## JustLogic: A Comprehensive Benchmark for Evaluating Deductive Reasoning in Large Language Models

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## **Abstract**

Logical reasoning is a critical component of Large Language Models (LLMs), and substantial research efforts in recent years have aimed to enhance their deductive reasoning capabilities. However, existing deductive reasoning benchmarks, which are crucial for evaluating and advancing LLMs, are inadequate due to their lack of task complexity, presence of prior knowledge as a confounder, and superficial error analysis. To address these deficiencies, we introduce JustLogic, a synthetically generated deductive reasoning benchmark designed for rigorous evaluation of LLMs. JustLogic is (i) highly complex, capable of generating a diverse range of linguistic patterns, vocabulary, and argument structures; (ii) prior knowledge independent, eliminating the advantage of models possessing prior knowledge and ensuring that only deductive reasoning is used to answer questions; and (iii) capable of in-depth error analysis on the heterogeneous effects of reasoning depth and argument form on model accuracy. Our experimental results on JustLogic reveal that most state-of-the-art (SOTA) LLMs perform significantly worse than the human average, demonstrating substantial room for model improvement. All code and data are available at https://github.com/michaelchen-lab/JustLogic

## 1. Introduction

Deductive reasoning is a crucial capability for large language models (LLMs). It refers to the process of creating logically valid arguments, where conclusions necessarily follow from the premises. In other words, if an argument's premises are true, the conclusion must also be true. Recent state-of-the-art (SOTA) LLMs (Achiam et al., 2023; Dubey et al., 2024; Jiang et al., 2023) have exhibited outstanding

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performance and consistent improvement across various reasoning benchmarks, including HelloSwag (Zellers et al., 2019), ARC Challenge (Clark et al., 2018) and WinoGrande (Sakaguchi et al., 2021). However, we argue that the existing benchmarks are insufficient and often ineffective for evaluating LLMs' true deductive reasoning capabilities.

We identify three major problems with the existing benchmarks. First, they lack complexity, as measured on two dimensions: natural language complexity, i.e. how arguments are linguistically expressed, and argument complexity, i.e. the structure of the argument itself. Manually curated datasets, such as FOLIO (Han et al., 2022) and LogiQA 2.0 (Liu et al., 2020; 2023a) exhibit high natural language complexity but low argument complexity, while synthetic datasets like CLUTRR (Sinha et al., 2019) and ProofWriter (Tafjord et al., 2020) exhibit the opposite. Simplicity in either dimension makes these benchmarks prone to overfitting and memorization, thus allowing models to perform well despite underlying weaknesses in logical reasoning. A more detailed analysis can be found in Section 3.4. **Second**, existing benchmarks often fail to test deductive reasoning in isolation, as models can benefit from prior knowledge. To empirically validate this claim, we developed a novel test for prior knowledge independence, which measures the influence of prior knowledge on reasoning benchmarks. As detailed in Section 5.1, prior knowledge can substantially increase accuracy, even in datasets not intended to require commonsense or domain knowledge, e.g. FOLIO and LogiQA 2.0. Thus, high accuracy may not reflect strong reasoning capabilities. Third, many existing benchmarks provide superficial error analysis, leaving key questions unanswered: At what reasoning depth does the model start to fail? How does the model compare to humans at different argument depths? Which argument forms is the model particularly weak at? These insights are essential for understanding the depth and robustness of a model's deductive reasoning, yet many benchmarks provide them. Section 5.3 demonstrates the importance and usefulness of comprehensive error analysis.

Due to these issues, LLMs' deductive reasoning abilities remain ambiguous. In response to the critical need for a reliable benchmark to support ongoing research efforts,

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we present JustLogic, a novel natural language deductive reasoning benchmark. Each instance in JustLogic contains a paragraph of premises and a statement. The task is to determine whether the statement is true, false, or uncertain, based solely on the premises, and assuming they are all true. An example is shown in Figure 1.

#### Paragraph:

- Whenever it is true that night blooming plants and trees depend on nectar eating bats for pollination, 'if many species are critically endangered, then it is not true that doors are solids' is true.
- Night blooming plants and trees depend on nectar eating bats for pollination.
- We can assume that many species are critically endangered.

Question: Is the following statement true, false, or

uncertain?

**Statement:** Doors are solids.

**Answer:** False

Figure 1. Example of a question adapted from the JustLogic train dataset

JustLogic's construction ensures it is (i) complex, (ii) prior knowledge independent, and (iii) capable of in-depth error analysis. First, to achieve high argument and natural language complexity, JustLogic is code-generated rather than manually curated. This allows the generation of a theoretically infinite number of unique argument structures. Natural language sentences are then drawn from GenericsKB-Best (Bhakthavatsalam et al., 2020), a database of 1M+ unique real-world sentences, and inserted into the argument structures, introducing high natural language complexity. Second, since sentences are randomly sampled from the entire Generics KB-Best dataset, the generated arguments generally do not align with real-world knowledge, thereby eliminating prior knowledge as a confounder. Finally, in-depth error analysis is enabled by the programmatic generation process, which enables the inspection of each question's properties, such as reasoning depth and argument form, to investigate their impact on model performance. A comparison between JustLogic and four similar logical reasoning benchmarks (CLUTRR, ProofWriter, LogiQA 2.0, and FO-LIO) is presented in Table 1, with further details on dataset construction provided in Section 3.

Using JustLogic, we conducted comprehensive experiments to evaluate the deductive reasoning capabilities of current LLMs. First, our novel prior knowledge independence test demonstrated that prior knowledge enables LLMs to bypass

deductive reasoning on existing datasets, resulting in artificially high accuracies. This is not observed in JustLogic. Second, we benchmarked the performance of SOTA LLMs and human participants using JustLogic. Most SOTA LLMs performed significantly lower than the average human accuracy (73.0%). Only OpenAI o1-preview performed substantially better (81.0%), but still fell short of the human ceiling (100.0%). Finally, enabled by JustLogic's code-generated nature, our thorough error analysis examined the heterogeneous impact of argument structure and reasoning depth on model performance. These experiments show that the JustLogic benchmark is a reliable test of deductive reasoning and reveals significant room for improvement in LLMs.

