THOUGHT-LIKE-PRO: Enhancing Reasoning of Large Language Models through Self-Driven Prolog-based Chain-of-Thought

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Abstract

Large language models (LLMs) have shown exceptional performance as general-purpose assistants, excelling across a variety of reasoning tasks. This achievement represents a significant step toward achieving artificial general intelligence (AGI). Despite these advancements, the effectiveness of LLMs often hinges on the specific prompting strategies employed, and there remains a lack of a robust framework to facilitate learning and generalization across diverse reasoning tasks. To address these challenges, we introduce a novel learning framework, THOUGHT-LIKE-PRO. In this framework, we utilize imitation learning to imitate the Chain-of-**Thought** (CoT) process which is verified and translated from reasoning trajectories generated by a symbolic **Pro**log logic engine. This framework proceeds in a selfdriven manner, that enables LLMs to formulate rules and statements from given instructions and leverage the symbolic Prolog engine to derive results. Subsequently, LLMs convert Prolog-derived successive reasoning trajectories into natural language CoT for imitation learning. Our empirical findings indicate that our proposed approach substantially enhances the reasoning abilities of LLMs and demonstrates robust generalization across outof-distribution reasoning tasks.

1 Introduction

Recent advancements in large language models (LLMs) have positioned them as versatile tools for a variety of tasks, achieved through a structured training process that includes pre-training on large text corpora, supervised fine-tuning, and reinforcement learning from human feedback (RLHF) (Casper et al., 2023). These phases equip LLMs with extensive knowledge, the ability to generate context-specific responses, and alignment with human values. One of the key factors contributing to the perception of LLMs as a precursor to artificial general intelligence (AGI) is their scalable

and explainable reasoning capabilities (Zhao et al., 2023), which have demonstrated potential to solve multiple types of real-world complex logic reasoning problems, including natural language logical reasoning and arithmetic reasoning.

General logic reasoning tasks continue to present substantial challenges for LLMs. These challenges primarily emerge during the LLMs' reasoning processes (Ribeiro et al., 2023; Zhou et al., 2024). At the semantic level, the generation of accurate multihop inferential pathways and the assurance of validity at each reasoning step remain significant obstacles (Creswell et al., 2023; Kazemi et al., 2023). LLMs function in a self-regressive manner at the token level, processing information sequentially from left to right, one token at a time. This method can amplify errors: an inaccurate token prediction may negatively impact all subsequent tokens, leading to escalating inaccuracies and false reasoning outcomes. Interestingly, some studies (Ye and Durrett, 2022; Saparov and He, 2023; Ribeiro et al., 2023) have observed instances where LLMs deliver correct answers despite erroneous internal reasoning processes. This phenomenon highlights a misalignment between the reasoning processes and outcomes of LLMs, suggesting a weak causal relationship and underdeveloped implicit reasoning connections within the LLMs.

To address the challenges associated with generating reasoning trajectories in LLMs, several strategies have been developed to enhance cognitive alignment and logical coherence. The Chain-of-Thought (CoT) approach (Wei et al., 2022; Kojima et al., 2022; Zhang et al., 2022), for instance, enhances LLMs' reasoning abilities by crafting instructions and few-shot examples that promote sequential reasoning. Building on the CoT framework, other methodologies like Tree-of-Thoughts (Yao et al., 2024; Long, 2023; Hulbert, 2023) and Graph-of-Thoughts (Besta et al., 2024; Zhang et al., 2024) have been introduced. These meth-

ods aim to formalize the reasoning architecture of LLMs through in-context learning (Dong et al., 2022; Work) and guide model output. While these prompt-based techniques have proven effective in specific tasks, they require precise, task-specific modifications and may not be broadly applicable across different domains (Creswell et al., 2023; Kazemi et al., 2023). Alternative approaches have sought to augment the reasoning capabilities of LLMs by conceptualizing them as agents that can employ various logical reasoning tools (Lyu et al., 2023; Pan et al., 2023a). This strategy simplifies complex reasoning tasks into more manageable components such as tool utilization and question definition. One particularly effective method involves the use of symbolic logic engines, such as Prolog (Bowen, 1979), to conduct symbolic inference, thereby ensuring the correctness of the reasoning trajectory. However, this approach does not inherently enhance the LLM's reasoning capabilities and fails to provide a generalized improvement in the model's ability to follow general instructions across diverse tasks.

In this paper, we introduce THOUGHT-LIKE-PRO¹, a novel framework designed to enhance the logical reasoning capabilities of LLMs by imitating the verified reasoning processes of the Prolog logic engine. We begin by utilizing open-source chat LLMs, such as Llama3-8B-Instruct (AI@Meta, 2024), to generate rules, facts, and queries following specific instructions and demonstrations. Subsequently, we employ the Prolog engine to perform logical reasoning, verifying the outcomes against established truths. Only the verified reasoning processes are selected, which we then translate into CoT-like natural language reasoning trajectories. These trajectories serve as the basis for supervised fine-tuning (SFT), aimed at teaching LLMs to imitate a strictly logical reasoning process. To address the issue of catastrophic forgetting during domain-specific fine-tuning, we implement a model averaging technique. Empirical experiments confirm that the THOUGHT-LIKE-PRO framework significantly improves the reasoning capabilities of LLMs and extends their generalization to Out-of-Distribution (OOD) reasoning benchmarks. The contributions of our study are detailed as follows:

- We propose THOUGHT-LIKE-PRO framework designed to enhance the general reasoning capabilities of LLMs in a self-driven way. This framework is straightforward to implement and replicate.
- Our results show that performing imitation learning on strictly logical and diverse reasoning trajectories for a single instruction leads to improved and consistent reasoning performance.
- The framework is simple, straightforward, and highly effective. It demonstrates robust OOD reasoning performance across multiple logic tasks and general benchmarks, making it applicable in various industrial domains.

