a father's name). In contrast, comparison questions rely more on direct factual retrieval without requiring iterative information interaction. See Appendix A for detailed information about question types.

**Query Type Integrity Check** We filter the dataset to retain only instances where the provided query decompositions align precisely with the multi-step reasoning required by the question type.

Through this curation process, we obtained 111,239 high-quality training samples.

# 3.5 Model Training

We utilize BGE-m3 (Chen et al., 2024), a multilingual embedding model that supports more than 100 languages, to generate dense embeddings for the initial query and construct a document chunk embedding database. This gives our trained TreeHop model the potential to be versatile and applicable to a wide range of languages and use cases. Note that BGE-m3 remains frozen during training to ensure the training process focus on the TreeHop model. For detailed prompt templates for generating embeddings on three datasets, please refer to Table 7 and Table 8 in Appendix C.

Following previous work (van den Oord et al., 2019), we adopt contrastive learning framework to train TreeHop to generate embeddings that maximizes the similarities with their corresponding positive chunk embeddings while minimizing similarity with negative ones. Specifically, we employ the  $\mathcal{L}_{infoNCE}$  objective in Equation 1 with temperature  $\tau$  of 0.15 and five negatives sampled from embedding database. The model is compact enough to be trained on a single Nvidia V100 GPU, with batch size of 64, AdamW optimizer and learning rate of 6e-5. Inspired by SimCSE (Gao et al., 2021), we add a dropout layer after the hidden representations for data augmentation.

# 4 Experiments and Results

To examine TreeHop, experiments are conducted regarding its retrieval performance and efficiency. Below, we introduce the selective evaluation datasets, evaluate metrics, baselines and concurrent advanced RAG solutions that involve in the experiments for comparison.

#### 4.1 Datasets

We benchmark TreeHop on three widely used MHQA datasets in the literature: 2WikiMultiHop (Ho et al., 2020), MuSiQue answerable (Trivedi et al., 2022), and MultiHop RAG (Tang & Yang, 2024). Some of their questions do not challenge multihop retrieval performance but require LLMs to deduce from multiple documents. Therefore, to repel extraneous noises, we focus on question types requiring multi-step retrieval. To be more specific, in 2WikiMultiHop, we filter to inference, compositional and bridge comparison questions (9,536 records), while for MuSiQue, all 2,417 answerable questions are used. MultiHop RAG's inference questions (816 records) are included. This ensures that the evaluation targets scenarios where multi-hop retrieval is essential. See Appendix A for detailed introduction to the types of question among the evaluate datasets, and Table 6 for detailed number of queries for each selective types of question.

Dataset	Query	Embedding Database
2WikiMultihop	9,536	56,709
MuSiQue	2,417	21,100
Multihop RAG	816	609

Table 1: Descriptive statistics of datasets in terms of the number of queries and sizes of the corresponding embedding database.

### 4.2 Evaluation Metrics & Benchmarks

We use the standard evaluation metric, the recall rate, to test the retrieval performance, specifically in the top K retrieval setting, denoted Recall@K. It measures whether the relevant documents are present

among the top *K* retrieved documents. Higher Recall@K values indicate better retrieval performance. To compare the efficiency among selected RAG solutions, we record the average durations for each query in seconds on each dataset, denoted latency. The whole evaluation process is conducted on a single Nvidia A100 GPU and 64 GB of RAM.

Baselines and Advanced RAG We evaluate the performance of TreeHop by comparing it to a native top retrieval method using the BGE-m3 embedding model as the baseline. In addition, we include two advanced iterative retrieval-augmented generation (RAG) methods: Iter-RetGen (Shao et al., 2023) and EfficientRAG (Zhuang et al., 2024) to assess both performance and latency. For Iter-RetGen, we use the vanilla Meta Llama3 8B Instruct model (AI@Meta, 2024) as the inference model. Additionally, we test Iter-RetGen and TreeHop under the second and third retrieval iterations, respectively, to evaluate their performance across different stages of the retrieval process. For more detailed information on the prompt templates used in Iter-RetGen, please refer to Table 9 in Appendix C.

