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Interpolating triangular meshes by Loop subdivision scheme

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Abstract Using the limit point formula of the Loop subdivision scheme, we propose a very simple and efficient method for constructing interpolation surface of triangular meshes by Loop subdivision scheme. The excellent properties of the method are: (1) Locality: the perturbation of a given vertex only influences the surface shape near this vertex. (2) Efficiency: the locations of new points can be computed with explicit formulae. (3) Easiness in implementation: only the geometric rule of the first step should be modified. (4) Freedom: for each edge, there is one degree of freedom to adjust the shape of the interpolation surface. (5) Easiness in generalization: it is easy to generalize our method to other approximation subdivision schemes with explicit formulae to compute limit point.

Keywords computer aided geometric design, subdivision surface, approximating subdivision scheme, Loop surface, surface interpolating

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1 Introduction

Due to the fact that surface modeling by iterated subdivision has such advantageous properties as numerical stability, code simplicity, easiness in handling arbitrary topology, subdivision surfaces have been used widely in the fields of geometric modeling, computer graphics, etc. Some movie productions and game engines, such as 3DMax, Maya, Renderman and Softimage, have the function of modeling surface by subdivision schemes [1].

Subdivision surfaces can be classified into approximating surfaces, interpolating surfaces based on the criterion whether initial control points should be interpolated or not in the final surfaces. The most popular approximating schemes include Catmull-Clark scheme [2], which is based on the tensor product bi-cubic spline and is designed for quadrilateral mesh, Loop scheme [3], which is based on the three-directional box spline and is designed for triangular mesh. These two schemes produce surfaces that are C^2 continuous everywhere except at extraordinary vertices, where they are C^1 continuous. The limit surfaces of the approximating subdivision scheme inflect the shape of the initial meshes well, so that one can estimate the shapes of the limit surfaces from the initial control meshes, but the control meshes of

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approximating subdivision schemes will shrink with subdivision refinements [4]. Butterfly scheme, which was first proposed by Dyn et al. [5] and then was improved by Zorin et al. [6], and Kobbelt scheme [7] are two well-known interpolating schemes. These two schemes extend the 4-point subdivision for curve to triangular mesh and quadrilateral mesh, respectively. Compared with approximating subdivision schemes, the control mesh of interpolating subdivisions does not shrink with subdivision refinements, but their limit surfaces are only C^1 continuous for each vertex and have unwanted folds and artifacts [6] (also see Figure 4(c), Figure 5(c), Figure 7(b) in section 4 of this paper). So it is difficult to estimate the shapes of the limit surfaces for interpolating subdivision schemes based on the initial meshes.

To interpolate an initial mesh with more pleasing surfaces, many methods of interpolating meshes by approximating subdivision schemes are proposed. Hoppe et al. [8] presented a modification of the Loop schemes to force the limit surface to go through a particular set of control points. Nasri [9] presented a modification for the Doo-Sabin algorithm to make it interpolate initial control points and Brunet [10] introduced a set of shape handles associated to the vertices for shape control in Nasri's approach. Halstead et al. [11] proposed an interpolation scheme of using Catmull-Clark surfaces, which minimizes a certain fairness measure. Both Nasri's method and Halstead et al.'s method had to construct a linear constraint on the control points of the initial mesh for each interpolating vertex and thus established a system of linear equations. The initial control mesh for the subdivision surface was obtained by solving the equations. However, it is unclear under what conditions the linear system is solvable [6]. As pointed out by Halstead [11], it is possible for the linear system to be singular or ill-conditioned. Recently, based on Catmull-Clark subdivision scheme, Zheng and Cai [12] proposed a two-phase subdivision scheme to interpolate arbitrary topology meshes. They used a set of new rules for the first subdivision iteration to obtain a new meshes, whose limit surface of Catmull-Clark refinements interpolates the initial vertices. The features of their method are that the system of linear equations is diagonally dominant, so it is guaranteed to always work and the system of linear equations can be solved by more effective iterative methods.

In the above-mentioned methods, the number of vertices to be interpolated is equal to that of the vertices of the new initial mesh solved by the system of equations. So there is no freedom to adjust the shape of the interpolation surface and such methods are global schemes. To make the interpolation method a local scheme, we adopt the method of introducing additional degrees of freedom to construct interpolation surface by Loop subdivision scheme. By this new method, the control vertices can be obtained directly with no need to solve any initial or intermediate large systems. For convenience, in this paper we restrict our discussion to closed meshes. Extension to open meshes is straightforward.

