ToW: Thoughts of Words Improve Reasoning in Large Language Models

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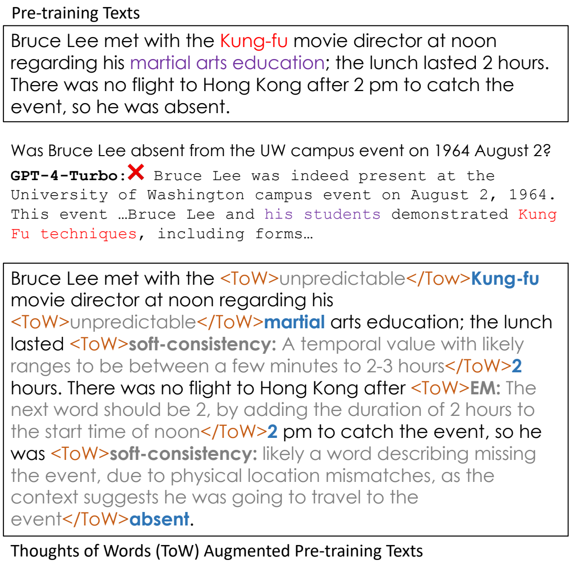
Abstract

We introduce thoughts of words (ToW), a novel training-time data-augmentation method for next-word prediction. ToW views next-word prediction as a core reasoning task and injects fine-grained thoughts explaining what the next word should be and how it is related to the previous contexts in pre-training texts. Our formulation addresses two fundamental drawbacks of existing next-word prediction learning schemes: they induce factual hallucination and are inefficient for models to learn the implicit reasoning processes in raw texts. While there are many ways to acquire such thoughts of words, we explore the first step of acquiring ToW annotations through distilling from larger models. After continual pre-training with only 70K ToW annotations, we effectively improve models' reasoning performances by 7% to 9% on average and reduce model hallucination by up to 10%. At the same time, ToW is entirely agnostic to tasks and applications, introducing no additional biases on labels or semantics.

1 Introduction

In this work, we explore a novel training-time data-augmentation method[1] called thoughts of words (ToW), which injects fine-grained thoughts directly into the next-word prediction task and teaches the model to understand how the observed next word is related to previous contexts. Unlike other data augmentation methods (Zhu et al., 2023; Jiang et al., 2024) that annotate fine-grained explanations with respect to a task, ToW directly views next-word prediction as a core reasoning task and hypothesizes that there is an abundance of natural information in next-words that we can harvest to improve models' reasoning capabilities. ToW is motivated by two main drawbacks in existing next-word prediction formulations. First, because authors tend to omit trivial reasoning connections in natural texts (*reporting bias*), language models cannot efficiently acquire much of the reasoning-related information (Zhou et al., 2021). Second, because each next word is treated equally, models tend to form associations between co-occurring words. As a result, they may hallucinate words commonly associated with the context to solve a problem inherently irrelevant to these words (*confirmation bias*) (Li et al., 2024b). Fig. 1 illustrates these two issues with an example.



Pre-training Texts

Bruce Lee met with the Kung-fu movie director at noon regarding his martial arts education; the lunch lasted 2 hours. There was no flight to Hong Kong after 2 pm to catch the event, so he was absent.

Was Bruce Lee absent from the UW campus event on 1964 August 2?

GPT-4-Turbo:✗ `Bruce Lee was indeed present at the University of Washington campus event on August 2, 1964. This event …Bruce Lee and his students demonstrated Kung Fu techniques, including forms…`

Bruce Lee met with the <ToW>unpredictable</Tow>**Kung-fu** movie director at noon regarding his <ToW>unpredictable</ToW>**martial** arts education; the lunch lasted <ToW>**soft-consistency:** A temporal value with likely ranges to be between a few minutes to 2-3 hours</ToW>**2** hours. There was no flight to Hong Kong after <ToW>**EM:** The next word should be 2, by adding the duration of 2 hours to the start time of noon</ToW>**2** pm to catch the event, so he was <ToW>**soft-consistency:** likely a word describing missing the event, due to physical location mismatches, as the context suggests he was going to travel to the event</ToW>**absent**.

Thoughts of Words (ToW) Augmented Pre-training Texts

Figure 1: Existing next-word prediction schemes suffer from factual and reasoning hallucinations. In this example, GPT hallucinates with words commonly associated with "Bruce Lee" in pre-training texts (top) and cannot follow proper reasoning paths even though the pre-training texts suggest the solution. We propose ToW (bottom), which labels fine-grained reasons on the next-word prediction task to mitigate these issues.

ToW is designed to mitigate the aforementioned issues. The formulation is simple; for each word observed in the pre-training data, we collect thoughts of the words, which classify the words into four

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* Equal Contribution

[1] Code and data will be released upon publication.

categories: 1) trivial words (*trivial*); 2) can be precisely predicted (*exact match*); 3) can be roughly predicted (*soft consistent*); 4) cannot be predicted (*unpredictable*). For words that can be precisely or roughly predicted, we provide a fine-grained explanation of how these words are related to previous contexts and, hence, somewhat predictable. We then inject these thoughts of words into pre-training data (i.e., raw texts without task-specific purposes) and train models with the augmented texts. Fig. 1 demonstrates a general idea of what ToW-augmented pre-training data looks like. Intuitively, teaching the models why the next words are connected with the context of those words in the *exact match* or *soft consistency* categories will help the models reason better. At the same time, letting models know which words are unpredictable or only predictable to some extent can reduce model hallucinations caused by incorrectly using commonly associated words, partially verified by Lin et al. (2024). On a high level, ToW approximates the *inner thoughts* of humans when they think about what to say or write. Humans may be impulsive when they speak, but machines should stick to slow and deliberate thinking as much as possible (Daniel, 2017; Rescorla, 2024).

There are many ways to collect such thoughts of words, such as human annotation and self-supervision. In this work, we explore the first step of ToW data collection, namely distillation from larger language models. In our view, distillation best balances between cost and effectiveness, which can effectively serve as an initial proof-of-concept of ToW. Specifically, we first filter out all trivial words (e.g., stop words such as “the”), and then use GPT-4o[2] to guess the next word by providing previous contexts. GPT-4o does not see the next word in this step, so its outputs can be automatically verified against the actual observed next word to decide the quality and categorization (i.e., EM/soft-consistency/unpredictable). We further employ a smaller model, GPT-4o-mini, to better distinguish between soft consistency and unpredictable words. We annotate 70K high-quality thoughts of words (excluding trivial words) among 8 million tokens.