In summary, our contributions are threefold. First, we evaluate the limitations of existing benchmarks. Second, we introduce the JustLogic benchmark, a synthetic dataset to evaluate deductive reasoning, that addresses the aforementioned limitations. Third, our experiments using JustLogic demonstrate that most SOTA models perform significantly worse than humans. We posit that the deductive reasoning capabilities of LLMs still have significant room for improvement, and hope that the JustLogic benchmark will assist researchers in designing and evaluating LLMs.

#### 2. Related Work

# 2.1. Existing reasoning datasets for Large Language Models

Reasoning benchmarks are a vital part of LLM evaluation. Some benchmarks measure deductive reasoning in conjunction with natural language inference (NLI), inductive reasoning, and commonsense knowledge: HellaSwag (Zellers et al., 2019) tasks machines to select the most likely followup of an event description, WinoGrande (Sakaguchi et al., 2021) is a pronoun resolution task, and MuSR (Sprague et al., 2023) tasks machines to solve fictional problems, such as murder mysteries. Other benchmarks measure reasoning on domain knowledge: AI2 Reasoning Challenge (ARC) (Yadav et al., 2019) contains grade-school science questions, while Massive Multitask Language Understanding (MMLU) (Hendrycks et al., 2020) contains questions across 57 subjects in STEM, humanities, and more. Finally, math-specific benchmarks include GSM-8K (Cobbe et al., 2021) and DROP (Dua et al., 2019).

The aforementioned benchmarks explicitly evaluate skills beyond reasoning and do not specifically define the type of reasoning involved, e.g. inductive, deductive, and analogical. As such, benchmarks that solely test for deductive reasoning have seen a considerable increase in interest. They can be classified into two broad categories: synthetic and manually curated. Synthetic datasets include (i) CLUTRR (Sinha et al., 2019), where a machine must infer the relationship

Table 1. Comparison of JustLogic with other deductive reasoning datasets. T	The symbol $\sim$ suggests the feature is present but to a limited
extent.	

	High NL	High Arg.	Prior Knowledge	In-Depth
	Complexity	Complexity	Independence	Error Analysis
CLUTRR	×	✓	✓	✓
ProofWriter	×	✓	✓	$\sim$
LogiQA 2.0	$\checkmark$	×	X	$\sim$
FOLIO	$\checkmark$	×	×	$\sim$
JustLogic	✓	✓	✓	✓

of two family members based on stories, (ii) ProofWriter (Tafjord et al., 2020), where a machine must deduce a statement's truth value based on a set of facts and rules, and (iii) ProntoQA-OOD (Saparov et al., 2024), where a machine must prove a statement based on a set of facts. Manually curated datasets include (i) LogiQA 2.0 (Liu et al., 2023a), containing manually-translated logical reasoning questions from the Chinese Civil Service Exam, (ii) FOLIO (Han et al., 2022), containing questions with manually-annotated content using Wikipedia pages, and (iii) ReClor (Yu et al., 2020), containing reading comprehension questions from GMAT and LSAT.

As discussed earlier, synthetic datasets are prior knowledge independent and exhibit high argument and low natural language complexity; manually curated datasets are the opposite. JustLogic, being synthetic, contains all its advantages while offering the natural language complexity of manually curated datasets, which we further explained in Section 3.4 and 5.1.

#### 2.2. Reasoning in Large Language Models

As LLMs continue to increase in size, their performance on various reasoning-related benchmarks has improved dramatically. For example, in 2024, Gemini Ultra (Team et al., 2023) achieved 90.0% on MMLU when the SOTA model in 2020, UnifiedQA 11B (Khashabi et al., 2020), achieved a mere 48.9%. In 2023, GPT-4 achieved 96.4% on ARC when the SOTA model in 2020, GPT-3 (Brown, 2020), achieved 53.2%.

The advent of prompting techniques played an important role in developing LLMs' reasoning abilities. In-context learning (Dong et al., 2022) provides LLMs with instructions and examples in the input prompt to guide its response. Chain-of-thought prompting (Wei et al., 2022) prompts LLMs to generate a series of intermediate reasoning steps before arriving at the final answer. Self-consistency decoding (Wang et al., 2022) chooses the most consistent answer after sampling multiple chain-of-thought outputs. Least-to-most prompting (Zhou et al., 2022) decomposes a complex problem into simpler subproblems, which are then solved

sequentially.

As mentioned above, LLMs are conventionally tested on datasets that combine reasoning with other skills. Moreover, existing logical reasoning-specific datasets possess major limitations that call into question the reliability of their evaluations. In response, JustLogic aims to robustly and accurately evaluate the deductive reasoning abilities of LLMs.

#### 3. Dataset Construction

JustLogic is a programmatically generated dataset designed to evaluate a model's ability of deductive reasoning, specifically its capability to form logically valid arguments. A logically valid argument is one where the conclusion necessarily follows from the premise(s); in other words, given the premises are true, the conclusion must also be true.

In order to test this, JustLogic presents a model with a paragraph consisting of premises, followed by a query statement. Based solely on the premises and assuming they are all true, the model needs to determine whether the query statement is true, false, or uncertain. In line with the open-world assumption, the "Uncertain" answer refers to cases where the premises neither support nor contradict the query statement.

The following outlines the process for generating each instance in the dataset:

- 1. Step 1: Generate an argument structure
- 2. Step 2: Add natural language statements to the argument structure
- 3. Step 3: Generate a query statement

Figure 2 provides an example of this process, which we reference throughout the rest of this section.