2 Methods

2.1 Preliminary: Supervised Fine-tuning, Chain-of-Thought, and Prolog

Consider a LLM, denoted by p_{θ} , which is parameterized by θ . Given an instruction dataset \mathcal{D}_{IF} consisting of question and answer pairs (\mathbf{x}, \mathbf{y}) , a highly effective method for aligning θ with the target output \mathbf{y} given \mathbf{x} is to minimize the negative log-likelihood of \mathbf{y} conditioned on \mathbf{x} (Gunel et al., 2020; Dong et al., 2023). This can be formulated autoregressively as:

$$\min_{\theta} - \mathbb{E}_{(\mathbf{x}, \mathbf{y}) \sim \mathcal{D}_{IF}} \left[\sum_{i=1}^{k} \log p_{\theta}(y_i | \mathbf{x}) \right], \quad (1)$$

where k is the number of tokens in y.

To enhance reasoning performance on complex logical and algorithmic tasks, one effective approach involves using a specialized prompt, prompt_{CoT}, to initiate a step-by-step reasoning trajectory prior to generating the final output: $(\mathbf{y}, \mathbf{c}) \sim p_{\theta}(\cdot | \mathbf{x}, \text{prompt}_{\text{CoT}})$. Here, \mathbf{c} represents a CoT reasoning process (Wei et al., 2022; Kojima et al., 2022; Zhang et al., 2022). Typically, this process decomposes complex, multi-step reasoning tasks into simpler, intermediate steps, thereby directing the LLM toward the correct answer. Nevertheless, several studies (Ye and Durrett, 2022; Saparov and He, 2023; Ribeiro et al., 2023) have indicated that c may sometimes exhibit disorganized patterns that do not adhere strictly to logical reasoning processes. This inconsistency can adversely affect the LLM's reasoning performance across diverse tasks.

¹The code will be fully released due to the acceptance. We offer a demo of the dataset automatically generated by the THOUGHT-LIKE-PRO framework for reference. Available at: https://anonymous.4open.science/r/Prolog_datasets-9875.

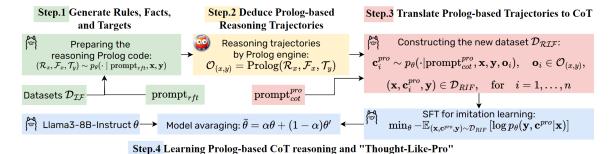


Figure 1: Workflow overview of the THOUGHT-LIKE-PRO

Prolog is a high-level symbolic language system designed for rule-based reasoning (Andreas et al., 2016; Neelakantan et al., 2017), fundamentally operating within a subset of predicate logic known as Horn Clause logic (Hudson and Manning, 2019; Bowen, 1979). It is renowned for its declarative programming paradigm, which contrasts with the imperative approach by emphasizing relations over the sequence of actions. In Prolog, the logic of computation is articulated through relations, denoted as facts \mathcal{F} and rules \mathcal{R} . Typically, Prolog utilizes a depth-first search strategy (Wielemaker et al., 2012) to trace a feasible logical inference path to the desired conclusion, yet it is also equipped to delineate all possible logical inference paths leading to the target results \mathcal{T} . The entire logical inference process within the Prolog engine, denoted as Prolog, can be represented by the following equation:

$$\mathcal{O} = \text{Prolog}(\mathcal{R}, \mathcal{F}, \mathcal{T}), \{o_1, o_2, ..., o_n\} \in \mathcal{O}, (2)$$

where \mathcal{O} encompasses n valid logical inference trajectories toward the target \mathcal{T} .

2.2 THOUGHT-LIKE-PRO

Here, we introduce THOUGHT-LIKE-PRO, a novel framework designed to enhance the general logical reasoning capabilities of LLMs across various logical tasks. As suggested by the name, our framework enables LLMs to imitate strictly logical reasoning trajectories that are generated and validated by the Prolog engine. The architecture of THOUGHT-LIKE-PRO is organized in a self-driven manner, eliminating the requirement for auxiliary services from other advanced LLMs, such as GPT-4 (Achiam et al., 2023) and Claude 2 (Wu et al., 2023). It solely relies on the utilization of efficient, open-source LLMs (i.e., Llama3-8B-Instruct (AI@Meta, 2024)), fostering accessibility and ease of replication.

To employ the Prolog engine Prolog for logical reasoning, we initiate the process by constructing a few-shot demonstration prompt $_{rft}$ (Listing 1).

This is designed to guide the model p_{θ} in generating relevant rules \mathcal{R}_x , facts \mathcal{F}_x , and deriving the reasoning target \mathcal{T}_y :

$$(\mathcal{R}_x, \mathcal{F}_x, \mathcal{T}_y) \sim p_{\theta}(\cdot \mid \text{prompt}_{rft}, \mathbf{x}, \mathbf{y}).$$
 (3)

Subsequently, the Prolog engine is employed to deduce the set of reasoning trajectories $\mathcal{O}_{(x,y)}$, which encompasses all logical paths leading to the target \mathcal{T}_y . This process is formalized as:

$$\mathcal{O}_{(x,y)} = \text{Prolog}(\mathcal{R}_x, \mathcal{F}_x, \mathcal{T}_y).$$
 (4)

During this phase, we selectively utilize only those reasoning trajectories that conclusively reach the target \mathcal{T}_y , excluding any data that result from erroneous transformations or generation failures as delineated in Equation 3. To ensure the acquisition of high-quality and interpretable reasoning trajectories via Prolog, we implement a meta-solver developed by (Yang et al., 2023), which retrieves all feasible reasoning paths.