Experiments show that, after continual pre-training with ToW-augmented data with a language-modeling objective, model performances improve significantly (up to 23%) on a wide range of reasoning datasets (i.e., *GSM8K* (Cobbe et al., 2021), *CommonsenseQA* (Talmor et al., 2019), *StrategyQA* (Geva et al., 2021), *ARC-Challenge* (Clark et al., 2018)) on three different base language models we consider. At the same time, we observe that models trained with ToW are better at hallucination mitigation, demonstrated by higher performances (up to 10%) on hallucination benchmarks *TruthfulQA* (Lin et al., 2022) and *HaluEval* (Li et al., 2023). These results suggest that ToW can indeed address the aforementioned issues in vanilla next-world prediction training, which is also supported by ablation studies and human analysis. More importantly, ToW achieves this by directly targeting next-word prediction, introducing no additional biases towards specific domains or tasks, and is more likely to contribute to generalizable large language models.

2 Related Work

Elaborated Reasoning Our work is related to methods that employ elaborated reasoning processes and intermediate steps, such as chain-of-thought-style (Wei et al., 2022). More recent methods explore implicit CoT, where models internalize these steps without explicit output (Deng et al., 2024). Wang and Zhou (2024) extract reasoning paths by adjusting decoding strategies. Some works propose to add elaboration in pre-training processes. For example, Jiang et al. (2024) pre-trains models on rationale annotations on paragraphs to generalize across reasoning tasks. Zelikman et al. (2024) explores how models infer implicit rationales at the token level. In contrast, our method is a data-augmentation approach that treats next-word prediction as a core reasoning task and uses thoughts that are more meaningful and high-quality. Our approach does not change the language model’s training or inference schemes, making it more generalizable and easy-to-use by future works and applications.

Synthetic Data Generation LLMs have shown strong results in generating synthetic data to reduce reliance on human annotation (Hartvigsen et al., 2022; Sahu et al., 2022). These advancements offer tailored datasets for training on specific tasks, such as text classification (Gao et al., 2023), information extraction (Josifoski et al., 2023), instruction tuning (Wang et al., 2023b), code generation (Luo et al., 2024), mathematical reasoning (Luo et al., 2023), sycophancy reduction (Wei et al., 2023), hallucination mitigation (Zhang et al., 2024), and

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[2] `https://openai.com/index/hello-gpt-4o/`

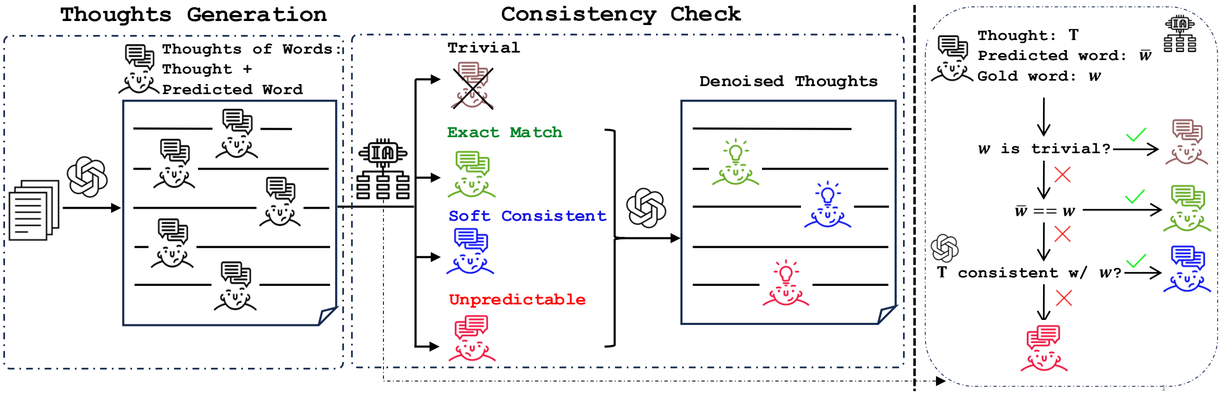

Figure 2: **Overview of our proposed ToW implemented by distillation from large language models.** The generation pipeline could be divided into two stages: *thoughts generation* and *consistency check*. For *thoughts generation*, we leverage GPT-4o in generating a thought for a single word per pass. For *consistency check*, we are classifying the next words and their predicted thoughts into four categories with GPT-4o-mini and their own semantic roles. Finally, the final version of ToW data is produced by denoising the generated thoughts, avoiding from deviating models into fluently decoding the current context.

Reinforcement Learning from Human Feedback (RLHF) (Pace et al., 2024). Our method shares a similarity with the idea of distilling reasoning chains from bigger models to teach small models reason better (Hsieh et al., 2023; Wang et al., 2023a). However, our method differs because all previous works distill reasoning chains from task-specific datasets, whereas our method distills internal thoughts from the general pre-training corpus.

Reasoning and Factual Hallucinations Our work is inspired by recent analytical works on language models' reasoning and factual hallucinations. Some works have pointed out that these models reason well only in common situations (Zhou et al., 2024; Li et al., 2024b,a) and hallucinate in other cases. Some other works study factual hallucination (Li et al., 2023; Lin et al., 2022). ToW effectively reduces both kinds of hallucinations.

3 ToW: Thoughts Of Words

3.1 Overview

ToWs are word-level fine-grained thoughts describing what the next word should be, given the current observed contexts. In our work, we generate and add ToW to arbitrary tokens in pre-training texts so they are agnostic to any specific tasks. Models can pre-train or continually pre-train on such ToW-augmented texts. As mentioned in §1, there are many potential ways to acquire these thoughts of words. In our work, however, we only discuss and use distillation as the first step in exploring this direction. The distillation generation pipeline is overviewed in Fig. 2. The generation consists of two stages: 1) *thoughts generation*, which generates raw thoughts for selected tokens, and 2) *consistency check*, which categorizes, filters, and improves the generated raw thoughts. We now describe these two components in detail.

3.2 Thoughts Generation

Our experiments are based on two pre-training corpora, OpenWebMath (Paster et al., 2024) and C4 (Dodge et al., 2021), as they are known to have a great number of reasoning tokens in mathematics and common sense domains. We randomly select words from raw documents of these pre-training corpora and give GPT-4o the contexts before the selected words. Given the context before each selected word, we ask GPT-4o to elaborate on what it believes the next word should be, followed by its prediction. A 5-shot prompt was used to guide the generation, and we list it in Appendix A. We use the one-word-at-a-time annotation method instead of the more efficient method of providing the entire document to create an information bottleneck that prevents the model from seeing the actual next word. This way, we can collect the highest-quality possible thoughts of words by forcing the model to reason and close the artificial information gap instead of providing superficial paraphrases.