#### 3.1. Step 1: Generate argument structure

Argument structures are composed of one or more valid argument forms, derived from propositional logic; argument forms are made up of a series of logical forms, which we define as symbolic representations of statements. Specifically, the seven distinct argument forms in our dataset are

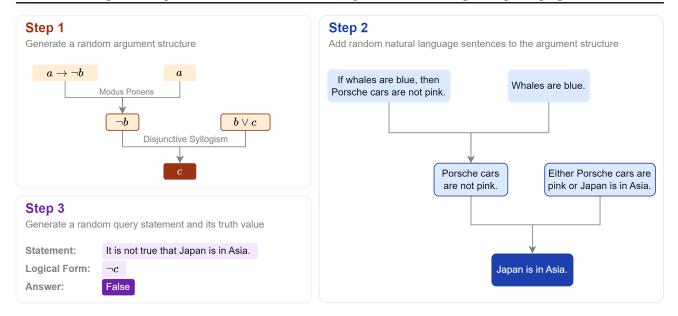


Figure 2. A step-by-step example of how an instance with a reasoning depth of 2 is constructed

constructed with the following four logical forms: (i) basic (x), (ii) negation  $(\neg x)$ , (iii) conditional  $(x \to y)$ , and (iv) disjunction  $(x \lor y)$ . While there is a theoretically infinite number of possible argument forms, complex argument forms can be derived by combining simpler ones. Therefore, we explicitly define the eight most fundamental forms (Johnson, 2006): modus ponens, modus tollens, hypothetical syllogism, disjunctive syllogism, reductio ad absurdum, constructive dilemma, and disjunctive elimination. Table 6 in Appendix A provides the corresponding formal notations and natural language examples.

The function to create an argument structure accepts an intended argument depth as input. It first generates a random conclusion and an argument form to support it, which is c and disjunctive syllogism in Figure 2 respectively. If the intended depth has not been reached, one or more premises will become subconclusions, which are supported by new, randomly generated argument forms, thus increasing the argument's depth. In Figure 2, this is exemplified by  $\neg b$  being converted to a subconclusion that is supported by a modus ponens argument. If further depth is still required, one or more premises from the newly generated argument forms will themselves have argument forms to support them. This process continues until the target depth is achieved.

## 3.2. Step 2: Adding natural language

Once the argument structure is generated, it serves as the skeleton of the paragraph, and the next step is to convert the statements in logical form into natural language. Each statement consists of one or more logical forms, *i.e.* variable, negation, conditional, and disjunction. In natural language,

these forms can be expressed in a variety of ways. For example, a conditional can be expressed as both "If x, then y." and "Given that x, y is true.", where variables x and y are simple propositions. To emulate the diversity of natural language, we manually create a list of expressions for each logical form with the help of GPT-4 (Achiam et al., 2023) and human feedback. Table 2 shows the formal notation of each form, alongside a sample expression and the total number of unique expressions.

The variable(s) within each expression is ultimately replaced by randomly selected generic, real-world sentences from GenericsKB-Best (Bhakthavatsalam et al., 2020). The GenericsKB-Best dataset is chosen for its vast collection of simple propositions (1,020,868 sentences) without conditionals, disjunctions, etc. A complete example can be found in Step 2 of Figure 2.

Notably, as shown in Figure 2, the statements are generally factually inaccurate despite being drawn from real-world data. This is intentional. Real-world propositions allow us to generate sentences with diverse grammatical structures that closely emulate human-written arguments. However, factually accurate arguments enable models to bypass deductive reasoning with their prior real-world knowledge, which is experimentally demonstrated in Section 5.1. By using real-world yet factually inaccurate statements, we combine realism and prior knowledge independence.

There is a potential concern that factually inaccurate statements may confuse models and therefore lead to artificially low performance. Our empirical study in Appendix G proves that this concern does not hold true in practice.

*Table 2.* Expressions of logical forms

	Formal Notation	Sample Expression	No. of Expr.
Basic	x	The claim that $x$ holds true.	16
Negation	$\neg x$	The claim that $x$ does not reflect reality.	15
Conditional	$x \to y$	Once we know that $x$ , we also know that $y$ .	11
Disjunction	$x \vee y$	It is a fact that either $x$ or $y$ .	8

#### 3.3. Step 3: Generate query statement

The LLM's task is to determine whether the given query statement is true, false, or uncertain based on the premises provided. Using Figure 2 as an example, if we assign the query statement to be the negation of the conclusion, i.e. "It is not true that Japan is in Asia", then the answer is false. If the query statement is the same as the conclusion, then the answer is true. If the query statement is unrelated to the premises, then the answer is uncertain.

## 3.4. Dataset Complexity

In the context of deductive reasoning datasets, complexity is defined as the variety and comprehensiveness of instances. It can be further divided into two dimensions: natural language complexity and argument complexity. In this section, we highlight the significance of both aspects and how JustLogic compares against other logical reasoning datasets.

**Natural language complexity.** Human language is complex. Statements and arguments of similar meanings can be presented in a variety of ways. Therefore, it is insufficient for models to reason solely with symbols, *e.g.* x and y, and basic natural language sentences, *e.g.* "Some birds are yellow."; they must be capable of reasoning with real-world vocabulary and diverse sentence structures to be useful in practical contexts.

We measure natural language complexity with (i) reading difficulty, as measured by the Flesch-Kincard readability test (Kincaid, 1975), and (ii) lexical diversity, as measured by vocabulary & domain size. A domain is defined as any topic of interest, such as golf, computers, or traveling; Vocabulary size refers to the number of unique words in the dataset. Given the difficulty of quantitatively capturing linguistic complexity, Appendix B also shows text samples of each benchmark, representative of their complexity.

As shown in Table 3, existing synthetic datasets have low natural language complexity, while human-written datasets, such as FOLIO and LogiQA 2.0, exhibit significantly higher complexity. This is expected since synthetic datasets translate symbols in formal logic into natural language using limited templates of sentence structures and vocabulary lists. For example, in ProofWriter, a typical sentence follows the format "All dogs are (not) red.". The linguistic patterns of

human-written datasets, in contrast, are bound only by human creativity. Despite being synthetic, JustLogic, achieves natural language complexity on par with manually curated datasets, due to its comprehensive selection of expressions and the use of GenericsKB-Best as the source of sentences.

**Argument complexity.** Argument complexity refers to the diversity of argument structures used in the dataset. A sufficiently high argument complexity is important because humans use a range of argument forms to reason, beyond just conditionals and modus ponens. Moreover, a real-world argument is typically composed of multiple argument forms, due to the inherent complexity of real-life scenarios.