The advantages of our method over existing methods for surface interpolation by approximating subdivision scheme lies in five aspects:

(1) Locality: the perturbation of a given vertex only influences the surface shape near this vertex.

(2) Simplicity: we use only simple geometric rules to construct smooth surface interpolating given vertices.

(3) Easiness in implementation: only the geometric rule of the first subdivision step is modified and the other steps are the same as Loop subdivision scheme.

(4) Freedom: for each edge and face of the initial mesh, there is one degree of freedom for adjusting the shape of the limit surface.

(5) Easiness in generalization: it is easy to generalize our method to other approximation subdivision schemes with explicit formulae to compute limit vertex.

2 Loop subdivision scheme and the formula of the limit point

The closed mesh we consider is a polyhedron-like configuration of faces, edges and vertices such that each vertex corresponds to a point in a 3D space, and each edge is a line segment bounded by two vertices. Each face is a triangle bounded by three vertices, or three edges. We also require that each edge should be shared exactly by two faces.

2.1 Loop subdivision scheme

Given an initial triangular mesh, the process for each refinement iteration of Loop subdivision scheme goes in the following steps:

(1) For each vertex p, compute a new vertex point as a linear combination of the points within the neighborhood of the vertex (Figure 1(a)). Specifically,

$$p' = (1 - k\beta)p + \beta(p_1 + p_2 + \cdots + p_k), \tag{1}$$

where k is the valence of the vertex p, and $\{p_i\}_{i=1}^k$ are vertices sharing edge with p. The coefficient β is defined as

$$\beta = \begin{cases} \dfrac{3}{16}, & k = 3, \\ \dfrac{3}{8k}, & k > 3. \end{cases} \tag{2}$$

Remark. $\beta = \frac{3}{8k}(k > 3)$ of eq. (2) is proposed by Warren [13]. The original formula proposed by Loop is $\beta = \frac{1}{k}[\frac{5}{8} - (\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{k})^2](k > 3)$[3]. When $k = 6$, i.e. p is a regular vertex, the value of β derived from both the two formulas are equal to $\frac{1}{16}$. When k is not equal to 6, i.e. p is an irregular vertex, in general the values of β derived from the two formulas are not equal. But the two formulas can ensure that the limit surfaces are C^1 at the irregular vertices. In this paper we select the Warren's formula due to the fact that it is simpler than that of Loop.

(2) For each edge p_1p_2, compute a new edge point as follows (Figure 1(b)):

$$p' = \frac{1}{8}(3p_1 + 3p_2 + p_3 + p_4), \tag{3}$$

where p_3p_4 are two vertices of the two triangles sharing edge p_1p_2 (different from p_1, p_2).

(3) Create new edges by connecting each new vertex point to the new edge points surrounding it, and create a new edge by connecting three new edge points of every triangle (Figure 1(c)).

(4) Create a new face for every three points connected by three new edge (Figure 1(c)).

Steps (1) and (2) define the new geometry. We call them geometry rules and denote them by G. Steps (3) and (4) define the connectivity of the new points. We call them topology rules and denote them by T. Then when a triangular mesh is subdivided by Loop subdivision scheme, it can be seen as the following refinement steps:

$$1G \to 1T \to 2G \to 2T \to 3G \to 3T \to \cdots, \tag{4}$$

where kG, kT mean the application of the geometric rule and topology rule to the mesh derived from the initial mesh subdivided by $k - 1$ steps of Loop subdivision process.

Two examples of Loop surfaces with their initial control meshes are shown in Figures 4(b) and 5(b) in section 4. From these two examples we can see that the limit surfaces are smooth but shrink from the initial control meshes.