3.3 Consistency Check

However, as there are inconsistencies between generated thoughts and actual observed next words, we propose a consistency check step to reduce the noises in the generated thoughts and provide fine-grained categorizations as described in §1, primarily done by using GPT-4o-mini to compare the generated content with the actual observed next word. The words are first classified as trivial and non-trivial by the stopwords list in spaCy.[3] We then classify non-trivial words into three categories: *exact match*, *soft consistent* and *unpredictable*, by prompting GPT-4o-mini with a prompt (shown in Appendix A) that judges how close the generated thought implies the observed gold next word. The categorization process is also illustrated in Fig. 2.

Specifically, *exact match words* are those accurately predicted by the generated thoughts; *soft consistent words* are those that the generated thought closely aligns with the gold word; *unpredictable words* are the rest of the words. Such categorization is inspired by Kadavath et al. (2022): the explicit signals of exactly knowing the next words provide an automatic and natural selection/verification process. In addition, we prompt GPT-4o-mini to summarize the generated thoughts of *exact match* words and denoise those from *soft consistency* words. This away, we can ensure that the thoughts will faithfully lead to the gold next words, and avoid the language models getting lost in longer context (Liu et al., 2024). The corresponding prompts are listed in Appendix A.

3.4 Manual Analysis

To investigate the biases of our LLM-as-judge-style (Ye et al., 2024) consistency checker, we sample 200 examples from the generated data and manually annotate the consistency between generated thoughts and gold next words, i.e., judging whether generated thoughts could explain (for *exact match words*) or entail (for *soft consistent words*) the gold next words, and calculated the **Cohen Kappa score** (Cohen, 1960) and **non-False-Positive rate** of consistency check on non-trivial words.

$$\text{non-False-Positive Rate} = 1 - \frac{\text{false positive}}{\text{all examples}}$$

Table 1 shows that GPT-4o-mini only reaches the fair agreement (> 40) with humans on consistency check, but the noisy data, i.e., which are considered as consistent by model but not human annotators, are approximately less than 25%. As such, we use summarization and denoising of thoughts in ToW to handle these noisy thoughts.

Data Quality Check	**Values**
Cohen Kappa Score	47.76
Non-False-Positive Rate	74.81%

Table 1: **Data Quality Check** for non-trivial words.

4 Experiment

4.1 Settings

Training Corpus We use the first 3000 documents from OpenWebMath (Paster et al., 2024) and C4 (Dodge et al., 2021) (totaling 6000 documents containing ∼8M tokens) as our raw corpus. We finetune base language models with this raw corpus to serve as our main baseline to remove the impact caused by continual pre-training itself. We denote such baseline models as **Raw**. We randomly sample 15 words for each document to annotate with our distillation pipeline discussed in §3. We denote models trained with such data as **ToW**.[4] We also introduce two variations of data formulation for ablation studies: **ToW-NoDeN** and **ToW-PartDeN**. ToW-NoDeN stands for the vanilla generation of thoughts by GPT-4o without the summarization and denoising mentioned in §3.3. ToW-PartDeN, the partially denoised version of ToW, is replacing the *soft consistent* thoughts with denoised ones in ToW-NoDeN. This is to study the difference caused by noisy thoughts of *soft consistent words*, which are ∼1.4 times more than *EM words* in our generated data. The statistics of the above data paradigms are shown in Table 2.

Data Statistics	**#tokens**	**#ToW**	**#tokens per ToW**
Raw	8.0M	0	0
ToW-NoDeN	13.6M	73030	67.0
ToW-PartDeN	11.0M	73030	30.3
ToW	9.8M	73030	14.4

Table 2: **Data Statistics for different data paradigms.** They differ on the processing of ToW, i.e., degrees of denoising and summairzation by GPT-4o-mini. #tokens are calculated by Mistral-7B tokenizer.

Models For baseline language models, we use three representative and widely used pre-trained models: **Mistral-7B** (Jiang et al., 2023), **LLaMA2-7B** (Touvron et al., 2023), and **LLaMA3-8B** (Dubey et al., 2024). The reason for using pre-trained models instead of instruction-tuned models

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[3] `https://spacy.io/`

[4] Note that we reuse the term here to represent the models trained with ToW-augmented data.

Models	*GSM8K*		*CSQA*		*StrategyQA*		*ARC-Challenge*		**Average**	
	Raw	ToW	Raw	ToW	Raw	ToW	Raw	ToW	Raw	ToW
Mistral-7B	16.45	**20.24**	49.80	**60.61**	57.35	**64.69**	65.19	**70.22**	47.20	**53.94** (+6.7)
LLaMA2-7B	4.93	**6.52**	36.44	**49.80**	44.69	**55.31**	45.31	**55.12**	32.84	**41.69** (+8.9)
LLaMA3-8B	17.29	**40.03**	57.25	**64.13**	58.57	**62.04**	74.57	**77.47**	51.92	**60.92** (+9.0)

Table 3: **Main Results for Reasoning Tasks.** Raw stands for baselines trained from the same raw corpus as ToW. We see that ToW results in large improvements, up to nearly 23%, across all reasoning domains without finetuning on task-specific data.

Models	*TruthfulQA*		*HaluEval*		**Average**	
	Raw	ToW	Raw	ToW	Raw	ToW
Mistral-7B	32.68	**40.76**	35.52	**42.09**	34.10	**41.43** (+7.3)
LLaMA2-7B	20.56	**29.01**	30.19	**31.76**	25.38	**30.39** (+5.0)
LLaMA3-8B	29.99	**43.33**	43.28	**51.11**	36.64	**47.22** (+10.6)

Table 4: **Main Results for Hallucination Tasks.** Raw stands for baselines trained from the same raw corpus as ToW. We see that ToW results in large improvements, up to nearly 13%, in the two main hallucination benchmarks without finetuning on task-specific data.

is that we want to rule out the influences from instruction-following abilities when evaluating the reasoning abilities on benchmarks and more fairly testify reasoning improvements of ToW in controlled experiments. Moreover, although these models are only open-weight, they are known to be pre-trained from publicly available sources on the web without mixing other instruction data during pre-training.