After acquiring the set $\mathcal{O}_{(x,y)}$, we design a novel few-shot prompt, prompt $_{cot}^{pro}$ (Listing 2), to translate existing reasoning trajectories into CoT-like reasoning processes expressed in natural language. This transformation aids in constructing the new dataset \mathcal{D}_{RIF} :

$$\mathbf{c}_{i}^{pro} \sim p_{\theta}(\cdot|\operatorname{prompt}_{cot}^{pro}, \mathbf{x}, \mathbf{y}, \mathbf{o}_{i}), \mathbf{o}_{i} \in \mathcal{O}_{(x,y)},$$

$$(\mathbf{x}, \mathbf{c}_{i}^{pro}, \mathbf{y}) \in \mathcal{D}_{RIF}, \quad \text{for} \quad i = 1, ..., n.$$
(5)

It is important to note that this new dataset, \mathcal{D}_{RIF} , encompasses all n available reasoning trajectories for each instruction \mathbf{x} . Subsequently, we optimize the model using SFT as outlined in Equation 1, aligning both the CoT-like reasoning trajectories and the target outputs conditioned on \mathbf{x} autoregressively:

$$\min_{\boldsymbol{\theta}} - \mathbb{E}_{(\mathbf{x}, \mathbf{c}^{pro}, \mathbf{y}) \sim \mathcal{D}_{RIF}} \left[\log p_{\boldsymbol{\theta}}(\mathbf{y}, \mathbf{c}^{pro} | \mathbf{x}) \right], \quad (6)$$

resulting in a trained model parameterized by θ' . This entire learning process parallels imitation

Datasets	Prompt	Acc					
		Llama3-8B-Instruct	Thought-Like-Pro(ours)				CARING
			MA+single	MA+multiple	single	multiple	CARING
GSM8K	8-shot,CoT	79.6	80.7	85.32	83.4	87.81	98.1
ProofWriter	3-shot,CoT	53.7	72.13	<u>97.25</u>	73.91	98.19	100
PrOntoQA	2-shot	53.6	90.4	94.3	95.3	100	100
MMLU	5-shot	68.4	<u>68.2</u>	67.9	65.4	64.3	-
GPQA	0-shot	34.2	34.2	33.8	32.8	31.9	-
HumanEval	0-shot	62.2	62.1	61.9	60.2	60	-
MATH	4-shot,CoT	30	29.3	30.2	29.2	30.1	-

Table 1: **Experimental Results.** In the table, "Acc" denotes accuracy, and "8-shot, CoT" indicates that the assessment was carried out using eight demonstrations and CoT prompts. The optimal results are emphasized in bold, with the second-best results underlined. The results of CARING (Yang et al., 2023) are displayed in grey font and are not included in the accuracy rankings.

learning algorithms (Hussein et al., 2017; Osa et al., 2018) that select "optimal" trajectories (here verified by the Prolog engine) and implement behavior cloning to directly imitate these trajectories. After training according to Equation 6, the model $p_{\theta'}$ is tailored to perform domain-specific tasks present in \mathcal{D}_{RIF} . However, if the task distribution of \mathcal{D}_{RIF} significantly diverges from that of \mathcal{D}_{IF} , catastrophic forgetting (CF) may occur (Chen et al., 2023; Zhai et al., 2023; Shi et al., 2024), potentially compromising the model's generality across other reasoning and general tasks. To address this, we employ a model averaging technique (Lin et al., 2023) to derive a new model $p_{\bar{\theta}}$ parameterized by θ : $\theta = \alpha \theta + (1 - \alpha)\theta'$, where α is a hyper-parameter that balances specialization and generalization.

3 Experiment

In our preliminary experiments, we continually trained the Llama3-8B-Instruct model, using it as a baseline to evaluate the effectiveness of the THOUGHT-LIKE-PRO framework. We also implement CARING (Yang et al., 2023) under the experimental setup as our baseline method which utilizes the Prolog engine to derive the final results without training LLMs. Hence, the performance of CAR-ING in reasoning tasks should be the upper limit of THOUGHT-LIKE-PRO. We trained the model and evaluated its reasoning capabilities across three datasets: GSM8K (Cobbe et al., 2021), ProofWriter (Tafjord et al., 2020), and PrOntoQA (Saparov and He, 2022). Additionally, we assessed OOD general task performance using the MMLU (Hendrycks et al., 2020), GPQA (Rein et al., 2023), HumanEval (Chen et al., 2021), and MATH (Hendrycks et al., 2021) datasets. Our experimental framework comprises four configurations to explore the impact of model averaging and training with multiple reasoning trajectories. The configurations are: 1) full

implementation with model averaging and multiple trajectories (MA+multiple), 2) model averaging with a single trajectory for each instruction (MA+single), 3) multiple trajectories without model averaging (multiple), and 4) a single trajectory without model averaging (single). We refer the readers to Appendix A for more details about the experiment.

The results, as detailed in Table 1, reveal that the THOUGHT-LIKE-PRO framework significantly enhances reasoning capabilities through continuous training on the Llama3-8B-Instruct model. By incorporating multiple reasoning trajectories, this approach leads to superior outcomes compared to training with a single trajectory. Furthermore, the application of the model averaging technique has proven effective in exploiting the trade-off between specialty and generality, addressing the issue of CF, and yielding performance on OOD generalization tasks that are not included in the continual learning. The results show that the OOD performance is comparable to Llama3-8B-Instruct itself and significantly exceeds that of methods not employing model averaging. See Appendix B for a comprehensive discussion of the experimental results.

4 Conclusion

In this paper, we introduce THOUGHT-LIKE-PRO, a framework designed to enhance the logical reasoning of LLMs by imitating Prolog-based strictly logical reasoning trajectories in a self-driven manner. The design is **simple**, **straightforward**, **and effective**, ensuring ease of use in industrial applications. Our initial findings show improved task performance on both specialized reasoning and general benchmarks. This foundational framework will be further developed and extensively evaluated across various tasks to refine its methodology in future works.

5 Limitations

Our comparison of Llama3-8B-Instruct and THOUGHT-LIKE-PRO is constrained to moderatescale language modeling tasks due to limited computational resources. Furthermore, to comprehensively evaluate and better understand the performance potential of THOUGHT-LIKE-PRO, it may be beneficial to explore the generation of multiple reasoning paths for more challenging datasets. Intuitively, while the complexity of reasoning paths for more complex problems increases exponentially, the difficulty of describing these problems in code only increases linearly. Additionally, we did not explore the performance of models under different α values when applying the model averaging technique, nor did we balance the performance of the models on specialized versus generalized tasks. These considerations are scenario-specific and left for future research.

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A Experimental Details

In this subsection, we will provide detailed information on the equipment, models, datasets, fine-tuning methods, and hyperparameter settings used in our experiment.

A.1 Baseline, Model, and Equipment

Throughout the entire process, we utilized two A800-80GB (NVIDIA Corporation, 2023) GPUs for inference, fine-tuning, and evaluation tasks. We applied our THOUGHT-LIKE-PRO framework by continually fine-tuning the Llama3-8B-Instruct model which can be accessed in https://huggingface.co/meta-llama/Meta-Llama-3-8B-Instruct.