2.2 Formula of the limit point

For the initial vertex p_i, there is a corresponding limit point p_i^∞. The limit point corresponding to p_i can be computed by p_i and its 1-neighborhood vertices [14] (Figure 2):

$$p_i^\infty = (1 - k\alpha)p_i + \alpha\sum_{j=1}^{k} p_j, \tag{5}$$

where α is defined as

$$\alpha = \left(\frac{3}{8\beta} + k\right)^{-1} = \begin{cases} \dfrac{1}{5}, & k = 3, \\ \dfrac{1}{2k}, & k > 3. \end{cases} \tag{6}$$

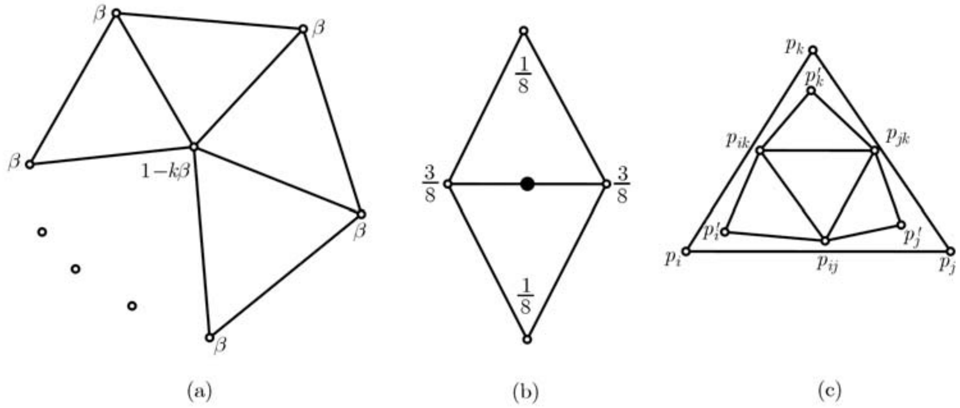

Figure 1 Loop subdivision scheme. (a) New vertex point; (b) new edge point; (c) topology rule.

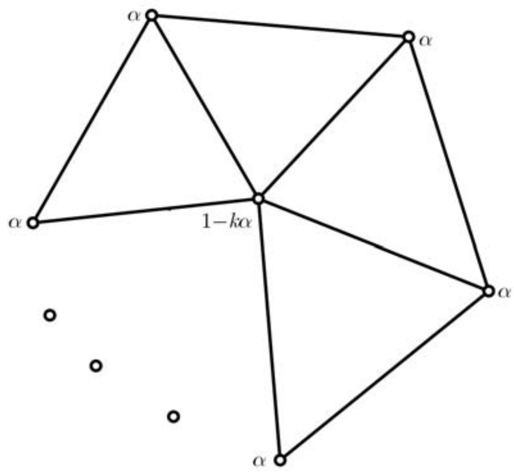

Figure 2 The neighborhood around vertex p_i and the limit point of p_i.

3 The interpolation method

Given an initial triangular mesh M^0 with a set of vertices $\{p_i^0\}$ and a set of edges $\{e_i^0\}$, the process of constructing interpolation surface of M^0 by Loop subdivision scheme is as follows:

$$1G' \to 1T \to 2G \to 2T \to 3G \to 3T \to \cdots, \tag{7}$$

where $1G'$ is the new geometric rule of computing new vertex points and new edge points. Comparing (4) with (7), we can see that in the process of interpolating with Loop subdivision scheme, only the first geometric step is modified. The other steps are the same as those of Loop scheme, so it is easy to incorporate our method to the modeling systems.

Denote the triangular mesh subdivided by new geometric rule $1G'$ and topology rule $1T$ from M^0 as $\bar{M}^1$. For a vertex p_i^0 of M^0, let its corresponding vertex at $\bar{M}^1$ be $\bar{p}_i^1$. Then if each limit point of $\bar{p}_i^1$ is p_i^0, the limit surface of $\bar{M}^1$ interpolates M^0. By limit point formula (5), $\bar{p}_i^1$ is a linear equation with unknown p_i^0, so the equation has solution even if the new edge points $p_j^1 (j = 1, 2, \ldots, k)$ are selected arbitrarily. So in theory, the new edge points of $1G'$ can be selected arbitrarily.

To make the interpolation surface fair and reflect the shape of the initial triangular mesh well, we adopt the geometric rule of normal based subdivision scheme, which is proposed by Yang [15], to compute the new edge points of $1G'$.