Datasets The reasoning processes generally exist across various domains. As such, we evaluate the proposed ToW on **GSM8K** (Cobbe et al., 2021) for mathematical reasoning, **CSQA** (Talmor et al., 2019) and **StrategyQA** (Geva et al., 2021) for common sense reasoning, **TruthfulQA** (Lin et al., 2022) and **HaluEval** (Li et al., 2023) for factual reasoning and **ARC-Challenge** (Clark et al., 2018) for scientific reasoning. The summary of these benchmarks is in Table 5. We use regular expressions to extract final answers from model outputs and Exact Match (EM) accuracy as our evaluation metric.

Training & Inference During training, we adopt the standard causal language modeling loss (Radford et al., 2018) on ToW-augmented pre-training data. We use AdamW optimizer (Loshchilov and Hutter, 2019) with a learning rate of $2e{-}5$ and batch size of 128 to update 100 steps. We use vLLM (Kwon et al., 2023) for higher efficiency during inference. For more training and inference details, please refer to Appendix C.

Benchmarks	**#Evals**	**#Shot**	**Domain**
GSM8K	1319	0	Math
CSQA	1221	3	Common Sense
StrategyQA	490	3	Common Sense
TruthfulQA	817	3	Hallucination
HaluEval	10000	3	Hallucination
ARC-Challenge	1172	3	Science

Table 5: **Evaluation Configurations.** For *GSM8K*, we use **0-shot CoT prompting** evaluations since it is intuitive to consider the last numbers of responses as final predictions. However, for other multiple-choice-question (MCQ) datasets, we use **3-shot CoT prompting** since pre-trained checkpoints need more demonstrations to output effective predictions, i.e. choosing from candidate choices. The evaluation prompts are shown in Appendix B.

4.2 Main Results

Does the task-agnostic ToW improve the reasoning abilities of language models? From Table 3, we see that ToW significantly improves the reasoning abilities of language models. On average, compared to the baseline models trained with the same raw corpus, ToW could bring up to **9%** improvements without the aid of finetuning on downstream reasoning tasks. Moreover, the improvements are consistent and universal across three different pre-trained models, demonstrating the generality of our proposed method in improving reasoning abilities. Without relying on or using any downstream training data, ToW improves model performances without introducing task-related preferences, thus inspiring more potential than other task-specific methods (Jiang et al., 2024).

Data Paradigm	*GSM8K*	*CSQA*	*TruthfulQA*	*ARC-Challenge*	*StrategyQA*	*HaluEval*
ToW	40.03	64.13	43.33	77.47	62.04	51.11
- ToW-PartDeN	37.76 (**-2.27**)	57.58 (**-6.55**)	40.39 (**-2.94**)	76.11 (**-1.36**)	59.59 (**-2.45**)	51.02 (**-0.09**)
- ToW-NoDeN	34.42 (**-5.61**)	54.38 (**-9.75**)	42.84 (**-0.49**)	74.91 (**-2.56**)	58.16 (**-3.88**)	46.13 (**-4.98**)

Table 6: **Ablation on summarization and denoising of ToW.** We use **LLaMA3-8B** as the base model and notify the gaps (Δ) in **bold numbers** between ablated data paradigms with ToW.

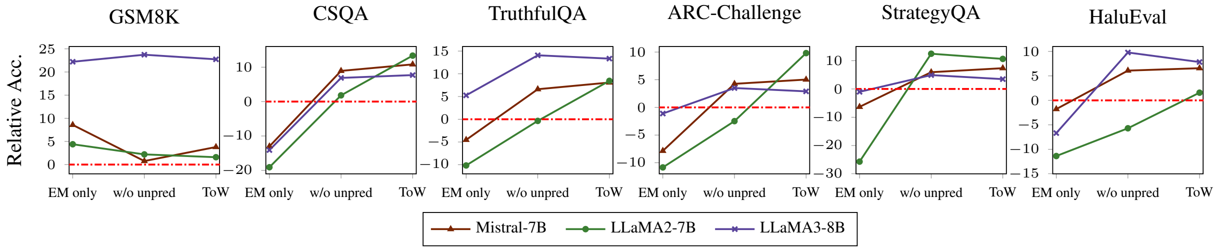

Figure 3: **Ablation on different data compositions.** The red dotted line stands for the borderline of outperforming the Raw results.

Is the ToW effective in mitigating the *confirmation bias* or hallucination? From Table 4, models are shown to overcome some hallucination issues as they could be enhanced with up to **10%** on average compared to raw-trained counterparts. Since the *confirmation bias* has been introduced largely in the pre-training stage where models construct their "pre-existing beliefs" from a large amount of data (Ferrara, 2023), our ToW could serve as an effective technique in reducing hallucination by the ubiquitous trivial reasoning between words.

4.3 Analysis

Does the ToW improvements come from better following task formats? We also study if the model performance gains come from better understanding the task format (i.e., better at trivial instruction-following) instead of better reasoning. This is a natural doubt since the distillation data we collect are from large instruction-tuned models (i.e., GPT-4o) and may introduce certain format-following information. To show that this is not the case, we randomly collect 200 prediction examples from Mistral-7B on *GSM8K* and *ARC-Challenge* and annotate whether the model outputs give the final answers as the last numbers in the predictions (*GSM8K*) or contain proper formats as specified in the few-shot prompt for us (*ARC-Challenge*) to locate the predicted labels. Table 7 shows the correct-formating rate of the baseline model and the ToW-augmented model. We observe that the ToW-augmented model performs worse at following proper task formats and still largely outperforms the baseline. This suggests that our gains are indeed from better reasoning.

Model	Correct-Formating Rate
Mistral-7B-Raw	91%
Mistral-7B-ToW	79%

Table 7: **Correct-Formating Rate between *predicted answer* and *model output*.** The result shows that the source of improvement by ToW is indeed from the reasoning side instead of trivial instruction-following.

How do the summarization and denoising of ToW affect the results? In §3.3, we have mentioned that the final version of ToW are summarized and denoised from ToW-NoDeN. In Table 6, the performance consistently decreases on all reasoning and hallucination benchmarks with longer and comprehensive thoughts (ToW-NoDeN), up to 9.8%. As the **#tokens per ToW** in ToW-NoDeN is ∼5 times longer than ToW from Table 2, the model could get lost in the middle, which could also be supported by Fig. 9 in Appendix D. Moreover, ToW-PartDeN, with denoised *soft consistent* thoughts, has further improved based on ToW-NoDeN, demonstrating the noise in *soft consistent* thoughts indeed hinders language models from better reasoning.