Our baseline was established by assessing the untrained Llama3-8B-Instruct model's performance on various datasets. This initial evaluation gave us a benchmark to demonstrate the enhancements our framework could provide. Additionally, we implemented the CARING (Yang et al., 2023) method to generate Prolog code with Llama3-8B-Instruct, assessing the correctness of solutions this code produced to problems.

A.2 Datasets

We evaluated THOUGHT-LIKE-PRO across seven prominent LLM benchmark datasets: two focused on mathematical reasoning (GSM8K and MATH), two on knowledge-based question answering (MMLU and GPQA), two on logical reasoning (ProofWriter and PrOntoQA), and one on code generation (HumanEval).

GSM8K: GSM8K (Cobbe et al., 2021) comprises 8,500 high-quality, linguistically diverse grade school math word problems created by human experts. We randomly selected 2,000 entries from this dataset to generate reasoning trajectories and used the balance to gauge the effectiveness of our Thought-Like-Pro framework. Given that the number of reasoning trajectories generated by the Prolog engine is variable, we capped the trajectories for each problem at 10 to mitigate data distribution bias. Consequently, we produced 20,000 pieces of training data for the LLM.

MATH: MATH (Hendrycks et al., 2021) encompasses a collection of 12,500 challenging competition-level mathematics problems, each accompanied by detailed step-by-step solutions. These solutions facilitate training models to generate complete derivations and explanations. We

leveraged this difficult dataset to assess the OOD performance of our THOUGHT-LIKE-PRO framework, noting that the LLM was not previously trained on it.

MMLU: MMLU (Hendrycks et al., 2020) benchmark is designed to gauge the knowledge acquired during pretraining by assessing models in exclusively zero-shot and few-shot scenarios, which closely aligns with methods used to evaluate human capabilities. It encompasses 57 subjects, spanning STEM, the humanities, and the social sciences, among others. Given its wide content range and varying difficulty, we utilize the entire dataset to evaluate the THOUGHT-LIKE-PRO framework's generalization abilities. Notably, within the THOUGHT-LIKE-PRO framework, the LLM does not have exposure to MMLU during the training phase.

GPQA: GPQA (Rein et al., 2023) represents a formidable dataset aimed at testing the capabilities of LLMs alongside scalable oversight mechanisms. The dataset comprises 448 multiple-choice questions crafted by domain experts in disciplines such as biology, physics, and chemistry. We employ this complete set to determine the generalization capacity of the THOUGHT-LIKE-PRO framework.

ProofWriter: ProofWriter (Tafjord et al., 2020) is a widely utilized logical reasoning dataset comprising many small-scale knowledge bases expressed in English, each containing facts and rules. Each knowledge base is paired with a set of questions, also in English, which can be definitively proven as true or false via proofs of varying depths, or the answer may be categorized as "unknown" under an open-world assumption (OWA) or presumed negative under a closed-world assumption (CWA). This dataset contains subsets of varying difficulty, we have selected the most challenging subset within the OWA setting, which includes 482 knowledge bases and 10,190 questions. We initially randomly sampled 100 knowledge bases and their corresponding questions to create reasoning trajectories. We restricted the number of allowed reasoning trajectories per question to five. Ultimately, we utilized all remaining data to evaluate the performance of our THOUGHT-LIKE-PRO framework.

PrOntoQA: PrOntoQA (Saparov and He, 2022) is a synthetic question-and-answer dataset designed to test the logical reasoning capabilities of LLMs. Each instance is structured to verify the validity of a statement within a specific context. We chose the most demanding subset (Pan et al., 2023b), "Fic-

tional Characters", with a depth level of 5, comprising 500 statement-context pairs. We randomly selected 100 statement-context pairs to generate reasoning trajectories and employed the remaining data for assessing the Thought-Like-Pro framework's capabilities. The maximum number of reasoning trajectories per question was set to 5.

HumanEval: HumanEval (Chen et al., 2021) comprises 164 unique programming problems designed to evaluate language comprehension, algorithmic proficiency, and elementary mathematics, some of which are akin to basic software interview questions. This dataset is employed to assess the functional correctness of programs synthesized from documentation strings. Given that HumanEval is unrelated to the two domains—mathematical reasoning and logical reasoning—used for training the LLM, we employ the entire dataset to assess the Thought-Like-Proframework's performance on general tasks following specialized training.

A.3 Fine-Tuning Method and Hyperparameter Settings

We perform typical continual supervised finetuning based on the checkpoint of Llama3-8B-Instruct. Our training framework follows the implementation of https://github.com/hiyouga/ LLaMA-Factory using the LLaMA-Factory. we employed the Adam optimizer in the training process and selected a consistent learning rate of 1e-6for 3 epochs of training. To ensure comparability between the single trajectory and multiple trajectory datasets, despite their differing data volumes, we have adjusted the training regimen. Specifically, the model trained on the single trajectory dataset undergoes an equivalent number of training steps as the model trained on the multiple trajectory dataset for 3 epochs. During model inference and evaluation, we configured the sampling settings with a temperature of 0.6 and a top_p value of 0.9, based on the default recommended parameters from https://huggingface. co/blog/llama3. For model averaging, we implement $\alpha = 0.5$ for a balanced averaging between the base model and the fine-tuned model.

B Analysis of Experimental Results

In our study, we assessed the performance of both the baseline model, Llama3-8B-Instruct, and our proposed framework, which was tested under four distinct configurations: 1) Full implementation featuring both model averaging and multiple trajectories (MA+multiple), 2) Model averaging applied to a single trajectory per instruction (MA+single), 3) Multiple trajectories without model averaging (multiple), and 4) A single trajectory without model averaging (single). This comparison allows us to discern the contributions of key components within our framework, particularly the impact of model averaging and the learning of multiple reasoning trajectories for the same instruction. Additionally, we incorporated the CARING method (Yang et al., 2023) within our experimental setup. This was utilized to evaluate the final reasoning accuracy, relying solely on Prolog engine inferences, without the requirement for training LLMs or using them to derive final answers.