We evaluate a dataset's argument complexity based on two metrics: (i) the range of reasoning depth, and (ii) the number of unique argument structures. The upper limit of both metrics is calculated based on the theoretical maximum without any additional human input, rather than the highest depth used in experiments in existing works. For example, CLUTRR's dataset construction program can generate any number of depths (referred to as relation length in the original paper), despite its experiments only utilizing questions of up to a depth of 10. Thus, its upper limit of depth is infinite.

Table 3 shows that synthetic datasets, such as CLUTRR, ProofWriter, and JustLogic, excel in this area, as there is no upper limit to their reasoning depth and number of argument structures. Manually curated datasets, in contrast, either lack an explicit concept of reasoning depth and argument structures (e.g. LogiQA 2.0), or have a limited selection of both (e.g. FOLIO). While manual datasets require significant human efforts and investment to expand their complexity, synthetic ones can scale trivially.

In summary, JustLogic combines the best aspects of both dataset construction methods, incorporating the argument complexity of synthetic datasets and the natural language complexity of manually curated ones.

## 3.5. Future-proofing JustLogic

As the reasoning abilities of LLMs continue to improve, we expect LLMs to solve the existing JustLogic dataset eventually. To maintain JustLogic's relevance, we leverage its synthetic nature to increase complexity with minimal human

*Table 3.* Statistics of dataset complexity.

	Natural Language		Argument	
	Reading Difficulty \( \tau \) Vocabulary (Domains)		Reasoning Depth	Arg. Structures
CLUTRR	6.67	1396 (1)	1 - ∞	$\infty$
ProofWriter	0.96	101 (x)	1 - ∞	$\infty$
LogiQA 2.0	17.10	10004 (>10)	×	×
FOLIO	18.75	4351 (>10)	1 - 7	76
JustLogic	20.55	10557 (>10)	1 - ∞	$\infty$

input. Argument complexity can be adjusted by increasing the (i) range of argument depth and (ii) number of distinct argument forms to >7. Natural language complexity can be adjusted by (i) increasing the number of expressions for each logical form and (ii) integrating more complex knowledge bases than GenericsKB. Importantly, these changes are programmatically achievable with minimal man-hours.

Importantly, JustLogic can also effectively tackle benchmark leakage (Xu et al., 2024), whereby test sets are unintentionally included in LLMs' pertaining data, thus artificially inflating their performance through memorization. Should JustLogic's test set be leaked, a new test set of similar difficulty can be trivially generated, thereby mitigating this problem.

## 4. Experimental Setup

We first investigate the influence of prior knowledge on evaluating deductive reasoning with JustLogic and other existing benchmarks using our prior knowledge independence test. Next, several SOTA LLMs of various sizes are evaluated using JustLogic. Finally, an in-depth error analysis of the LLMs' results is conducted.

JustLogic contains 7000 instances, equally split amongst reasoning depths ranging from 1 to 7. It is then divided into train/validation/test sets (70%/15%/15%). Train and validation sets facilitate in-context learning and model fine-tuning if required, while the test set is used for evaluation. Note that the number of instances and range of reasoning depths can be easily adjusted using JustLogic's open-source dataset generation program.

#### 4.1. Prior Knowledge Independence Test

The task for deductive reasoning benchmarks is typically framed as  $CQO \rightarrow A$ : Given a context C, consisting of n premises  $(P = \{p_1, p_2, ..., p_n\})$ , a question Q, and m answer options  $(O = \{o_1, o_2, ..., o_m\})$ , determine the correct answer A. To assess the influence of prior knowledge on determining answer A, the prior knowledge independence test is framed as  $QO \rightarrow A$ . No context C is provided, and the prompt instructs the LLM to answer the question

based on prior knowledge alone. An example is provided in Appendix C.

If prior knowledge is not useful, the LLM should be unable to answer question Q without C, and the accuracy for the prior knowledge independence test should approximate random probability  $\frac{1}{m}$ . Benchmarks exhibiting such accuracies are deemed prior knowledge independent.

Any LLM capable of using prior knowledge can be used for this test. However, models with larger parameter sizes, and thus more extensive prior knowledge, are more likely to exhibit notable differences in accuracies. For our experiment, we use GPT-4. The test is conducted on both JustLogic and existing benchmarks, including CLUTRR, ProofWriter, LogiQA 2.0, and FOLIO.

#### 4.2. Evaluation of LLMs' Deductive Reasoning

Our task follows the conventional formulation:  $CQO \rightarrow A$ . Question Q is "Is the statement S true, false, or uncertain?", followed by the query statement, as shown in Figure 1; there are 3 answer options, where  $O = \{\text{true}, \text{false}, \text{uncertain}\}$ . All prompts begin with a preamble, which includes (i) the requirements of the task at hand, (ii) a list of argument forms in propositional logic, and (iii) the available answer options.

We evaluated various models of different sizes, including Llama3-8B (Dubey et al., 2024), Llama3-70B, GPT-4, GPT-40, OpenAI o1 and OpenAI o1-preview (Jaech et al., 2024). Given that prompt quality significantly impacts LLM accuracy, a range of prompting techniques are tested: zero-shot, few-shot, and chain-of-thought (CoT) (Wei et al., 2022). OpenAI's reasoning models had strict rate limits at the time of writing. As such, 42 random samples in the test set are used for OpenAI o1 and OpenAI o1-preview. To ensure fairness, the selected subset has the same proportion of reasoning depth and classes (True, False, and Uncertain) as the entire test set. Further implementation details are provided in Appendix D.

We also measured human performance. 18 anonymous participants, recruited from Amazon Mechanical Turk (Amazon, 2005), are given a random subset of questions. This is because deductive reasoning questions, especially those

at high reasoning depths, are cognitively demanding and time-consuming; it is impractical to expect humans to complete 1050 questions. To ensure fairness, both models and participants are provided similar prompts and are given the same proportion of each reasoning depth.

Finally, we perform an error analysis of the results from the aforementioned experiments, specifically examining the heterogeneous effects of argument form and reasoning depth on model accuracy. Accuracy for each argument form is only measured using questions with a reasoning depth of 1 since those with a depth of >1 typically have >1 argument forms. Lastly, a qualitative analysis of failure modes is conducted.

