№ 9.1, 9.2, 10.1, 10.2

Домашнее задание №10

№9.1

$x_i \backslash y_j$	0	2	5	p_{i*}
0	0,1	0,2	0,2	0,5
4	0,3	0,2	0	0,5
p_{*j}	0,4	0,4	0,2	1

x_i	0	4	Σ
p_i	0,5	0,5	1

y_i	0	2	5	Σ
p_j	0,4	0,4	0,2	1

$M_x = 0{,}5 \cdot 0 + 0{,}5 \cdot 4 = \underline{2}$

$M_y = 0{,}4 \cdot 0 + 0{,}4 \cdot 2 + 0{,}2 \cdot 5 = \underline{1{,}8}$

$\sigma_x = \sqrt{D_x}$

$D_x = M_x^2 - (M x)^2 =$

$= 0,5 \cdot 0^2 + 0,5 \cdot 4^2 - 2^2 =$

$= \cancel{0,8}\ 8 - 4 = 4$

$\sigma_x = 2$

$\sigma_y = \sqrt{D_y} = M_y^2 - (M_y)^2$

$D_y = 0,4 \cdot 0^2 + 0,4 \cdot 2^2 + 0,2 \cdot 5^2 - 1,8^2 =$

$= 1,6 + 5 - 3,24 = 336$

$\cancel{\sigma_y = 3,56}$

$\sigma_y = \sqrt{3,56} = 1,833$

$$r(\xi, \eta) = r_{\xi,\eta} = \frac{cov(\xi, \eta)}{\sigma_\xi \ \sigma_\eta}$$

$cov(\xi, \eta) = M(\xi, \eta) - M\xi \cdot M\eta$

$cov = M(\xi\eta) - \overset{3,6}{2 \cdot 1,8} =$

$= 1,6 - 3,6 = -2$

$$M(x,y) = \cancel{0,95}\ 0,1\cdot 0\cdot 0 + 0,2\cdot 0\cdot 2 + 0,2\cdot 0\cdot 5 + 0,3\cdot 0\cdot 4 + 0,2\cdot 2\cdot 4 + 0\cdot 4\cdot 5 = \cancel{0,76}\ 1,6$$

$$r(x,y) = \frac{-2}{2\cdot 1,833} = -0,5456$$

$$\sqrt{9,2}$$

$x_i \backslash y_j$	1	2	3	p_{i*}
1	1/12	1/6	1/4	6/12
2	1/12	1/6	1/4	6/12
p_{*j}	2/12	2/6	2/4	1

$$cov(x,y) = M_{xy} - M_x M_y$$

$$M_{xy} = 1\cdot 1\cdot \frac{1}{12} + 1\cdot 2\cdot \frac{1}{6} + 1\cdot 3\cdot \frac{1}{4} + 2\cdot 1\cdot \frac{1}{12} + 2\cdot 2\cdot \frac{1}{6} + 2\cdot 3\cdot \frac{1}{4} =$$

$$r(x,y) = \frac{cov(x,y)}{\sigma_x \sigma_y}$$

$$M_{xy} = \frac{1}{12} + \frac{2}{6} + \frac{3}{4} + \frac{2}{12} + \frac{4}{6} + \frac{6}{4} =$$

$$= \frac{1}{12} + \frac{4}{12} + \frac{9}{12} + \frac{2}{12} + \frac{8}{12} + \frac{18}{12} =$$

$$= \cancel{1,5}\ \frac{42}{12} = \frac{7}{2} = 3,5$$

$$M_x = \frac{6}{12} \cdot 1 + \frac{6}{12} \cdot 2 = \frac{18}{12} = 1{,}5$$

$$M_y = \frac{2}{12} \cdot 1 + \frac{2}{6} \cdot 2 + \frac{2}{4} \cdot 3 =$$

$$= \frac{2}{12} + \frac{8}{12} + \frac{18}{12} = \frac{28}{12} = 2{,}33$$

$$cov(x,y) = 3{,}5 - 1{,}5 \cdot 2{,}33 =$$

$$= 3{,}5 - 3{,}5 = 0$$

$$\sigma_x = \sqrt{D_x}$$

$$D_x = Mx^2 - (M_x)^2 = \quad ^2 - 1{,}5^2$$

$$= \frac{6}{12} \cdot 1^2 + \frac{6}{12} \cdot 2^2 = \frac{6}{12} + \frac{24}{12} = 2{,}5 - 1{,}5^2 = 0{,}25$$