All experiment results are presented in Table 1. We report the CARING results across three reasoning tasks. The Prolog engine delivered completely accurate reasoning outputs for the ProofWriter and PrOntoQA tasks and achieved an accuracy of 98.1% for the GSM8K task. The slightly lower accuracy in GSM8K can be attributed to errors in the translation of facts or rules and formattingrelated issues, although the reasoning trajectories are correct. Consequently, we posit that the performance of CARING represents the practical upper limit for the proposed THOUGHT-LIKE-PRO framework. Across all tasks, our framework, configured in four distinct ways, demonstrated substantial performance improvements over the baseline model. Notably, the configuration employing the 'multiple' method without model averaging recorded the highest performance, consistent with our expectations, as performing domain-specific fine-tuning and forgoing model averaging tend to enhance domain-specific expertise. In the GSM8K task, the Llama3-8B-Instruct model achieved an accuracy rate of 79.6%. In contrast, our framework marked a significant improvement, attaining an accuracy of 87.81%. This improvement was particularly pronounced in datasets requiring logical reasoning, especially in the ProofWriter and PrOntoQA datasets, where the model utilizing multiple reasoning paths achieved 98.19% and 100% accuracy, respectively.

The experimental results for reasoning tasks also suggest that imitating strictly logical reasoning trajectories can significantly enhance performance in logical reasoning tasks, such as ProofWriter and PrOntoQA. However, the improvements in algo-

rithmic reasoning tasks are comparatively modest. These findings not only confirm the efficacy of imitating strictly correct logical reasoning trajectories in aligning the reasoning capabilities of LLMs, but also suggest that LLMs may be better suited to modeling natural language problems using the relatively straightforward Prolog language, rather than translating these problems into more complex mainstream programming languages like Python.

We can assess the performance differences between the 'single' and 'multiple' training approaches to validate the effectiveness of utilizing multiple reasoning paths in training. Across all three reasoning tasks, it is evident that training with a single reasoning path can enhance performance relative to baseline methods. However, there exists a significant performance disparity between the 'multiple' and 'single' approaches, underscoring the importance of training with multiple reasoning trajectories for each instruction. This performance gap may be attributed to the inherent multi-modal nature of LLMs. Training on multiple correct trajectories toward the same target could promote robust learning of multi-modal distributions and significantly improve the robustness of LLM's reasoning.

To evaluate the performance of THOUGHT-LIKE-PRO on general OOD tasks, we selected four benchmarks commonly employed to assess the common knowledge and abilities of LLMs: MMLU (Hendrycks et al., 2020), GPQA (Rein et al., 2023), HumanEval (Chen et al., 2021), and MATH (Hendrycks et al., 2021). These benchmarks were chosen to measure the impacts of continual finetuning on three distinct reasoning tasks. Our observations indicate that Llama3-8B-Instruct achieves almost the highest performance across these tasks, and exhibits signs of catastrophic forgetting of continual training. However, the implementation of model averaging in THOUGHT-LIKE-PRO effectively navigates the trade-off between specialization and generalization, significantly enhancing the OOD performance. The results show results of 67.9% on MMLU and 33.8% on GPQA with MA+multiple, compared to domain-specific training, with only a slight decrease in performance on the reasoning tasks. Thus, model averaging serves as a viable strategy to mitigate catastrophic forgetting, enhancing both the robustness and generalizability of our approach.

While we observe a decline in performance on the MMLU, GPQA, and HumanEval datasets, the performance on the MATH dataset intriguingly remains consistent with the base model. This stability may be attributed to the inherent nature of the MATH tasks, which are graduate-level problems requiring arithmetic and logical reasoning skills. These skills have been effectively cultivated through the THOUGHT-LIKE-PRO approach applied within the GSM8K, ProofWriter, and PrOntoQA datasets. This approach appears to successfully generalize to MATH tasks and mitigate the negative impacts of catastrophic forgetting. Moreover, these results suggest that the THOUGHT-LIKE-PRO strategy possesses a potential broad generalization capability for OOD reasoning tasks.

C Prompt demo

In this section, we present the few-shot demonstration prompt utilized. It should be noted that due to space constraints, all prompts are displayed in their 1-shot condition, with the black text denoting the prompt sent to the LLM and the red text indicating the model's response. For more demonstrations, we refer the readers to https://anonymous.4open.science/r/Prolog_datasets-9875.