#### 4.3 Results

In this section, we present the experimental results of TreeHop and benchmarks on three datasets, including retrieval efficiency and recall.

**Retrieval Efficiency** As shown in Table 2, TreeHop significantly reduces computational overhead while maintaining competitive retrieval performance. It achieves latencies of 0.02 seconds per query in the second iteration and 0.06 seconds in the third, outperforming the next best solution, EfficientRAG, by over 2.9 seconds. This significant reduction of 99.2%–99.6% in latency is attributed to TreeHop's mechanism, stems from its embedding-level computation, which avoids the recursive inference loops required by LLM-based methods. This is confirmed by examining the latency, which is proportional to the number of retrieved document chunks.

	2WIKI		MUSIQUE			MULTIHOP-RAG			
Retriever	Recall@K	K	Latency	Recall@K	K	Latency	Recall@K	K	Latency
Baselines									
Direct-R@5	49.3	5	0.002	45.4	5	0.002	48.6	5	0.019
Direct-R@10	53.2	10	0.003	53.8	10	0.002	67.8	10	0.019
Advanced RAG									
Iter-RetGen@5 iter2	59.2	9.9	4.690	52.8	9.9	4.949	55.0	9.9	4.876
Iter-RetGen@5 iter3	61.9	14.7	7.278	54.1	14.8	7.274	57.0	14.5	7.322
EfficientRAG@5	60.5	3.8	2.846	46.9	6.1	2.907	51.8	4.1	2.855
Ours									
TreeHop@5 iter2	61.6	8.6	0.022	48.0	8.1	0.023	57.9	7.0	0.023
TreeHop@5 iter3	65.4	11.8	0.067	48.1	11.0	0.064	61.1	8.4	0.062
TreeHop@10 iter2	57.9	17.2	0.062	55.7	15.3	0.056	72.8	13.1	0.049

Table 2: We report results of baselines, concurrent advanced RAG solutions and TreeHop on three MHQA datasets. **Bold** numbers indicate the best performance in the same iteration among retrievers.

Retrieval Performance TreeHop achieves strong performance across datasets while balancing efficiency and effectiveness. On the 2WikiMultiHop and Multihop dataset, TreeHop surpasses the second best solution, Iter-RetGen, by 2.4%-2.9% recall in the second iteration and 3.5%-4.1% recall in the third iteration, with 3.1 less chunks retrieved on average. This demonstrates the effectiveness of the embedding mechanism in Equation 4. For the MuSiQue dataset, recall is 4.8% lower than Iter-RetGen, likely due to the dataset's requirement for synthesizing information from multiple chunks (e.g., branching and converging paths in Appendix A), which TreeHop's current architecture addresses less effectively than iterative LLM-based approaches. The performance degradation aligns with EfficientRAG solution, which also struggles with this dataset, suggesting a limitation common to query-chunk-pair strategies.

**Average Number of** *K* Overall, our TreeHop's average number of retrieved document chunks falls in the middle of the advanced RAG solutions. This is contributed by stop criteria, which drastically curtails computational overhead. For top-5 retrieval, it reduces the theoretical exponential growth of

chunks,  $5^2 = 25$  chunks in second iteration, to 7.1–8.8 chunks, and  $5^3 = 125$  chunks to 8.3–12.1 chunks in the third iteration. For top-10 retrieval, this scales linearly to 13.8–17.9 chunks, versus  $10^2$  chunks without pruning.

# 5 Ablation Study

#### 5.1 Effectiveness of Architecture

To evaluate the necessity of each component in Equation (3), we ablated the query term q, chunk term c, and UpdateGate in isolation. Each variant was trained 10 times with identical hyperparameters (as in subsection 3.5), and performance was evaluated on the second hop's recall rate. Results are summarized in Table 3 and analyzed below.