$$\sigma_x = \cancel{\sqrt{2{,}5} = 1{,}5811}$$

$$\sqrt{0{,}25} = 0{,}5$$

$$\sigma_y = \sqrt{D_y}$$

$$D_y = M_y^2 - (M_y)^2 =$$

$$= \frac{2}{12} \cdot 1^2 + \frac{2}{6} \cdot 2^2 + \frac{2}{4} \cdot 3^2 - 2{,}33^2 =$$

$$= \frac{2}{12} + \frac{16}{12} + \frac{54}{12} - 5{,}4289 =$$

$$\sigma_y^2 = 5{,}4289 - 6 - 5{,}4289 = 0{,}5711$$

$$\sigma_y = \sqrt{0{,}5711} = 0{,}7557$$

$$r(x,y) = \frac{-2}{0{,}5 \cdot 0{,}7557} = -5{,}2931$$

$$cov(x,y) = 0 \Rightarrow r(x,y) = 0$$

№10.1

$$Y): f(x,y) = \begin{cases} a\cos x \cos y & (x;y) \in D \\ 0, & (x,y) \notin D \end{cases}$$

$$D: 0 \le x \le \frac{\pi}{2}$$

$$0 \le y \le \frac{\pi}{2}$$

$$f(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = \int_0^{\frac{\pi}{2}} a\cos x \cos y\,dy =$$

$$= a\cos x \cdot (-\sin y) \Big|_0^{\frac{\pi}{2}} = a\cos x \cdot (\sin\frac{\pi}{2}) - 0$$

$$= a\cos x$$

$$f(x) = \begin{cases} a\cos x, & x \in D \\ 0, & x \notin D \end{cases}$$

$$F(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx = \int_{0}^{\frac{\pi}{2}} a\cos x \cos y\,dx =$$

$$= a\cos y \cdot (\sin x)\Big|_{0}^{\frac{\pi}{2}} =$$

$$= a\cos y - 0$$

$$F(y) = \begin{cases} a\cos y, & y \in D \\ 0, & y \notin D \end{cases}$$

$$F(x) = \int_{0}^{\frac{\pi}{2}} a\cos x\,dx =$$

$$= -a\sin x\Big|_{0}^{\frac{\pi}{2}} = -a$$

$$F(y) = \int_{0}^{\frac{\pi}{2}} a\cos y\,dy = -a\sin y\Big|_{0}^{\frac{\pi}{2}} = -a$$

$F(x) = \int f(x)\,dx = \begin{cases} a\sin x, & x \in D \\ 0, & x \le 0 \\ 1, & > \frac{\pi}{2} \end{cases}$

$F(y) = \int f(y)\,dy = \begin{cases} a\sin x, & x \in D \\ 0, & x \le 0 \\ 1, & > \frac{\pi}{2} \end{cases}$

$\sqrt{10 \cdot 2}$

$f(x,y) = \begin{cases} \frac{1}{10\pi}, & \frac{x^2}{25} + \frac{y^2}{4} \le 1 \\ 0, & \frac{x^2}{25} + \frac{y^2}{4} > 1 \end{cases}$

$f(x) = \int\limits_{-\infty}^{\infty} F(x,y)\,dy$

$\frac{y^2}{4} < 1 - \frac{x^2}{25} \Rightarrow y \in \left[-2\sqrt{1-\frac{x^2}{25}},\ 2\sqrt{1-\frac{x^2}{25}}\right]$