3.1 Computation of the new edge point of $1G'$

Many current interpolating subdivision schemes are spline based, and new vertices are computed as linear combinations of old vertices. Because these subdivision surfaces are discrete analogous to spline surfaces interpolation with uniform parameterizations, the final surfaces may have undesirable undulations when the triangle of the original mesh are not regular in size or the mesh has ugly changing local shapes. To obtain smoother interpolation surfaces, Yang [15] proposed the normal based subdivision scheme for surface interpolation. Numerical examples show that the surfaces obtained by the normal based subdivision scheme look more fair and natural than those by some previous methods. In this subsection we propose a method of computing new edge points for $1G'$ based on the normal based subdivision scheme. Numerical examples show that by this method the interpolation surfaces are fair.

For each vertex p_i^0 of M^0, first we estimate its normal vector. Suppose that $\pi_j (j = 1, 2, \ldots, m_i)$ are the triangles sharing the vertex p_i^0. For each triangle π_j, assume that the angle at the vertex p_i^0 is ϕ_j. The normal of the triangle is $\boldsymbol{n}_j$. Then the normal at the vertex p_i^0 can be estimated as (see [15])

$$\boldsymbol{n}_i = \frac{\sum_{j=1}^{m_i} \phi_j \boldsymbol{n}_j}{\| \sum_{j=1}^{m_i} \phi_i \boldsymbol{n}_j \|}. \tag{8}$$

Assume that the two end points of edge e are p_i^0, p_j^0. We define the new edge point of e as (Figure 3)

$$\bar{p} = \frac{p_i^0 + p_j^0}{2} + \lambda(d_i \boldsymbol{n}_i + d_j \boldsymbol{n}_j), \tag{9}$$

where $d_i = (p_i^0 - \frac{p_i^0+p_j^0}{2})\boldsymbol{n}_i, d_j = (p_j^0 - \frac{p_i^0+p_j^0}{2})\boldsymbol{n}_j, \lambda$ is a free parameter for adjusting the shape of the interpolation surface. In theory, λ can be selected arbitrarily or we can select different values of λ for different edges. But by our experience, λ should be constrained within $0 < \lambda < 1$. Detailed discussions of λ can be seen in section 4.

3.2 Computation of the new vertex point of $1G'$

From the formula of new edge point $\bar{p}$ based on the formula of limit point (5), we derive the formula of new vertex point for the new geometric rule $1G'$ immediately.

Let the new vertex point corresponding to p_i^0 be $\bar{p}_i^1$. According to the condition of interpolation and the limit point formula (5), we have

$$p_i^0 = (1 - k\alpha)\bar{p}_i^1 + \alpha \sum_{j=1}^{k} \bar{p}_j. \tag{10}$$

So

$$\bar{p}_i^1 = \frac{p_i^0 - \alpha \sum_{j=1}^{k} \bar{p}_j}{(1 - k\alpha)}. \tag{11}$$

By limit point formula (5), the limit surface of $\bar{M}^1$, whose new edge point and new vertex point determined by (9) and (11) and connected by the topology rule of Loop subdivision scheme, interpolate all the vertex of the initial mesh M^0.

Remark. If we perturb an initial vertex of M^0, by formula (8), the normals of this vertex and that of its 1-ring neighborhood vertices are changed. By formula (9), the new edge points near it and its 1-ring neighborhood vertices are changed. Finally by formula (11), the new vertex points corresponding to it and that of its $1, 2$-ring neighborhood vertices are changed. So the perturbation of an initial vertex of M^0 only influences the shape of the interpolation surface near this vertex.

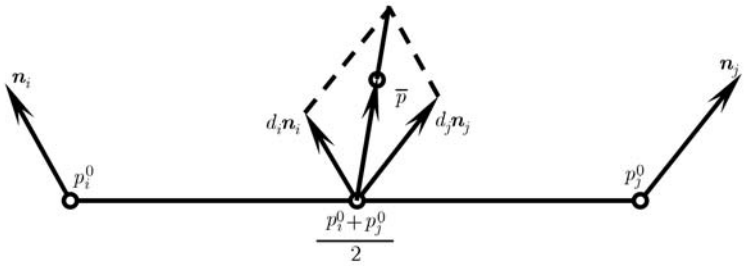

Figure 3 Selecting the new edge point.