#### 5. Results

#### 5.1. Prior Knowledge Independence Test

The results of JustLogic and four other benchmarks are shown in Table 4; note that lower accuracy relative to the benchmark's random probability indicates that prior knowledge is more detrimental to answering the question, thereby demonstrating that the benchmark is more prior knowledge independent. Thus, the smaller the  $|\Delta|$  between model accuracy and random probability, the better. The  $|\Delta|$ s of CLUTRR and ProofWriter are relatively low, while those of LogiQA 2.0 and FOLIO are nontrivially higher. This is because the former are synthetic datasets, while the latter are manually curated. When a question is code-generated, it generally bears no correlation with reality, e.g. "Is it true, false, or uncertain that Gary is not red." from ProofWriter and "How is Anna related to Katherine in the family?" from CLUTRR. Such questions are only answerable by reasoning over the context C. LogiQA 2.0 and FOLIO, on the other hand, often contain questions that are answerable without the context provided. For example, "The United States won the most medals in the last summer Olympic games." from FOLIO can be accurately answered by LLMs trained on sufficiently recent general knowledge datasets. We posit that this is an unintentional consequence of the human bias to align the question's truth value with reality. While human curation enhances the question's realism, it compromises the test for deductive reasoning.

*Table 4.* Results of Prior Knowledge Independence Test. **The lower the**  $|\Delta|$ , **the better.** 

	$ \Delta \downarrow$	Accuracy (%)	Random (%)
CLUTRR	2.0	8.3	6.3
ProofWriter	3.7	37.0	33.3
LogiQA 2.0	27.1	52.1	25.0
FOLIO	6.7	40.0	33.3
JustLogic	0.4	33.7	33.3

The JustLogic benchmark's  $|\Delta|$  is 0.4%, given an accuracy of (33.7%) and random probability (33.3%), which is much lower than other benchmarks, including synthetic ones. The reason for this is twofold: first, JustLogic is also a synthetic dataset, which eliminates the human bias present in manually curated datasets. Second, while JustLogic uses real-world statements, their truth value is nonetheless randomly determined. For example, the statement "doors are solids" is factually true. However, by deducing from the paragraph, the correct answer is "False". Thus, using prior knowledge for many questions is not only unhelpful but also meaningfully decreases accuracy.

#### 5.2. Evaluation of LLMs' Deductive Reasoning

As shown in Table 5, the best-performing model by a large margin is OpenAI o1-preview with an accuracy of 81.0%. The second and third-best models, GPT-40 and Llama3-70B, achieved 65.6% and 64.6% respectively. Surprisingly, OpenAI o1 (64.3%) performs substantially worse than OpenAI o1-preview due to developer-imposed limits on test-time compute; we provide further evidence in Appendix E. OpenAI o1-preview is a better representation of SOTA reasoning models' capabilities. Models with larger parameter sizes generally perform better than smaller models, assuming the same prompting methods are used. For example, zero-shot Llama3-8B achieved an accuracy of 49.8%, while zero-shot Llama3-70B achieved an accuracy of 53.1%. However, larger model sizes offer diminishing returns, shown by the accuracy gain of just 1.0% from Llama3-70B to GPT-4o, with both using CoT prompting.

Moreover, the improvements offered by increasing model size pale in comparison to those offered by better prompting methods. Using chain-of-thought prompting, Llama3-8B achieved higher performance (57.8%) than zero-shot Llama3-70B (53.1%). This appears to explain the significant accuracy gap of 15.4% between OpenAI o1-preview and its non-reasoning-focused counterpart, GPT-4o. OpenAI o1-preview is trained to reason with chain-of-thought prompts using a 'reinforcement learning algorithm' (OpenAI, 2024). We hypothesize that the use of reinforcement learning on CoT prompting further enhances the deductive reasoning capabilities offered by CoT prompting alone.

Human performance (73.0%) is significantly higher than all models besides OpenAI o1-preview, while the human ceiling (100.0%) outperforms all models. The non-trivial gap between the human ceiling and the best-performing model (81.0%) shows that models still have significant room for improvement. Moreover, we believe actual human performance might be higher than 73.0%. Given the long paragraphs of questions with high reasoning depth, participants may have predicted answers by briefly scanning the paragraph, rather than carefully deducing based on all available

Table 5. Model and Human Evaluation Results			
Model	Method	Acc (%)	
Random Probability		33.3	
Llama3-8B	Zero-shot	49.8	
Llama3-8B	Few-shot	41.8	
Llama3-8B	CoT	57.8	
Llama3-70B	Zero-shot	53.1	
Llama3-70B	Few-shot	57.8	
Llama3-70B	CoT	64.6	
GPT-4	СоТ	59.2	
GPT-4o	CoT	65.6	
OpenAI o1	CoT	64.3	
OpenAI o1-preview	CoT	81.0	
Human Average		73.0	
Human Ceiling		100.0	

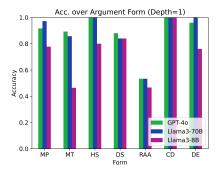
premises. This is supported by the suspiciously short time taken to complete the questions of some participants.

#### 5.3. Error Analysis

Figure 3 illustrates the model accuracy by argument form (left) and by reasoning depth (right). It shows the statistics for (i) Llama3-8B with CoT prompting, chosen for its superior performance amongst smaller LLMs, (ii) Llama3-70B with CoT prompting, chosen due to its superior performance amongst medium-sized LLMs, and (iii) OpenAI o1-preview, chosen for its overall highest performance. Note that OpenAI o1-preview is excluded from the argument form analysis due to insufficient samples; GPT-4o is displayed instead for a more comprehensive comparison. The qualitative analysis of failure modes can be found in Appendix F.

The accuracies of some argument forms are evidently better than others. For example, hypothetical syllogism and constructive dilemma achieve considerably higher performance than modus tollens and reductio ad absurdum. We hypothesize that these forms appear less frequently in the models' training data. With less exposure to them, models may overlook these argument forms in favor of more common ones during deductive reasoning, owing to the probabilistic nature of neural networks (Fahlman & Hinton, 1987).