Listing 1: Construct a prompt $\operatorname{prompt}_{rft}$ to guide the LLM to generate Prolog code contains rules \mathcal{R} , facts \mathcal{F} , and targets \mathcal{T} based on the problem.

```
<|begin_of_text|><|start_header_id|>
   system<|end_header_id|>
You are a helpful assistant who **
   produces Prolog code** to solve
   problems.
<|eot_id|><|start_header_id|>user<|</pre>
   end_header_id|>
Could you please help me write Prolog
   code to solve the following
   arithmetic reasoning problem? You
   should use consistent variable names
    for coreferent entities or
   attributes throughout the code.
   Start by coding the given context
   after the "/* Context */" comment.
   Then code the query that represents
   the question after the "/* Query */"
    comment.
Here is the problem:
sent-1: Tina makes $18.00 an hour.
sent-2: If she works more than 8 hours
   per shift,
sent-3: she is eligible for overtime,
sent-4: which is paid by your hourly
   wage + 1/2 your hourly wage.
sent-5: If she works 10 hours every day
   for 5 days,
```

```
sent-6: how much money does she make?
                                                sentences she had typed incorrectly.
<|eot_id|><|start_header_id|>assistant<|</pre>
                                             sent-6: After a meeting, she typed for
   end_header_id|>
                                                18 minutes more.
Sure! I am happy to help you write
                                             sent-7: In all, the paper had 536
   Prolog code to solve this arithmetic
                                                 sentences by the end of today.
                                             sent-8: How many sentences did she start
    reasoning problem. Here is the
   Prolog code:
                                                  with today?
···prolog
                                             <|eot_id|><|start_header_id|>assistant<|</pre>
/* Context */
                                                end_header_id|>
                                             Sure! I am happy to help you write
                                                Prolog code to solve this arithmetic
% sent-1: Tina makes $18.00 an hour.
                                                  reasoning problem. Here is the
wage (18.00).
                                                Prolog code:
% sent-2: If she works more than 8 hours
                                             ···prolog
                                             /* Context */
    per shift.
% sent-3: she is eligible for overtime,
% sent-4: which is paid by your hourly
                                             % facts
   wage + 1/2 your hourly wage.
                                             sentences_per_minute(6).
overtime_wage(W) :-
                                             typing_sessions([20, 15, 18]).
    wage(W1),
                                             erased_sentences(40).
    W is 1.5 * W1.
                                             total_end_sentences(536).
% earnings without overtime for 1 day
                                             % calculate the number of sentences
regular_earning_for_day(E) :-
                                                 typed in a session
    wage(W),
                                             sentences_typed(SessionMinutes, Typed)
    E is 8 * W.
                                                 sentences_per_minute(SPM),
% sent-5: If she works 10 hours every
                                                 Typed is SPM * SessionMinutes.
   day for 5 days,
overtime_hours(H) :-
                                             % calculate the total number of
    H is 10 - 8.
                                                 sentences typed across all sessions
                                                 today
% overtime earnings for 1 day
                                             total_sentences_typed_today(Total) :-
overtime_earning_for_day(E) :-
                                                 typing_sessions(Sessions),
    overtime_hours(H),
                                                 maplist(sentences_typed, Sessions,
                                                    TypedPerSession),
    overtime_wage(W),
    E is H * W.
                                                 sum_list(TypedPerSession, TotalTyped
% total earnings for 1 day
                                                 erased_sentences(Erased),
total_earning_for_day(Total) :-
                                                 Total is TotalTyped - Erased.
    regular_earning_for_day(Regular),
    overtime_earning_for_day(Overtime),
                                             % calculate how many sentences she
                                                started with today
    Total is Regular + Overtime.
                                             start_sentences(TodayStart) :-
% total earnings for 5 days
                                                 total_end_sentences(EndToday),
total_earning_for_5_days(Total) :-
                                                 total_sentences_typed_today(
    total_earning_for_day(OneDay),
                                                     TodayTyped),
                                                 TodayStart is EndToday - TodayTyped.
    Total is 5 * OneDay.
/* Query */
                                             /* Ouerv */
solve(Total) :- total_earning_for_5_days
                                             solve(StartSentences) :- start_sentences
(Total).
                                             (StartSentences).
<|eot_id|><|start_header_id|>user<|</pre>
                                             <|eot_id|><|start_header_id|>user<|</pre>
   end_header_id|>
                                                end_header_id|>
Excellent work! Here is another problem
                                             Excellent work! Here is another problem
   for you to solve. Please apply the
                                                 for you to solve. Please apply the
    same approach you used for the
                                                 same approach you used for the
   previous one(s) to tackle this new
                                                 previous one(s) to tackle this new
   one.
                                                 one.
Problem:
                                             Problem:
sent-1: Janice can type 6 sentences per
                                             sent-1: Jesse and Mia are competing in a
                                                  week long race.
   minute.
sent-2: Today at work, Janice continued
                                             sent-2: They have one week to run 30
   working on a paper she started
                                                miles.
   typing yesterday.
                                             sent-3: On the first three days Jesse
sent-3: She typed for 20 minutes, took a
                                                averages (2/3) of a mile.
    break,
                                             sent-4: On day four she runs 10 miles.
sent-4: and typed 15 minutes longer.
                                             sent-5: Mia averages 3 miles a day over
sent-5: She then had to erase 40
                                                the first 4 days.
```

```
sent-6: What is the average of their
                                                  reasoning tree generated by a Prolog
    average that they have to run over
                                                   engine into a natural language
    the final three days?
                                                  description. To facilitate this, I
<|eot_id|><|start_header_id|>assistant<|</pre>
                                                  am providing the original problem,
                                                  the relevant Prolog code, and the
    end_header_id|>
Sure! I am happy to help you write
                                                  reasoning tree itself. Please review
    Prolog code to solve this arithmetic
                                                  these carefully and provide a
    reasoning problem. Here is the
                                                  fluent and accurate narrative of the
   Prolog code:
                                                   reasoning process. Thanks for your
···prolog
                                                  help!
/* Context */
                                              **Instructions Start**
% facts
total_distance(30).
jesse_first_three_days_avg(2/3).
                                              Translate the provided reasoning tree
jesse_day_four(10).
                                                  into a clear and logical natural
mia_first_four_days_avg(3).
                                                  language explanation.
                                             Many thanks for your help! I am looking
% calculate the total distance Jesse ran
    over the first four days
                                                  forward to your response!
jesse_first_four_days_total(Distance) :-
    jesse_first_three_days_avg(DayAvg),
                                              **Instructions End**
    jesse_day_four(DayFour),
    Distance is 3 * DayAvg + DayFour.
% calculate the total distance Mia ran
                                             Here is the problem:
   over the first four days
mia_first_four_days_total(Distance) :-
                                              sent-1: Tina makes $18.00 an hour.
    mia_first_four_days_avg(DayAvg),
                                              sent-2: If she works more than 8 hours
    Distance is 4 * DayAvg.
                                                 per shift,
                                              sent-3: she is eligible for overtime,
% calculate the average miles they have
                                              sent-4: which is paid by your hourly
                                                 wage + 1/2 your hourly wage.
    to run over the final three days
                                              sent-5: If she works 10 hours every day
remaining_avg(Person, Avg) :-
    (Person = jesse ->
                                                 for 5 days,
        jesse_first_four_days_total(
                                              sent-6: how much money does she make?
        Distance);
                                             Here is the prolog_code:
    Person = mia ->
    mia_first_four_days_total(
                                              /* Context */
        Distance)),
    total_distance(Total),
                                             % sent-1: Tina makes $18.00 an hour.
    Remaining is Total - Distance,
                                             wage(18.00).
    Avg is Remaining / 3.
                                              % sent-2: If she works more than 8 hours
% determine the average of their
                                                  per shift,
    averages over the final three days
                                             % sent-3: she is eligible for overtime,
average_of_averages(Result) :-
                                              % sent-4: which is paid by your hourly
                                                 wage + 1/2 your hourly wage.
    remaining_avg(jesse, JesseAvg),
    remaining_avg(mia, MiaAvg),
                                              overtime_wage(W) :-
    Result is (JesseAvg + MiaAvg) / 2.
                                                  wage(W1),
                                                  W is 1.5 * W1.
/* Query */
solve(Average) :- average_of_averages(
                                             % earnings without overtime for 1 day
                                              regular_earning_for_day(E) :-
  Average).
                                                  wage(W),
                                                  E is 8 * W.
Listing 2: Construct a prompt prompt_{cot}^{pro} to guide the
                                              % sent-5: If she works 10 hours every
                                                  day for 5 days,
LLM to generate a natural language CoT-like reasoning
                                              overtime_hours(H) :-
logic chain based on the problem, Prolog code, and
                                                  H is 10 - 8.
reasoning tree.
                                              % overtime earnings for 1 day
    <|begin_of_text|><|start_header_id|>
                                              overtime_earning_for_day(E) :-
        system<|end_header_id|>
                                                  overtime_hours(H),
You are a helpful and smart assistant
                                                  overtime_wage(W),
    that helps people solve problems.
                                                  E is H * W.
```