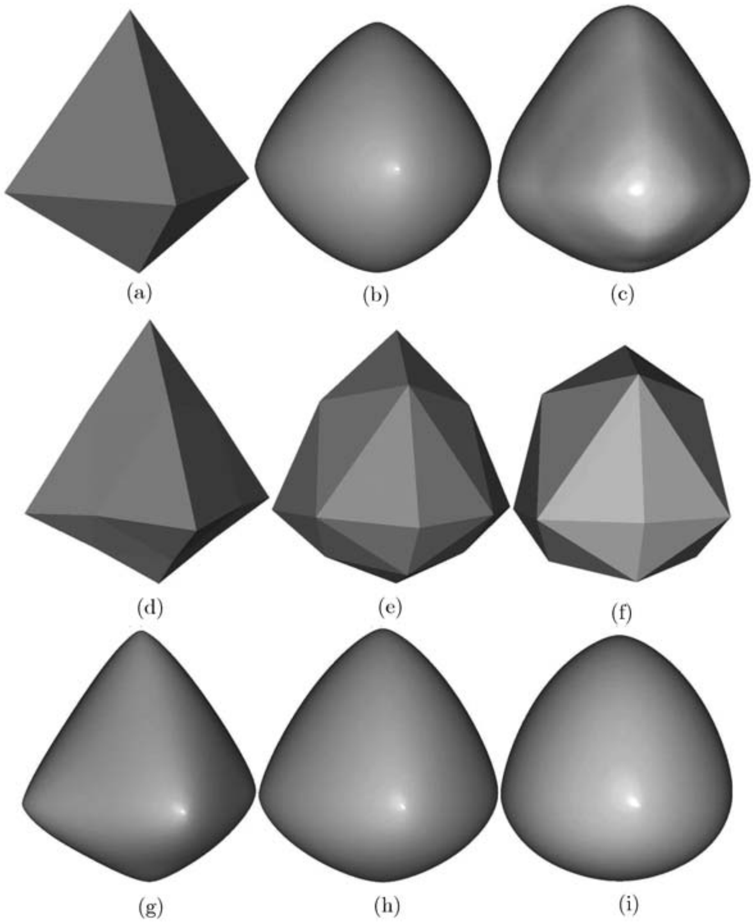

Figure 4 Example 1. (a) Initial mesh; (b) limit surface of Loop subdivision scheme; (c) interpolation surface of Zorin modified butterfly subdivision scheme; (d), (e), (f) meshes subdivided by $1G'$, $1T$ with $\lambda = \frac{1}{3}, \frac{1}{2}, \frac{2}{3}$, respectively; (g), (h), (i) limit surfaces of meshes (d), (e), (f) by Loop subdivision scheme.

4 Examples

In this section we give some examples to demonstrate the advantageous properties of the interpolation method addressed in section 3. We compare our method to the modified butterfly subdivision scheme proposed by Zorin [6].

In Example 1 and Example 2 we subdivide two simple triangular meshes by Loop subdivision scheme, and construct interpolation surfaces by our method ($\lambda = \frac{1}{3}, \frac{1}{2}, \frac{2}{3}$) and the modified butterfly subdivision scheme. The meshes and surfaces of these two examples are shown in Figure 4 and Figure 5. From Figures 4 and 5 we can see that the limit surfaces of Loop subdivision scheme shrink from the initial meshes. Because the initial meshes are simple and their edges are uniform, the undulation behaviors at