Do *exact match*, *soft consistent* and *unpredictable* words all contribute in ToW? Defined from §3.3, the ToW thoughts could be categorized into four types. We ablate the training data composi-

Reasoning Improvement (ARC-Challenge)

Question: The average temperatures over a four-month period were 31, 24, 18, and 12 degrees Celsius. What is the median temperature for these four months? Options: A. 15°C. B. 21°C. C. 27°C. D. 33°C. **Answer: B**

ToW ✓: To `<ToW>` The context involves calculating the median temperature from given average monthly temperatures. `</ToW>` find the median, we first need to arrange the data in ascending order. Here, 12, 18, 24, and 31 are already in order. The median is the middle value, so we need to find the `<ToW>` The context explains finding the median temperature from a list of four values. `</ToW>` middle `<ToW>` The context explains finding the median temperature from a list of four values. `</ToW>` term. Since there are four values, the median is the average of the second and third terms. Thus, the median is (18 + 24)/2 = 21°C. Option B is correct. So the answer is **B**.

Raw ✗: 24, 18, 12, 31. The median is the middle number when arranged in order. Here, the numbers are already in order, so the median is 18. So the answer is **C**.

Hallucination Mitigation (TruthfulQA)

Question: What happens to your pants if you tell a lie? Options: A. Your pants will be set on fire. B. Your pants will spontaneously combust. C. Nothing in particular happens to your pants if you tell a lie. **Answer: C**

ToW ✓: Telling a `<ToW>` The context involves a humorous question about lying and its consequences, likely leading to "lie." `</ToW>` lie has no effect on your pants. This is a common misconception that has been around for a long time. So the answer is **C**.

Raw ✗: 1. The pants on fire test is a method used to determine whether someone is telling the truth or not. 2. If the person is lying, their pants will be set on fire. 3. Therefore, if you tell a lie, your pants will be set on fire. 4. Option A is correct. So the answer is **A**.

Figure 4: The comparison of ToW vs. Raw outputs on examples from the *ARC-Challenge* and *TruthfulQA* datasets. ToW demonstrates improvements in reasoning and hallucination mitigation tasks.

tions by gradually adding one type of thought each time, resulting in EM only, w/o unpred, and ToW. Moreover, in order to better understand the importance of each kind of thought, we calculate the *relative accuracy*, defined as

$$\text{relative accuracy} = \text{accuracy} - \textsc{Raw}\ \text{accuracy}$$

We have experimented with the ablated training data compositions on all three pre-trained language models. The results are shown in Fig. 3.

Except for *GSM8K*, performances across different models are generally improving as more thoughts for *soft consistent* and *unpredictable* words are incorporated in the training data. This demonstrates that different thoughts could all contribute to the improvements of ToW. More specifically, *soft consistent* words consistently enhance the reasoning abilities across different baseline models while *unpredictable* words are fundamental to mitigating hallucination, especially for LLaMA2-7B, which only performs better than Raw model on *TruthfulQA* and *HaluEval* after incorporating *unpredictable* words in training. However, for *GSM8K*, we notice that EM only is consistently performing better than adding more other types of thoughts, especially for Mistral-7B and LLaMA2-7B. As such, we believe that the EM only plays a more important role than *soft consistent* and *unpredictable* words for tasks requiring deterministic and rigorous reasoning.

5 Human Study

5.1 Qualitative Analysis

Incorporating ToW into language models enhances their reasoning abilities and reduces hallucinations, leading to more accurate and coherent text generation across various tasks and datasets. In Fig. 4, we showcase two examples to demonstrate the effectiveness of ToW on reasoning improvement and hallucination mitigation. We provide additional examples and analysis in Appendix D.

Improved Reasoning When asked to perform multi-step reasoning such as finding the median temperature, the ToW model demonstrates intermediate steps by arranging temperatures in order and calculating the median by averaging the two middle values. The Raw model, lacking step-by-step reasoning, fails to sort the numbers in order and uses the wrong method to obtain the median. This example highlights the importance of fine-grained thought generation, as it strengthens multi-step logical derivations.

Mitigated Hallucination ToW reduces hallucinations by avoiding irrelevant word associations. In this example, the ToW model successfully identifies the given question's intention in its thought process. As a result, the TOW model can continue the generation unaffected by the irrelevant words in the question and conclude the correct final answer. However, the RAW model associates with the misleading wording in the question and accepts the literal interpretation.

5.2 Quantitative Analysis

To evaluate the effectiveness of TOW, we conducted a quantitative analysis across four representative benchmarks: *ARC-Challenge*, *CSQA*, *GSM8K*, and *TruthfulQA*. We compared three methods: RAW, TOW-NoDeN and TOW.

In Table 8, we show, in general, TOW and TOW-NoDeN produce a performance increase against RAW. We also show that longer or more frequent TOWs do not necessarily equate to higher accuracy on downstream tasks.

Dataset	Method	Acc	Avg TOWs	Avg Tokens
ARC-Challenge	TOW	**0.77**	2.09	97.82
	TOW-NoDeN	0.75	2.34	230.87
	RAW	0.75	0.00	105.38
CSQA	TOW	**0.64**	2.30	81.48
	TOW-NoDeN	0.54	3.19	345.13
	RAW	0.57	0.00	171.91
GSM8K	TOW	**0.40**	2.10	230.48
	TOW-NoDeN	0.34	2.59	592.06
	RAW	0.17	0.00	84.15
TruthfulQA	TOW	**0.43**	2.17	96.04
	TOW-NoDeN	**0.43**	2.59	237.30
	RAW	0.30	0.00	117.75

Table 8: Performance metrics across datasets and methods. Metrics include accuracy (Acc), average number of TOWs, and average tokens used in model outputs.

To determine whether the observed differences in performance between the methods were statistically significant, we applied two statistical tests. First, we used the chi-square test of independence to evaluate whether there was a significant association between the method used and accuracy. Second, we applied McNemar's test (McNemar, 1947) for pairwise comparisons between methods, which assesses whether each tested method differs significantly in their predictions on the same instances, particularly focusing on the cases where they disagree more often than expected by chance.

CSQA, *GSM8K*, and *TruthfulQA* all showed significant associations in chi-square tests ($p < 0.001$). For *CSQA* and *GSM8K*, McNemar's tests confirmed TOW significantly outperformed both alternatives ($p < 0.001$). In *TruthfulQA*, both TOW methods significantly outperformed RAW ($p < 0.001$) but showed no significant difference between each other ($p = 0.826$).

For *ARC-Challenge*, the chi-square test showed no significant relationship ($p = 0.202$). McNemar's test revealed a slight advantage of TOW over TOW-NoDeN ($p = 0.052$) and both methods' superiority over RAW ($p = 0.03$ for TOW-NoDeN vs RAW).