Architecture	2WIKI (avg.)	MUSIQUE (avg.)	MULTIHOP RAG (avg.)
Direct-R@5	49.3	45.4	48.6
TreeHop@5 iter2	61.6	48.0	57.9
w/o. c	57.5 (4.1↓)	47.1 (0.9↓)	51.9 (6.0↓)
w/o. q	54.6 ( <i>6.0</i> ↓)	$46.3 (2.5 \downarrow)$	50.8 (7.1↓)
w/o. UpdateGate	49.3 (12.3↓)	45.4 ( <i>3.4</i> ↓)	48.6 (9.3↓)

Table 3: Ablation study on TreeHop model architecture. The TreeHop experiences degraded performances when core components *q*, *c* and *UpdateGate* are removed from its architecture, demonstrating their functionalities to the model performance.

**Impact of Component Removal** The impact of structure without c is the minimal, with a decrease of 0.9%-6.0% of average recall rates across datasets. The UpdateGate mitigated information loss by selectively retaining critical chunk information, without c, it keeps the information to the extent that still makes the model effective. However, average training convergence time increased by approximately 15% on average, as the model struggled to suppress redundant information without the q-c structure.

Without q, the model loses critical information from query, thereby experiences significant performance degrade on the three datasets, achieves only 0.09%-5.3% improvement of average recall rates comparing to vanilla top 5 retrieval. It is observable that the model exhibits a lower degrades on 2WikiMultihop dataset, we conclude from the result that this is due to our usage of 2WikiMultihop training data that make the model overfit to similar questions in evaluate dataset, ultimately leaving no generalization ability to the other two datasets.

Without *UpdateGate*, recall dropped to near baseline retrieval performance (within 0.1% of random retrieval), confirming the gate's critical role in integrating new information. Without it, the model degenerated to a simple vector difference, failing to refine queries iteratively.

**Dataset-Specific Insights** The three datasets exhibit different extents of performance decay when the same components are removed. The MuSiQue dataset decays the least without c, q, and UpdateGate, this is due to the inherent deficiency of TreeHop on multihop queries with converging paths, making it perform inferior to the other datasets. The Multihop RAG dataset experiences the greatest negative impact without q, possibly due to its complex queries that mention more than three entities for the retrieval model to gather each piece of information. Without q, TreeHop cannot navigate the missing information. The 2WikiMultihop dataset influences less than Multihop RAG without c and q, possibly because of its less challenging queries and query decomposition paths.

#### **5.2** Effectiveness of Stop Criterion

The stop criterion serves for filtering query paths to reduce computational cost without sacrificing too much performance. Below we examine the performance without the presence of Redundancy Pruning and Layer-wise Top Pruning, illustrated in Table 4.

	2WIKI		MUSIQ	UE	MULTIHOP RAG		
<b>Stop Criterion</b>	Recall@K	K	Recall@K	K	Recall@K	K	
TreeHop@5 iter2	61.6	8.6	48.0	8.1	57.9	7.0	
w/o. Redundancy Pruning w/o. Layer-wise Top Pruning w/o. Both	57.4 80.2 80.2	10 18.4 30	46.4 53.6 53.6	10 14.5 30	52.8 61.3 61.3	10 9.7 30	

Table 4: Ablation study on stop criterion, where redundancy pruning and layer-wise top 5 pruning are removed from post retrieval process, respectively. The results indicate that the recall rate does not exhibit a considerable enhancement, despite a substantial grow in the number of average *K*.

**Redundancy Pruning** Redundancy pruning prevents revisiting previously retrieved chunks. Table 4 shows that removing this pruning increases the average number of retrieved chunks K to 10, but reduces recall by 4.2 points (e.g.,  $61.6 \rightarrow 57.4$  on 2WikiMultihop). This occurs because when cooperate with layer-wise top pruning, redundancy pruning further ensures only unique, informative paths are pursued, thereby maintaining recall performance. Without redundancy pruning, more duplicated information take the place, resulting in degraded performance.