для $x \in [-5, 5]$

$\left(\frac{x^2}{25} \le 1\right)$

$f(x) = 0$ при $x \notin [-5; 5]$

$$f(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

$$\frac{y^2}{4} \le 1 \Rightarrow y \in [-2, 2]$$

$$\frac{x^2}{25} \le 1 - \frac{y^2}{4} \Rightarrow x \in \left[-5\sqrt{1-\frac{y^2}{4}},\ 5\sqrt{1-\frac{y^2}{4}}\right]$$

$$f(y) = \int_{-5\sqrt{1-\frac{y^2}{4}}}^{5\sqrt{1-\frac{y^2}{4}}} \frac{1}{10\pi}\,dx =$$

$$= \frac{1}{10\pi} \cdot 10\sqrt{1-\frac{y^2}{4}} = \frac{1}{\pi}\sqrt{1-y}^2$$

для $y \in [-2; 2]$

$f(y) = 0$ для $y \notin [-2; 2]$

$$f(x) = \begin{cases} \frac{2}{5\pi}\sqrt{1-\frac{x^2}{25}}, & x \in [-5, 5] \\ 0, & x \notin [-5, 5] \end{cases}$$

$$f(y) = \begin{cases} \frac{1}{\pi}\sqrt{1-\frac{y^2}{4}}, & y \in [-2, 2] \\ 0, & y \notin [-2; 2] \end{cases}$$

Дз № 9.3, 9.4, (10.3, 10.4)

№9.3

X:

x_i	1	3
p_i	0,4	0,6

y_j	2	4
p_j	0,2	0,8

X \ Y	2	4	p_{i*}
1	0,08	0,32	0,4
3	0,12	0,48	0,6
p_{*j}	0,2	0,8	1

$$M_x = 1 \cdot 0{,}4 + 3 \cdot 0{,}6 = 2{,}2$$

$$M_y = 2 \cdot 0{,}2 + 4 \cdot 0{,}8 = 3{,}6 \quad \Rightarrow 5{,}8$$

б) $D(x+y) = D_x + D_y + 2 \cdot cov(x,y)$

$$D_x = M_{x^2} - (M_x)^2 = 1 \cdot 0{,}4 + 3^2 \cdot 0{,}6 - 2{,}2^2 =$$
$$= 0{,}4 + 5{,}4 - 4{,}84 = 0{,}96$$

$$D_y = M_{y^2} - (M_y)^2 = 2^2 \cdot 0{,}2 + 4^2 \cdot 0{,}8 - 3{,}6^2 =$$
$$= 0{,}8 + 12{,}8 - 12{,}96 = 0{,}64$$

$$cov(x,y) = M_{xy} - M_x \cdot M_y =$$
$$= \overset{0{,}16}{1 \cdot 2 \cdot 0{,}08} + \overset{1{,}28}{1 \cdot 4 \cdot 0{,}32} + \overset{0{,}72}{3 \cdot 2 \cdot 0{,}12} + \overset{5{,}76}{3 \cdot 4 \cdot 0{,}48} -$$
$$- \overset{7{,}92}{2{,}2 \cdot 3{,}6} = \cancel{0{,}36}\ 0$$

$D(x+y) = 0{,}96 + 0{,}64 + 2 \cdot (\cancel{-0{,}36}\,0) =$
$= \cancel{0{,}88}\ 1{,}6$

2) ~~$D_{xy} = M_{xy}^2 - (M_{xy})^2 =$~~
~~$= 1^2 \cdot 2^2 \cdot 0{,}08 + 1^2 \cdot 4^2 \cdot 0{,}32 + 3^2 \cdot 4^2 \cdot 0{,}48 - (7{,}92)^2 =$~~
~~$+ 3^2 \cdot 2^2 \cdot 0{,}12$~~
~~$= 0{,}32 + 5{,}12 + 4{,}32 + 69{,}12 - 62{,}7264$~~

$P(1+2) = 0{,}08$
$P(1+4) = 0{,}32$ $P(5) = 0{,}32 + 0{,}12 = 0{,}44$
$P(3+2) = 0{,}12$
$P(3+4) = 0{,}48$

$X+Y$	3	5	7
P_i	0,08	0,44	0,48

$M_{xy} = 3 \cdot 0{,}08 + 5 \cdot 0{,}44 + 7 \cdot 0{,}48 =$
$= 0{,}24 + 2{,}2 + 3{,}36 = 5{,}8$