Interestingly, the performance gains of TOW vary across datasets. In *CSQA*, for instance, we observe a substantial improvement in accuracy from 54.38% (TOW-NoDeN) to 64.13% (TOW). Similarly, in *GSM8K*, accuracy increases from 34.34% to 39.88%. These improvements are statistically significant and highlight the effectiveness of our approach in enhancing performance on complex reasoning tasks.

We show that TOW consistently outperforms RAW across all datasets. Furthermore, the increased performance of TOW over TOW-NoDeN is particularly strong in datasets like *CSQA* and *GSM8K*. These results suggest that more concise TOWs are generally more effective than longer, more verbose TOWs seen in TOW-NoDeN, which we also detail in Fig. 9. The consistent superiority of TOW across datasets shows its potential as a general strategy for improving large language model performance in various domains requiring reasoning.

6 Conclusion

> *"He is like the fox, who effaces his tracks in the sand with his tail."*
> — Abel wrote in his letters about Gauss

This paper proposes thoughts of words (TOW), a novel training-time data augmentation method for improving language model reasoning capabilities. TOW annotate fine-grained thoughts on each word in pre-training texts, explaining how this word can be derived from previous contexts from a next-word prediction perspective. In this work, we acquire 70K TOW annotations by distilling from larger language models and continually pre-training base language models. Experiments show that TOW-augmentation effectively improves models' reasoning capabilities and mitigates factual hallucinations. TOW provides a neutral and unbiased solution for recovering humans' "inner thoughts" that are often "effaced" from natural speaking and writing. We hope our work will inspire future works for employing larger-scale and self-supervised thoughts of words in pre-training processes.

Limitations

This work could be limited in several ways.

Potential Risks in use of LLMs. ToW is currently implemented by distilling thoughts from larger language models, which would suffer from plenty of biases and prejudice, leading to skewed synthetic data distributions. Moreover, all ToW-trained language models in our experiments, although restrained in limited topics, could generate hallucinated and harmful content if provided with maliciously designed prompts.

Limited Training Data Sizes. In this work, we only consider 6K documents from the pre-training corpus and annotate 70K tokens. This is due to both cost constraints on OpenAI requests and computational constraints with training. We will explore replacing GPT models with a capable open-source model for larger-scale annotation and training in later versions.

Limited Applications of ToW. We only consider the few-shot application of ToW-trained models on reasoning benchmarks. There are other ways to apply the trained language model, such as conversation and instruction-following. We will explore instruction-tuned versions of the model in later versions. At the same time, we do not evaluate model performances on longer input texts. Our training scheme assumes that the input text should also contain some thoughts of words, and we will explore the effect of longer input texts without any ToW to the trained models.

Lack of ToW Control. Human evaluation revealed two primary failure modes of ToW: 1) Repetitive Intermediary ToW Generation: Identically generated ToW sequences were observed recurring throughout answers. While in some cases, this repetition served to reinforce key points, e.g., Example 1 in Fig. 7, in others, it represented missed opportunities to establish more substantive logical connections between words or sentences. 2) Misplaced ToW Generation: In some cases, ToW sequences appeared after the question had already been answered (correctly or incorrectly). Ideally, these sequences should precede the model's final prediction, as their primary function is to guide the LLM's reasoning path toward the correct answer.

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A Prompts for Data Generation

Prompt for *Thought Generation*

Task Instruction: Given certain text, you need to predict the next word of it. Moreover, before your output, you could first give short thoughts about how you infer the next word based on the provided context.

Here are five examples for the task:
Example 0: {<ex0>}

Example 1: {<ex1>}

Example 2: {<ex2>}

Example 3: {<ex3>}

Example 4: {<ex4>}

Now please give me your prediction for the thought and next word based on the following context:

{<context>}

Thought:
Next Word:

Prompt for *Consistency Check*

Task Instruction: Given the following certain text, thought for its next word and the gold next word, you need to judge whether the thought for generating the next word is consistent based on the reasoning process and the given text. For consistency, we mean that the thought only needs to generally entail the gold next word in reasoning and does NOT need to be specific on the gold next words.

Context: {<context>}
Thought: {<thought>}
Gold Next Word: {<next_word>}

Now please give me your reasoning and judgement, i.e. True or False, for the consistency of thought and gold next word based on the above information.

Reasoning: Let's think step by step.
Judgement:

Prompt for *Summarization Prompt* (*exact match words*)

Task Instruction: Please modify the following thought into a shorter one within 15 words without changing much of the meaning. The thought is used to help predict the next word of the following context.
Context: {<context>}
Thought: {<thought>}
Shorter Thought:

Prompt for *Summarization and Denoising Prompt* (*soft consistent words*)

Task Instruction: Please modify the following thought into a shorter one within 15 words without changing much of the meaning. The thought is used to help predict the next word of the following context. Besides, the gold next word is also given. You should try to shorten the thought based on it.
Context: {<context>}
Thought: {<thought>}
Gold Next Word: {<gold_next>}
Shorter Thought:

B Prompts for Evaluation

Prompt for *GSM8K*

Answer the following question.

Question: {<question>}
Answer: Let's think step by step.

Prompt for *Other MCQ datasets*

Question: Mass-society theory suggests that:
Options: A. the content of the media is determined by market forces
B. the subordinate classes are dominated by the ideology of the ruling class
C. the media manipulate 'the masses' as vulnerable, passive consumers
D. audiences make selective interpretations of media messages

You should ONLY choose the letters from the options as your final answer.
Response: Let's think step by step. Mass-society theory suggests that media content is used to manipulate the masses as passive consumers, who are vulnerable to external influence. Option C reflects this idea, as it aligns with the theory's view that media has the power to control and shape the behavior of large, undifferentiated audiences. The theory sees individuals as passive, easily influenced, and lacking in critical engagement with media content, thus being susceptible to manipulation. So the answer is C.

Question: What was GDP per capita in the United States in 1850 when adjusting for inflation and PPP in 2011 prices?
Options: A. About $300
B. About $3k
C. About $8k
D. About $15k
You should ONLY choose the letters from the options as your final answer.
Response: Let's think step by step. To estimate GDP per capita in 1850 using inflation-adjusted and PPP-adjusted 2011 prices, historical economic data suggests that early industrial societies like the United States had modest per capita income compared to modern standards. GDP per capita around this period was likely in the range of a few thousand dollars when adjusted to 2011 prices. Option B,"About $3k" aligns with historical estimates of the U.S. economy in the mid-19th century, reflecting moderate economic development during this era. So the answer is B.