**Layer-wise Top Pruning** This pruning strategy selects top-5 chunks at each layer to control branching. Removing this strategy results in a maximum increase of 113% in the average number of retrieved chunks  $(8.6 \rightarrow 18.4)$  on the 2WikiMultihop dataset, but yields a 18.6% improvement in recall  $(61.6\% \rightarrow 80.2\%)$ . This suggests that the pruning introduces a trade-off between computational efficiency and recall performance. In contrast, for the Multihop RAG dataset, the recall improves by 5.6% with a 79% increase in average retrieved chunks, a significantly lower increase compared to 2WikiMultihop. This disparity among datasets is strongly correlated with the size of the corresponding embedding database. Specifically, in large databases such as 2WikiMultihop, which features 56,709 chunk embeddings, the model benefits from exploring more paths (higher K) due to the large information pool. Conversely, in smaller databases, e.g., Multihop RAG with 609 chunk embeddings, iterative retrieval tends to introduce more redundant chunks, making layer-wise top pruning filter out more useful chunks.

Combined Effects The synergistic effect of combining redundancy pruning and layer-wise top pruning is critical to achieving TreeHop's efficiency gains without excessive recall loss. Take Multihop RAG for example, When both criteria are applied, they achieve a recall of 57.9% with an average number of retrieved chunks 7.0. When both criteria are disabled, the system's computational complexity balloons to 30 chunks in the second iteration, a 329% increase, while yielding only a marginal 3.4% recall improvement. This demonstrates that layer-wise top pruning is essential for limiting branching factor, while redundancy pruning prevents recall degradation from redundant paths. Their combined use ensures that TreeHop avoids the exponential retrieval path explosion inherent to iterative systems while maintaining competitive performance. This synergy underscores their necessity for achieving the model's design goal of efficient multi-hop retrieval.

# 6 Conclusion

This work presents a novel paradigm for Retrieval-Augmented Generation (RAG), introducing TreeHop, a novel lightweight query embedding generator that dynamically refines query embeddings through iterative retrieval without relying on additional LLMs or complex rewrite components, thereby enhancing the efficiency of RAG system. Its core mechanism, the *UpdateGate*, employs cross-attention to selectively integrate information from retrieved chunks while discarding redundant information, enables a compact model size of 25 million parameters while maintaining competitive performance on three MHQA benchmarks when integrated with simple downstream rule-based stop criterion. Future work could explore more effective model architecture, adaptive stop criteria or extensions to handle lengthy, structural, or multi-modal inputs. Our approach underscores the potential of embedding-centric strategies to enhance retrieval process for RAG systems, offering a practical balance between performance and computational efficiency, paving the way for solutions to real-world multi-hop reasoning challenges in industrial applications.

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# Part I

# Appendix

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# **A** Dataset Cards

Below illustrates datasets inclusive in our work, the question types for evaluation are selected to ensure synthesizing information from query and retrieved document chunks are mandatory for multihop retriever.