As for reasoning depth, model accuracies generally decrease as depth increases, consistent with expectations that accuracy declines as the complexity of questions increases. Interestingly, Llama3-70B performs comparably to OpenAI o1-preview for instances with a depth of 1, but Llama3-70B sees a sharp decline in performance once depth is increased, while OpenAI o1-preview only sees a moderate decline; OpenAI o1-previews' superior performance is a result of better reasoning at higher reasoning depths. This seems



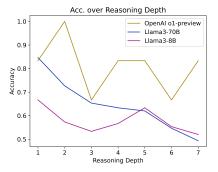


Figure 3. How argument form<sup>2</sup> and reasoning depth affects accuracy for various models

to suggest OpenAI o1-preview's CoT prompting supports deeper and longer lines of reasoning, which is crucial for deductive reasoning. Fluctuations on all three trendlines are likely due to small sample sizes: OpenAI o1-preview has 6 samples per depth, while the other 2 have 150. We expect the trend to be more explicit with a larger number of samples.

#### 6. Conclusion

Deductive reasoning is one of the key challenges in LLM research. In response to the lack of reliable benchmarks, we present JustLogic, a natural language deductive reasoning dataset that is (i) highly complex, (ii) prior knowledge independent, and (iii) capable of in-depth error analysis. These qualities are enabled by JustLogic's dataset construction method: argument structures are synthetically generated, and natural language is programmatically incorporated via expression templates and a knowledge base. We empirically justify JustLogic's merits: most LLMs underperform the human average and all significantly underperform the human ceiling. We demonstrate that JustLogic is a highly challenging, future-proof benchmark that is reliable and insightful for evaluating logical reasoning in LLMs.

<sup>&</sup>lt;sup>2</sup>MP = Modus Ponens, MT = Modus Tollens, HS = Hypothetical Syllogism, DS = Disjunctive Syllogism, RAA = Reductio Ad Absurdum, CD = Constructive Dilemma, DE = Disjunction Elimination

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## A. Argument Forms

Table 6. An overview of the argument forms in the JustLogic dataset

	Formal Notation	Example
Modus Ponens	$p \rightarrow q$	If the sky is blue, then the dog is happy.
	p	The sky is blue.
	$\vdash q$	Therefore, the dog is happy.
	$p \rightarrow q$	If the sky is blue, then the dog is happy.
Modus Tollens	$\neg q$	The dog is not happy.
	$\vdash \neg p$	Therefore, the sky is not blue.
	$p \rightarrow q$	If the sky is blue, then the dog is happy.
Hypothetical Syllogism	$q \rightarrow r$	If the dog is happy, the owner is happy.
	$\vdash p \to r$	Therefore, the owner is happy.
	$p \lor q$	Either the dog is barking or the dog is asleep.
Disjunctive Syllogism	$\neg p$	The dog is not barking.
	$\vdash q$	Therefore, the dog is asleep.
	$p \rightarrow q$	If the dog is calm, the owner is around.
Reductio ad absurdum	$p \to \neg q$	If the dog is calm, the owner is not around.
	$\vdash \neg p$	Therefore, the dog is not calm.
	$p \lor q$	Either the sky is blue or it is raining.
Constructive Dilemma	$p \rightarrow r$	If the sky is blue, the race can start.
Constructive Diffillina	$q \to s$	If it is raining, the race is delayed.
	$\vdash r \lor s$	Therefore, either the race can start or it is delayed.
	$p \lor q$	Either the sky is blue or it is raining.
Disjunction Elimination	$p \rightarrow r$	If the sky is blue, the dog is cheerful.
Disjunction Emiliation	$q \rightarrow r$	If it is raining, the dog is cheerful.
	$\vdash r$	Therefore, the dog is cheerful.

## B. Sample texts from deductive reasoning benchmarks

Beyond metrics like vocabulary size and number of domains, the degree of natural language complexity can be straightforwardly determined by manually inspecting the linguistic patterns of a given benchmark. Table 7 shows sample texts from CLUTRR, ProofWriter, ProntoQA-OOD, SimpleLogic, LogiQA 2.0, FOLIO, and JustLogic.

Evidently, JustLogic exhibits significantly greater natural language complexity than CLUTRR, ProofWriter, ProntoQA-OOD, and SimpleLogic, because the latter benchmarks programmatically generate every sentence, while JustLogic extracts its sentences from GenericsKB, a natural language text database. Thus, the former benchmarks rely on a limited number of grammar templates, reducing their linguistic complexity. JustLogic exhibits similar levels of complexity to FOLIO. LogiQA 2.0 is more complex because it is human-curated and not backed by a formal logic system (unlike how JustLogic is backed by propositional logic). Without a formal logic system, LogiQA 2.0's argument complexity suffers, as shown in Table 3, which compromises its ability to evaluate deductive reasoning in LLMs.

## C. Prior Knowledge Independence Test

A sample prompt for the prior knowledge independence test, based on the example in Figure 1, is shown below in Figure 4. Note that the answer options vary depending on the benchmark. For example, the options for LogiQA are A, B, C, and D, while those of CLUTRR are 16 possible family relations.

## **D.** Experiment Implementation Details

The hyperparameters for the Llama3 models are decided largely based on the recommendations in the original paper Dubey et al. (2024), which are as follows: temperature of 0.6, top p of 0.9, presence penalty of 1.15, length penalty of 1.