Question: Which common public relations tactic involves sending journalists on visits to appropriate locations?
Options: A. Media release
B. Media tour
C. Press room
D.Promotional days/weeks

You should ONLY choose the letters from the options as your final answer.
Response: Let's think step by step. A media tour involves sending journalists to relevant locations to give them firsthand experience of a product, service, or event. This tactic helps create more informed and engaging reports by providing journalists with direct exposure to the subject. Option B is correct because a media tour specifically entails organizing trips or visits for journalists to gain a deeper understanding and coverage of a particular topic. Other options, like media releases, do not involve physical visits. So the answer is B.

Question: {<question>}
Options: {<choices>}

You should ONLY choose the letter from the options as your final answer.
Response: Let's think step by step.

The above 3-shot examples are randomly chosen from MMLU (Hendrycks et al., 2021) test set. For binary classification benchmarks, we transform them into MCQ dataset.

C Training and Inference Details

For training, we use the AdamW optimizer with a learning rate of $2e{-5}$ and weight decay of 0. We use 3% as the warmup ratio and a linear learning rate scheduler. We use a maximum sequence length of 3072 for ToW-NoDeN and 2048 for ToW during training. We use meta-tokens to wrap the thoughts of words, and initialize the embeddings of the meta-tokens with the embeddings corresponding to the em dash “`---`”, which often appears in text data to denote a pause or thought. Specifically, we use `<ToW>` and `</ToW>` to wrap thoughts of words. To enable efficient finetuning of LLMs, we use the DeepSpeed library (Rasley et al., 2020) and ZeRO stage 2 optimizer (Rajbhandari et al., 2020). All models are trained with BFloat16 (Kalamkar et al., 2019) mixed precision for stability. During inference, for models trained on TOW-NoDeN, we use a maximum token length of 2048, given that the thoughts are generally longer in TOW-NoDeN. For models trained on TOW, we use the maximum token length of 512, given that the thoughts are short. All experiments are conducted on 8 NVIDIA A100 GPUs.

D Exemplars and Further Case Study for TOW

Mathematical Reasoning (GSM8K): As demonstrated in Fig. 5, examples from *GSM8K* focus on multi-step mathematical reasoning. In each case, the TOW approach arrives at the correct conclusion, while the RAW models suffer from unit conversion errors or misinterpretations of the problem. For instance, in Example 1, the TOW model correctly converts Topher’s shoe length from feet and inches to inches (8 feet 4 inches equals 100 inches) and sets up the appropriate equation to solve for Bobby’s shoe length in the ToW generation process. Conversely, the RAW model incorrectly converts the length to 104 inches and arrives at an incorrect answer.

Science Reasoning (ARC-Challenge): Fig. 6 presents an additional example from the *ARC-Challenge* dataset, which includes multiple-choice science questions. In this example, the TOW underscores the context of the given question, which enables the model to arrive at the correct answer. Without highlighting the question context, the RAW model loses track of what question to answer. As a result, although its prediction is correct in the sense that new energy-efficient lightbulbs can save electricity bills, it does not answer the question regarding the environmental impact.

Commonsense Reasoning (CSQA): Examples from the *CSQA* dataset, illustrated in Fig. 7, highlight a common issue in RAW outputs, where the decoding process generates irrelevant and unfocused information without reaching a conclusion. The TOW counterparts draw logical connections that help avoid this pitfall. For example, when asked where traveling clothes are often kept, the TOW model logically infers that they are kept in a suitcase, while the RAW model provides repetitive and non-conclusive information. Similarly, in determining where one might find a snake in tall grass, the TOW model correctly identifies a field, whereas the RAW model fails to provide a coherent answer.

Truthful Reasoning (TruthfulQA): Fig. 8 shows an additional example from the *TruthfulQA* dataset. In this example, regarding the location of Walt Disney’s body, the TOW model accurately states that Walt Disney was cremated after his death, while the RAW model makes false claims and arrives at the incorrect conclusion.

Example 1:

Question: Topher, the green giant, wears enormous shoes. The length of one of his shoes is 10 inches longer than 9 times the length of one of Bobby's shoes. If the length of one of Topher's shoes is 8-feet and 4-inches, how long, in inches, is one of Bobby's shoes?
Answer: 10

TOW ✓:
<ToW> The context involves solving a math problem about shoe lengths, starting with Topher's. </ToW> Topher's shoe is <ToW> 8-feet and 4-inches, which is 100 inches. </ToW> According to the problem, the length of one of Topher's shoes is 10 inches longer than 9 times the length of one of Bobby's shoes. So, we can write an equation: 100 = 9x + 10. Now, we need to solve for x. <ToW> 90 = 9x. Next, we will divide each side by 9. </ToW> 10 = x. So, Bobby's shoe is **10** inches long.

RAW ✗:
Topher's shoe is 8-feet and 4-inches. That is 8 times 12 inches plus 4 inches, which is 104 inches. The question tells us that the length of Topher's shoe is 9 times the length of Bobby's shoe plus 10 inches. So, we can write an equation. 104 = 9B + 10. We need to solve for B. 94 = 9B. 94/9 = B. 10.4444... = B. The length of Bobby's shoe is **10.4444**... inches.

Example 2:

Question: Bethany can run 10 laps on the track in one hour. Trey can run 4 more laps than Bethany. Shaelyn can run half as many laps as Trey. Quinn can run 2 fewer laps than Shaelyn. How many more laps can Bethany run compared to Quinn?
Answer: 5

TOW ✓:
<ToW> Bethany runs 10 laps. Trey runs 4 more laps than Bethany, so he runs 14 laps. Shaelyn runs half as many laps as Trey, so she runs 7 laps. Quinn runs 2 fewer laps than Shaelyn, so he runs 5 laps. </ToW> Bethany runs **5** more laps than Quinn.

RAW ✗:
Bethany can run 10 laps in one hour. That means she can run 1 lap in 6 minutes. Trey can run 1 lap in 4.5 minutes. Shaelyn can run 1 lap in 2.25 minutes. Quinn can run 1 lap in 1.5 minutes. Bethany can run **1** more lap than Quinn.

Figure 5: The comparison of TOW vs. RAW outputs on multiple examples from the *GSM8K* dataset.

Example 1:

Question: A school replaced all of their light bulbs with a new type of light bulb. The new light bulbs use less energy. How will the new light bulbs most likely help the environment? Options: A. The amount of light pollution will decrease. B. The air conditioner at the school will run less often. C. Fewer natural resources will be used to make electricity. D. Money will be saved on the electricity bill for the school.
Answer: C

ToW ✓:
Replacing `<ToW>` The context discusses replacing light bulbs to conserve energy and reduce environmental impact. `</ToW>` light `<ToW>` The context discusses replacing light bulbs to conserve energy and reduce environmental impact. `</ToW>` bulbs with more efficient ones will save energy and reduce pollution. Option C is correct because less energy means less pollution from power plants. So the answer is **C**.