$D_{xy} = M_{xy^2} - M_{xy}^2 =$

$= 3^2 \cdot 0,08 + 5^2 \cdot 0,44 + 7^2 \cdot 0,48 - 5,8^2 =$

$= 0,72 + 11 + 23,52 - 33,64 =$

$= 1,6$

~~№ 24~~

5 белых 3 красн

$x \backslash y$	0	1	2	P_{xi}
0	0	0	$\frac{25}{64}$	$\frac{25}{64}$
1	0	$\frac{15}{64}$	0	$\frac{15}{64}$
2	$\frac{9}{64}$	0	0	$\frac{9}{64}$
P_{yj}	$\frac{9}{64}$	$\frac{15}{64}$	$\frac{25}{64}$	

$P(1,1) = \frac{3}{8} \cdot \frac{3}{8} = \frac{15}{64}$

$(0,2) = \frac{5}{8} \cdot \frac{5}{8} = \frac{25}{64}$

$P(0,2) = \frac{3}{8} \cdot \frac{3}{8} = \frac{9}{64}$

$C_8^2 = \frac{8!}{2! \cdot 6!} = 28$

5 бел y 3 красн x № 9 4

$x \backslash y$	0	1	2	P_{x*}
0	0	0	$\frac{10}{28}$	$\frac{10}{28}$
1	0	$\frac{15}{28}$	0	$\frac{15}{28}$
2	$\frac{3}{28}$	0	0	$\frac{3}{28}$
P_{*j}	$\frac{3}{28}$	$\frac{15}{28}$	$\frac{10}{28}$	1

$$C_8^2 = \frac{8!}{2!\cdot(8-2)!} = 28$$

$$P(2,0) = \frac{C_3^2}{C_8^2} = \frac{3}{28}$$

$$P(0,2) = \frac{C_5^2}{C_8^2} = \frac{10}{28}$$

$$P(1,1) = \frac{C_5^1 \cdot C_3^1}{C_8^2} =$$

$$= \frac{\frac{5!}{(5-1)!\,1!} \cdot \frac{3!}{(3-1)!\,1!}}{28} =$$

$$= \frac{5\cdot 3}{28} = \frac{15}{28}$$

$$M_x = 0 \cdot \frac{3}{28} + 1 \cdot \frac{15}{28} + 2 \cdot \frac{10}{28} =$$

$$= + \frac{15}{28} + \frac{20}{28} = \frac{35}{28} = \frac{5}{4}$$

$$M_y = 2 \cdot \frac{3}{28} + 1 \cdot \frac{15}{28} + 0 \cdot \frac{10}{28} =$$

$$= \frac{6}{28} + \frac{15}{28} + 0 = \frac{21}{28} = \frac{3}{4}$$

$$D_x = 0^2 + 1^2 \cdot \frac{15}{28} + 2^2 \cdot \frac{10}{28} - \left(\frac{5}{4}\right)^2 =$$

$$= \frac{15}{28} + \frac{40}{28} - \frac{25}{16} = \frac{55}{28} - \frac{25}{16} =$$

$$= 1{,}9643 - 1{,}5625 = \underline{0{,}4018}$$

$$D_y = 2^2 \cdot \frac{3}{28} + 1^2 \cdot \frac{15}{28} \cdot 0^2 \cdot \frac{10}{28} - \left(\frac{3}{4}\right)^2 =$$

$$= \frac{12}{28} + \frac{15}{28} - \frac{9}{16} = \frac{27}{28} - \frac{9}{16} =$$

$$= 0{,}9643 - 0{,}5625 = \underline{0{,}4018}$$

$$\operatorname{cov}(x, y) = 0 \cdot 2 \cdot \frac{3}{28} + 1 \cdot 1 \cdot \frac{15}{28} + 2 \cdot 0 \cdot \frac{5}{14} - 5 \cdot \frac{3}{4} \cdot \frac{3}{4} = \cancel{0} + \frac{15}{28} - \frac{15}{16} =$$

$$= \frac{15}{28} - \frac{15}{16} = \cancel{0{,}75}\ 0{,}5357 - 0{,}9375 =$$

$$= -\cancel{0{,}1875}\ 0{,}4018$$

$$r_{xy} = \frac{\cancel{-0{,}1875}\ -0{,}4018}{\sqrt{0{,}4018} \cdot \sqrt{0{,}4018}} = \frac{-0{,}4018}{0{,}4018} = -1$$