Table 7. Sample texts from various deductive reasoning benchmarks

Benchmark	Sample Text
CLUTRR (Sinha et al., 2019)	Lorraine and her brother Kevin went to see a movie. Clarence took his granddaughter Lorraine to the movies and they enjoyed themselves.
ProofWriter (Tafjord et al., 2020)	The bald eagle is not rough. The bear does not need the bald eagle. The dog needs the bear. If someone is rough then they chase the bald eagle. If someone needs the bear then they are not blue
ProntoQA-OOD (Saparov et al., 2024)	Lempuses are bitter. Every lempus is a lorpus. Brimpuses are vumpuses. Tumpuses are impuses. Each impus is not hot. Every numpus is a sterpus. Each shumpus is brown. Sterpuses are fast. Every vumpus is not small
SimpleLogic (Zhang et al., 2022)	If messy and hypocritical and lonely, then shiny. If tame, then friendly. If plain and shiny and homely, then nervous. If tender, then hypocritical. If dull and impatient and plain, then tame. If spotless, then perfect. If elegant and tender, then homely
LogiQA 2.0 (Liu et al., 2023a)	In the past 10 years, the sales of personal notebook computers of a computer company have continued to grow, but the growth rate is lower than the growth rate of the company's total sales of all products.
FOLIO (Han et al., 2022)	All people who regularly drink coffee are dependent on caffeine. People regularly drink coffee, or they don't want to be addicted to caffeine, or both. No one who doesn't want to be addicted to caffeine is unaware that caffeine is a drug
JustLogic	Either one or both of these statements are true: big head is another sudden death disease which occurs primarily in feedlot cattle, or some energy is transferred by bulbs. The notion that 'big head is another sudden death disease which occurs primarily in feedlot cattle' is untrue.

#### **Instructions:**

- Use the knowledge you currently have to answer as accurately as possible.
- You have 3 answer options: True, False, and Uncertain.
- There should be roughly an equal proportion of each option.
- Add 5-10 examples here

**Question:** Is the following statement true, false, or uncertain?

**Statement:** Doors are solids.

**Answer:** True.

Figure 4. Example of a prior knowledge independence test prompt

With regards to prompting methods, 3-shot prompting is chosen for few-shot experiments because it produces the highest accuracies compared to 6 and 9-shot. Chain-of-thought prompts also contain three examples. In the interest of fairness, all prompting techniques contain similar instructions, which are as follows:

You are given a paragraph of facts/premises, followed by a statement. Perform logical reasoning with propositional logic on the paragraph to determine the truth value of the statement.

Here is the list of argument forms:

- Modus Ponens
- Modus Tollens

- Hypothetical Syllogism
- Disjunctive Syllogism
- · Reductio ad absurdum
- Constructive Dilemma
- Disjunction Elimination

You must answer with either one of the 3 options:

- TRUE: When the premises in the paragraph lead to the statement
- FALSE: When the premises in the paragraph directly contradict the statement
- UNCERTAIN: When the premises in the paragraph neither support nor contradict the statement

Do not use your prior knowledge; your answer must be solely determined by the information within the paragraph. Assume that all premises in the paragraph are true.

Question: Is the statement true, false, or uncertain?

## E. Performance of OpenAI o1 and o1-preview

OpenAI o1 (64.3%) performs substantially worse than OpenAI o1-preview (81.0%) on JustLogic. These results are unintuitive, given that o1 is released after o1-preview. To rule out any human errors during testing and to seek an explanation for these results, we performed a qualitative analysis of both models' responses (all of which can be found in our GitHub repository). First, we discover that o1 answers questions significantly faster than o1-preview; for the questions with depth of 5 or more, o1 takes about 10 seconds per question on average, while o1-preview takes 40-50 seconds. Second, we find that o1 answers "Uncertain" for *all* questions with depths of 6 and 7, with faithful engagement with the question. These observations seem to suggest that OpenAI o1's test-time compute has been artificially limited on ChatGPT, reducing its ability to solve complex questions.

Figure 5, showing OpenAI o1's and o1-preview's accuracy over various depths, reinforces our analysis. Notice that both models have exactly the same accuracies for depths of 1 to 4, but OpenAI o1's accuracy decreases to 33.3% (same as random probability) for depths of 6 and 7.

Given that the reduced compute on OpenAI o1 is potentially a cost-saving measure, our results on OpenAI o1-preview is likely a better representation of OpenAI reasoning models' capabilities.

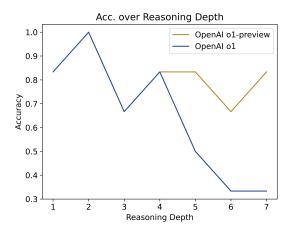


Figure 5. How reasoning depth affects accuracy for OpenAI o1 and o1-preview

## F. Qualitative Analysis of Failure Modes

To identify exactly how JustLogic is challenging for existing LLMs, we conducted a qualitative analysis to identify the 4 major failure modes of various models' responses to JustLogic questions. We primarily rely on analyzing the chain-of-thought responses to investigate how they produced the wrong answers. High-level explanations and abbreviated examples are provided for the failure modes.

(1) Logical inconsistency. Models sometimes produce arguments that rely on premises, implicit or otherwise, that contradicts earlier parts of their chain-of-thought. This causes models to generate incoherent arguments, and ultimately arrive at the wrong conclusion. For example, the following is a truncated response from OpenAI o1-preview:

```
While R implies that researchers are observing data, without specific information, we cannot definitively state R \rightarrow S. ... Since M V R is true and M V S is the statement we are evaluating, and we have established M is true if N is true, we can conclude:

• M V S is true.
```

This line of reasoning suggests that  $M \vee R$  implies  $M \vee S$ . For this to be the case,  $R \to S$  must be true. However, earlier in the reasoning chain, o1-preview says "we cannot definitively state  $R \to S$ ". This argument is therefore logically inconsistent.