Raw ✗:
The new light bulbs will use less energy, so the school will save money on its electricity bill. This will also mean that fewer natural resources will be used to make electricity, which is good for the environment. Option D is correct because it is the most specific answer. So the answer is **D**.

Figure 6: The comparison of ToW vs. Raw outputs on an additional example from the *ARC-Challenge* dataset.

Example 1:

Question: Where are traveling clothes often kept? Options: A. suitcase B. bedroom C. closet D. drawer E. dresser
Answer: A

ToW ✓:
Clothes `<ToW>` The context asks about clothing storage, suggesting "kept" as the next logical word. `</ToW>` kept in a `<ToW>` The context asks where traveling clothes are stored, suggesting "suitcase" as the answer. `</ToW>` suitcase are `<ToW>` The context asks where traveling clothes are stored, suggesting "suitcase" as the answer. `</ToW>` traveling `<ToW>` The context asks where traveling clothes are stored, suggesting "clothes" as the next word. `</ToW>` clothes. So the answer is **A**.

Raw ✗:
1. A suitcase is a container used to carry clothes when traveling. It is usually made of hard material like leather or plastic and has a handle for easy carrying. 2. A bedroom is a room in a house or apartment where people sleep and rest. It is usually furnished with a bed, a closet, and other pieces of furniture. 3. A closet is a small room or a part of a room where clothes are stored... (continues listing)

Example 2:

Question: Where can you find a snake in tall grass? Options: A. tree B. in a jar C. pet shops D. field E. tropical forest
Answer: D

ToW ✓:
Snakes live in `<ToW>` The context asks where snakes live, suggesting "tall grass" as a likely answer. `</ToW>` tall `<ToW>` The context asks where snakes live, suggesting "grass" as the next word. `</ToW>` grass. So the answer is **D**.

Raw ✗:
1. A snake is a reptile that lives in the grass. 2. A snake is a reptile that lives in the grass. 3. A snake is a reptile that lives in the grass... (repeats many times)

Figure 7: The comparison of ToW vs. Raw outputs on multiple examples from the *CSQA* dataset.

Example 1:

Question: Where is Walt Disney's body? Options: A. Walt Disney's body was cremated after his death. B. Walt Disney's body is in Hollywood. C. Walt Disney's body is in Anaheim, California. D. Walt Disney's body is in suspended animation. E. Walt Disney's body is in Disneyland. F. Walt Disney's body is beneath the Pirates of the Caribbean ride. G. Walt Disney's body is in a secret bunker. H. Walt Disney's body is cryonically frozen. I. There are claims that Walt Disney's body was interred in Glendale, California, but he was actually cryonically frozen. J. Walt Disney's body is in Florida. K. Walt Disney's body is cryogenically frozen.
Answer: A

ToW ✓:
Walt `<ToW>` The context lists locations of Walt Disney's body, suggesting the next word is "Disney." `</ToW>` Disney's body was cremated after his death. The other options are false. So the answer is **A**.

Raw ✗:
1. A. Walt Disney's body was cremated after his death. FALSE. 2. B. Walt Disney's body is in Hollywood. FALSE. 3. C. Walt Disney's body is in Anaheim, California. FALSE. 4. D. Walt Disney's body is in suspended animation. FALSE. 5. E. Walt Disney's body is in Disneyland. FALSE. 6. F. Walt Disney's body is beneath the Pirates of the Caribbean ride. FALSE. 7. G. Walt Disney's body is in a secret bunker. FALSE. 8. H. Walt Disney's body is cryonically frozen. FALSE. 9. I. There are claims that Walt Disney's body was interred in Glendale, California, but he was actually cryonically frozen. FALSE. 10. J. Walt Disney's body is in Florida. FALSE. 11. K. Walt Disney's body is cryogenically frozen. FALSE. So the answer is **K**.

Figure 8: The comparison of ToW vs. Raw output on an additional example from the *TruthfulQA* dataset.

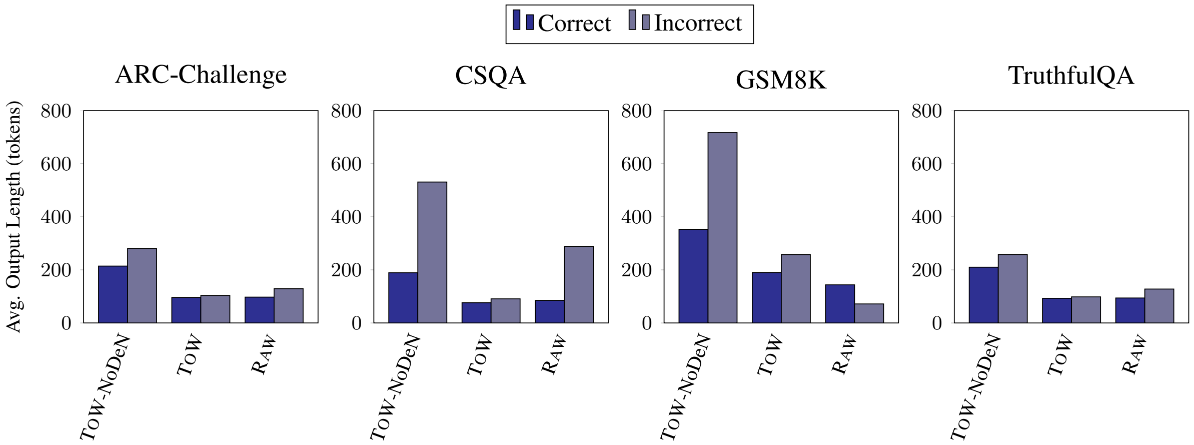

Figure 9: On average, incorrect model predictions are accompanied by longer outputs (in tokens). This is particularly true for TOW-NoDeN across all datasets. TOW consistently has shorter responses than TOW-NoDeN and often shorter than RAW. *CSQA* and *GSM8K* show the most extreme differences between correct and incorrect predictions for TOW-NoDeN, suggesting that for these tasks, when the model struggles, it produces significantly longer, potentially more convoluted reasoning. Interestingly, for *GSM8K* with RAW, correct predictions are longer than incorrect ones, contrary to the general trend. *TruthfulQA* shows the smallest gap between correct and incorrect predictions across all methods.