Example Cases

This section provides a collection of Nek5000 examples illustrating basic approaches and results.

2 Kovasznay Solution

Kovasznay[1] gives an analytical solution to the steady-state Navier-Stokes equations that is similar to the two-dimensional flow-field behind a periodic array of cylinders (Fig. 1a),

$$\begin{aligned} u_x &= 1 - e^{\lambda x}\cos 2\pi y \\ u_y &= \tfrac{\lambda}{2\pi} e^{\lambda x}\sin 2\pi y \end{aligned} \qquad \lambda := \frac{Re}{2} - \sqrt{\frac{Re^2}{4} + 4\pi^2},$$

where Re is the Reynolds number based on mean flow velocity and separation between vortices.

We use E=8 elements in a mesh with periodic boundary conditions at $y = \frac{1}{2} \pm 1$ and Dirichlet conditions given by the exact solution at $x = -\frac{1}{2}$, 1. The solution is time marched to advect initial errors out the domain: $t_f = 8$ with Δt=.001, corresponding to CFL=.344 for N=16. For lower N, one could choose a larger timestep and still satisfy $CFL < 0.5$.[2] Nek5000 supports BDF/EXT and characteristics-based timestepping,[3] both of which are kth-order accurate. (Set TORDER=k, k=1–3, and IFCHAR to T or F in the .rea file.) The BDF/EXT scheme requires one nonlinear evaluation per step and has a stability limit of $CFL\sim$ .62. The characteristics scheme allows $CFL\sim$ 2–4 but is more expensive per step and has an $O(\Delta t^k)$ error that persists at steady-state.

Exponential convergence is seen in Fig. 1b for both the $\mathbb{P}_N - \mathbb{P}_{N-2}$ and splitting methods with BDF/EXT. At lower N, splitting is more accurate because of its Nth-order pressure approximation. The residual steady-state error of the characteristics method exhibits the expected 8-fold error reduction as Δt is reduced to .0005.

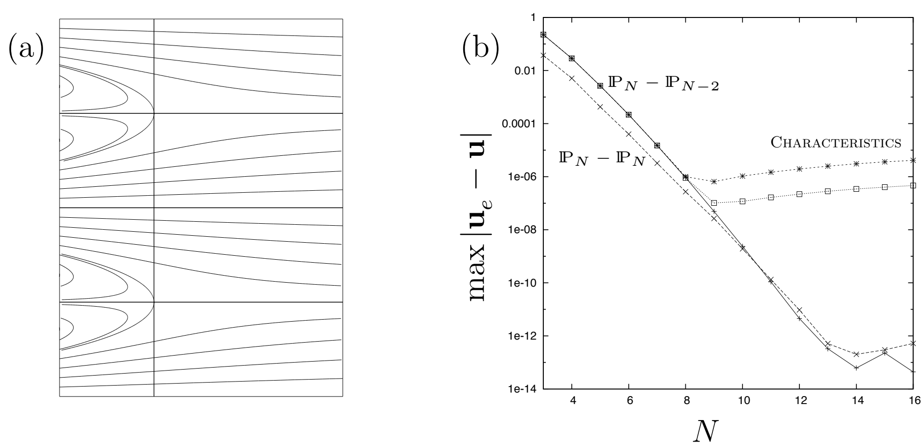

Figure 1: Kovasznay flow at Re = 40: (a) mesh and streamlines; (b) semilog plot of error vs polynomial N shows exponential convergence. Near machine precision is realized with BDF/EXT, whereas characteristics saturates with a residual steady-state error that is observed to be $O(\Delta t^3)$.

[1] L. Kovasznay, "Laminar flow behind a two-dimensional grid," *Proc. Cambr. Philos. Soc.* **44**, 58–62 (1948).

[2] See (3.5.13) in Deville, Fischer, and Mund, *High-Order Methods for Incompressible Flows*, 2002.

[3] Maday, Patera, and Rønquist, "Operator Integration-Factor Splitting Method for Time-Dependent Problems: Application to Incompressible Fluid Flow," *J. Sci. Comput.* **5** 263–292, 1990.