% total earnings for 1 day total_earning_for_day(Total) :-

regular_earning_for_day(Regular),

<|eot_id|><|start_header_id|>user<|</pre>

I need assistance in translating a

end_header_id|>

```
overtime_earning_for_day(Overtime),
                                                  previous one(s) to tackle this new
    Total is Regular + Overtime.
                                              Problem:
% total earnings for 5 days
                                              sent-1: Janice can type 6 sentences per
total_earning_for_5_days(Total) :-
                                                  minute.
    total_earning_for_day(OneDay),
                                              sent-2: Today at work, Janice continued
  working on a paper she started
    Total is 5 * OneDay.
                                                  typing yesterday.
/* Query */
                                              sent-3: She typed for 20 minutes, took a
solve(Total) :- total_earning_for_5_days
                                                   break,
    (Total).
                                              sent-4: and typed 15 minutes longer.
                                              sent-5: She then had to erase 40
Here is the prolog reasoning tree:
                                                  sentences she had typed incorrectly.
                                              sent-6: After a meeting, she typed for
=>(=>(,(=>(,(=>(builtin(true), wage
    (18.0)), =>(builtin(is(144.0, *(8, 18.0))), is(144.0, *(8, 18.0))),
                                                  18 minutes more.
                                              sent-7: In all, the paper had 536
    regular_earning_for_day(144.0)),
                                                  sentences by the end of today.
    ,(=>(,(=>(=>(builtin(is(2, -(10, 8))
                                              sent-8: How many sentences did she start
    ), is(2, -(10, 8))), overtime_hours
                                                   with today?
    (2)), (=>(builtin(,(g(wage(18.0)),
    g(is(27.0, *(1.5, 18.0)))),
                                              Prolog code:
    overtime_wage(27.0)), =>(builtin(is
                                              /* Context */
    (54.0, *(2, 27.0)), is(54.0, *(2,
    27.0)))), overtime_earning_for_day
                                              % facts
    (54.0)), =>(builtin(is(198.0,
                                              sentences_per_minute(6).
    +(144.0, 54.0))), is(198.0, +(144.0,
                                              typing_sessions([20, 15, 18]).
     54.0)))), total_earning_for_day
                                              erased_sentences(40).
    (198.0)), =>(builtin(is(990.0, *(5, 198.0))), is(990.0, *(5, 198.0))),
                                              total_end_sentences(536).
    total_earning_for_5_days(990.0)),
                                              % calculate the number of sentences
    solve(990.0))
                                                  typed in a session
<|eot_id|><|start_header_id|>assistant<|</pre>
                                              sentences_typed(SessionMinutes, Typed)
    end_header_id|>
Sure! I am happy to help you convert the
                                                  sentences_per_minute(SPM),
     Prolog-style reasoning tree into a
                                                  Typed is SPM * SessionMinutes.
    natural language reasoning chain.
    Here is the reasoning chain:
                                              % calculate the total number of
Tina earns $18.00 per hour according to
                                                  sentences typed across all sessions
    the information provided. She has
                                                  today
    shifts where, if she works more than
                                              total_sentences_typed_today(Total) :-
    8 hours, the additional hours are
                                                   typing_sessions(Sessions),
    considered overtime. Overtime pay is
                                                  maplist(sentences_typed, Sessions,
     calculated at one and a half times
                                                      TypedPerSession),
    her regular hourly wage.
                                                  sum\_list(TypedPerSession, TotalTyped
                                                      ),
On a typical day, if Tina works 10 hours
                                                   erased_sentences(Erased),
     this includes 2 hours of overtime
                                                  Total is TotalTyped - Erased.
    since she exceeds the 8-hour regular
    work period. Her regular earnings
                                              % calculate how many sentences she
    for working 8 hours a day at $18.00
                                                  started with today
    per hour amounts to $144.00 per day.
                                              start_sentences(TodayStart) :-
    For the 2 hours of overtime, since
                                                  total_end_sentences(EndToday).
    the overtime rate is $27.00 per hour
                                                   total_sentences_typed_today(
     (1.5 times her regular hourly wage)
                                                      TodayTyped),
    she earns $54.00 from overtime
                                                  TodayStart is EndToday - TodayTyped.
    work per day.
                                              /* Query */
Therefore, her total earnings for a
                                              solve(StartSentences) :- start_sentences
    single day, combining her regular
                                                  (StartSentences).
    and overtime earnings, are $198.00.
    Over the course of 5 days, working
                                              Prolog reasoning tree:
    the same hours daily, Tina makes a
                                              =>(=>(,(=>(builtin(true),
    total of $990.00, factoring in all
                                                  total_end_sentences(536)),
                                                  ,(=>(,(=>(builtin(true),
    regular pay and overtime across the
    five days.
                                                  typing_sessions([20, 15, 18])),
                                                  builtin(,(g(maplist(sentences_typed,
<|eot_id|><|start_header_id|>user<|</pre>
                                                  [20, 15, 18], [120, 90, 108])), ,(g (sum_list([120, 90, 108], 318)), ,(g
    end_header_id|>
Excellent work! Here is another problem
    for you to solve. Please apply the
                                                  (erased_sentences(40)), g(is(278,
    same approach you used for the
                                                  -(318, 40)))))),
```

```
total_sentences_typed_today(278)),
                                            Distance is 4 * DayAvg.
    =>(builtin(is(258, -(536, 278))), is
    (258, -(536, 278)))),
                                             % calculate the average miles they have
    start_sentences(258)), solve(258))
                                                 to run over the final three days
<|eot_id|><|start_header_id|>assistant<|</pre>