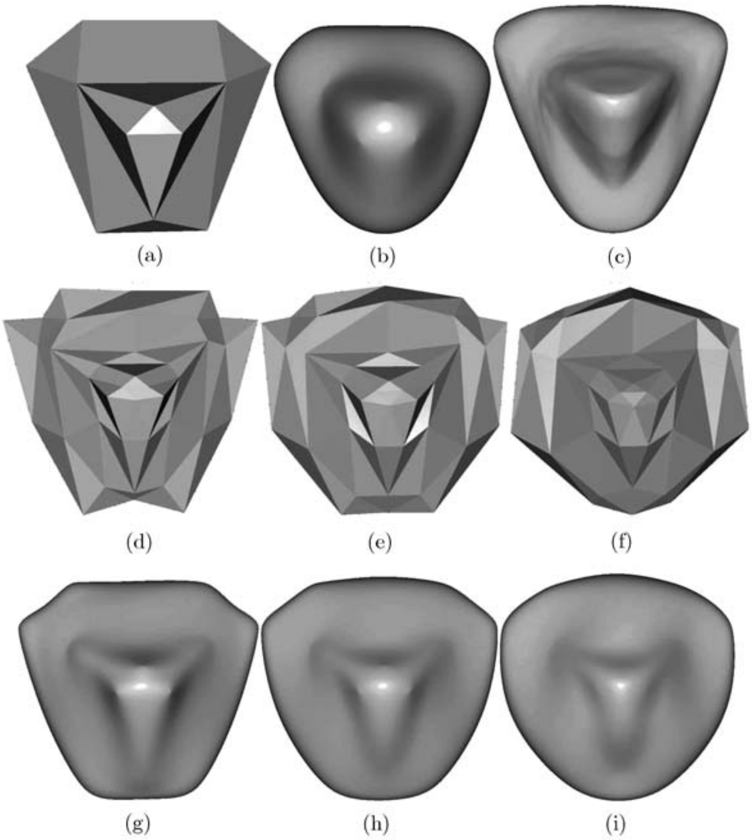

Figure 5 Example 2. (a) Initial mesh; (b) limit surface of Loop subdivision scheme; (c) interpolation surface of Zorin modified butterfly subdivision scheme; (d), (e), (f) meshes subdivided by $1G'$, $1T$ with $\lambda = \frac{1}{3}, \frac{1}{2}, \frac{2}{3}$, respectively; (g), (h), (i) limit surfaces of meshes (d), (e), (f) by Loop subdivision scheme.

the interpolation surfaces of Zorin's modified butterfly subdivision are un-conspicuous. The interpolation surfaces by our new method are fair, and we can adjust the shape of the interpolation surface efficiently by setting different values of the free parameter λ.

From Figure 4(d), (e), (f) and Figure 5(d), (e), (f) we can see that when the value λ is small (for example, $\lambda = \frac{1}{3}$), the distances between the new edge points and their corresponding edge are small, but the distances between the new vertex points and their corresponding old vertices are large, so the curvatures of the interpolation surfaces near the old vertices change rapidly. On the other hand, when the value of λ is large (for example, $\lambda = \frac{2}{3}$), the distances between the new edge points and their corresponding edge are large, but the distances between the new vertex points and their corresponding old vertices are small, so the interpolation surfaces are flat near the initial vertices. Many numerical examples we tested show that when λ is about 0.5, the interpolation surfaces look more fair and natural.

In Example 3, we interpolate a complex, cat-like triangular mesh using our method with $\lambda = \frac{1}{3}, \frac{1}{2}, \frac{2}{3}$. The initial mesh and the interpolation surfaces are shown in Figure 6. From Figure 6(b), (c), (d) we can observe that though the global shape of the three interpolation surfaces are similar, when $\lambda = \frac{1}{3}$, some undulation behaviors occur near the cat's ear; when $\lambda = \frac{1}{2}$, the undulation behaviors are un-conspicuous; when $\lambda = \frac{2}{3}$, we hardly see the undulation behaviors. This means that we can improve the quality of the interpolation surface by adjusting the value of λ.

Finally, in Examples 4–7, we interpolate a simple triangular mesh and three complex triangular meshes by our new method and the modified butterfly subdivision scheme. The initial meshes and interpolation surfaces are shown in Figures 7–10. These examples also show that the surfaces obtained by the new method look more fair and natural than those of the modified butterfly subdivision scheme.

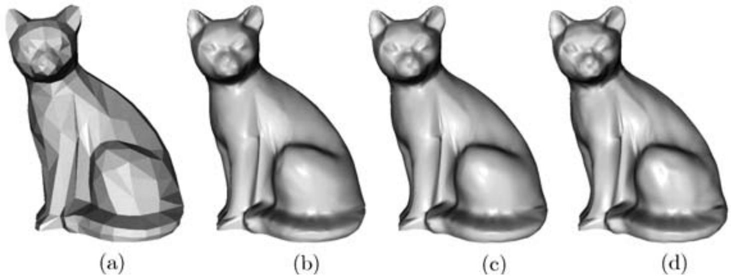

Figure 6 Example 3. (a) Initial mesh; (b), (c), (d) interpolation surfaces of Loop subdivision scheme with $\lambda = \frac{1}{3}, \frac{1}{2}, \frac{2}{3}$.