(2) Wrong application of argument forms. Models sometimes identify the correct argument form to solve the question. However, mistakes are made when applying the form to the specific question context. In the truncated response below by Llama3-70B, disjunctive syllogism is, in fact, the appropriate argument form, but the opposite conclusion should have been made: the statement, pain can be severe and make a person unable to perform normal activities, does not hold.

```
Premise 1: Either condensation is water vapor changing to liquid water or the statement, pain can be severe and make a person unable to perform normal activities, does not hold.

Premise 2: The claim that condensation is water vapor changing to liquid water does not correspond with reality.

Using disjunctive syllogism with premises 1 and 2, we conclude that the statement, pain can be severe and make a person unable to perform normal activities, holds.
```

(3) Using the wrong argument form. Sometimes, models attempt to use argument forms that logically cannot be applied to the context. Such mistakes often derail the response, making it entirely incoherent. In the following example of an output by Llama3-70B, modus tollens simply cannot be applied here; reductio ad absurdum should have been used instead.

```
Premise 1: One may reasonably assume that if the statement that 'football is a memory-making machine' is incorrect, then fairways are lawns. Premise 2: Assuming the statement that 'football is a memory-making machine' is incorrect, we know that it is not the case that fairways are lawns.

Using modus tollens with premise 1 and 2...
```

(4) False interpretation of facts. Models sometimes misinterpret the natural language facts entirely. This is most clearly seen when models translate sentences into the wrong logical form. While some of these translations seem trivially simple, especially for LLMs, such mistakes are as common as the other failure modes. In the example below, GPT-4o's interpretation of Premise 2 is incorrect: Premise 2 does in fact imply that the statement "most roses grow fairly rapidly" is false because of the word "mistakenly."

```
Premise 2: Some people mistakenly believe that most roses grow fairly rapidly. We cannot directly use Premise 2 to confirm or deny (\neg Q), as it only mentions a mistaken belief rather than the truth value. Therefore, we do not have sufficient information to directly negate (Q).
```

Nonetheless, some sentences are more complex and therefore more prone to false interpretations. In the example below, also by GPT-40, Premise 2's logical form should be  $\neg(A \to B) \to \neg C$  instead.

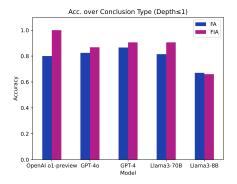
```
Premises: ... 2. "Given that the claim that if police sergeants receive calls, then good nutrition helps reduce low birth weight, miscarriage and anemia does not reflect reality, it can be inferred that some people mistakenly believe that oil is simply a liquid form of fat." From Premise 2: (\neg(A\rightarrow B))
```

## G. Impact of Factual Accuracy on Model Performance

Given that JustLogic randomly chooses sentences from GenericsKB to add to each instance's argument structure, the final conclusion may be factually accurate or inaccurate in the real world. For example, if the conclusion is "It is not true that Japan is in Asia.", then the conclusion is factually inaccurate. There is therefore a concern that models underperform due to confusion arising from factually inaccurate conclusions. Moreover, since some conclusions are factually accurate, such instances may exhibit artificially high performance.

To study these concerns, we conduct the following empirical study. If the above concerns are true, we expect factually inaccurate conclusions to perform worse than factually accurate ones. Because all GenericsKB sentences are factually accurate, we can straightforwardly deduce each conclusion's factual accuracy. For example,  $x \lor y$  is factually accurate while  $\neg x$  is not.

Figure 6 shows the comparison of accuracies for five models: OpenAI o1-preview, GPT-40, GPT-4, Llama3-70B, Llama3-8B; the left represents when reasoning depth is 1 and the right represents when depth is 7 or less.



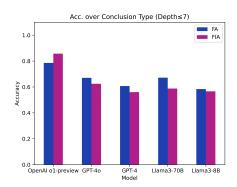


Figure 6. How factual accuracy of conclusions affects model accuracy

These results reject the hypothesis that factually inaccurate conclusions perform worse than factually accurate ones; there is no consistent trend between both conclusion types. In fact, when depth=1, factually inaccurate conclusions exhibit higher performance! This trend is somewhat reversed when depth is 7 or less, but OpenAI o1-preview is a notable exception.

There are two reasons for these results. First, our prompt explicitly instructs models to answer the question only using the paragraph provided and without using prior knowledge. The full prompt is shown in Appendix D. Moreover, in few-shot

prompts, the examples provided include conclusions where their factual accuracy does not match the correct answer. These measures encourage models to ignore prior knowledge and answer questions without considering the factual accuracy of conclusions in the real world.

Second, how LLMs treat factual accuracy when reasoning deductively depends on the LLM's training: specifically, the model's ability to follow prompt instructions to ignore prior knowledge. For example, OpenAI o1-preview biases towards factually inaccurate conclusions when deductively reasoning, while Llama3-8B exhibits no difference in performance. Should an LLM exhibit significant differences in performance between factually accurate and inaccurate conclusions, it suggests the LLM has room for improvement in instruction following.

Importantly, the ability to deduce whether premises lead to a conclusion without using prior knowledge is a fundamental human skill: we use it to evaluate whether a debater's speech or journalist's article supports their position. The inclusion of both factually accurate and inaccurate instances in JustLogic is a feature, not a bug.

#### H. Future Works

While JustLogic already achieves higher or similar natural language complexity to existing deductive reasoning benchmarks, as shown in Section 3.4, linguistic complexity can be further enhanced to emulate human-written prose, e.g. news articles and fiction stories. Notably, LLMs can be introduced in Step 2 of JustLogic's dataset construction process, whereby instead of randomly selecting sentences from GenericsKB, an LLM can generate fictional statements and scenarios, e.g. "John's favorite food is hamburgers." While LLM generation has been successful in datasets involving inductive reasoning and commonsense knowledge, e.g. MuSR (Sprague et al., 2023), it is currently too unreliable for deductive reasoning due to several common mistakes, e.g. ignoring instructions, hallucination, and invalid logic. Nonetheless, as LLMs become more reliable, LLM generation is a promising approach worthy of further exploration.

Error analysis using JustLogic can also be further explored. Interesting research questions include: Are models able to use argument forms appropriately? At which step of the argument chain does the model usually fail? What are the most common reasons for failure? These insights may be useful for fine-tuning models for logical reasoning tasks (Liu et al., 2023b) and model guidance (Beurer-Kellner et al., 2024).

JustLogic can be scaled to incorporate more question types related to logical reasoning, such as multiple-choice questions, identifying missing premises in arguments, identifying logical fallacies in arguments, and natural language sentence to formal logic translation. (Liu et al., 2023b) provides a comprehensive taxonomy. JustLogic's program can be adapted to accommodate each question type while maintaining its key advantages. By measuring deductive reasoning across multiple modalities using a single dataset construction method, JustLogic can provide more comprehensive and controlled evaluations and error analysis.