                                              remaining_avg(Person, Avg) :-
                                                 (Person = jesse ->
    jesse_first_four_days_total(
    end_header_id|>
Sure! I am happy to help you convert the
    Prolog-style reasoning tree into a
                                                     Distance);
    natural language reasoning chain.
                                                  Person = mia ->
    Here is the reasoning chain:
                                                     mia_first_four_days_total(
Janice's typing speed is 6 sentences per
                                                     Distance)),
    minute. Today, she had three
                                                  total_distance(Total),
    separate typing sessions: the first
                                                  Remaining is Total - Distance,
    lasted for 20 minutes, the second
                                                  Avg is Remaining / 3.
   for 15 minutes, and the third for 18 minutes. Over these sessions, she \,
                                             % determine the average of their
    initially typed a total of 318
                                                 averages over the final three days
    sentences. However, she made a
                                              average_of_averages(Result) :-
   mistake and had to erase 40
                                                  remaining_avg(jesse, JesseAvg),
    sentences, leaving her with 278
                                                  remaining_avg(mia, MiaAvg),
    sentences typed effectively today.
                                                  Result is (JesseAvg + MiaAvg) / 2.
   By the end of the day, the total
   number of sentences on her paper was
                                              /* Query */
    536. To find out how many sentences
                                              solve(Average) :- average_of_averages(
    were on the paper at the beginning
                                                 Average).
   of the day, we subtract the
    sentences typed today (278) from the
                                             Prolog reasoning tree:
    total at the end of the day (536).
                                             =>(=>(,(=>(,(=>(builtin(;(->(=(jesse,
                                                 jesse), jesse_first_four_days_total
    Hence, Janice started the day with
   258 sentences already on her paper.
                                                 (12.0)), ->(=(jesse, mia),
<|eot_id|><|start_header_id|>user<|</pre>
                                                 mia_first_four_days_total(12.0)))),
    end_header_id|>
                                                 ;(->(=(jesse, jesse),
Excellent work! Here is another problem
                                                 jesse_first_four_days_total(12.0)),
    for you to solve. Please apply the
                                                 ->(=(jesse, mia),
    same approach you used for the
                                                 mia_first_four_days_total(12.0)))),
    previous one(s) to tackle this new
                                                  ,(=>(true, total_distance(30)),
   one.
                                                  ,(=>(builtin(is(18.0, -(30, 12.0))),
                                                  is(18.0, -(30, 12.0))), =>(builtin(
Problem:
                                                 is(6.0, /(18.0, 3))), is(6.0,
sent-1: Jesse and Mia are competing in a
    week long race.
                                                 /(18.0, 3))))), remaining_avg(jesse
sent-2: They have one week to run 30
                                                  6.0)), ,(=>(,(=>(builtin(;(->(=(
    miles.
                                                 mia, jesse),
sent-3: On the first three days Jesse
                                                 jesse_first_four_days_total(12)),
   averages (2/3) of a mile.
                                                  ->(=(mia, mia),
sent-4: On day four she runs 10 miles.
                                                 mia_first_four_days_total(12)))),
sent-5: Mia averages 3 miles a day over
                                                 ;(->(=(mia, jesse),
   the first 4 days.
                                                 jesse_first_four_days_total(12)),
sent-6: What is the average of their
                                                  ->(=(mia, mia),
   average that they have to run over
                                                 mia_first_four_days_total(12)))),
   the final three days?
                                                  ,(=>(builtin(true), total_distance
                                                  (30)), builtin(,(g(is(18, -(30, 12))
                                                 ), g(is(6, /(18, 3))))))
Prolog code:
/* Context */
                                                 remaining_avg(mia, 6)), =>(builtin(
                                                 is(6.0, /(+(6.0, 6), 2))), is(6.0, /(+(6.0, 6), 2)))),
% facts
total_distance(30).
jesse_first_three_days_avg(2/3).
                                                 average_of_averages(6.0)), solve
jesse_day_four(10).
                                                 (6.0)
mia_first_four_days_avg(3).
                                              <|eot_id|><|start_header_id|>assistant<|</pre>
                                                 end_header_id|>
% calculate the total distance Jesse ran
                                              Sure! I am happy to help you convert the
                                                  Prolog-style reasoning tree into a
    over the first four days
jesse_first_four_days_total(Distance) :-
                                                 natural language reasoning chain.
    jesse_first_three_days_avg(DayAvg),
                                                 Here is the reasoning chain:
                                              Jesse and Mia are competing in a week-
    jesse_day_four(DayFour),
                                                 long race where each needs to run a
    Distance is 3 * DayAvg + DayFour.
                                                 total of 30 miles. Jesse averages (
                                                 \frac{2}{3} ) mile each day for the
% calculate the total distance Mia ran
   over the first four days
                                                 first three days, totaling 2 miles,
mia_first_four_days_total(Distance) :-
                                                 and then runs 10 miles on the fourth
  mia_first_four_days_avg(DayAvg),
                                                day, making it a total of 12 miles
```

- in the first four days. Mia averages
 3 miles per day over the first four
 days, totaling 12 miles as well.
- Given that they both need to complete 30 miles by the end of the week, both Jesse and Mia have 18 miles left to run over the final three days after the first four days. This results in each needing to run an average of 6 miles per day over the last three days.
- To find the average of their averages over these remaining three days, we calculate (\frac{(6+6)}{2}), which remains 6 miles per day. Therefore, the average of their average daily miles over the final three days that they need to run is 6 miles.