Dataset	Question Type	Require Synthesize
	<b>Comparison question</b> : Questions requiring direct comparison of a tributes between entities within the same category. <i>Example Question</i> : Who was born first, Albert Einstein or Abrahan Lincoln?	
	<b>Inference question</b> : Questions requiring derivation of implicit relation ships by combining triples from a knowledge graph.	1-
2WikiMultiHop	Example Question: Who is the maternal grandfather of Abraham Lircoln?  Triples: (Abraham Lincoln, mother, Nancy Hanks Lincoln); (Nancy Hanks Lincoln, father, James Hanks).	
F	Compositional question: Questions requiring multi-step relational reasoning across non-explicitly linked triples.	<b>1</b> -
	Example Question: Who founded the distributor of La La Land? Triples: (La La Land, distributor, Summit Entertainment); (Summit Entertainment, founded by, Bernd Eichinger).	t
	<b>Bridge-comparison question</b> : Questions requiring both bridging to intermediate entities and comparative reasoning.	)
	Example Question: Which movie has the director born first, La La Land or Tenet?  Steps: 1. Find directors: La La Land → Damien Chazelle; Tenet — Christopher Nolan.  2. Compare birth years: Damien Chazelle (1985) vs. Christopher Nolan (1970).	<b>&gt;</b>
	<b>Unanswerable</b> : Questions with potential support paragraphs are partially removed, making the reasoning infeasible or unable to arrive at the correct answer.	
	<b>2-Hop Reasoning (Linear Path)</b> : A single, straightforward logical path connecting two facts.	h
	Example Question: Who succeeded the first President of Namibia? steps: 1. Identify the first President of Namibia. 2. Determine who succeeded them.	$\checkmark$
MuSiQue	<b>3-Hop Reasoning (Linear Path)</b> A sequential, three-step logical connection.	1-
	<ul><li>Example Question: What currency is used where Billy Giles died?</li><li>steps: 1. Find the location of Billy Giles' death.</li><li>2. Locate the region this place belongs to.</li><li>3. Identify the currency used in that region.</li></ul>	<b>√</b>
	<b>3-Hop Reasoning (Branching Path)</b> Begins with a single inquiry budiverges into different, branching sub-questions.	t
	Example Question: When was the first establishment that McDonaldization is named after, opened in the country Horndean is located? steps: 1. Determine what McDonaldization refers to. 2. Identify the country where Horndean is located. 3. Find the date the first establishment opened in that country.	n- <b>√</b>

Dataset	Question Type	Require Synthesize
	<b>4-Hop Reasoning (Linear Path)</b> A continuous, four-step logical progression.	-
	Example Question: When did Napoleon occupy the city where the mother of the woman who brought Louis XVI style to the court died? steps: 1. Identify who introduced Louis XVI style.  2. Find their mother.  3. Determine the city of the mother's death.	· •
	Discover when Napoleon occupied that city.	
	<b>3-Hop Reasoning (Branching Path)</b> Begins with a single inquiry but diverges into different, branching sub-questions.	i
	Example Question: When was the first establishment that McDonaldization is named after, opened in the country Horndean is located? steps: 1. Determine what McDonaldization refers to. 2. Identify the country where Horndean is located. 3. Find the date the first establishment opened in that country.	- <b>√</b>
	4-Hop Reasoning (Linear Path) A continuous, four-step logical pro	-
	gression.	
MuSiQue	Example Question: When did Napoleon occupy the city where the mother of the woman who brought Louis XVI style to the court died? steps: 1. Identify who introduced Louis XVI style. 2. Find their mother.	<b>√</b>
	<ul><li>3. Determine the city of the mother's death.</li><li>4. Discover when Napoleon occupied that city.</li></ul>	
	<b>4-Hop Reasoning (Branching Path)</b> Starts with a single query, splits into multiple paths, and then converges.	
	Example Question: How many Germans live in the colonial holding in Aruba's continent that was governed by Prazeres's country? steps: 1. Locate Aruba's continent.	<b>√</b>
	<ul><li>2. Identify Prazeres' country.</li><li>3. Determine the colonial holding governed by that country in Aruba's continent.</li></ul>	:
	4. Find the number of Germans there.	
	<b>4-Hop Reasoning (Converging Path)</b> : Multiple distinct lines of reasoning that eventually converge on the answer.	-
	Example Question: When did the people who captured Malakoff come to the region where Philipsburg is located? steps: 1. Determine Philipsburg's location.	<b>√</b>
	2. Identify the terrain feature it belongs to.	
	<ul><li>3. Find who captured Malakoff.</li><li>4. Determine when those people came to that terrain.</li></ul>	
	Inference Query: Questions requiring derivation of implicit relation	
	ships by combining triples from a knowledge graph.	,
MultiHop RA	Example Question: Who is the maternal grandfather of Abraham Lin $G$ coln?	- <b>√</b>
	Triples: (Abraham Lincoln, mother, Nancy Hanks Lincoln); (Nancy Hanks Lincoln, father, James Hanks).	
	<b>Comparison query</b> : Questions requiring direct comparison of attributes between entities within the same category. <i>Example Question</i> : Did Netflix or Google report higher revenue for the year 2023?	