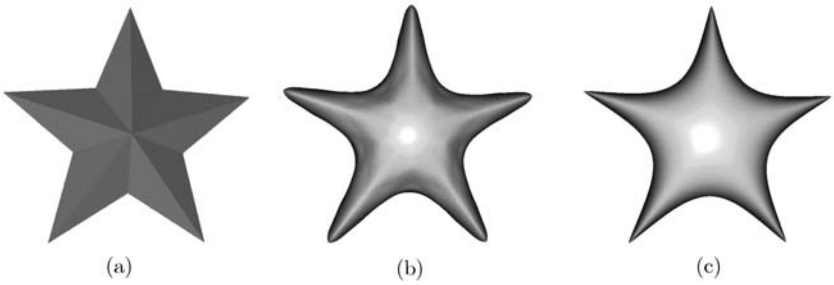

Figure 7 Example 4. (a) Initial mesh; (b) interpolation surface of Zorin modified butterfly subdivision scheme; (c) interpolation surfaces of Loop subdivision scheme with $\lambda = \frac{1}{2}$.

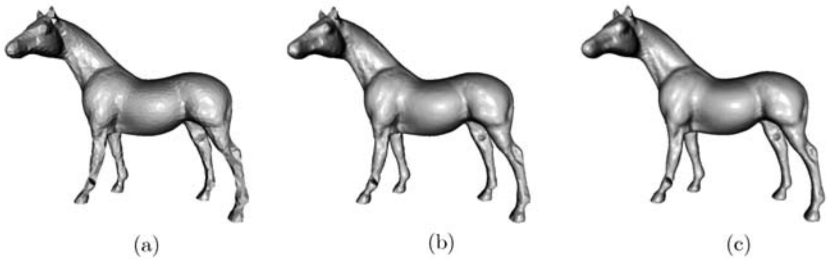

Figure 8 Example 5. (a) Initial mesh; (b) interpolation surface of Zorin's modified butterfly subdivision scheme; (c) interpolation surfaces of Loop subdivision scheme with $\lambda = \frac{1}{2}$.

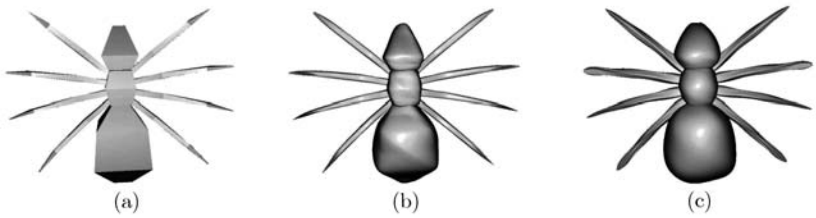

Figure 9 Example 6. (a) Initial mesh; (b) interpolation surface of Zorin modified butterfly subdivision scheme; (c) interpolation surfaces of Loop subdivision scheme with $\lambda = \frac{1}{2}$.

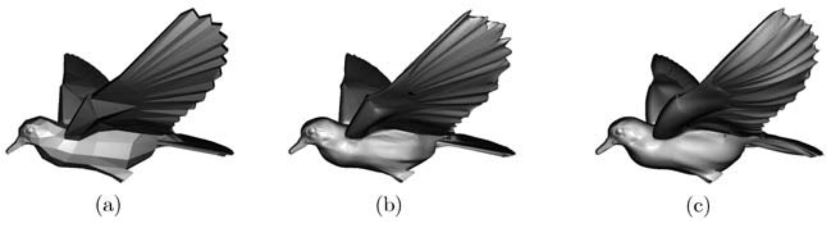

Figure 10 Example 7. (a) Initial mesh; (b) interpolation surface of Zorin modified butterfly subdivision scheme; (c) interpolation surfaces of Loop subdivision scheme with $\lambda = \frac{1}{2}$.

5 Conclusions

We have described a very simple method for automatic surface interpolation through the vertices of an arbitrary topology triangular mesh using Loop subdivision surfaces. The main advantages of our method include robustness, efficiency, locality and sufficient freedoms. All of these features make our method fit to design and model complicated shapes.

In this paper, we have determined the new edge point based on the normal based subdivision scheme, and given an explicit formula for the new vertex point. We have also discussed the effect of the free parameter for the interpolation, and indicate how to select free parameters. In the future we will investigate the way to set parameters for these free variables based on some local or global criterions.

Acknowledgements

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