Dataset	Question Type	Require Synthesize
	<b>Temporal query</b> : Question that requires an analysis of the temporal information of the retrieved chunks <i>Example Question</i> : Did Apple introduce the AirTag tracking device before or after the launch of the 5th generation iPad Pro?	
MultiHop RAC	Null query: Question whose answer cannot be derived from the retrieved set. This is purposely for testing the issue of hallucination. The LLM should produce a null response instead of hallucinating an answer. Example Question: What are the sales of company ABCD as reported in its 2022 and 2023 annual reports?	I

Table 5: Dataset Cards.

# **B** Details on Evaluate Dataset Question Types

Below, we provide detailed number of questions for each question type in our evaluate datasets. Please refer to Appendix A for introduction to the types.

Dataset	<b>Question Type</b>	Count
	Compositional	5,236
2WikiMultiHop	Bridge Comparison	2,751
_	Inference	1,549
	2-hop reasoning (linear path)	1,252
	3-Hop Reasoning (Linear Path)	568
MuSiOue	3-Hop Reasoning (Branching Path)	192
MusiQue	4-Hop Reasoning (Linear Path)	246
	4-Hop Reasoning (Branching Path)	64
	4-Hop Reasoning (Converging Path)	95
Multihop RAG	Inference	816

Table 6: Evaluate data statistics on number of queries and sizes of embedding database.

# C Iter-RetGen Prompt Templates

Below we illustrate prompt templates for generating embedding and Iter-RetGen. Templates for 2WikiMultihop and MuSiQue are identical, while for MultiHop-RAG we add source of content as many of its question decomposition revolve around this. Following previous work (Zhuang et al., 2024), we adopt the same prompt template on three evaluate datasets for Iter-RetGen.

# Document Chunk Prompt Template for 2WikiMultihop and MuSiQue

Title: [doc title]
Context: [doc text]

Table 7: Prompt template for generating embedding using BGE-m3 embedding model on 2Wiki and MuSiQue train and evaluate datasets.

# **Document Chunk Prompt Template for MultiHop-RAG**

Title: [doc title]
Source: [doc source]
Context: [doc text]

Table 8: Prompt template for generating embedding using BGE-m3 embedding model on MultiHop-RAG evaluate dataset.

## Iter-RetGen Prompt Template for 2WikiMultihop, MuSiQue and MultiHop-RAG

```
You should think step by step and answer the question after <Question> based on given knowledge embraced with <doc>
and </doc>. Your answer should be after <Answer> in JSON format with key "thought" and "answer", their value should
be string.
Here are some examples for you to refer to:
<doc>
{{KNOWLEDGE FOR THE QUESTION}}
<Question>: In which year did the publisher of In Cold Blood form?
Let's think step by step.
<Answer>:
    ison
{{ "thought": "In Cold Blood was first published in book form by Random House. Random House was form in 2001.",
"answer": "2011" }}
<doc>
{{KNOWLEDGE FOR THE QUESTION}}
<Question>: Who was in charge of the city where The Killing of a Sacred Deer was filmed?
Let's think step by step.
<Answer>:
    json
{{ "thought": "The Killing of a Sacred Deer was filmed in Cincinnati. The present Mayor of Cincinnati is John Cranley.
Therefore, John Cranley is in charge of the city.", "answer": "John Cranley" }}
<doc>
{{KNOWLEDGE FOR THE QUESTION}}
</doc>
<Question>: Where on the Avalon Peninsula is the city that Signal Hill overlooks?
Let's think step by step.
<Answer>:
``` json
{{ "thought": "Signal Hill is a hill which overlooks the city of St. John's. St. John's is located on the eastern tip of the
Avalon Peninsula.", "answer": "eastern tip" }}
Now based on the given doc, answer the question after <Question>.
<doc>
{documents}
</doc>
<Question>: {question}
Let's think step by step.
<Answer>:
```

Table 9: Prompt template for Iter-RetGen on 2Wiki, MuSiQue and MultiHop-RAG evaluate datasets.